The dissipative Landau-Zener problem

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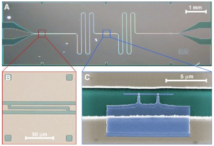
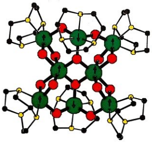
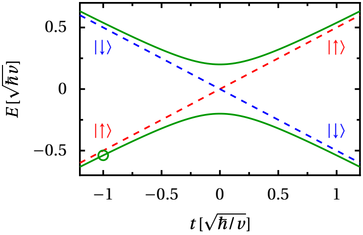




in collaboration with

M. Wubs (Augsburg ➔ Copenhagen), P. Hänggi (Augsburg),
K. Saito (Tokyo), Y. Kayanuma (Osaka)

“standard” Landau-Zener problem



time-dependent two-level system

$$H(t) = -\frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x$$

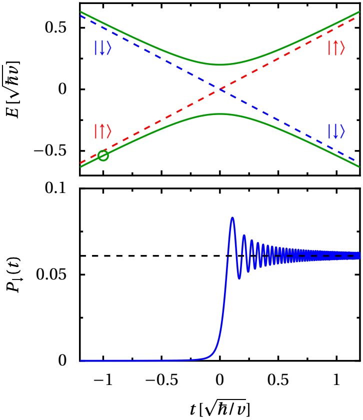
- diabatic states: $|\uparrow\rangle, |\downarrow\rangle$
- adiabatic states

initial state: $|\psi(t=-\infty)\rangle = |\uparrow\rangle$

- ? time evolution
- ? spin-flip probability $P_{\uparrow\to\downarrow}$

“standard” Landau-Zener problem

finite times: numerical



$t \to \infty$: analytical

- transition probability

$$P_{\uparrow\to\downarrow}(\infty) = 1 - \exp\left(-\frac{\pi\Delta^2}{2\hbar v}\right)$$

- large splitting Δ:
  adiabatic following, $P_{\uparrow\to\downarrow}(\infty) = 1$

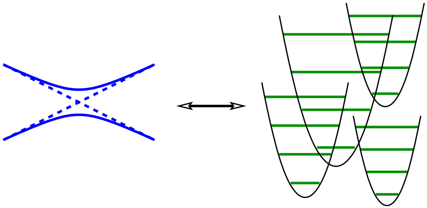
Landau, Zener, Stückelberg (1932)

alternative: complete summation of a perturbation series in $\frac{\Delta}{2}\sigma_x$

Kayanuma (1984)

The dissipative Landau-Zener problem

two-level system coupled to an environment:



harmonic-oscillator bath

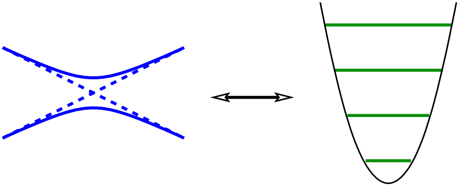
previous work:

- classical noise — Kayanuma, J.Phys.Soc.Jpn.'84
- Ohmic phase noise, weak coupling — Ao & Rammer, PRL'89

here:

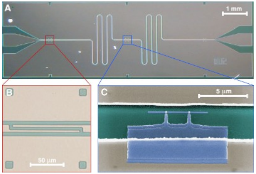
- arbitrary coupling, zero temperature ➔ exact solution

coupling to a single quantum oscillator

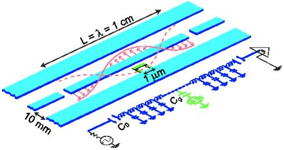


Saito, Wubs, SK, Hänggi, Kayanuma, EPL **76**, 22 (2006)

circuit QED



A. Wallraff *et al.*, Nature'04



A. Blais *et al.*, PRA'04

solid-state version of
two-level atom in an
optical resonator

tunable parameters

- gate voltage V_g
- magnetic flux ϕ_{ext}

two particular charge states:

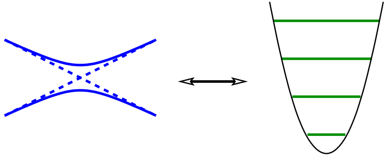
$|N_0\rangle, \; |N_0 + 1\rangle$ "the qubit"

at charge-degeneracy point

$$H = -\frac{E_J(t)}{2}\sigma_z + \gamma(b^\dagger + b)\sigma_x + \hbar\Omega b^\dagger b$$

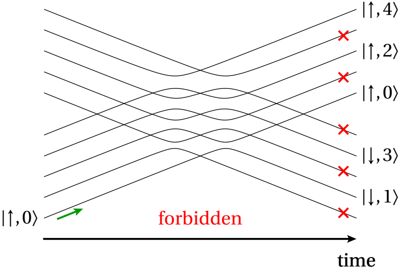
$$E_J(t) \longrightarrow vt$$

couling to single quantum oscillator



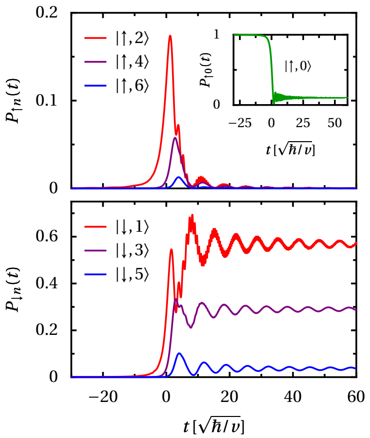
$$H(t) = -\frac{vt}{2}\sigma_z + \gamma\sigma_x(b^\dagger + b) + \hbar\Omega b^\dagger b$$

adiabatic energies:

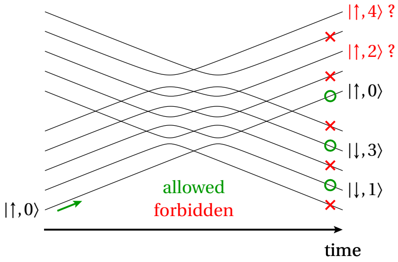


- diabatic states: $|\uparrow, n\rangle, |\downarrow, n\rangle$
- initial state: $|\uparrow, 0\rangle$ (ground state for $t \to -\infty$)
- coupling $\sigma_x(b^\dagger + b)$ selection rule: $|\uparrow, 2\ell + 1\rangle$ and $|\downarrow, 2\ell\rangle$ never polulated

numerical solution

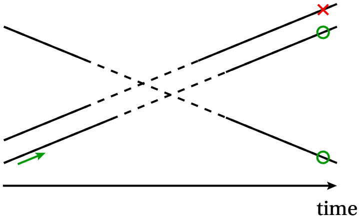


- all $|\uparrow, n \neq 0\rangle$ finally unpopulated
- selection rule for odd n only



? hidden selection rule **?**

Landau-Zener problem for three levels



- upper level for $t \to \infty$ never populated ➔ "no-go theorem"

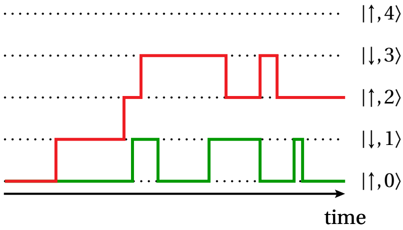
  Brundobler & Elser (1993)

  Shytov (2004); Volkov & Ostrovsky (2005)

- here:
  - generalization to (infinitely) many levels
  - corollary: "no-go theorem" for perturbation series

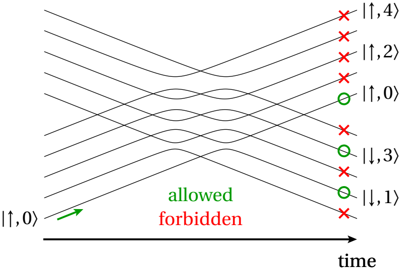
no-go theorem

series for $\langle\uparrow, n|U(\infty,-\infty)|\uparrow,0\rangle$ with perturbation $H_{\text{int}} = \gamma\sigma_x(b^\dagger + b)$



- no contribution
- only contribution: repeated jumps between $|\uparrow,0\rangle$ and $|\downarrow,1\rangle$

no-go theorem

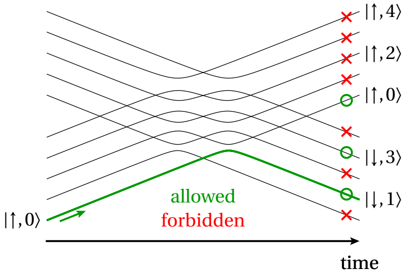


consequences:

1. no-go theorem for $t \to \infty$:
2. perturbation series for $P_{\uparrow\to\uparrow}$ consists of only the states $|\uparrow,0\rangle$ and $|\downarrow,1\rangle$
   - ➔ local no-go theorem
   - ➔ same perturbation series as for standard LZ problem with $\Delta/2 \longrightarrow \gamma$

$$P_{\uparrow\to\uparrow}(\infty) = \exp\left(-\frac{2\pi\gamma^2}{\hbar v}\right)$$

applications



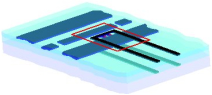
quantum state preparation:

- slow switching, $v \ll \gamma^2/\hbar$:
  single-photon generation
- "weak" coupling: $\gamma \ll \hbar\Omega$:

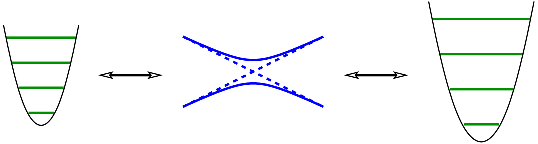
$$\psi(\infty) = \alpha(v)|\uparrow, 0\rangle + \beta(v)|\downarrow, 1\rangle$$

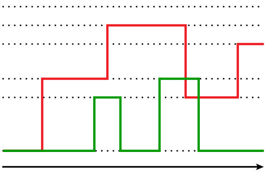
controlled qubit-oscillator
entanglement, Bell states

generalization to two oscillators



R. Gross *et al.*, WMI Garching





no-go theorem for initial state $|\uparrow,0,0\rangle$

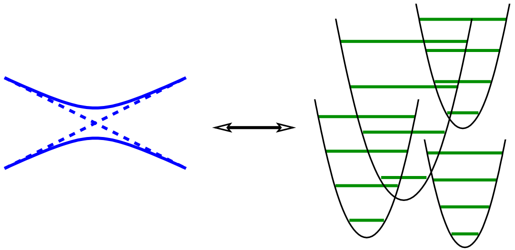
- forbidden: many-photon states
- allowed: jumps between ground state and single-photon states

$$P_{\uparrow\to\uparrow}(\infty) = \exp\left(-\frac{2\pi(\gamma_1^2+\gamma_2^2)}{\hbar v}\right)$$

Wubs, SK, Hänggi, Physica E, in press; cond-mat/0703425

The dissipative Landau-Zener problem

two-level system coupled to a harmonic-oscillator bath



➔ quantum dissipation

Wubs, Saito, SK, Hänggi, Kayanuma, PRL **97**, 200404 (2006) general coupling qubit-bath coupling:

$$H = -\frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + (\sigma_z \cos\theta + \sigma_x \sin\theta)\sum_\nu \gamma_\nu (a_\nu^\dagger + a_\nu) + \sum_\nu \hbar\omega_\nu a_\nu^\dagger a_\nu$$

- $\cos\theta \neq 0$: displaced oscillator ground states

  ➔ diabatic states $|\uparrow, \boldsymbol{n}_+\rangle$ and $|\downarrow, \boldsymbol{n}_-\rangle$, note: generally $|\boldsymbol{n}_+\rangle \neq |\boldsymbol{n}_-\rangle$

  ➔ reorganization energy $E_0 = \sum_\nu \frac{\gamma_\nu^2}{4\hbar\omega_\nu}$

- effective coupling strength $S = \sum_\nu \gamma_\nu^2$

- temperature $T = 0$: initially in the (adiabatic) ground state

Landau-Zener transition probability

- no-go theorem
- transition probability

$$P_{\uparrow\to\uparrow}(\infty) = \exp\left(-\frac{\pi W^2}{2\hbar v}\right), \quad W^2 = \left(\Delta - E_0 \sin\theta\cos\theta\right)^2 + S\sin^2\theta$$

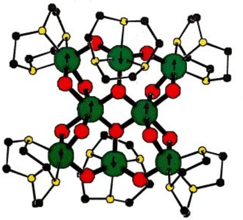
exact solution for a dissipative quantum system

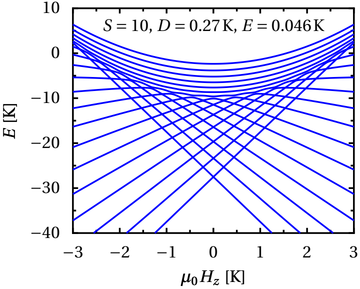
➔ $\theta \neq 0$: experimental determination of

- reorganization energy E_0
- effective coupling strength S

➔ $\theta = 0$: bath has no influence

application: nanomagnets





- molecular Fe_8 cluster, $S = 10$

$$H = -DS_z^2 + E(S_x^2 - S_y^2) + g\mu_0 \vec{S} \cdot \vec{H}(t)$$

- determination of the very small tunnel splittings via LZ transitions

  Wernsdorfer & Sessoli, Science (1999)

- hypothesis:

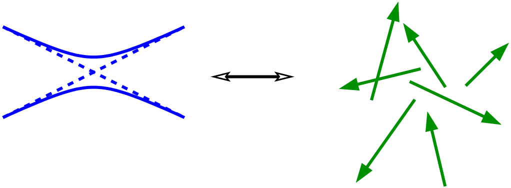
  all LZ transitions are independent of environment and dephasing

  Leuenberger & Loss, PRB (2000)

- proof: see above

The dissipative Landau-Zener problem

in the presence of a spin bath



Saito, Wubs, SK, Kayanuma, Hänggi, PRB **75**, 214308 (2007)

spin bath

general coupling to a spin bath

$$H = -\frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + \sum_{i=x,y,z} \sigma_i \sum_\nu \gamma_\nu^i \tau_\nu^i + \sum_\nu \sum_{i=x,y,z} B_\nu^i \tau_\nu^i$$

- no-go theorem ➔ … ➔ exact spin-flip probability

special case:

- $\gamma_\nu^x = \gamma_\nu^y = 0$ corresponds to $\theta = 0$

$$H_{\text{qubit–env}} = \sigma_z \sum_\nu \gamma_\nu^i \tau_z^i$$

➔ $W^2 = \Delta^2$, i.e. Landau-Zener probability bath-independent

summary

- qubit coupled to …
  - … quantum oscillator
  - … quantum heat bath at $T = 0$
  - … spin bath
- applications
  - quantum state preparation
  - determination of system-bath coupling
- current projects
  - qubit–oscillator–bath models
  - finite temperature
  - intuitive understanding of “no-go theorems”

thanks to . . .

- Roland Doll
  David Zueco
  Peter Hänggi (Augsburg)
- Martijn Wubs (Copenhagen)
- Keiji Saito (Tokyo)
- Yosuke Kayanuma (Osaka)

no-go theorem: some hint on a derivation

- perturbation series in qubit-oscillator coupling $\sigma_x(b^\dagger + b)$: terms of the structure

$$\int_{-\infty}^{\infty} \mathrm{d}t_1 \int_{t_1}^{\infty} \mathrm{d}t_2 \int_{t_2}^{\infty} \mathrm{d}t_3 \ldots \exp\left[\mathrm{i}\sum\left(\lambda_\ell \Omega t_\ell + \frac{v}{2\hbar}(t_{2\ell}^2 - t_{2\ell-1}^2)\right)\right]$$

  where $\lambda_\ell = \pm 1$ (from $b^\dagger$ and b)

- substitution to time differences

$$\int_{-\infty}^{\infty} \mathrm{d}t_1 \int_{0}^{\infty} \mathrm{d}\tau_2 \mathrm{d}\tau_3 \ldots$$

- first integral provides $\delta\left(v \sum_\ell \tau_\ell + \Omega \sum_{\ell=1}^{2k} \lambda_\ell\right)$

- since all $\tau_\ell \geq 0 \rightarrow \sum_{\ell=1}^{2k} \lambda_\ell \leq 0$

- all terms with "$<$" have vanishing prefactor $\rightarrow \lambda_\ell = (-1)^\ell$