How to catch a wave packet...

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Dresden, 06/08/2005

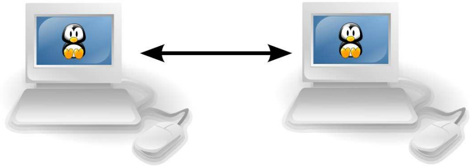
1. Introduction
   - Connecting quantum computers
   - Dispersion

2. Catching the information
   - Classical case
   - Quantum case
   - Convergence

3. Conclusion

Transfer of quantum information through permanently coupled systems

- How can we connect two parts of a quantum computer?



- Classical wires cannot be used for unknown quantum states
- Converting stationary solid-state qubits to flying qubits (photons, electrons) may be difficult and not worthy for short distances.
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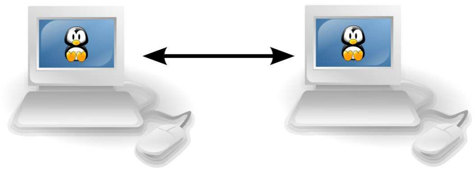

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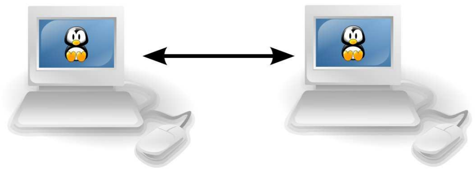

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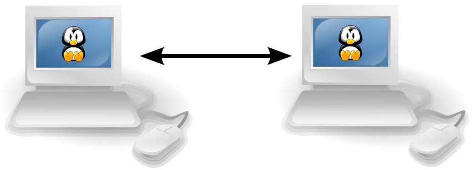

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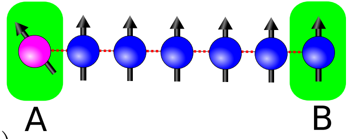
- Linear chain of spin-$1/2$ particles equally coupled by Heisenberg interaction, i.e.



$$H = -J \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1} + Z_n Z_{n+1})$$

- Encode unknown state $|\psi_A\rangle$ in the first spin, leave the rest in the ground state $|00\ldots00\rangle$.

- The excitation is forming a *spin wave* that travels along the chain and finally reaches Bob. BUT...

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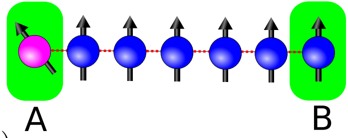

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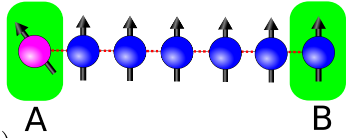

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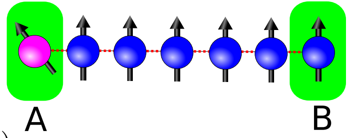

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- Even without external noise, the fidelity $|\langle\psi_A|\rho_B|\psi_A\rangle|$ is not perfect!!!
- DISPERSION: Fidelity decreases as $\frac{1}{(\text{LENGTH})^{2/3}}$
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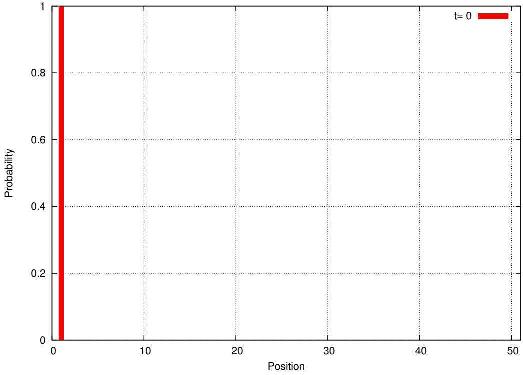

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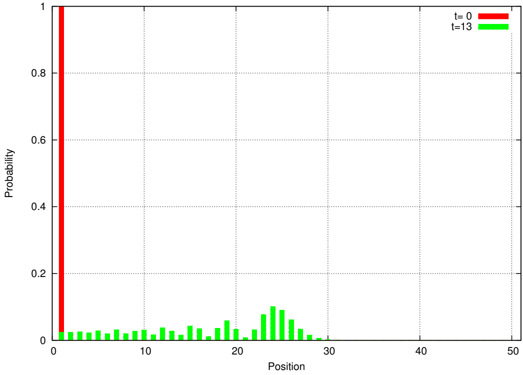

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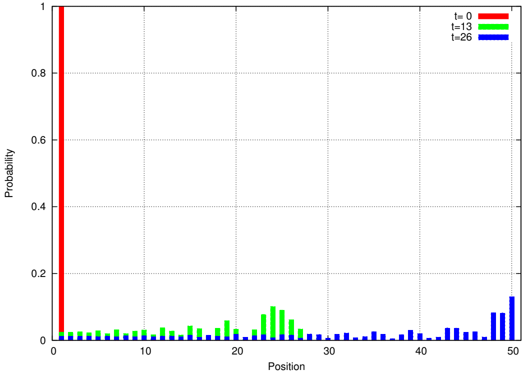

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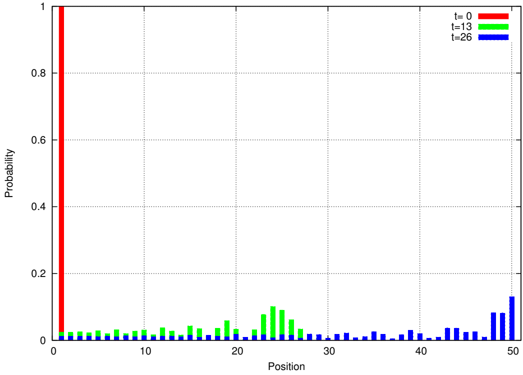

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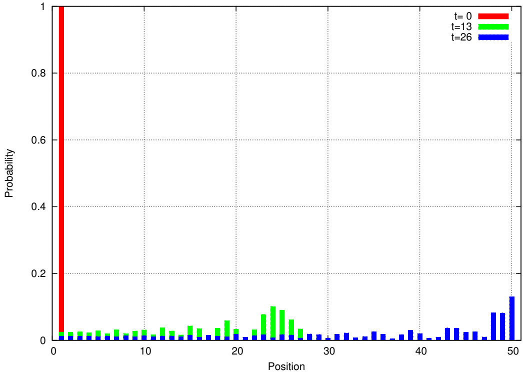

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Catching the dispersed information

- In principal one can overcome dispersion by increasing the control along the channel, by choosing specific hamiltonians or by encoding/decoding the information in a more complicated way
- Idea here: make use of the resources of the receiver to increase the fidelity of the transfer
- In particular, we will use the *memory* and *processor* of the receiving party to gradually store the wave packet and to decode it by a unitary transformation
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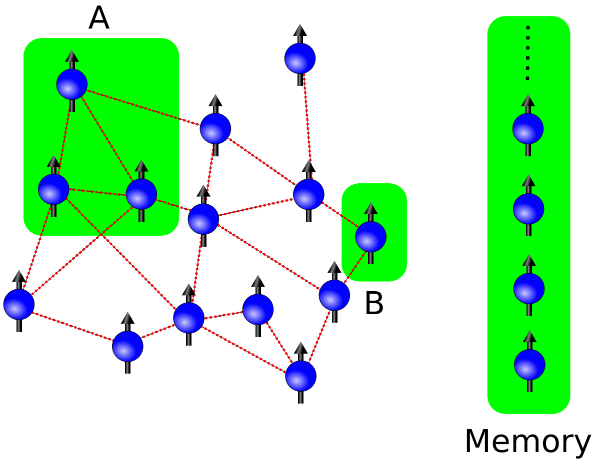

- A and B control some regions, and B has a large memory.

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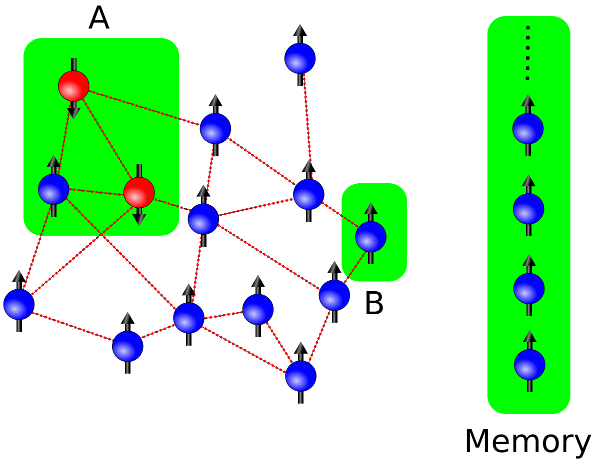

- A encodes { 1, 2, 3 } by flipping as many arrows (Here: "2").

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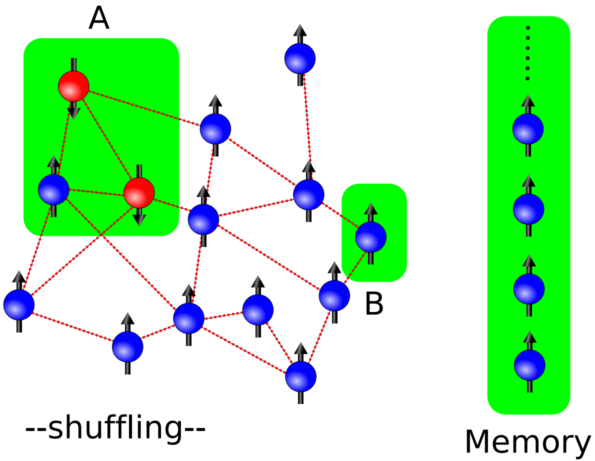

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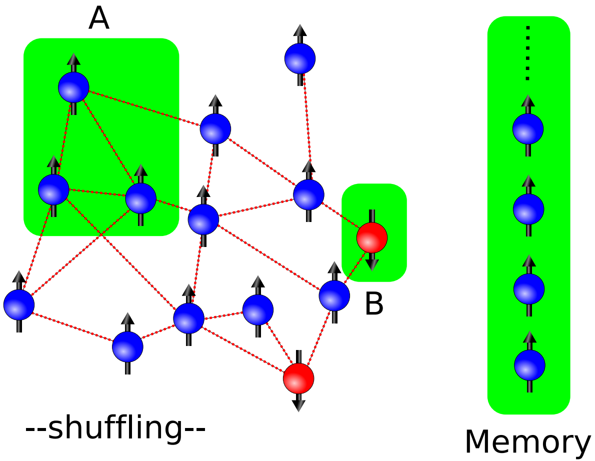

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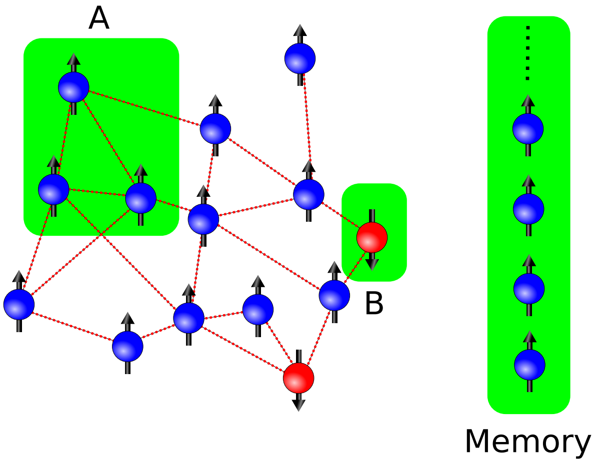

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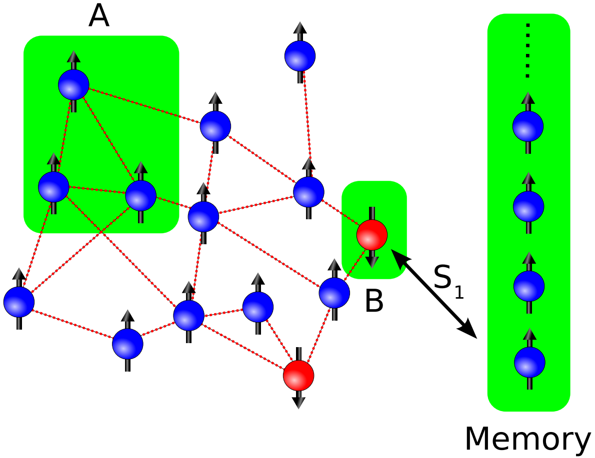

- After some time, Bob swaps his arrow(s) with the memory

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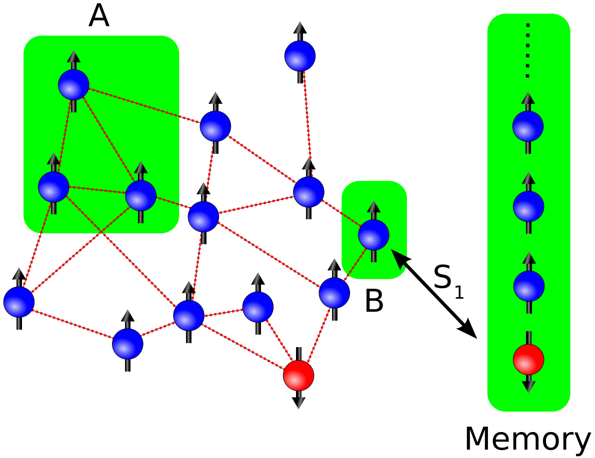

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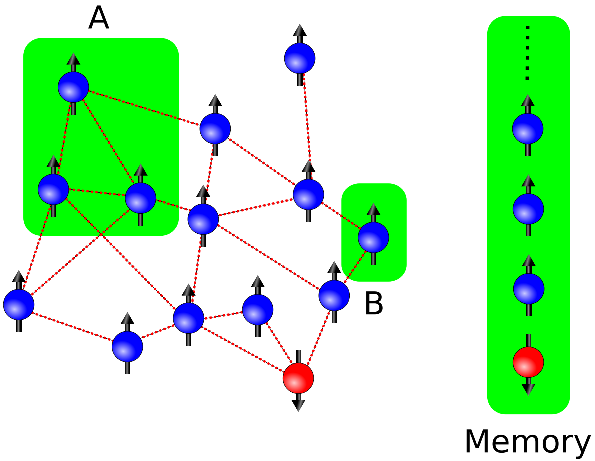

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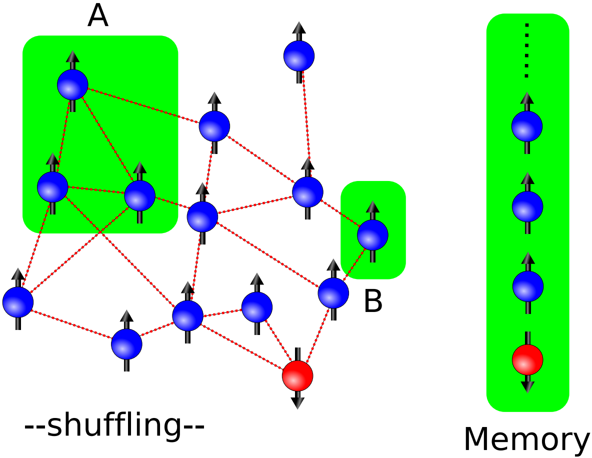

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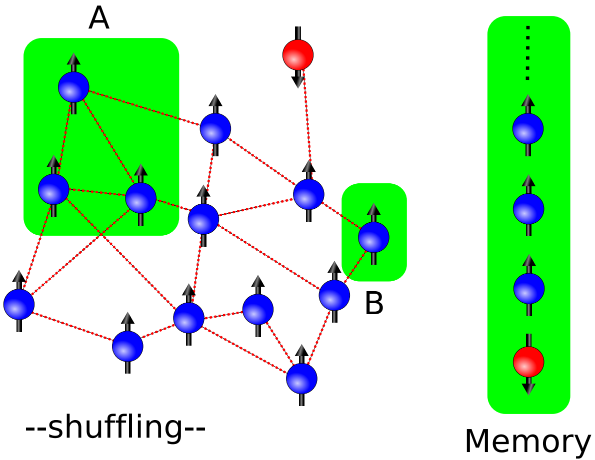

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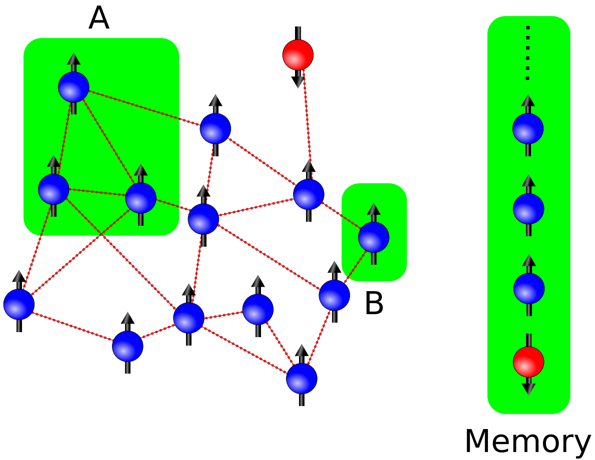

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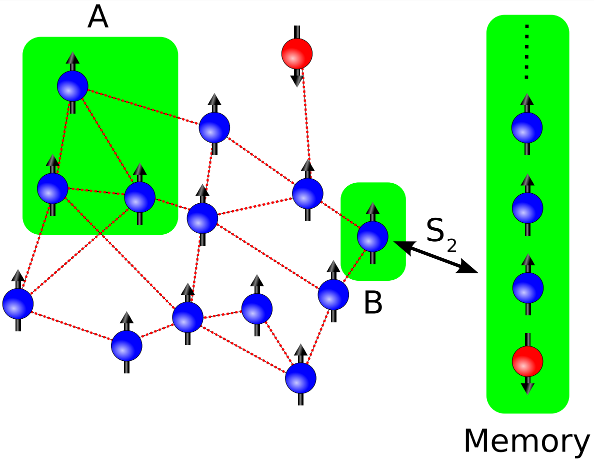

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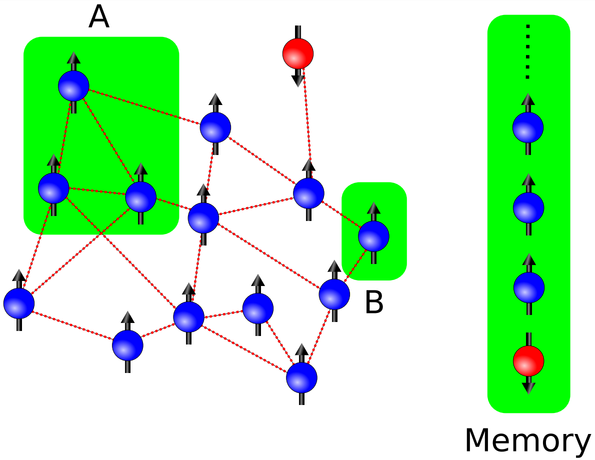

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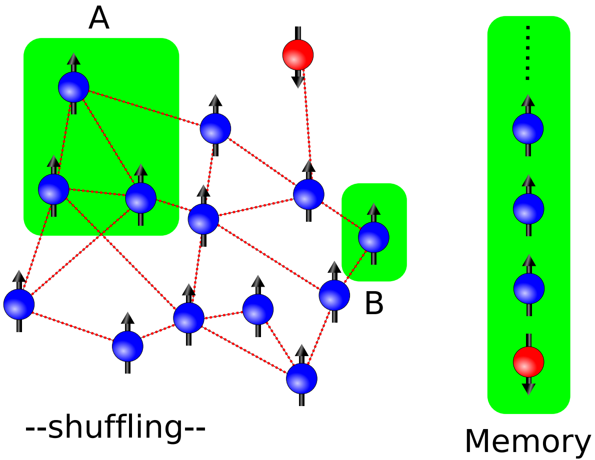

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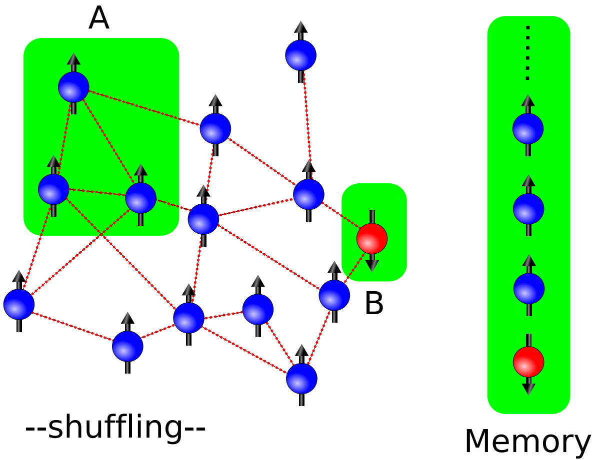

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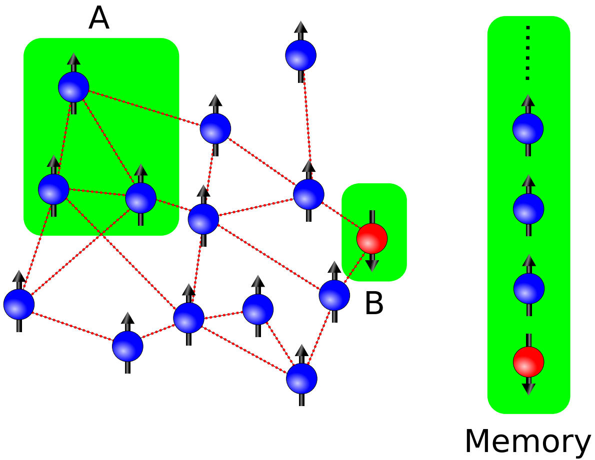

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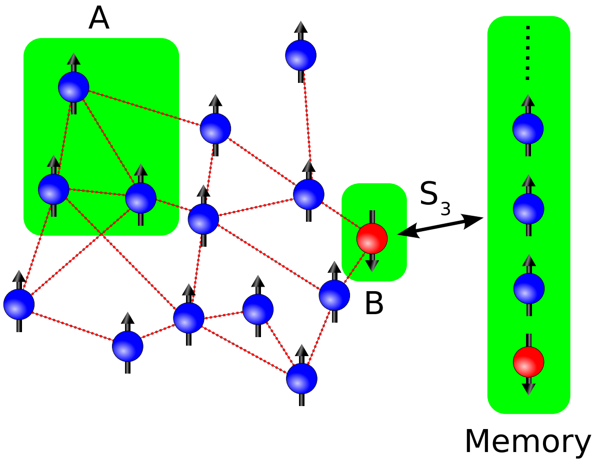

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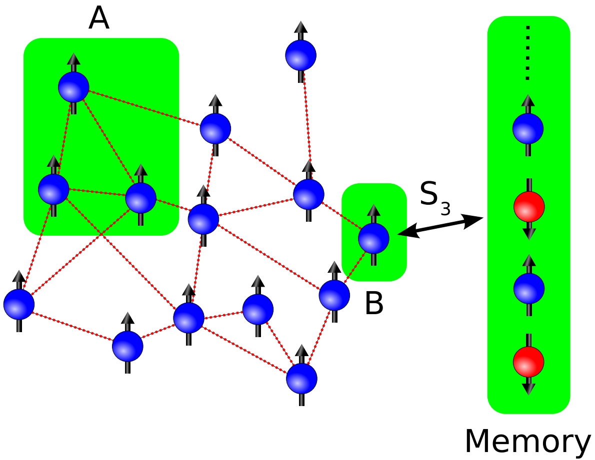

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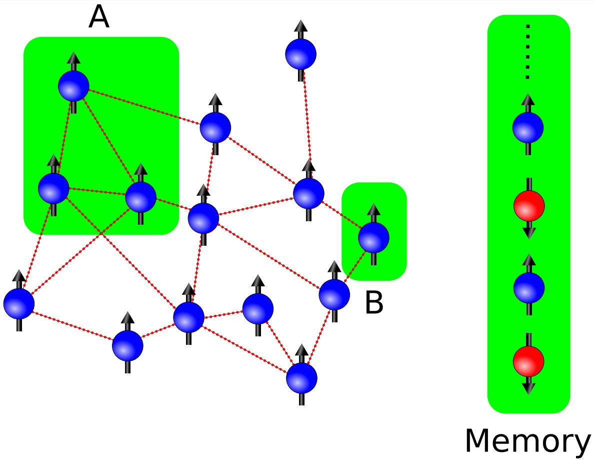

- Gradually, all flips leave the system and Bob decodes Alice's message by counting the flips in his memory

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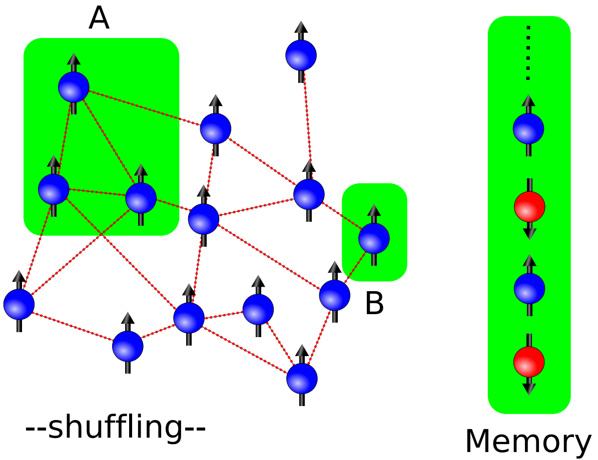

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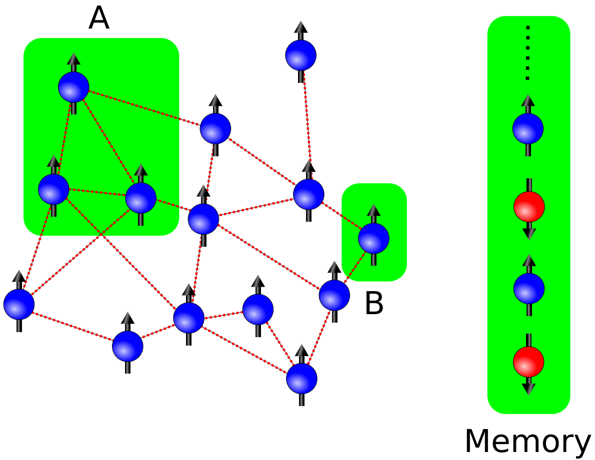

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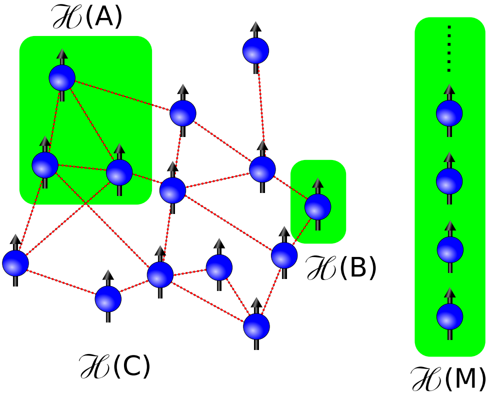

- The graph consists of qubits with a Hilbert space given by $\mathcal{H}(A) \otimes \mathcal{H}(C) \otimes \mathcal{H}(B) \otimes \mathcal{H}(M)$

Quantum case

- The “shuffling” is replaced by a coherent unitary dyamics for a fixed time τ

$$U = \exp\left(-iH\tau\right) \tag{1}$$

induced by the Hamiltonian

$$H = \sum J_{kl}\left(X_k X_l + Y_k Y_l\right) + \text{diagonal part} \tag{2}$$

- This Hamiltonian conserves the number of spin flips
- There is no dynamics in the memory
- The swaps are unitary operations S_j within the region of Bob's control
- The initial state is

$$|\psi\rangle_A \otimes |0\rangle_B \otimes |0\rangle_C \otimes |0\rangle_M \tag{3}$$

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Protocol

- As in the classical case, Bob performs swaps S_j to his memory at time intervals τ
- The dynamics is described by

$$W_j \equiv S_j U S_{j-1} \cdots S_1 U \tag{4}$$

- W_j tends to decrease the number of excitations in the system $A + C + B$ by swapping them in the memory
- Note that

$$W_j |E\rangle_{A,C} \otimes |0\rangle_B \otimes |0\rangle_M \propto W_j |E\rangle_{A,C} \otimes |0\rangle_B \otimes |0\rangle_M \tag{5}$$

for any eigenstate $|E\rangle_{A,C} \otimes |0\rangle_B$ of H. Example:

$$W_j |000\rangle_{ACB} \otimes |0\rangle_M = |000\rangle_{ACB} \otimes |0\rangle_M \tag{6}$$

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$$W_j |000\rangle_{ACB} \otimes |0\rangle_M = |000\rangle_{ACB} \otimes |0\rangle_M \tag{6}$$

Convergence theorem

Theorem

If there exists no other eigenstate of the Hamiltonian H of the form

$$|E\rangle_{A,C} \otimes |0\rangle_B$$

then

$$\lim_{j\to\infty} W_j \, |\psi 0 0\rangle_{ACB} \otimes |0\rangle_M = |0 0 0\rangle_{ACB} \otimes |\Phi(\psi)\rangle_M$$

for arbitrary ψ
(V. Giovannetti and DB, quant-ph/0508022)

- Note that since W_j is unitary and independent of ψ for all j, there exists a unitary transformation on the memory that Bob can apply to recover the message with perfect fidelity
- If Bob's memory is only finite,one can still *improve* the fidelity

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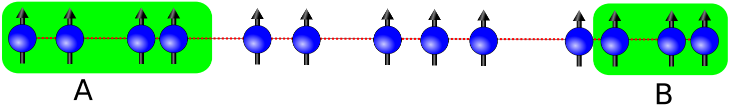

- Perfect state transfer is possible using a finite Heisenberg spin chain with arbitrary nearest-neighbor couplings
- The regions of A and B can be choosen arbitrarily, but the transfer is more efficient if Alice controls a larger region to send multipartite states
- Anderson localisation is irrelevant since the scheme aims at short chains only. The fingerprint of Anderson localisation for longer chains is a substancial slowdown of the convergence

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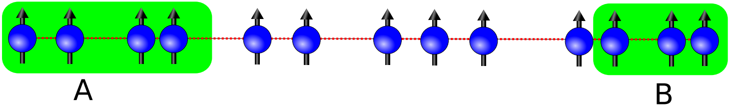

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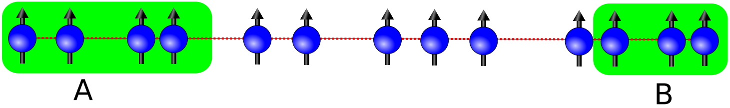

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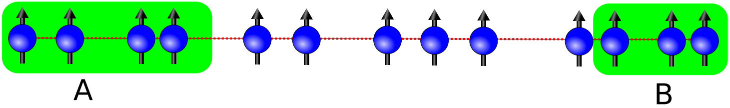

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Conclusion

- Solid state quantum wires of permanently coupled chains are very promising because they require low control
- But usually there is dispersion
- By making use of the resources of the receiving party, one can overcome this
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