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2D Fluctuations: Pictures From Exhibition

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Outline

- Specifics of fluctuations in 2D superconductor
- The width of critical region in magnetic field
- Relation between T_{BKT} and T_{BCS}
- Fluctuation renormalization of the Josephson current
- Strong vortex-antivortex fluctuations in type II superconducting films

Modulus and Phase fluctuations

$$\mathcal{F}[\Psi(\mathbf{r})] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}.$$

$$Z = \prod_{\mathbf{k}} \int d^2\Psi_{\mathbf{k}} \exp\left\{ -\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})|\Psi_{\mathbf{k}}|^2 \right\}.$$

$$F = -\frac{T}{2}\sum_{\mathbf{k}} \left\{ \ln\frac{\pi T_c}{3b\widetilde{\Psi}^2 + a + \frac{\mathbf{k}^2}{4m}} + \ln\frac{\pi T_c}{b\widetilde{\Psi}^2 + a + \frac{\mathbf{k}^2}{4m}} \right\}.$$

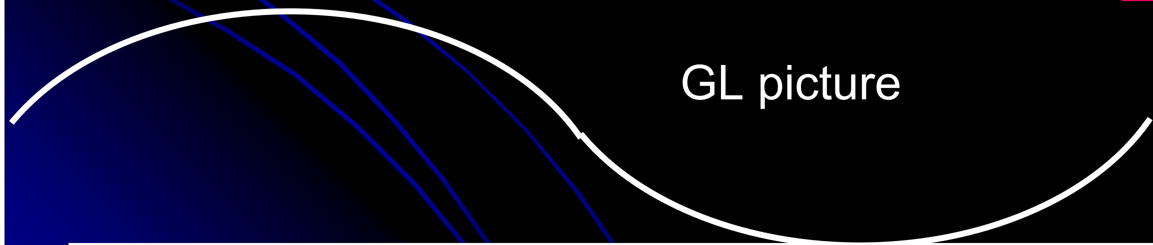
$$|\widetilde{\Psi}|^2 = \begin{cases} |a|/b, & \epsilon < 0 \\ 0, & \epsilon > 0 \end{cases}.$$

$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})}.$$

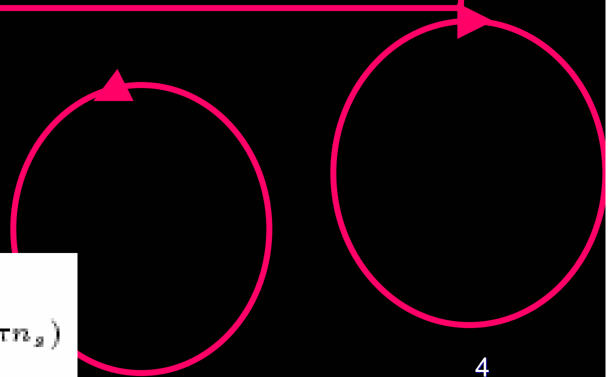
$$F(\epsilon < 0) = -\frac{T}{2} \sum_{\mathbf{k}} \left\{ \ln \frac{\pi T_c}{2|a| + \frac{\mathbf{k}^2}{4m}} + \ln \frac{\pi T_c}{\mathbf{k}^2/4m} \right\}.$$

Modulus fluctuations

Phase fluctuations



GL picture



BKT picture

$$\langle \Psi^*(0)\Psi(\mathbf{r})\rangle_{|\mathbf{r}|\gg\xi}(T) = |\Psi|^2 \exp\left(-\frac{mT}{\pi n_s} \ln \frac{r}{\xi}\right) = |\Psi|^2 \left(\frac{r}{\xi}\right)^{-mT/(\pi n_s)}.$$

The width of critical region in magnetic field

a. H=0

$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})}.$$

$$Gi_{(D)} = \left[\frac{7\zeta(3)\vartheta_D}{8\pi^2} \left(\frac{V_D}{V} \right) \frac{1}{\nu_D T_c \xi^D} \right]^{2/(4-D)}$$

$$Gi_{(2)} = \frac{7\zeta(3)}{32\pi^3} \frac{1}{\nu_2 T_c \xi^2}.$$

b. H≠0, T-T_c<< T_c LLL approximation

$$F(\epsilon, H) = -\frac{SH}{\Phi_0} T \int_{-\pi/s}^{\pi/s} \frac{dk}{2\pi} \ln \frac{\pi T}{\alpha T_c \epsilon + \frac{H}{4m\Phi_0} + J\left(1 - \cos(k_z s)\right)} \;,$$

$$\delta C(\epsilon, H) = -\frac{1}{VT_c}\left(\frac{\partial^2 F(\epsilon, H)}{\partial \epsilon^2}\right) = \frac{H}{\Phi_0 s}\left(-\frac{\partial}{\partial \epsilon}\right)\frac{1}{\sqrt{\epsilon(H)\left[\epsilon(H) + r\right]}} \,.$$

$$\delta C_{(2)}(r \ll \epsilon, H) = \frac{H}{\Phi_0 s}\frac{1}{\epsilon^2(H)}.$$

$$Gi_{(2)}(H) = \epsilon_{cr}(H) = \sqrt{Gi_{(2)}(T_c, 0)\frac{2H}{\widetilde{H}_{c2}(0)}}.$$

c. H - Hc2(0) << Hc2(0)

$$\chi(T) = \frac{\partial M}{\partial H} = -\frac{1}{4\pi\beta_A\left[2\varkappa_2^2(T) - 1\right]}$$

$$\Delta\chi(0) \approx \frac{v_F^2}{c^2}(p_F l)^2 \frac{\pi^2}{63\zeta(3)\cdot 2.88\cdot\beta_A}\frac{e^2}{v_F} \approx 0.034\frac{v_F^2}{c^2}(p_F l)^2\frac{e^2}{v_F}.$$

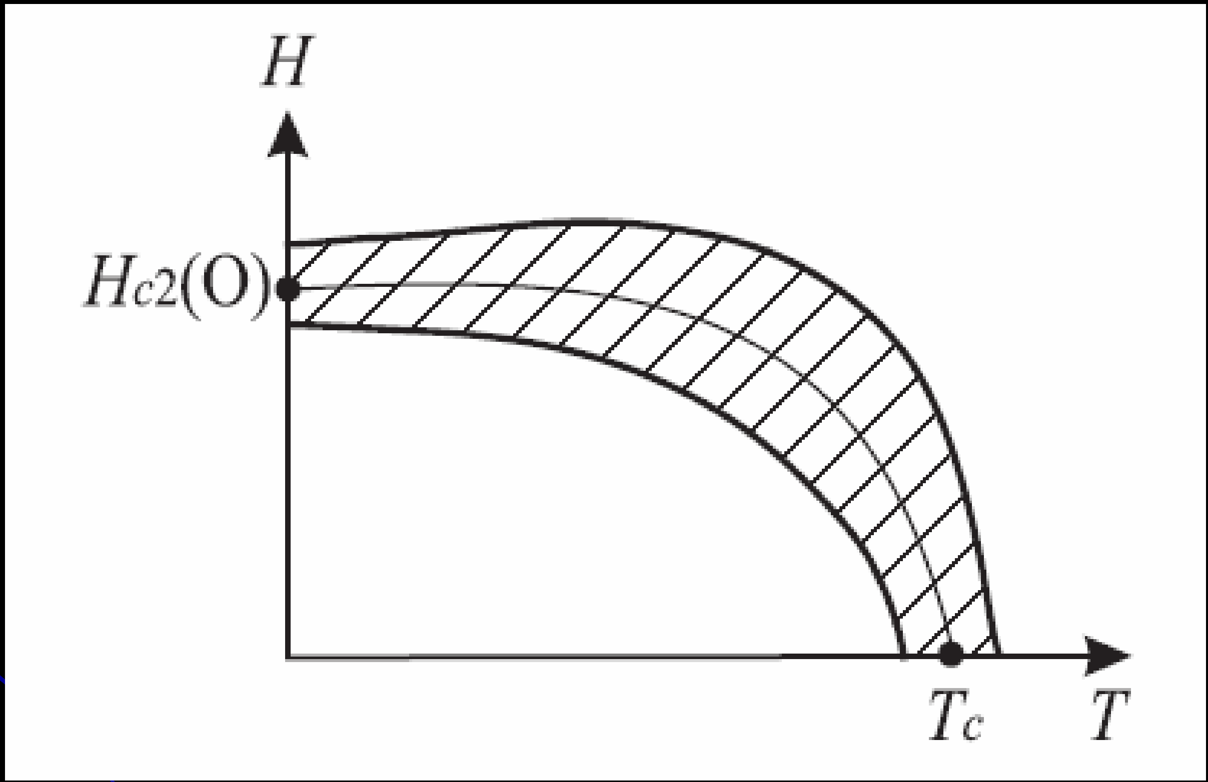
Naturally, this value is considerably less than $(4\pi)^{-1}$.

$$\chi_{\text{fl}}\left(t \ll \widetilde{h}\right) = -\frac{e^2}{\pi^2 c^2}\frac{v_F^2\tau}{d}\frac{1}{\widetilde{h}},$$

$$\widetilde{h} = \frac{H - H_{c2}(T)}{H_{c2}(T)}$$

$$Gi_{(\text{film})}\left(t \ll \widetilde{h}\right) = \widetilde{h}_{cr}\left(t \ll \widetilde{h}\right) = \frac{2.65}{p_F^2 l d} \approx 2Gi_{(\text{film})}(T_c, H = 0).$$

Critical region width



Superconducting transition in 2D film

a. Mean field theory

$$\mathbf{j} = -n_s \frac{2e^2}{m}\mathbf{A}$$ and $$n_s = \widetilde{\Psi}^2 = -\frac{a}{b}$$ $\longrightarrow T_{c0}$

b. Fluctuation GL theory

$$\langle \mathbf{j}_{\text{fl}} \rangle = -\frac{\partial F_{\text{fl}}(\mathbf{A})}{\partial \mathbf{A}} = T\frac{\partial \ln Z(\mathbf{A})}{\partial \mathbf{A}}$$

$$= -\frac{\int \mathfrak{D}\Psi(\mathbf{r})\mathfrak{D}\Psi^*(\mathbf{r})\frac{\partial \mathcal{F}[\Psi(\mathbf{r}),\mathbf{A}])}{\partial \mathbf{A}} \exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r}),\mathbf{A}]}{T}\right)}{\int \mathfrak{D}\Psi(\mathbf{r})\mathfrak{D}\Psi^*(\mathbf{r}) \exp\left(-\frac{\mathcal{F}[\Psi(\mathbf{r}),0]}{T}\right)}.$$

$$\langle \mathbf{j}_{\text{fl}} \rangle = \mathbf{j}_1 + \mathbf{j}_2 = -\frac{2e^2}{m}\mathbf{A}\left\langle |\Psi(\mathbf{r})|^2 \right\rangle_{\mathbf{A}=0} - \frac{e}{m}\left\langle \operatorname{Im}[\Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})]\right\rangle_{\mathbf{A}}$$

$$\langle \mathbf{j}_{\text{fl}} \rangle = -\frac{2e^2}{m}\mathbf{A}\left[\left\langle |\Psi(\mathbf{r})|^2 \right\rangle_{\mathbf{A}=0} - \sum_{\mathbf{k}} \frac{T}{(2\alpha T_c|\epsilon| + \mathbf{k}^2/4m)}\right].$$

$$\left\langle |\Psi(\mathbf{r})|^2 \right\rangle = \widetilde{\Psi}^2 + \left\langle \psi_r^2 \right\rangle + \left\langle \psi_i^2 \right\rangle + 2\widetilde{\Psi} \left\langle \psi_r \right\rangle$$

$$\left\langle \psi_r^2 \right\rangle = \frac{T}{2} \sum_{\mathbf{k}} \frac{1}{2\alpha T_c |\epsilon| + \mathbf{k}^2/4m},$$

$$\left\langle \psi_i^2 \right\rangle = \frac{T}{2} \sum_{\mathbf{k}} \frac{1}{\mathbf{k}^2/4m}.$$

$$\left\langle \psi_r \right\rangle = -\frac{1}{2\widetilde{\Psi}} \left(3\left\langle \psi_r^2 \right\rangle + \left\langle \psi_i^2 \right\rangle \right).$$

$$\left\langle \mathbf{j}_{\mathrm{fl}} \right\rangle = -\frac{2e^2}{m} \mathbf{A} \left[\widetilde{\Psi}^2 - 2 \sum_{\mathbf{k}} \frac{T}{(2\alpha T_c |\epsilon| + k^2/4m)} \right].$$

$$n_s(T) = \frac{\alpha}{b} \left[T_c|\epsilon| - \frac{2b}{\alpha^2} \sum_{\mathbf{k}} \frac{1}{(2|\epsilon| + k^2/4m\alpha T_c)} \right].$$

$$T_c = T_{c0} \left(1 - 2Gi_{(2d)} \ln Gi_{(2d)}^{-1} \right),$$

c. BKT theory

$$E_v = \frac{\pi n_{s2}(T)}{2m} \ln \frac{R}{\xi}.$$

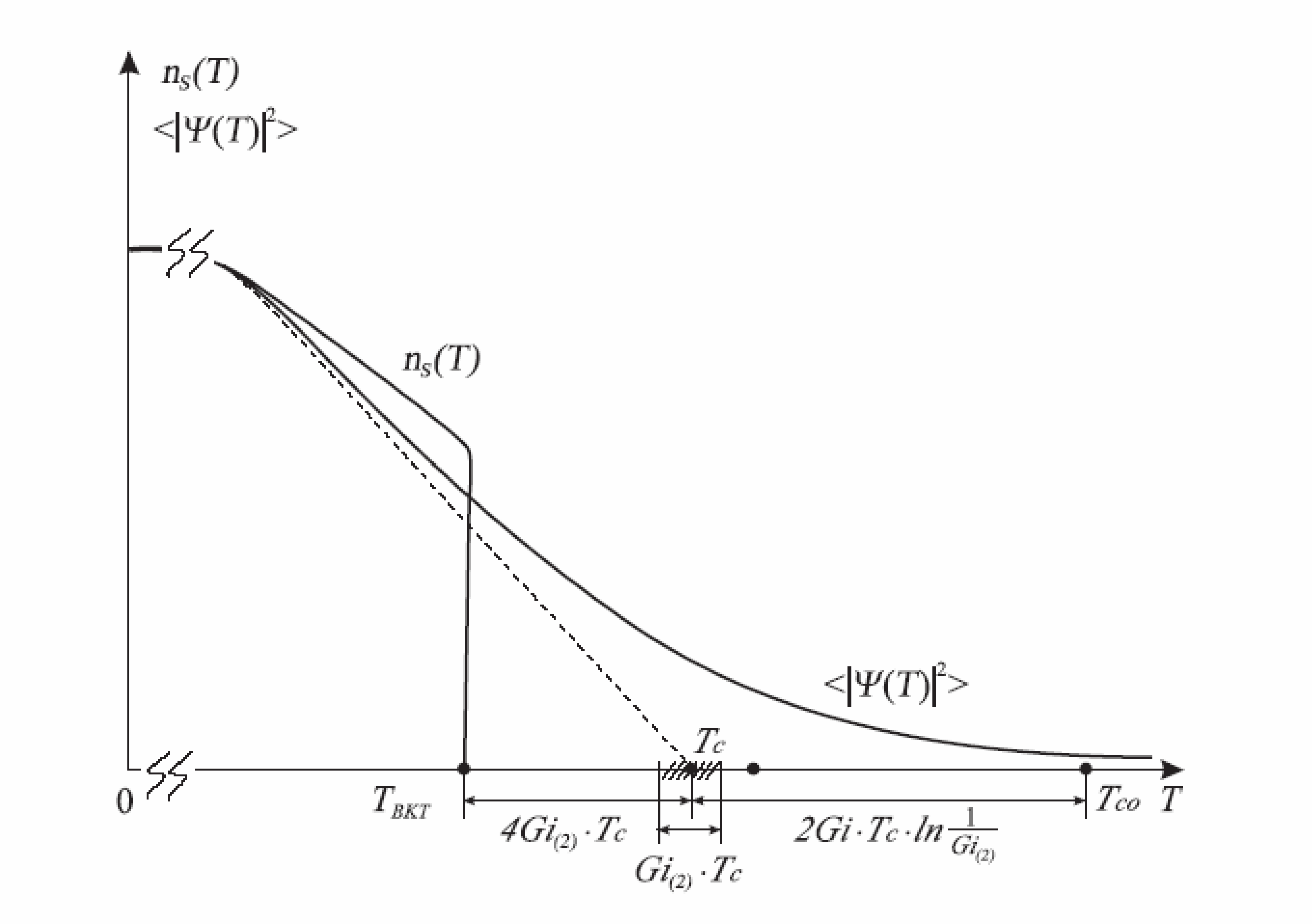
$$F = E - TS = \left[\frac{\pi n_{s2}(T)}{2m} - 2T \right] \ln \frac{R}{\xi}.$$

One can see that when $T < \pi n_{s2}(T)/4m$ the free energy is minimal for the state without vortices, while for $T > \pi n_{s2}(T)/4m$ the free energy is minimized when in system there are vortices. Vortices destroy stiffness so it is natural to identify the critical temperature of the transition with T_{BKT} determined by Eq. (15.14).

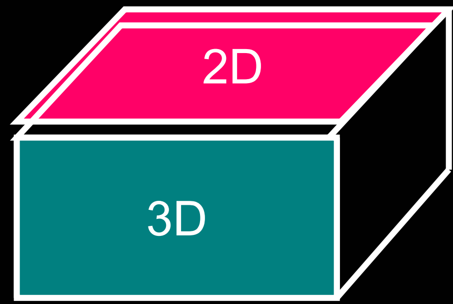
$$n_{s2}(T_{\mathrm{BKT}}) = \frac{4mT_{\mathrm{BKT}}}{\pi}.$$

Interpolation formula:

$$n_{s2}(T) = \frac{mT_{\mathrm{BKT}}}{\pi} \left[\frac{|\epsilon|}{Gi_{(2)}} - 2\ln \frac{Gi_{(2)}}{|\epsilon| + Gi_{(2)}} + 4 \right],$$


$n_S(T)$
$<|\Psi(T)|^2>$
$n_S(T)$
$<|\Psi(T)|^2>$
T_c
0
T_{BKT}
$4Gi_{(2)} \cdot Tc$
$Gi_{(2)} \cdot Tc$
$2Gi \cdot Tc \cdot ln \frac{1}{Gi_{(2)}}$
Tco
T

Fluctuation renormalization of the Josephson current



$$I_c = \frac{\pi \Delta^2(T)}{4eTR_n}.$$

Mean field in the vicinity of T_c gives:

$$\Delta^2(T) \to \Delta_0^2(T) = \frac{8\pi^2 T_{c0}^2}{7\zeta(3)} |\epsilon_0|,$$

$$I_c^{(0)}(|\epsilon_0|) = \frac{2\pi^3 T_{c0}}{7\zeta(3)\, eR_n} |\epsilon_0|$$

where $|\epsilon_0| = (T_{c0} - T)/T_{c0}$

GL fluctuation theory gives:

$$\Delta^2(T) \to \langle \Delta(T) \rangle^2.$$

$$\langle \Delta(T) \rangle^2_{(\mathrm{fl})} = \Delta_0^2 - \frac{1}{4mC_{(D)}}\left[3\langle \psi_r^2 \rangle + \langle \psi_i^2 \rangle\right]$$

$$I_c(T) = \frac{\pi \langle \Delta(T) \rangle^2_{(\mathrm{fl})}}{4eTR_n} = I_c^{(0)}(|\epsilon_0|) + \delta I_c^{(\mathrm{fl})}(|\epsilon_0|).$$

$$\delta I_{c(2)}(|\epsilon|) = \frac{4\pi^3 T_c}{7\zeta(3)\, eR_n} Gi_{(2)} \left[\ln \frac{|\epsilon|}{Gi_{(2)}} + \ln\left(\frac{\xi(T)}{L_J}\right)\right].$$

Exponential tail in Josephson junction close to T$_c$

$$\mathcal{F}[\varphi] = \frac{n_s}{4m}\int d^2\mathbf{r}\,[\nabla\varphi(\mathbf{r})]^2 = \frac{n_s}{4m}\sum_{\mathbf{k}} \mathbf{k}^2\varphi_{\mathbf{k}}^2.$$

+

$$\delta E_J^{(\mathrm{fl})}(\epsilon) = \frac{E_J}{2S}\int d^2\mathbf{r}\left[\varphi^{(\mathrm{fl})}(\mathbf{r})\right]^2.$$

$$F\left(\varphi^{(\mathrm{fl})}\right) = \frac{n_s}{2m}\sum_{\mathbf{k}}\left(\mathbf{k}^2 + \frac{1}{L_J^2}\right)\varphi_{\mathbf{k}}^2,$$

where the Josephson length L_J is determined by relation

$$\frac{n_s}{2mL_J^2} = \frac{E_J}{2S}.$$

When I_c is small $L_J \gg \xi$ and the fluctuation correction (13.57) can become relatively large even for $\epsilon \gg Gi_{(2d)}$. In order to find the form of $I_c(\epsilon)$ in this nonperturbative region of temperatures let us calculate the average value

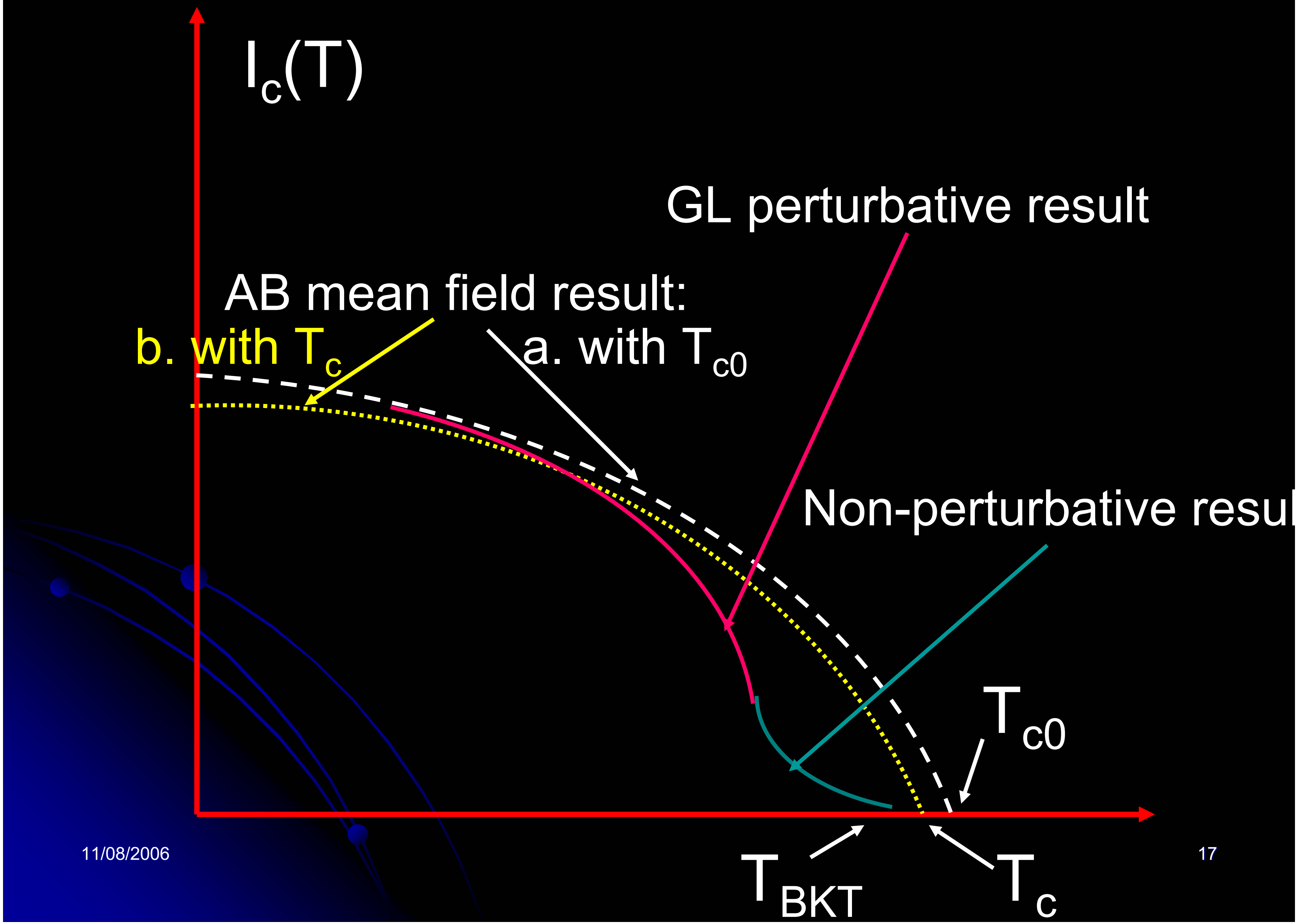
$$I_c(\varphi) = \frac{E_J}{2S}\left\langle \sin\left(\varphi + \varphi^{(\text{fl})}\right)\right\rangle \qquad (15.10)$$

without expansion in series over $\varphi_{(\text{fl})}$. The averaging in Eq. (15.10) is carried out with the free energy (15.1) over all possible phase fluctuations. Carrying out the Gauss integral in Eq. (15.10) analogously this was done in Eqs. (15.2)–(15.5) one can find

$$\delta E_J^{(\text{fl})}(\epsilon) = E_{J0} \exp\left(-\frac{2Gi_{(2d)}}{|\epsilon|}\ln\frac{L_J}{\xi(T)}\right). \qquad (15.11)$$

In the region $\epsilon \gg Gi_{(2d)}$ the superfluid density n_s can be taken in its unperturbed form (2.149) what gives

$$I_c(\epsilon) = I_c^{(AB)}(\epsilon)\exp\left(-\frac{\epsilon^*}{|\epsilon|}\right), \qquad (15.12)$$



$I_c(T)$
GL perturbative result
AB mean field result:
b. with T_c
a. with T_{c0}
Non-perturbative resul
T_{c0}
T_{BKT}
T_c

Strong vortex-antivortex pair fluctuations in type II SC film

- In the immediate vicinity of transition the thermodynamic and transport properties of the film are determined by the large size ($R>>\xi(T)$) vortex-antivortex pairs

- Beyond the critical region $Gi<\tau<<1$ usually the thermodynamic and transport properties are determined by the long wave-length fluctuation of the order parameter.

- The cornerstone of the presented theory is the fact that the energy of the vortex-antivortex pair tends zero when $R<\xi(T)$) and proliferation of such cheap pairs determines the fluctuation thermodynamics in the region of temperatures

$$Gi_{(2d)} \ll \tau \lesssim Gi_{(2d)} \ln^2 \left[p_F \xi \left(T \right) \right]$$

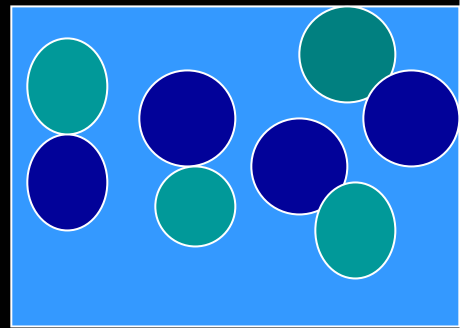
below transition.

Partition function

$$Z = \int \mathfrak{D}\Delta(\mathbf{r}) \int \mathfrak{D}\Delta^*(\mathbf{r}) \exp\left\{-\frac{F\left(\Delta(\mathbf{r}), \Delta^*(\mathbf{r})\right)}{T}\right\}.$$

Gas approximation:

$$Z_{(fl)} = Z_p^{S/\left[\pi\xi^2(T)\right]},$$



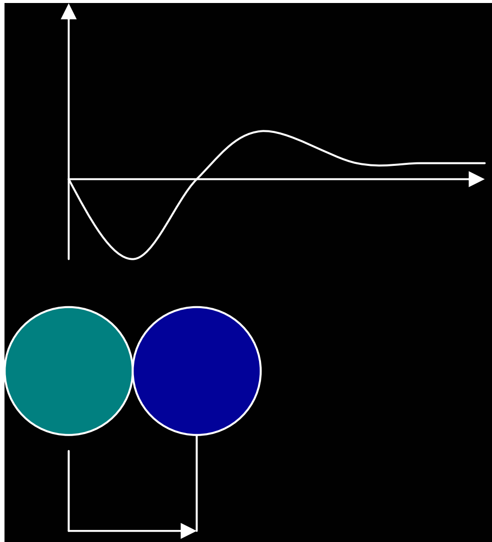
where

$$Z_p = \int d\Delta_\delta\left(\mathbf{r}\right) \int d\Delta_\delta^*\left(\mathbf{r}\right) \exp\left\{-\frac{F_p\left(\Delta_\delta\left(\mathbf{r}\right), \Delta_\delta^*\left(\mathbf{r}\right)\right)}{T}\right\}$$

is the contribution of the isolated pair of size $\delta\ \xi(T)$, with all $0<\delta<1$

Order parameter

The order parameter $\Delta_\delta(\mathbf{r})$ has two zeros of the opposite vorticity at the distance $2\,\delta\,\xi(T)$.



$2\,\delta\,\xi(T)$.

Corresponding free energy functional:

$$F_p = \nu d \int d^2\mathbf{r} \left\{ \left[-\tau |\Delta_\delta(\mathbf{r})|^2 + \frac{\pi \mathcal{D}}{8T_c} |\partial_- \Delta_\delta(\mathbf{r})|^2 + \frac{7\zeta(3)}{16\pi^2 T_c^2} |\Delta_\delta(\mathbf{r})|^4 \right] + \frac{\tau}{2} |\Delta_0(T)|^2 \right\}.$$

F_p is the difference between the state with one v-a pair and the ground state with the homogeneous order parameter

$$\Delta_\delta(\mathbf{r}) = |\Delta_\delta(\mathbf{r})| e^{i\chi(\mathbf{r})}$$

$$\chi(\mathbf{r}) = \varphi_1(\mathbf{r}) - \varphi_2(\mathbf{r}) + \tilde{\chi}(\mathbf{r}).$$

Solution of the GL equation

$$\Delta_\delta(\rho) = \Delta_0(T)\, f(\rho)$$

$$\rho = r/\xi(T)$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) f + \frac{1}{2} f - \frac{1}{2} f^3 = 0.$$

$$f(\rho) = C_1 J_0\left(\rho/\sqrt{2}\right) + C_2 N_0\left(\rho/\sqrt{2}\right)$$

$$\rho \sim \delta \ll 1$$

$$f(\rho) = 1 - C_3 K_0(\rho)$$

$$\rho \gtrsim 1$$

$$C_i = \delta_{i1} + \frac{1}{2\ln\frac{2}{\gamma\delta}}\begin{cases}\ln 2 \\ \pi \\ 2\end{cases}.$$

Contribution to the free energy of the pairs of size $2\delta\ \xi(T)$ is

$$F_p(\delta, T) = \frac{b(T)}{\ln\frac{2}{\gamma\delta}} \int_0^\infty x K_0(x)\, dx = \frac{b(T)}{\ln\frac{2}{\gamma\delta}},$$

with

$$b(T) = \frac{\pi^2 \nu d D \Delta_0^2(T)}{4T_c}.$$

Functional inegration in our model is equivalent to the account for contributions of all discs of sizes $0<\delta<1$

$$Z_p = \frac{1}{[p_F \xi(T)]^2} + 2\int_{1/[p_F\xi(T)]}^{1} \delta d\delta \exp\left\{-\frac{F_p(\delta, T)}{T}\right\}$$

Steepest descent approximation results in

$$Z_p = \frac{4 \cdot 2^{1/4}\pi^{1/2}}{\gamma^2} \left(\frac{b(T)}{T}\right)^{1/4} \exp\left\{-2\sqrt{\frac{2b(T)}{T}}\right\}.$$

The corresponding free energy

$$\widetilde{F}_p(T) = -T \ln Z_p^{S/\pi\xi^2}$$
$$= -\frac{TS}{\pi\xi^2(T)}\left[-2\sqrt{\frac{2b(T)}{T}} + \frac{1}{4}\ln\frac{b(T)}{T}\right]$$

The steepest descent approximation is valid when

$$b(T) \gg T \longleftrightarrow \tau \gg Gi_{(2d)}$$

i.e. beyond the critical region, where is valid the concept of small vortex-intivortex pairs itself

Fluctuation heat capacity

$$C_p\left(T\rightarrow T_c\right) = -\frac{48ST_c}{v_F l_{tr}}\left(\frac{\partial^2}{\partial\tau^2}\right)$$
$$\left[4\sqrt{\frac{\nu dD}{7\zeta\left(3\right)}}\tau^{3/2} - \frac{\tau}{4\pi^2}\ln\frac{2\pi^4\nu dD\tau}{7\zeta\left(3\right)}\right]$$

Differentiation of the second term results in the well known positive contribution which occurs due to the long-wavelength order parameter fluctuations:

$$C_{(fl)}^{(2)}\left(\tau\right) = \frac{8\pi^2 Sd}{7\zeta(3)}\nu T_c\left(\frac{Gi_{(2d)}}{\tau}\right)$$

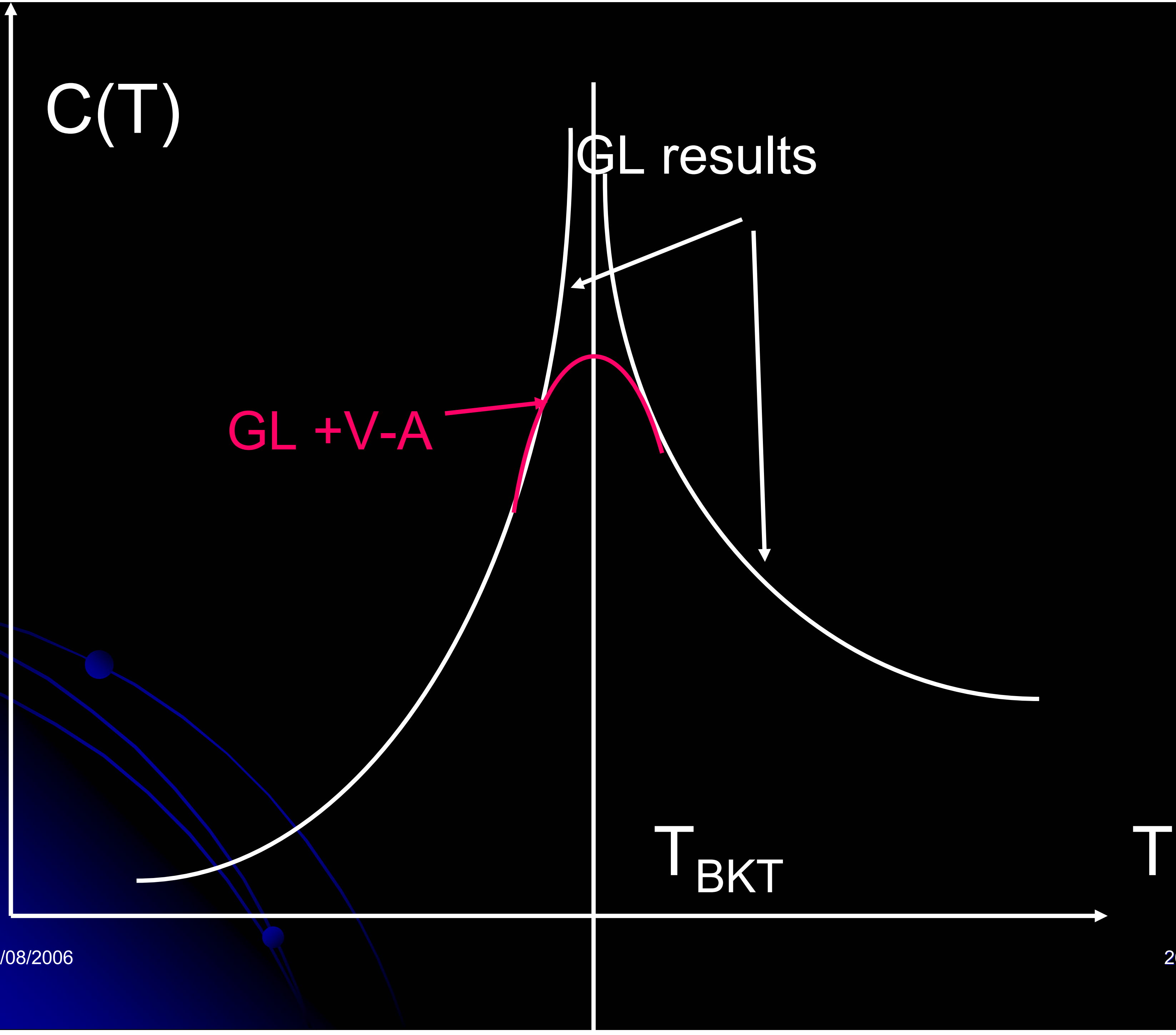
Differentiation of the first term gives the contribution of the small vortex-antivortex fluctuations:

$$C_p\left(\tau\right) = -\frac{48\pi^2 Sd}{7\zeta\left(3\right)}\nu T_c \left(\frac{2Gi_{(2d)}}{\tau}\right)^{1/2}$$

In the region of temperatures

$$Gi_{(2d)} \ll \tau \lesssim Gi_{(2d)} \ln^2 \left[p_F \xi\left(T\right)\right]$$

it is much larger than the former contribution and has the opposite sign with respect to it, smearing the heat capacity jump.


C(T)
GL results
GL +V-A
T
BKT
T