Time scale ratios and critical dynamics

R. Folk[1]

[1]Institut für Theoretische Physik, Johannes Kepler Universität Linz, A–4040 Linz, Austria

*supported by FWF

Path Integrals - New Trends and Perspectives
September 23 - 28, 2007, Dresden Germany

Cooperation with

M. Dudka, Institute for Condensed Matter Physics, Lviv

Yu. Holovatch, Lviv National University

G. Moser, University of Salzburg

M. Weiretmayer, R. F., University of Linz

Outline

1. Model C: Strong and weak scaling
2. Model C: Introducing disorder; effective dynamic critical behavior
3. Superconductor: Gauge dependence of dynamics
4. ^{3}He-^{4}He mixtures (Model F'): Tricritical dynamics
5. Antiferromagnet: Dynamical shape functions

Model C: Halperin, Hohenberg, Ma, Phys. Rev. B **10**, 139 (1974)

Dynamical Equations

Order parameter

$$\frac{\partial \vec{\phi}_0}{\partial t} = -\overset{o}{\Gamma} \frac{\delta H}{\delta \vec{\phi}_0} + \vec{\theta}_\phi$$

Conserved density

$$\frac{\partial m_0}{\partial t} = \overset{o}{\lambda} \nabla^2 \frac{\delta H}{\delta m_0} + \theta_m$$

Stochastic forces

$$\langle \theta_{\phi_i}(x,t)\ \theta_{\phi_j}(x',t')\rangle = 2\overset{o}{\Gamma}\ \delta_{ij}\delta(x-x')\delta(t-t')$$

$$\langle \theta_m(x,t)\ \theta_m(x',t')\rangle = -2\overset{o}{\lambda}\nabla^2\delta(x-x')\delta(t-t')$$

The important parameter is the time scale ratio $\overset{o}{w} = \frac{\overset{o}{\Gamma}}{\overset{o}{\lambda}}$

Model C

Static functional

$$H = \int d^d x \Bigg\{ \frac{1}{2} \overset{o}{\tau} (\vec{\phi}_0 \cdot \vec{\phi}_0) + \frac{1}{2} \sum_{i=1}^{n} \vec{\nabla} \phi_{i0} \cdot \vec{\nabla} \phi_{i0} + \frac{\overset{o}{\tilde{u}}}{4!} (\vec{\phi}_0 \cdot \vec{\phi}_0)^2 + \frac{1}{2} a_m m_0^2 + \frac{1}{2} \overset{o}{\gamma} m_0 (\vec{\phi}_0 \cdot \vec{\phi}_0) - \overset{o}{h}_m m_0 \Bigg\}$$

Model C: Strong and Weak Scaling

Strong scaling

$$w^\star = \text{nonzero, finite}$$

Same time scale for characteristic frequencies

$$\omega_\phi \sim k^{z} g_\phi(k\xi) \quad \omega_m \sim k^{z} g_m(k\xi)$$

$$z = 2 + \zeta_\Gamma^\star$$

Weak scaling

$$w^\star = 0 \text{ or } \infty$$

Different time scale for characteristic frequencies

$$\omega_\phi \sim k^{z_\phi} g_\phi(k\xi) \quad \omega_m \sim k^{z_m} g_m(k\xi)$$

$$z_\phi = 2 + \zeta_\Gamma^\star \qquad z_m = 2 + \zeta_\lambda^\star$$

Results for model C: The dynamical ζ-functions in two loop order; F., Moser, Phys. Rev. Lett. **91**, 030601 (2003)

$$\zeta_\Gamma = \rho\gamma^2 + \frac{(n+2)u^2}{36}\left(L - \frac{1}{2}\right) - \frac{n+2}{6}u\rho\gamma^2(1-L) - \frac{1}{2}\rho^2\gamma^4 b$$

$$\zeta_\lambda = \frac{n}{2}\gamma^2 \qquad \rho = \frac{w}{1+w}$$

$$b = \frac{1}{2}(n-(n+2)L) - \frac{1}{1+w}\left[w + (1+2w)\ln\frac{(1+w)^2}{1+2w}\right]$$

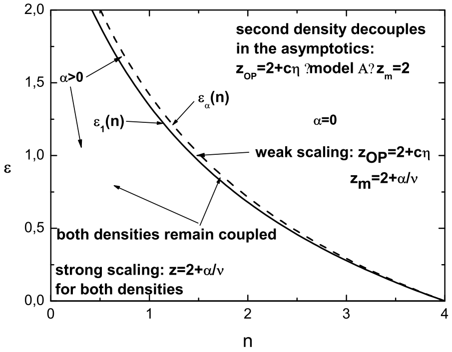
C: w real, $\quad L = 3\ln\frac{4}{3}$

C*: $w = w' + iw''$ $\quad L = 2\ln\frac{2}{1+\frac{\Gamma^+}{\Gamma}} + \left(2 + \frac{\Gamma}{\Gamma^+}\right)\ln\frac{(1+\frac{\Gamma^+}{\Gamma})^2}{1+2\frac{\Gamma^+}{\Gamma}}$

'Phase diagram' for model C

- Strong scaling: The fixed point value of w **is different from zero** then both densities have the same dynamical exponent $z_\phi = z_m = 2 + \alpha/\nu$.
- Weak scaling: The fixed point value of w **is zero** then the densities have different nontrivial dynamical exponents the OP $z_\phi = 2 + c\eta$ (model A) and the conserved density $z_m = 2 + \alpha/\nu$.

Stability boundaries



- Decoupling: The fixed point value of w **is zero** and the densities have different dynamical exponents The OP $z_\phi = 2 + c\eta$ (model A) and the

Disorder and Model C

Harris criterion

If the pure system has a diverging specific heat then the critical exponents may be changed by disorder and a new universality class is obtained. Otherwise disorder does not change the universality class of the pure system.

One concludes

If there is a change the new disordered unversality class is characterized by a non diverging specific heat

One knows for model C

If the static critical behavior is characterized by a non diverging specific heat the coupling of a conserved density does not change the critcal dynamics universality class, it remains model A

CONCLUSION

The critical dynamics of a disordered model is represented by model A. The coupling of a conserved density is in any case irrelevant.

ONLY VALID FOR THE ASYMPTOTICS!

Types of Disorder

$$\mathcal{H} = -\frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} J(|\mathbf{R}-\mathbf{R}'|)c_{\mathbf{R}}c_{\mathbf{R}'}\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} - D_0\sum_{\mathbf{R}}(\hat{x}_{\mathbf{R}}\vec{S}_{\mathbf{R}})^2,$$

- bond disorder

$$p(J) = \exp\left(-J^2/\Delta\right)$$

- site disorder

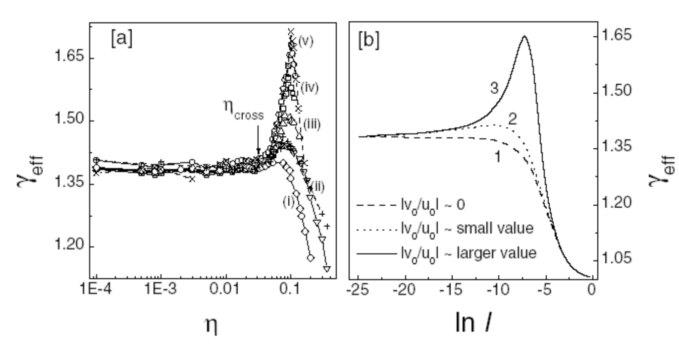
$$p(c=1)+p(c=0)=1$$

- anisotropic axis disorder

$$p(\hat{x}) = \frac{1}{2m}\sum_{i=1}^{m}\left[\delta^{(m)}(\hat{x}-\hat{k}_i)+\delta^{(m)}(\hat{x}+\hat{k}_i)\right]$$

Effective critical exponent γ_{eff}

Experiment and theory



Experiment for $Fe_{86}Mn_4Zr_{10}$ [1]; theory from [2].

[1] A. Perumal et al., Phys. Rev. Lett. **91**, 137202 (2003); [2] M. Dudka, et al., J. Magn. Magn. Mater. **256**, 243 (2003); [3] B.Berche, et al., Condensed Matter Physics **8**, 47 (2005)

Model C with disorder

Dynamical Equations

Order parameter
$$\frac{\partial \vec{\phi}_0}{\partial t} = -\mathring{\Gamma} \frac{\partial \mathcal{H}}{\partial \vec{\phi}_0} + \vec{\theta}_\phi$$

Conserved density
$$\frac{\partial m_0}{\partial t} = \mathring{\lambda} \nabla^2 \frac{\partial \mathcal{H}}{\partial m_0} + \theta_m$$

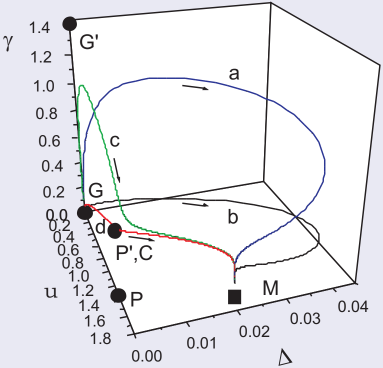
Static functional $\mathcal{H}$ of the quenched disordered magnetic system

$$\mathcal{H} = \int d^d x \Big\{ \frac{1}{2} \mathring{\tilde{\tau}} |\vec{\phi}_0|^2 + V(x) |\vec{\phi}_0|^2 + \frac{1}{2} \sum_{i=1}^{n} (\nabla \phi_{i,0})^2 + \frac{\mathring{\tilde{u}}}{4!} |\vec{\phi}_0|^4 + \frac{1}{2} a_m {m_0}^2 + \frac{1}{2} \mathring{\gamma} m_0 |\vec{\phi}_0|^2 - \mathring{h}_m m_0 \Big\}$$

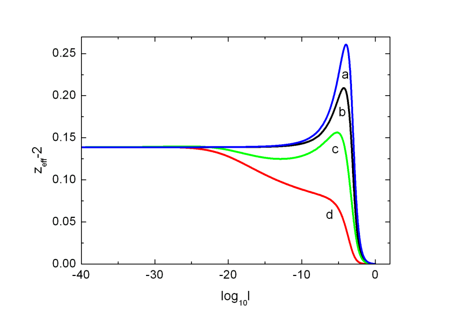
$$<< V(x) V(x') >>= 4 \mathring{\Delta} \delta(x - x')$$

Results for the diluted model C

Static flow; Ising case



Dynamical effective exponent z_{eff}



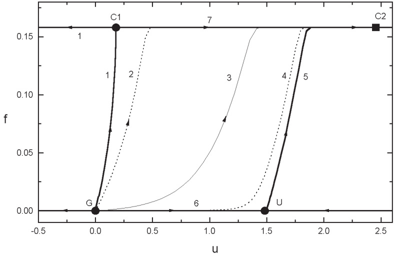
$$z_{eff}(l) = 2 + \zeta_\Gamma\big(u(l), \Delta(l), \gamma(l), \rho(l)\big) .$$

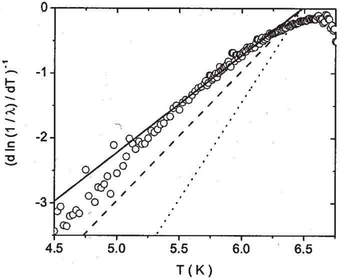
Dudka, F., Holovatch, Moser, J. Phys. A: Math. Gen. 39, 7943 (2006); 40, 8247 (2007)

Superconductor - Coupling to a gauge field

$$\mathcal{H} = \int d^d x \Big\{ \frac{1}{2} \mathring{r} |\vec{\psi}_0|^2 + \frac{1}{2} \sum_{i=1}^{n/2} |(\boldsymbol{\nabla} - i\mathring{e}\mathbf{A}_0)\psi_{0,i}|^2$$

$$+ \frac{\mathring{u}}{4!}(|\vec{\psi}_0|^2)^2 + \frac{1}{2}(\boldsymbol{\nabla} \times \mathbf{A}_0)^2 + \frac{1}{2\mathring{\varsigma}}(\boldsymbol{\nabla} \cdot \mathbf{A}_0) \Big\}$$





Pyrochlore oxide $RbOs_2O_6$ T_c: 6.3 K : T. Schneider, R. Khasanov, H. Keller (2005)

Dynamic model of Lannert et al. PRL **92** 097004 (2004)

Simpelst model: Two coupled (by the charge) relaxational equations

$$\frac{\partial\psi_{0,i}}{\partial t} = -2\mathring{\Gamma}_\psi \frac{\delta\mathcal{H}}{\delta\psi^+_{0,i}} + \theta_i$$

$$\frac{\partial\psi^+_{0,i}}{\partial t} = -2\mathring{\Gamma}_\psi \frac{\delta\mathcal{H}}{\delta\psi_{0,i}} + \theta^+_i$$

$$\frac{\partial A_{0,\alpha}}{\partial t} = -\mathring{\Gamma}_A \frac{\delta\mathcal{H}}{\delta A_{0,\alpha}} + \theta_\alpha .$$

$$< \theta_i(\mathbf{x},t)\theta^+_j(\mathbf{x}',t') > = 4\mathring{\Gamma}_\psi\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\delta_{ij},$$

$$< \theta_i(\mathbf{x},t) > = 0$$

$$< \theta_\alpha(\mathbf{x},t)\theta_\beta(\mathbf{x}',t') > = 2\mathring{\Gamma}_A\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\delta_{\alpha\beta}$$

$$< \theta_\alpha(\mathbf{x},t) > = 0 .$$

Strong and weak scaling

Important parameter - fixed point value of the time scale ratio w

$$w = \frac{\Gamma_\psi}{\Gamma_A}$$

Strong scaling

$$w^\star = \text{nonzero, finite}$$

Same time scale for characteristic frequencies

$$\omega_\psi \sim k^{z} g_\psi(k\xi) \quad \omega_A \sim k^{z} g_A(k\xi)$$

z measurable, gauge independent

Weak scaling

$$w^\star = 0 \text{ or } \infty$$

Different time scale for characteristic frequencies

$$\omega_\psi \sim k^{z_\psi} g_\psi(k\xi) \quad \omega_A \sim k^{z_A} g_A(k\xi)$$

z_A measurable, gauge independent

Dynamical critical exponents z_ψ and z_A

$$z_\psi = 2 + \zeta_{\Gamma_\psi}(u^\star, e^\star, w^\star, \varsigma) \quad z_A = 2 + \zeta_{\Gamma_A}(u^\star, e^\star, w^\star)$$

Strong Scaling

$$z_\psi = z = 2 + \frac{18}{n}\varepsilon - \varsigma\frac{6}{n}\,\frac{\varepsilon}{1 + w^\star}$$

$$z_A = z = 2 - \varepsilon + \frac{3\varepsilon}{2w^\star}$$

z GAUGE DEPENDENT

STABLE FIXED POINT IN ONE LOOP

Weak Scaling

$$z_\psi = 2 + \frac{18}{n}\varepsilon$$

$$z_A = 2 - \epsilon$$

z_ψ, z_A GAUGE INDEPENDENT

DYNAMICALLY UNSTABLE FIXED POINT IN ONE LOOP

z_ψ same as in M. K. Bushev and D. I. Uzunov, Phys. Lett. A **76**, 306 (1980); Err., ibid. **78**, 491 (1980) calculating quantum fluctuation effects

Message

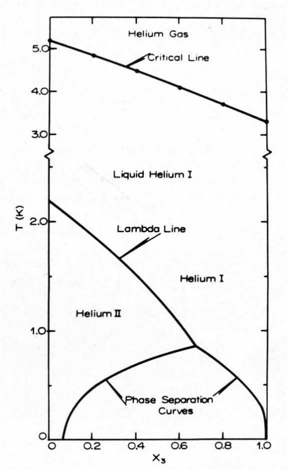
- If a strong scaling fixed point is stable its corresponding dynamic critical exponent z has to be independent of the choice of the gauge
- otherwise a weak scaling fixed point has to be the stable fixed point and only the dynamic critical exponent of the gauge field z_A has to be independent of the choice of the gauge

Questions

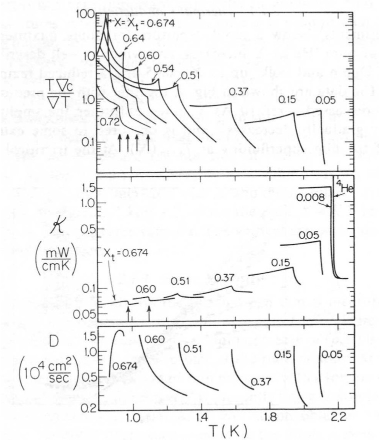
- What happens in two loop?
- What is the result of a nonperturbative RG treatment?
- Modification of the model: are there reversible couplings?
- Are there other slow variables?
- ...

Dudka, F., Moser, Condensed Matter Physics (Ukraine) **10**, 189 (2007)

Phase diagram of ^{3}He-^{4}He mixtures and transport coefficients



E.H. Graf, D.M. Lee, and J.D. Reppy Phys. Rev. Lett. **19**, 417 (1967)



H. Meyer et al. J. Low Temp. Phys. **70**, 219 (1987)

Siggia and Nelson set up model F′ describing critical dynamics

Order parameter

$$\frac{\partial \psi_0}{\partial t} = -2\mathring{\Gamma} \frac{\delta \mathcal{H}}{\delta \psi_0^+} + i\psi_0 \mathring{\mathbf{g}} \cdot \frac{\delta \mathcal{H}}{\delta \mathbf{m}_0} + \theta_\psi \, ,$$

$$\frac{\partial \psi_0^+}{\partial t} = -2\mathring{\Gamma}^+ \frac{\delta \mathcal{H}}{\delta \psi_0} - i\psi_0^+ \mathring{\mathbf{g}} \cdot \frac{\delta \mathcal{H}}{\delta \mathbf{m}_0} + \theta_\psi^+ \, ,$$

Conserved densities (entropy density - concentration)

$$\frac{\partial \mathbf{m}_0}{\partial t} = \begin{pmatrix} \mathring{\lambda} & \mathring{L} \\ \mathring{L} & \mathring{\mu} \end{pmatrix} \cdot \nabla^2 \frac{\delta \mathcal{H}}{\delta \mathbf{m}_0} + 2\mathring{\mathbf{g}} \Im \left[\psi_0 \frac{\delta \mathcal{H}}{\delta \psi_0} \right] + \boldsymbol{\theta}_m \, .$$

Siggia, Nelson, Phys. Rev. B **15**, 1427 (1977)
Folk, Moser, Phys. Rev. Lett. **89**, 125301 (2002); **93**, 229902 (E) (2004)

Static functional in the extended model - $n = 2$

$$
\begin{aligned}
\mathcal{H} &= \int d^d x \left\{ \frac{1}{2}\mathring{\tilde{\tau}}\psi_0^+\psi_0 + \frac{1}{2}\vec{\nabla}\psi_0^+\vec{\nabla}\psi_0 + \frac{\mathring{\tilde{u}}}{4!}(\psi_0^+\psi_0)^2 \right\} \\
&+ \int d^d x \left\{ \frac{1}{2}\mathbf{m}_0\cdot\mathbf{m}_0 + \frac{1}{2}\mathring{\gamma} m_{20}\psi_0^+\psi_0 - \mathring{h} m_{20} \right\} \\
\mathcal{H} &= \int d^d x \left\{ \frac{1}{2}\mathring{r}\psi_0^+\psi_0 + \frac{1}{2}\vec{\nabla}\psi_0^+\vec{\nabla}\psi_0 + \frac{\mathring{u}}{4!}(\psi_0^+\psi_0)^2 \right\}
\end{aligned}
$$

$$
\mathring{u} = \mathring{\tilde{u}} - 3\mathring{\gamma}^2
$$

Important model parameters

Static couplings

fourth order coupling: u asymmetric coupling: γ

Mode couplings ($\Gamma = \Gamma' + i\Gamma''$ is complex)

two couplings to the conserved densities: $f_1^2 = \frac{g_1^2}{\Gamma'\lambda}$ $f_2^2 = \frac{g_2^2}{\Gamma'\mu}$

Time scale ratios (two are complex)

$$w_1 = \frac{\Gamma}{\lambda} \qquad w_2 = \frac{\Gamma}{\mu} \qquad w_3^2 = \frac{L^2}{\lambda\mu}$$

One loop theory

Fixed point: exact in all loop orders

$$w_3^\star = \pm 1 \,, \qquad \pm f_1^\star = f_2^\star = f^\star \qquad \gamma^\star = 1$$

Fixed point: one loop order

$$\begin{aligned} w_1'^\star &= 0 \,, \qquad w_2'^\star = \infty \qquad f^{\star 2} = \frac{4}{3}\,\epsilon \\ w_1''^\star &= 0 \,, \qquad w_2''^\star = 0 \end{aligned}$$

Calculation of the exponent for the mass diffusion

$$\zeta_k^\star = -2/3 \qquad z_\psi = 5/3 \qquad z_{m_1} = 4/3 \qquad z_{m_2} = 7/3$$

$$D \sim t_X^{\alpha/\nu + \zeta_k^\star} \qquad \alpha = 1/2 \qquad \nu = 1/2 \qquad D \sim t_X^{1/3}$$

E.D. Siggia and D.R. Nelson, *Phys. Rev. B* **15**, 1427 (1977)
L. Peliti, in *Lecture Notes in Physics* ed. Ch. P. Enz (Springer

Two loop theory: F., Moser, subm. J. Low Temp. Phys.(2007)

In dynamic ζ-function terms proportional γ^4

One diverging in the ζ_Γ-function for $w_2' \to \infty$ (already in model C)

Fixed point found numerically

$$f^\star = 1.66 \qquad w_1^\star \;=\; 0 \qquad w_2^\star = 0$$

Calculation of the exponent for the mass diffusion

$$\zeta_k^\star \simeq -2 \qquad z_\psi \simeq 3 \qquad z_{m_1} \simeq 0 \qquad z_{m_2} \simeq 1$$

$D \sim t_X^{\alpha/\nu+\zeta_k^\star}$ $\qquad \alpha = 1/2 \qquad \nu = 1/2 \qquad$ $D \sim t_X^{-1}$ DIVERGES !!!

Way out: F., Moser, subm. J. Low Temp. Phys.(2007)

Reject asymmetry

$$u \equiv 0 \qquad \text{and} \qquad \gamma \equiv 0$$

Two loop fixed point found numerically

$$f^\star = 0.8256 \qquad w^\star = 0.0454$$

$$w_1^\star = w^\star(1 + g_2^2/g_1^2) \qquad w_2^\star = w^\star(1 + g_1^2/g_2^2)$$

Exponents

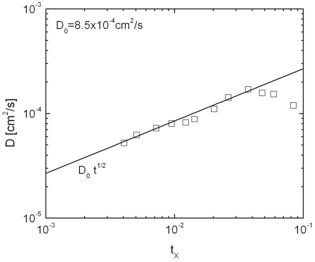
$$\zeta_k^\star = \zeta_\Gamma^\star = -\frac{\epsilon}{2} = -\frac{1}{2} \qquad z = 3/2$$

$$D \sim t_X^{\alpha/\nu + \zeta_k^\star} \qquad \alpha = 1/2 \qquad \nu = 1/2 \qquad D \sim t_X^{1/2}$$

Comparison with experiment

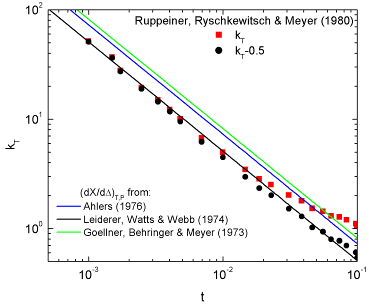
Mass diffusion

$$D = \frac{\mu(t_X) a_s^{\star 2}}{RT\chi_{T,P}(t_X)} G^\star \sim t_X^{1/2}$$



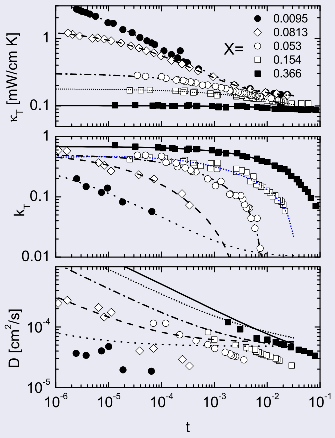
Thermal diffusion ratio

$$k_T^{T\to T_\lambda} = T_\lambda \left[\left(\frac{\partial c}{\partial \Delta}\right)_{PT} \frac{\sigma}{c} - \left(\frac{\partial c}{\partial T}\right)_{P\Delta} \right]$$

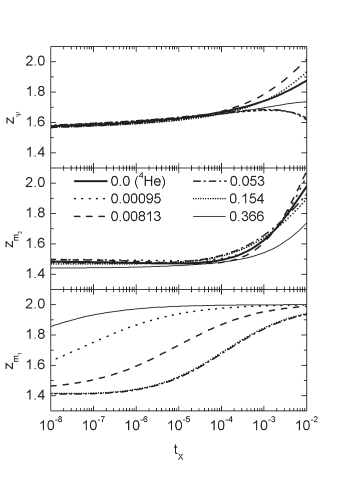


Effective exponents for $X \leq X_{tri}$

Experiments



Effective exponents



SSS-Model: Sasvari, Schwabl, Szepfalusy, Physica **81A**, 108 (1975)

Order parameter

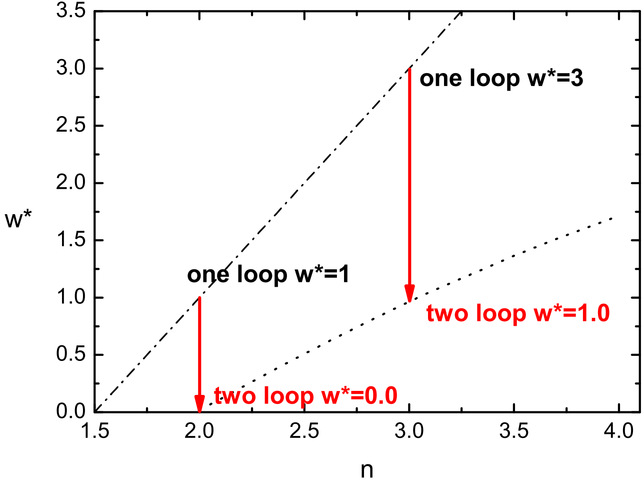
$$\frac{\partial \phi_{0\alpha}}{\partial t} = -\mathring{\Gamma}\frac{\delta \mathcal{H}}{\delta \phi_{0\alpha}} + \mathring{g}\sum_{\beta} \phi_{0\beta}\frac{\delta \mathcal{H}}{\delta m_{0\alpha\beta}} + \theta_{\psi_\alpha}$$

Conserved density

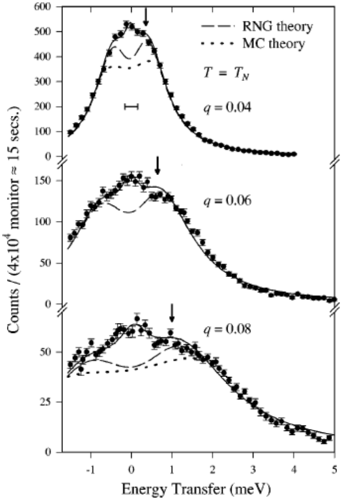
$$\frac{\partial m_{0\alpha\beta}}{\partial t} = \mathring{\lambda}\nabla^2 \frac{\delta \mathcal{H}}{\delta m_{0\alpha\beta}} + \mathring{g}\left\{\phi_{0\alpha}\frac{\delta \mathcal{H}}{\delta \phi_{0\beta}} - \phi_{0\beta}\frac{\delta \mathcal{H}}{\delta \phi_{0\alpha}}\right\} + \theta_{m_{\alpha\beta}}$$

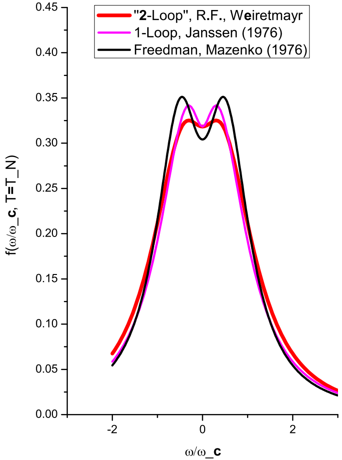
$$H = \int d^d x \left\{ \frac{1}{2}\mathring{\tau}(\vec{\phi}_0\cdot\vec{\phi}_0) + \frac{1}{2}\sum_{i=1}^{n}\vec{\nabla}\phi_{i0}\cdot\vec{\nabla}\phi_{i0} + \frac{\mathring{\tilde{u}}}{4!}(\vec{\phi}_0\cdot\vec{\phi}_0)^2 + \frac{1}{2}\sum_{\alpha\neq\beta} a_m m_{0\alpha\beta} m_{0\alpha\beta} \right\}$$

Time Scale Ratio of the SSS model



Shape function the OP; n=3 ($RbMgF_3$)





Exp.: Coldea, Cowley, Perring, McMorrow, Roessli (1998)