



# Signature of Collective Excitations close to the Metal-to-Insulator Transition

Michał Karski<sup>1</sup>, Carsten Raas<sup>2</sup>, Götz S. Uhrig<sup>2</sup>

<sup>1</sup>Institut für Theoretische Physik • Universität zu Köln

<sup>2</sup>Institut für Theoretische Physik • Universität des Saarlandes

## Introduction

### Mott-Hubbard metal-insulator transition

- correlation-driven transition from paramagnetic metallic to paramagnetic insulating phase
- essential features captured by the single-band Hubbard model

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \hat{c}_{j\sigma}^\dagger \hat{c}_{i\sigma}) + U \sum_i \left( \hat{n}_{i\uparrow} - \frac{1}{2} \right) \left( \hat{n}_{i\downarrow} - \frac{1}{2} \right)$$

### Dynamic mean-field theory (DMFT)

- non-perturbative approximation which becomes exact in the limit  $d \rightarrow \infty$
- neglects spatial correlations, retains the dynamic correlations
- lattice problem is mapped onto an effective single-impurity model

$$\hat{H} = \sum_{n,\sigma} \gamma_n (\hat{c}_{n\sigma}^\dagger \hat{c}_{n+1,\sigma} + \text{h.c.}) + V \sum_{\sigma} (\hat{c}_{d\sigma}^\dagger \hat{c}_{0\sigma} + \text{h.c.}) + U \left( \hat{n}_{d\uparrow} - \frac{1}{2} \right) \left( \hat{n}_{d\downarrow} - \frac{1}{2} \right)$$

embedded in a self-consistently determined medium

$$G_{\sigma}^{\text{And}}(\omega) = G_{\sigma}^{\text{Hub}}(\omega), \quad \Sigma_{\sigma}^{\text{And}}(\omega) = \Sigma_{\sigma}^{\text{Hub}}(\omega)$$

- effective single-impurity model must be solved for a single-particle Green function
- several numerical and analytical methods were applied as impurity solvers

### Focus of investigation

- spectral densities close to the metal-insulator transition
- interplay of electronic degrees of freedom with collective modes in a highly correlated metal

### Investigated model

- single-band Hubbard model at half band-filling and zero temperature on a Bethe lattice with infinite coordination

### Motivation

- Dynamic DMRG: well controlled energy resolution at all energy scales

## Methods

### Self-consistency cycle

- iterative determination of the hybridization function

$$\Gamma(z) = \frac{V^2}{z - \frac{\gamma_0^2}{z - \frac{\gamma_1^2}{z - \dots}}}$$

$\Gamma_S(z) \longrightarrow$   
 $\uparrow$  self-consistency ?  
 $\Gamma_E(z) \longleftarrow$

$\longrightarrow$   
 $\downarrow$

impurity-solver

$$\Gamma(z) \stackrel{\Gamma(z) \Rightarrow \Sigma^2 G_{\sigma}^{\text{Hub}}(z)}{\longrightarrow} G_{\sigma}^{\text{Hub}}(z) = G_{\sigma,S}^{\text{And}}(z)$$

### Impurity solver

- uses spin representation of the single-impurity Anderson Hamiltonian [1]

$$\hat{H} = V \left[ (\hat{S}_d^+ \hat{S}_0^- + \hat{T}_d^+ \hat{T}_0^-) + \text{h.c.} \right] + \sum_n \gamma_n \left[ (\hat{S}_n^+ \hat{S}_{n+1}^- + \hat{T}_n^+ \hat{T}_{n+1}^-) + \text{h.c.} \right] + U \hat{S}_d^z \hat{T}_d^z$$

- based on correction vector DMRG

$$|c(z_i)\rangle = \frac{1}{z_i - (\hat{H} - E_0)} \hat{S}_d^+ |\Psi_0\rangle \quad (z_i \equiv \omega_i + i\eta_i)$$

$$G_{\sigma}^{\geq}(z) = \langle \Psi_0 | \hat{S}_d^{\pm} | c(z) \rangle \quad \text{particle-hole symmetry} \rightarrow G_{\sigma}^{\geq}(z) = G_{\sigma}^{\geq}(z) - G_{\sigma}^{\geq}(-z)$$

$$\rho_{\sigma}^{(n_i)}(\omega_i) = -\frac{1}{\pi} \text{Im} G_{\sigma}(\omega_i + i\eta_i)$$

- the obtained spectral density is broadened and has to be deconvolved

### Least-bias deconvolution

- continuous and positive semi-definite ansatz for the spectral density [2]

$$\rho_{\sigma}(\omega) = \exp \left[ \mu + \sum_{i=1}^n \lambda_i \frac{\eta_i}{\pi [(\omega_i - \omega)^2 + \eta_i^2]} \right]$$

- maximize  $-S = \int_{-\infty}^{\infty} d\omega \rho_{\sigma}(\omega) \ln \rho_{\sigma}(\omega)$

$$\text{under the side conditions} \quad \rho_{\sigma}^{(n_i)}(\omega_i) = \int_{-\infty}^{\infty} d\zeta \frac{\eta_i \rho_{\sigma}(\zeta)}{\pi [(\omega_i - \zeta)^2 + \eta_i^2]}$$

## References

- [1] C. Raas, G. S. Uhrig, and F. B. Anders, Phys. Rev. B **69**, 041102(R) (2004).  
 [2] C. Raas and G. S. Uhrig, cond-mat/0412224, Eur. Phys. J. B (accepted)  
 [3] R. Bulla, T. A. Costi, and D. Vollhardt, Phys. Rev. B **64**, 045103 (2001).

- [4] M. P. Eastwood, F. Gebhard, E. Kalinowski, S. Nishimoto, and R. M. Noack, Eur. Phys. J. B **35**, 155 (2003).  
 [5] F. Gebhard, E. Jeckelmann, S. Mahlert, S. Nishimoto, and R. M. Noack, Eur. Phys. J. B **36**, 491 (2003).  
 [6] S. Nishimoto, F. Gebhard, and E. Jeckelmann, J. Phys.: Condens. Matter **16**, 7063-7081 (2004).

## Single particle spectra far from transition

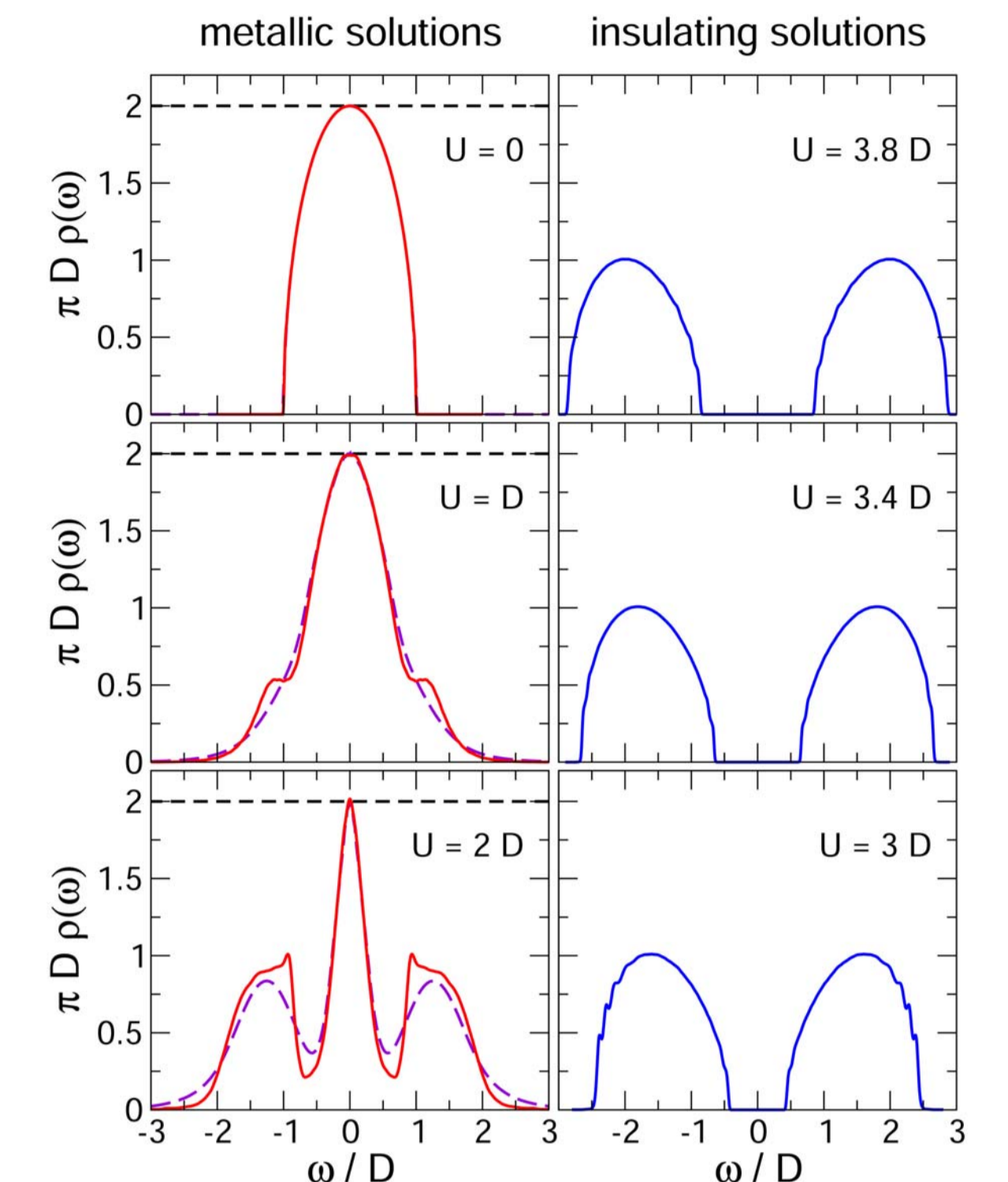
### Weak interaction, metallic region

- Hubbard bands appear already for  $U \approx D$
- no significant effects on collective modes (overdamped by Landau damping)

### Strong interaction, insulating region

- charge and collective modes are well separated in energy  
 $\rightarrow$  no significant interplay is to be expected

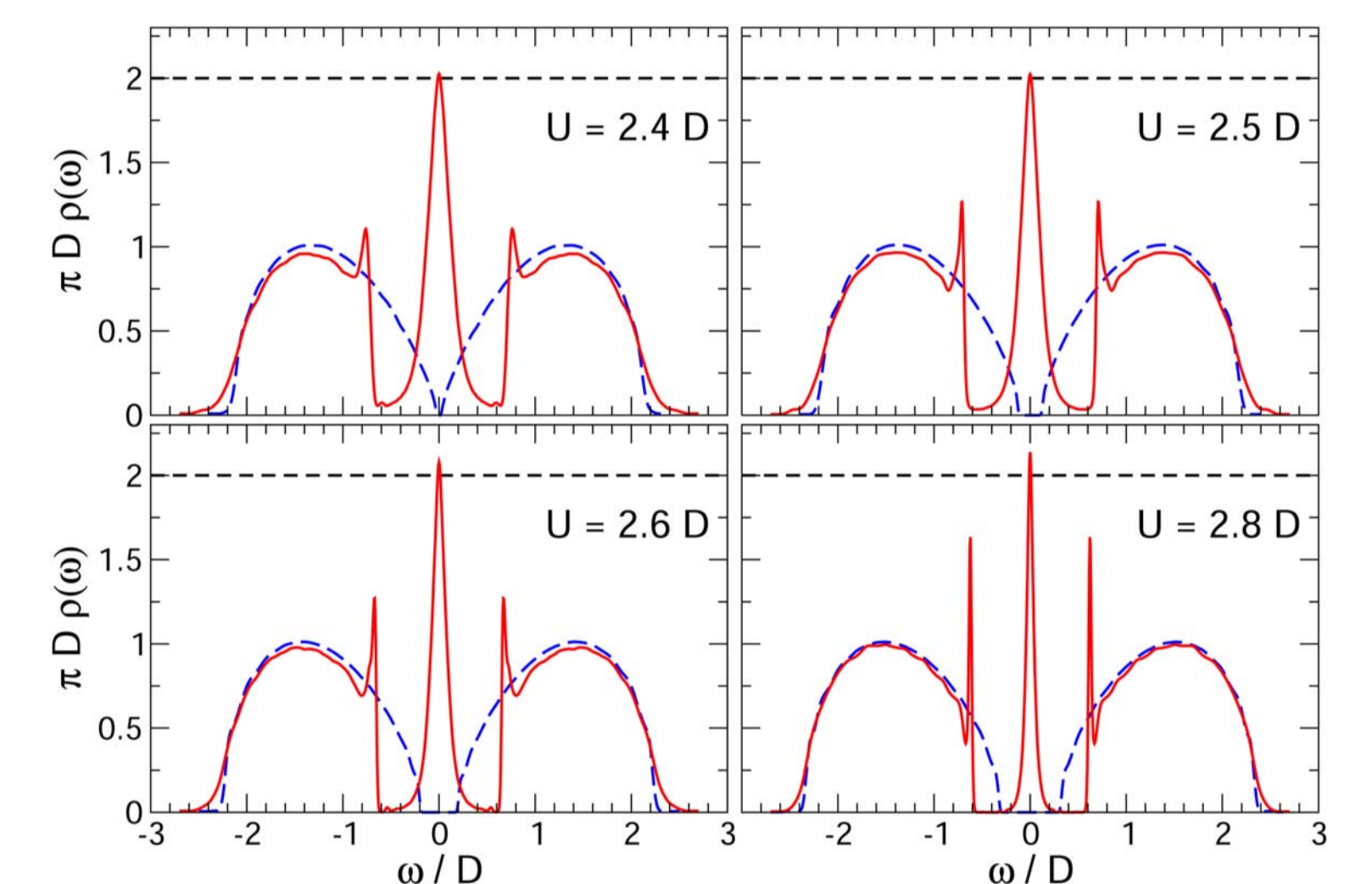
Spectral densities for the single-band Hubbard model at half band-filling and zero temperature deep in the metallic (red lines) and deep in the insulating regime (blue lines) calculated using a spin chain with 320 sites (160 fermionic sites). The dashed violet lines shows NRG-results [3].



## Single particle spectra close to transition

### Region close to transition

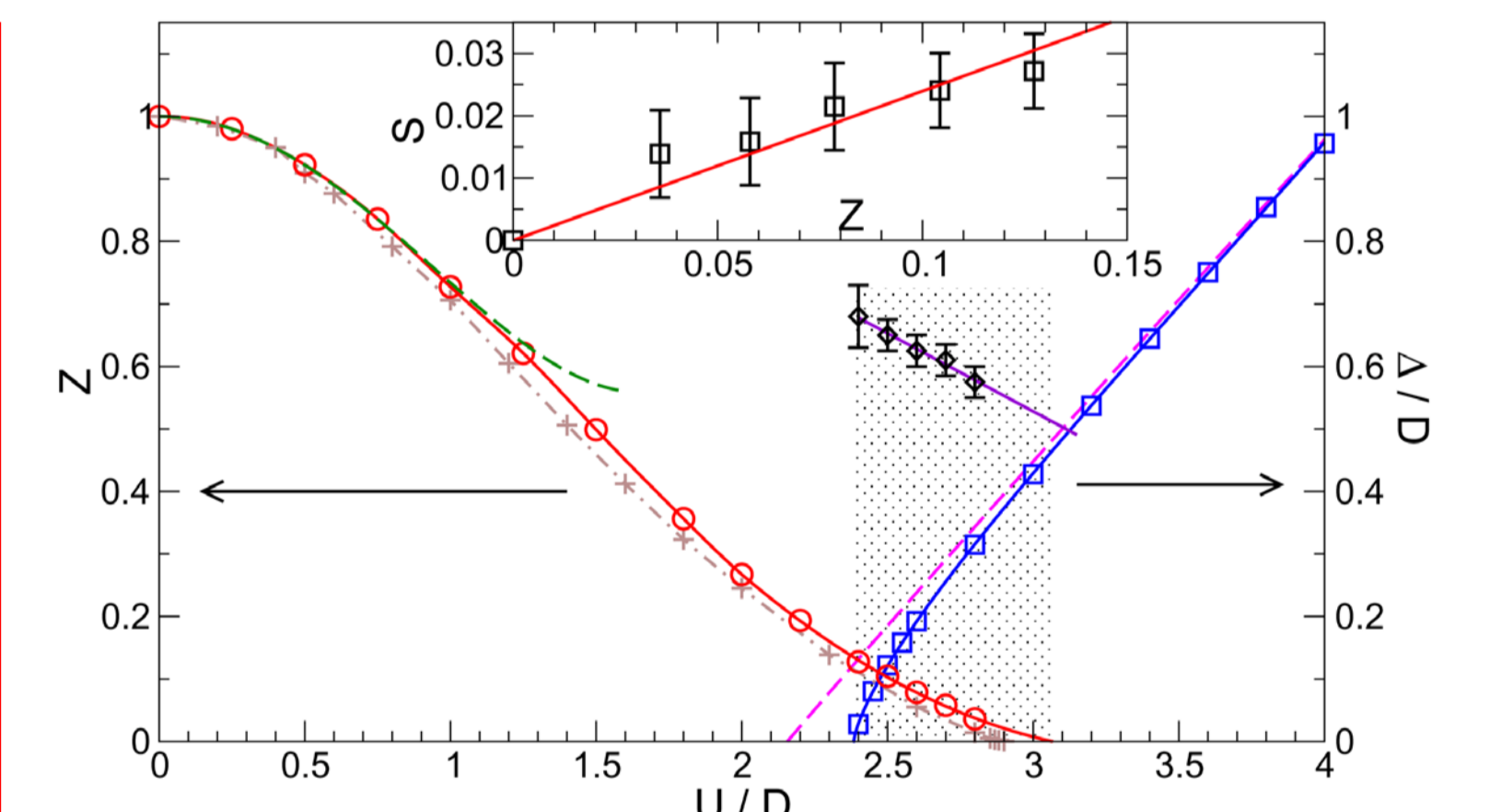
- no upturn [6] in the insulating solution close to  $U_{c1} = (2.38 \pm 0.02) D$
- sharp peaks at inner edges of the Hubbard bands in the metallic solution close to transition at  $U_{c2} = (3.07 \pm 0.1) D$
- preformed pseudo-gap in the metallic solution passes continuously into the insulating gap



Spectral densities for the single-band Hubbard model at half band-filling and zero temperature of the metallic (red solid) and the insulating (dashed blue) solution between  $U_{c1}$  and  $U_{c2}$ .

### Interpretation

- signature of collective excitations with heavy quasiparticles involved
- due to the energy an antibound state or resonance of heavy quasiparticles with a collective spin excitation is expected  
 $\rightarrow$  tentatively antipolaron
- scenario corroborated by the weight S of the side-peaks: S vanishes rather linearly with Z rather than quadratically or cubically



Dotted area: two-solution region. Left curves: metallic quasiparticle weight Z; red line with circles: interpolated DMRG, brown line with pluses: NRG [3]; dashed green line: perturbation up to  $U^4$  [4]. Right curves: insulating gap  $\Delta$  or pseudo-gap in the metal (violet line with diamonds); blue line with squares: DMRG; dashed magenta line: perturbation up to  $1/U^4$  [5]. Inset: weight S of the peaks at inner Hubbard band edges.

## Conclusions

- high-resolution calculation of the dynamic mean-field equations for the half-filled Hubbard model reveals a clear signature of collective excitations close to the metal-to-insulator transition
- effect of collective excitations is seen as sharp peaks at the inner edges of the Hubbard bands
- peaks evidence a strong interaction between charge and collective degrees of freedom
- tentative interpretation: antibound state (antipolaron)