Landau-Zener Transitions with quantum noise

Valery Pokrovsky

Department of Physics, Texas A&M University
and
Институт теоретической физики им. Л.Д. Ландау

Nikolai A. Sinitsyn

Department of Physics and Astronomy UT Austin

***Participants*:**

Stefan Scheidl

Institut für Theoretishe Physik, Universität zu Köln

Bogdan Dobrescu

Texas A&M Univesity

August 2006

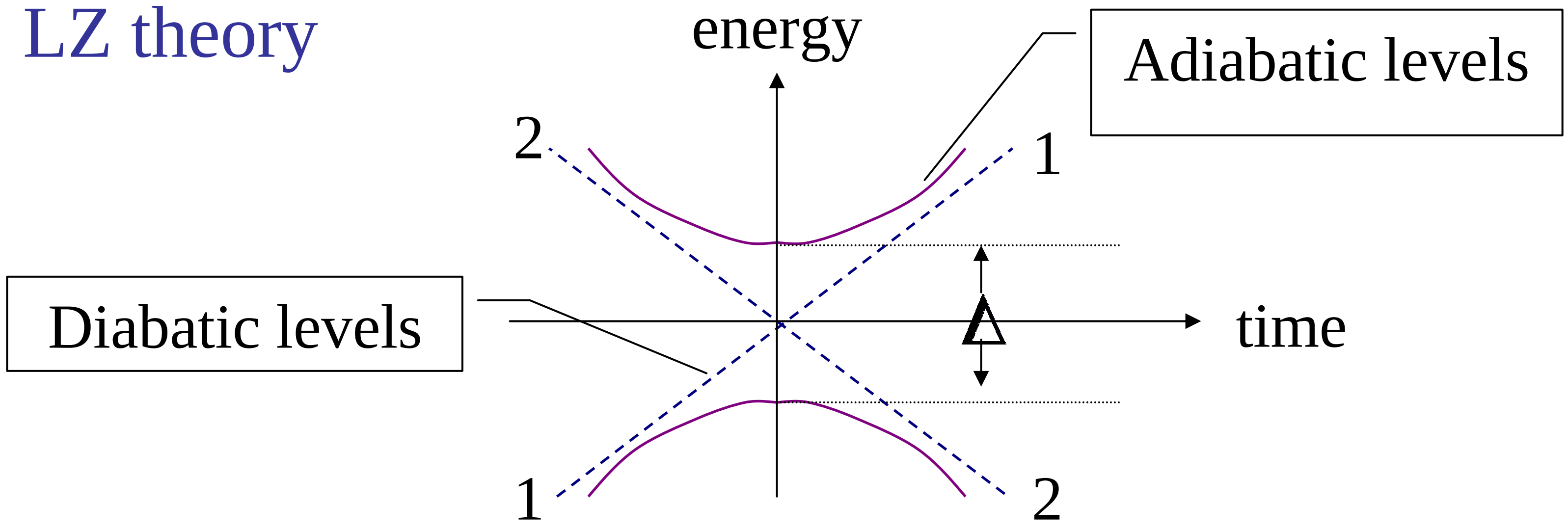
Dresden, 2006

Outline

- Introduction and motivation
- Landau-Zener problem for 2-level crossing
- Fast classical noise in 2-level systems
- Noise and regular transitions work together
- Quantum noise and its characterization
- Transitions due to quantum noise in the LZ system
- Production of molecules from atomic Fermi-gas at Feshbach resonance
- Conclusions

Introduction

LZ theory



Avoided level crossing (Wigner-Neumann theorem)

Schrödinger equations

$$i\dot{a}_1 = E_1(t)a_1 + \Delta a_2$$
$$i\dot{a}_2 = \Delta^* a_1 + E_2(t)a_2$$

$$E_2(t) - E_1(t) = \Omega(t); \quad \hbar = 1$$
$$\Omega(t) = \dot{\Omega}t$$

Adiabatic levels:

$$E_{\pm} = \frac{E_1 + E_2}{2} \pm \sqrt{\left(\frac{E_1 - E_2}{2}\right)^2 + |\Delta|^2}$$

Center-of mass energy = 0

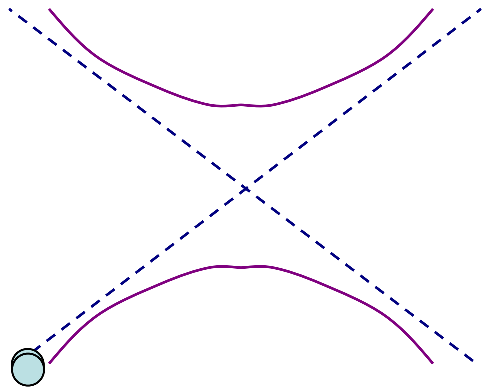
$$E_2 = -E_1 = \dot{\Omega} t / 2$$

LZ parameter:

$$\gamma = \frac{\Delta}{\hbar \sqrt{\dot{\Omega}}} \qquad \dot{\Omega} = \frac{g \mu_B \dot{B}_z}{\hbar}$$

γ □ 1

γ □ 1

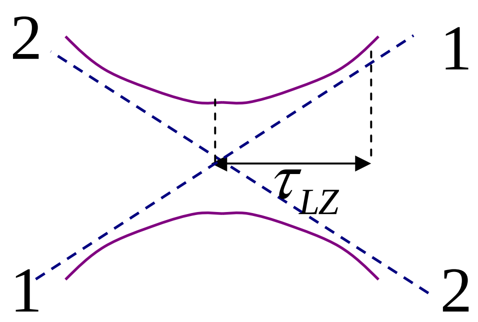


LZ transition matrix $$U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \qquad |\alpha|^2 + |\beta|^2 = 1$$

Amplitude to stay at the same d-level $$\alpha = e^{-\pi\gamma^2}$$

Amplitude of transition $$\beta = -\frac{\sqrt{2\pi}\exp\left(-\frac{\pi\gamma^2}{2} + i\frac{\pi}{4}\right)}{\gamma\Gamma(-i\gamma^2)}$$

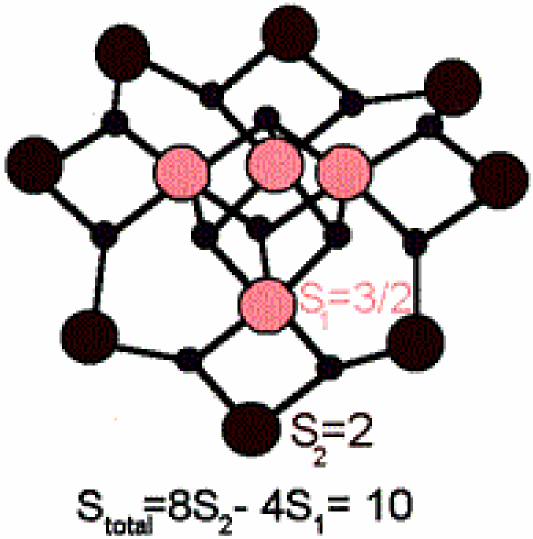
LZ transition time: $$\tau_{LZ} = \max\left(\frac{\Delta}{\dot{\Omega}}, \dot{\Omega}^{-1/2}\right)$$



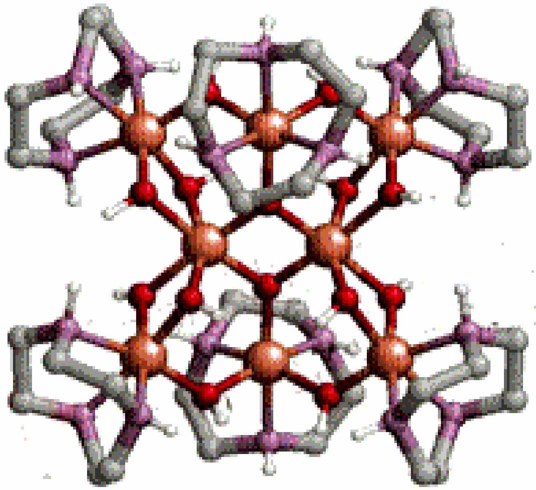
Condition of validity: $$\tau_{LZ} \ll \tau_{sat} = \left|\dot{\Omega}/\ddot{\Omega}\right|$$

Nanomagnets: Brief description

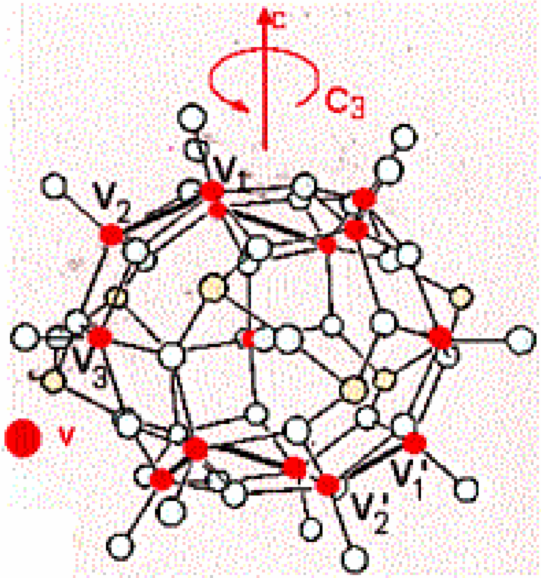
- $S = 10$: Mn_{12}, Fe_8. $S = 1/2$: V_{15}.



Mn_{12}

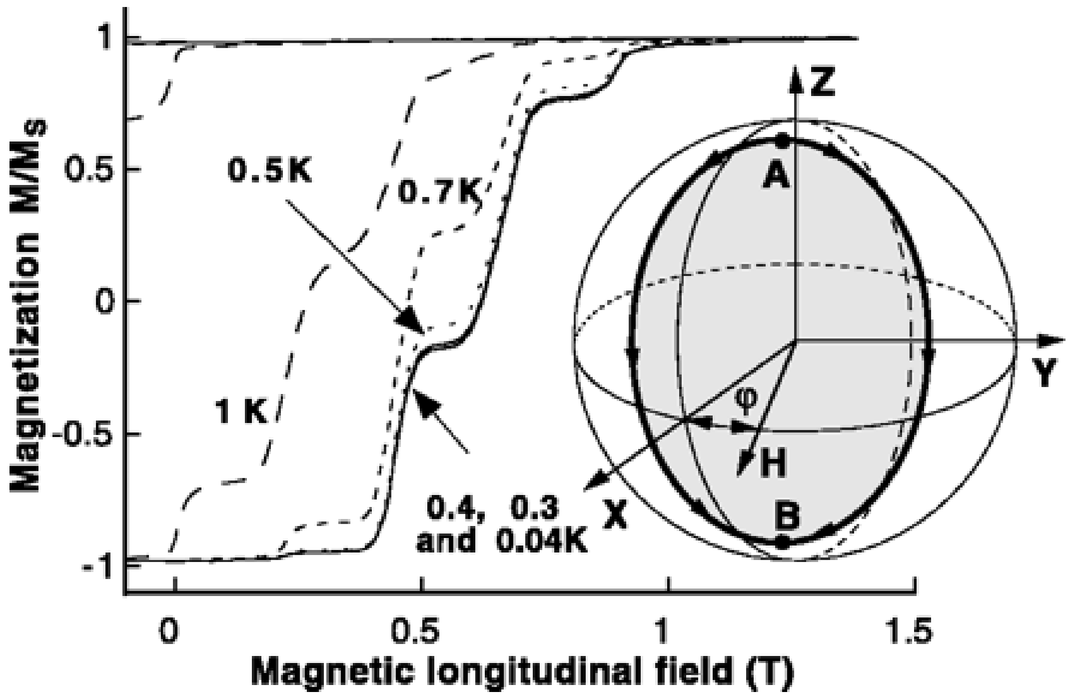


Fe_8



V_{15}

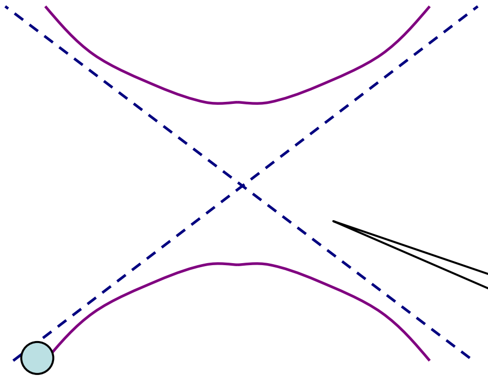
Spin reversal in nanomagnets



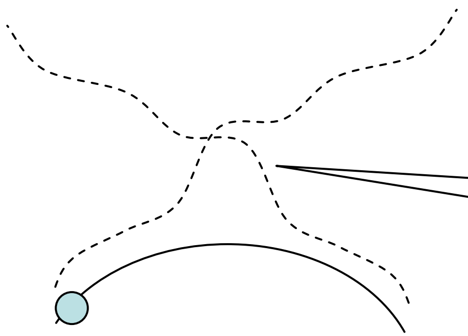
W. Wernsdorfer and R. Sessoli, Science **284**, 133 (1999)

Controllable switch between states for quantum computing:

The noise introduces mistakes to the switch work.



Transverse noise



Longitudinal noise creates decoherence

Landau-Zener tunneling in noisy environment

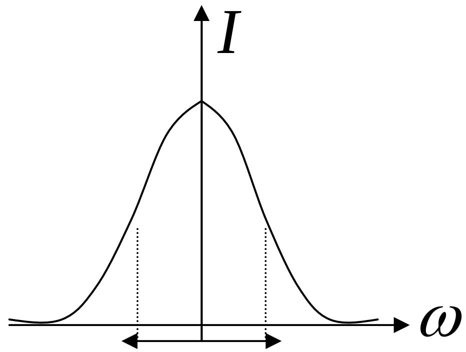
V. Pokrovsky and N. Sinitsyn, Phys. Rev. **B** 67, 144303, 2003

Classical fast noise in 2-level system

$$\mathbf{b}_{tot} = \mathbf{b}_{reg} + \boldsymbol{\eta}; \quad \mathbf{b}_{reg} = \hat{z}\dot{\Omega}t + \hat{x}\Delta$$

$\boldsymbol{\eta}(t)$ -- Gaussian noise $\quad \langle \eta_i(t)\eta_k(t')\rangle = f_{ik}\left(\frac{t-t'}{\tau_n}\right)$

Noise is fast if $\quad \tau_n \ll \dot{\Omega}^{-1/2}$



$\Delta\omega = 1/\tau_n$

Density matrix: $$\hat{\rho}(t) = \frac{1}{2} I + \mathbf{g}(t) \cdot \mathbf{s}$$

$\mathbf{g}(t) -$ ***Bloch vector***. It obeys Bloch equation:

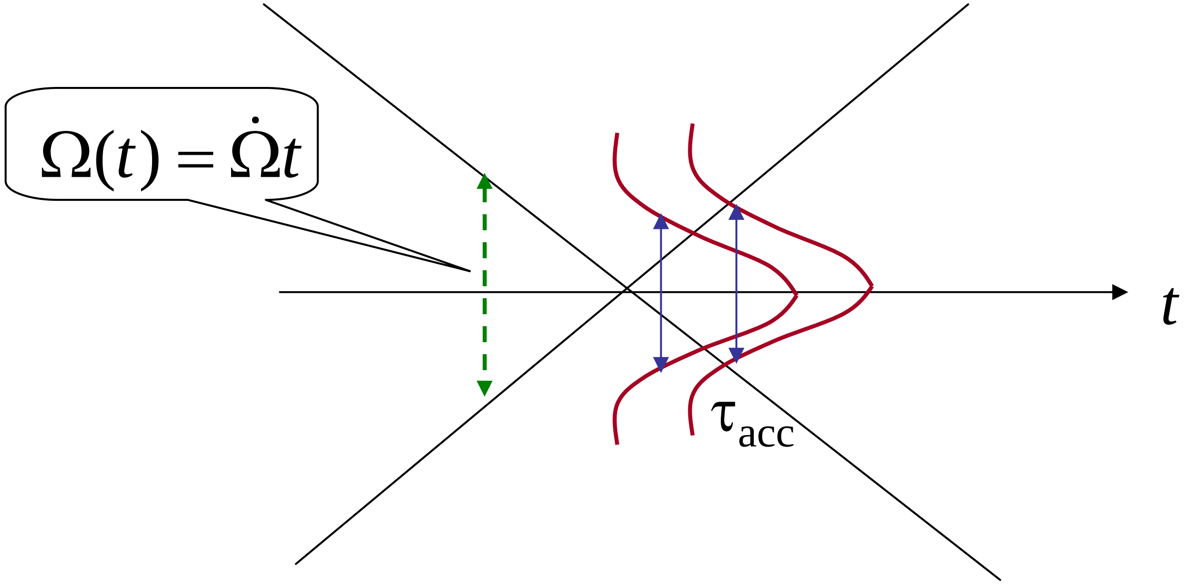
$$\dot{\mathbf{g}} = -\mathbf{b}_{tot} \times \mathbf{g}$$

$$g_z = \frac{1}{2}(n_1 - n_2) \equiv \frac{1}{2}(n_\uparrow - n_\downarrow)$$ Difference of populations

$$g_\pm = g_x \pm i g_y$$ Coherence amplitude

Integral of motion: $$\mathbf{g}^2 = const$$

Transitions produced by noise

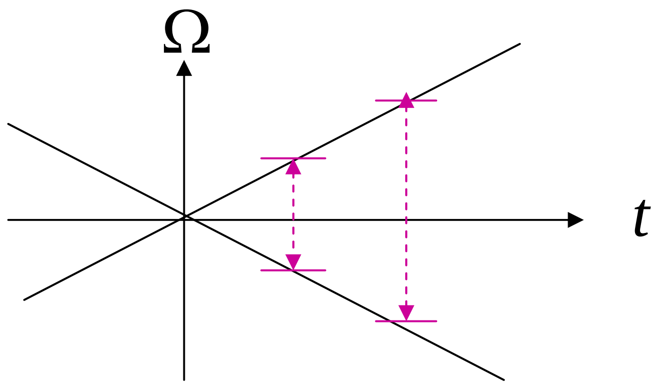


It produces transitions until $\Omega(t) = \dot{\Omega} t \le 1/\tau_n$

Accumulation time: $\tau_{acc} = \dfrac{1}{\dot{\Omega}\,\tau_n} \square \;\; \tau_n$

$\langle g_z(t) \rangle$ is slowly varying

Transition is produced by a spectral component of noise whose frequency equal to its instantaneous value in the LZ 2-level system.



$$\dot{n}_1 = -\left\langle \eta^*_{\Omega(t)} \eta_{\Omega(t)} \right\rangle n_1$$

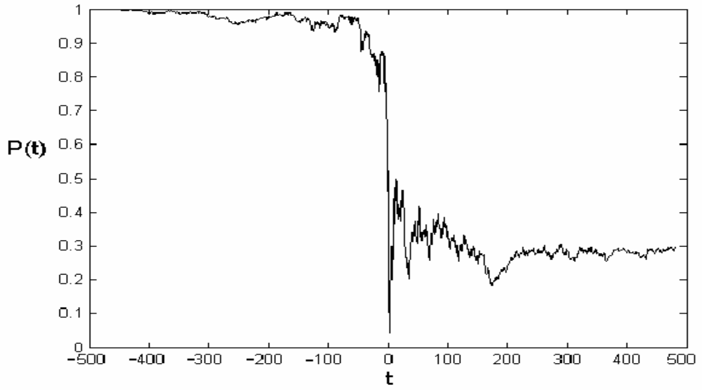
Transition probability measures the spectrum of noise

Transition probability for infinite time

$$P_{1\to 1} = \exp\left(-\frac{2\pi \left\langle |\eta|^2 \right\rangle}{\hbar^2 \dot{\Omega}} \right)$$

Fermi golden rule is exact for gaussian fast noise!

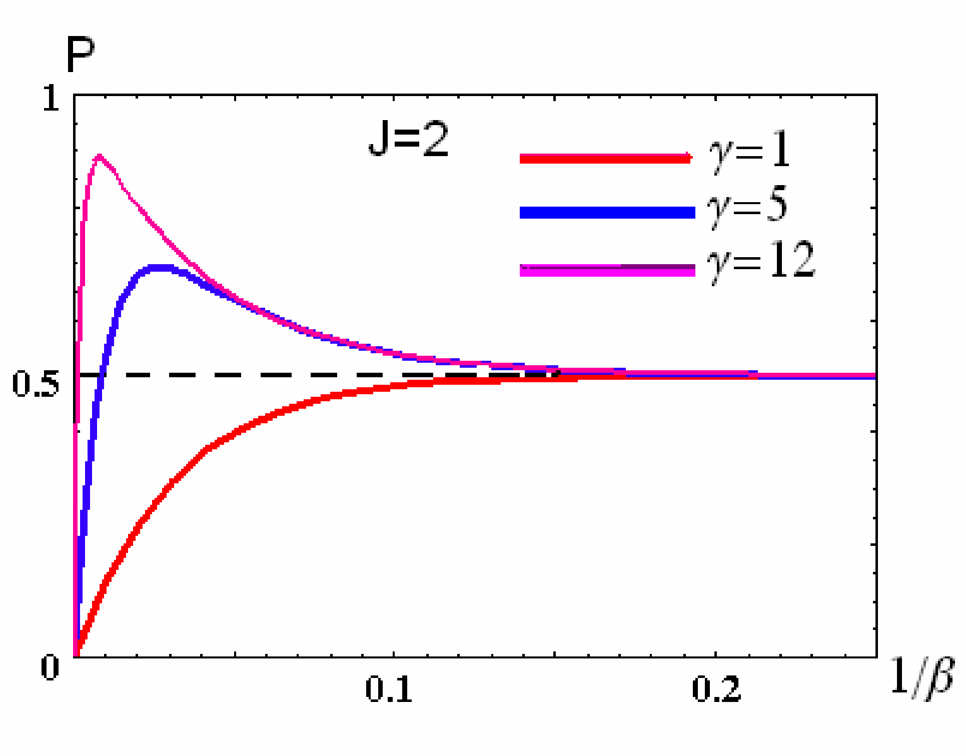
Regular and random field act together



τ_{acc} τ_{LZ} τ_{acc}

Separation of times: noise is essential only beyond τ_{LZ}

$$P_{1\to 2} = \frac{1}{2}\left[1 - e^{-\frac{2\pi\langle|\eta|^2\rangle}{\hbar^2\dot{\Omega}}}\left(2e^{-\frac{2\pi\Delta^2}{\hbar^2\dot{\Omega}}} - 1\right)\right]$$

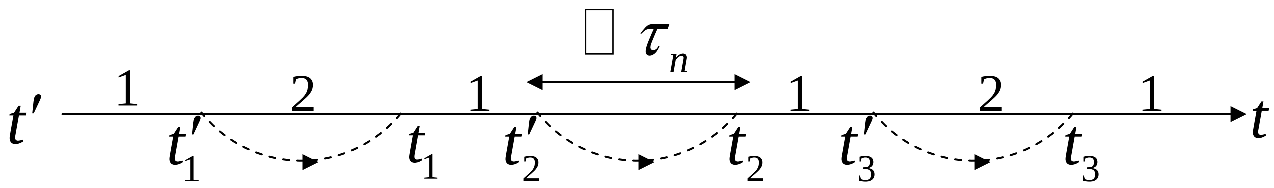


$$J=\frac{2\pi\langle|\eta|^2\rangle}{\dot{\omega}}$$

Plot of transition probability vs. inverse frequency rate. $\beta=\dot{\Omega}$

It is possible to get *P* larger than ½ at faster sweep rate or stopping the process at some specific field.

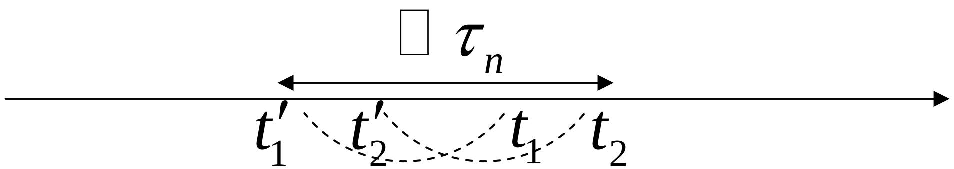
Graph representation



$$t' \xrightarrow{\;1\;} t \quad G(t,t') = \exp\left(\frac{i\dot{\Omega}\left(t^2 - t'^2\right)}{2}\right) \qquad t' \xrightarrow{\;2\;} t \qquad G^*(t,t')$$

$$t' \dashrightarrow t \qquad \left\langle \eta^\dagger(t)\eta(t')\right\rangle = \left\langle \eta(t')\eta^\dagger(t)\right\rangle$$

Chronological time order



4 times are close

What was omitted: Debye-Waller factor $W = \left\langle \exp\left[-i \int_{t'}^{t} \eta_z(t'')dt'' \right] \right\rangle$

$$t - t' \gg \tau_n$$

$W \approx 1$, if $\left\langle \eta_z^2 \right\rangle \tau_n^2 \ll 1$

Diagonal noise leads to decoherence for a long time

$$\left\langle g_{\pm}(+\infty) \right\rangle = 0$$

Decoherence time: $\tau_{dec} = \dfrac{1}{\left\langle \eta_z^2 \right\rangle \tau_n} \gg \tau_n$

Quantum noise and its characterization

$$\left\langle \eta^{\dagger}(t)\eta(t')\right\rangle \neq \left\langle \eta(t')\eta^{\dagger}(t)\right\rangle$$

Model of noise: phonons

$$H_{\text{int}} = u\left(a_1^{\dagger}a_2 + a_2^{\dagger}a_1\right);$$

$$u = \eta + \eta^{\dagger};\ \eta = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} g_{\mathbf{k}} b_{\mathbf{k}}$$

$$H_n = \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}$$

$$H_2 = \frac{\Omega(t)}{2}\left(a_1^{\dagger}a_1 - a_2^{\dagger}a_2\right);\ \Omega(t) = \dot{\Omega}t$$

$$\left\langle \eta_\omega^\dagger \eta_\omega \right\rangle = N_\omega \int d\mathbf{k}\,\delta\left(\omega - \omega_\mathbf{k}\right)\left|g_\mathbf{k}\right|^2;\; N_\omega = \left(e^{\omega/T} - 1\right)^{-1}$$

$$\left\langle \eta_\omega \eta_\omega^\dagger \right\rangle = \left(N_\omega + 1\right)\int d\mathbf{k}\,\delta\left(\omega - \omega_\mathbf{k}\right)\left|g_\mathbf{k}\right|^2 = e^{\omega/T}\left\langle \eta_\omega^\dagger \eta_\omega \right\rangle$$

$$\left\langle \eta^\dagger(t)\eta(t')\right\rangle = \int \frac{d\omega}{2\pi}\left\langle \eta_\omega^\dagger \eta_\omega \right\rangle e^{-i\omega(t-t')}$$

Different time scales for induced and spontaneous transitions

$$\tau_{ni} \sim T^{-1} \qquad\qquad \tau_{ns} \sim \omega_D^{-1}$$

Noise is fast if $T, \omega_D \gg \sqrt{\dot{\Omega}}$

Bloch equation for the fast quantum noise

$$\frac{ds_z}{dt} = \operatorname{sign}(t)\left\langle\left[\eta_{|\Omega|}, \eta^{\dagger}_{|\Omega|}\right]\right\rangle - \left(\left\langle \eta_{|\Omega|}\eta^{\dagger}_{|\Omega|}\right\rangle + \left\langle \eta^{\dagger}_{|\Omega|}\eta_{|\Omega|}\right\rangle\right) s_z$$

$$\Omega = \Omega(t) = \dot{\Omega} t$$

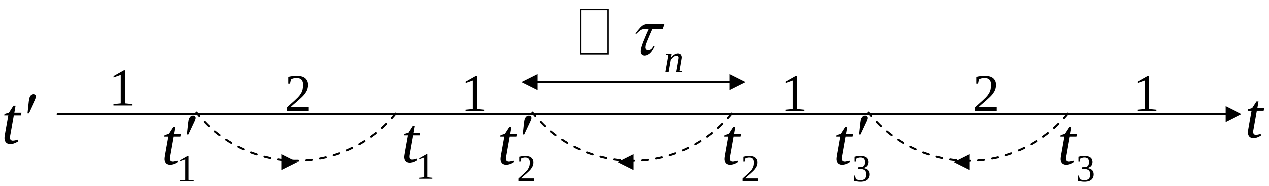
Solution

$$s_z(t) = s_z(t_0)\exp\left(-\int_{t_0}^{t}\left(\left\langle \eta_{|\Omega(t')|}, \eta^{\dagger}_{|\Omega(t')|}\right\rangle + \left\langle \eta^{\dagger}_{|\Omega(t')|}\eta_{|\Omega(t')|}\right\rangle\right)dt'\right) +$$

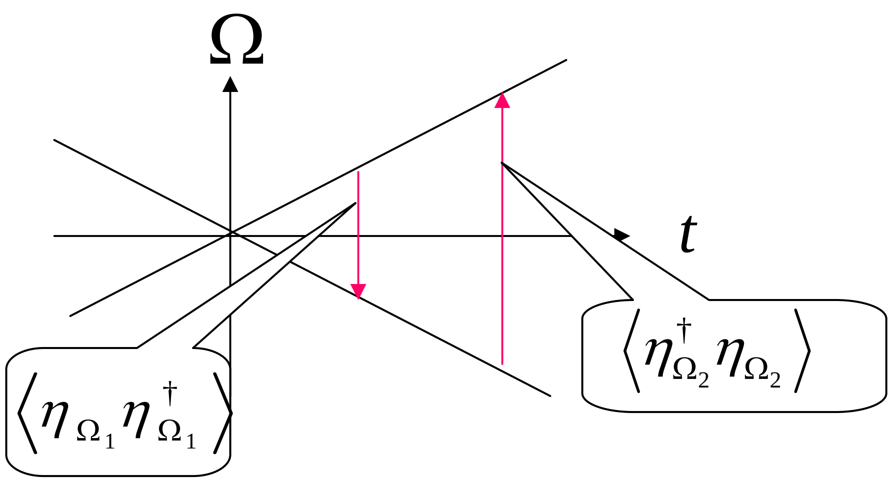
$$\int_{t_0}^{t} dt' \operatorname{sign}(t')\left\langle\left[\eta_{|\Omega(t')|}, \eta^{\dagger}_{|\Omega(t')|}\right]\right\rangle \exp\left(-\int_{t'}^{t}\left(\left\langle \eta_{|\Omega(t'')|}\eta^{\dagger}_{|\Omega(t'')|}\right\rangle + \left\langle \eta^{\dagger}_{|\Omega(t'')|}\eta_{|\Omega(t'')|}\right\rangle\right)dt''\right)$$

s_z turns into zero in the limit of strong noise, but not exponentially

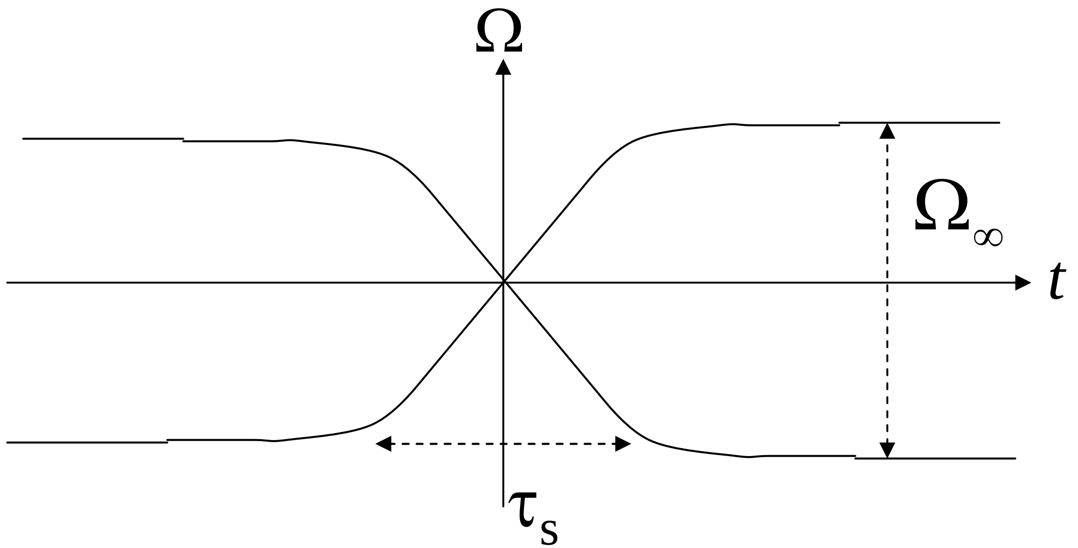
Essential graphs



Golden rule picture



Saturation of frequency



$$\tau_{acc} = \left(\dot{\omega}\tau_n\right)^{-1} \square \ \tau_s$$

Long time limit

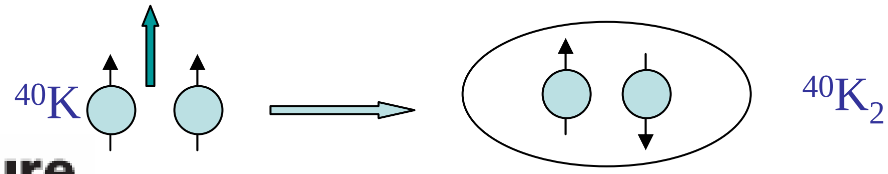
$$s_{z\infty} = -\frac{\left\langle \left[\eta_{\Omega_\infty}, \eta^\dagger_{\Omega_\infty} \right] \right\rangle}{\left\langle \eta_{\Omega_\infty} \eta_{\Omega_\infty}{}^\dagger + \eta_{\Omega_\infty}{}^\dagger \eta_{\Omega_\infty} \right\rangle}$$

Noise in thermal equilibrium

$$\left\langle \eta_\Omega \eta^\dagger_\Omega \right\rangle = e^{\frac{\hbar\Omega}{T}} \left\langle \eta^\dagger_\Omega \eta_\Omega \right\rangle$$

$$s_{z\infty} = \tanh \frac{\hbar\Omega_\infty}{2T}$$

Feshbach resonance



letters to nature

Creation of ultracold molecules from a Fermi gas of atoms

Cindy A. Regal*, Christopher Ticknor*, John L. Bohn* & Deborah S. Jin†

* *JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA*
† *Quantum Physics Division, National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA*

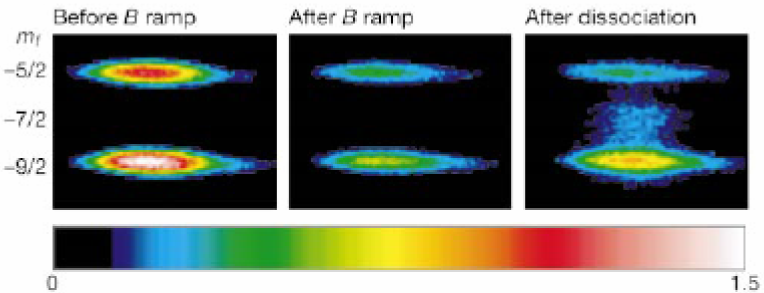
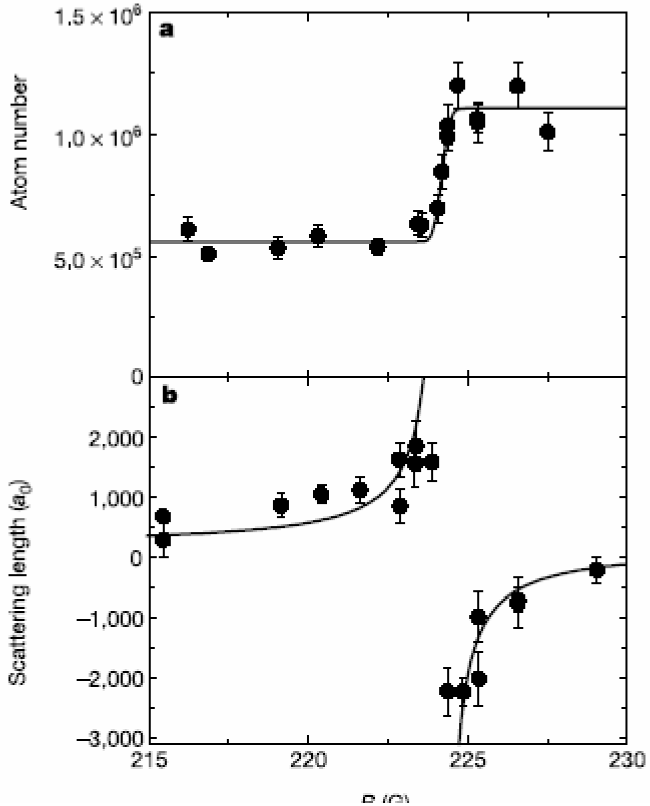
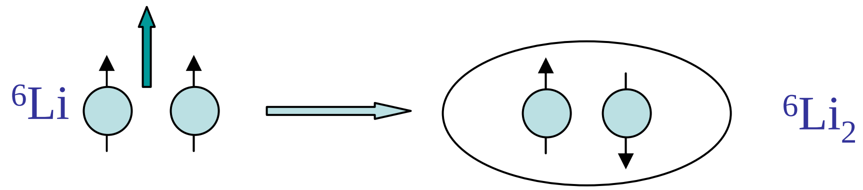


Figure 4 Absorption images of the quantum gas using a Stern–Gerlach technique. We start with ultracold fermionic atoms in the $m_f = -5/2$ and $m_f = -9/2$ states of ^{40}K. A magnetic field ramp through the Feshbach resonance causes 50% atom loss, owing to adiabatic conversion of atoms to diatomic molecules. To directly detect these bosonic molecules we apply an r.f. photodissociation pulse; the dissociated molecules then appear in the $m_f = -7/2$ and $m_f = -9/2$ atom states. The shaded bar indicates the optical depth.



Feshbach resonance



VOLUME 91, NUMBER 8 PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2003

Conversion of an Atomic Fermi Gas to a Long-Lived Molecular Bose Gas

Department of Physic ... *I, USA*

We have conv ... y
adiabatic passa ... t-
bound, $v = 38$, ... e
molecules remai ... e
adiabatic sweep ...

DOI: 10.1103/Phy ... s

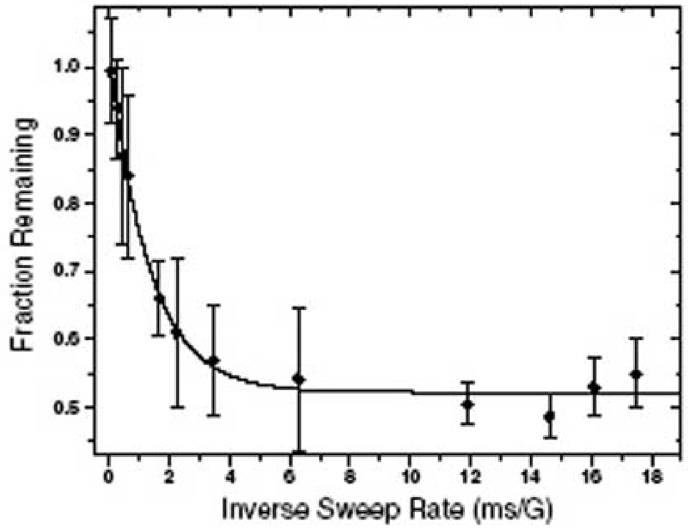
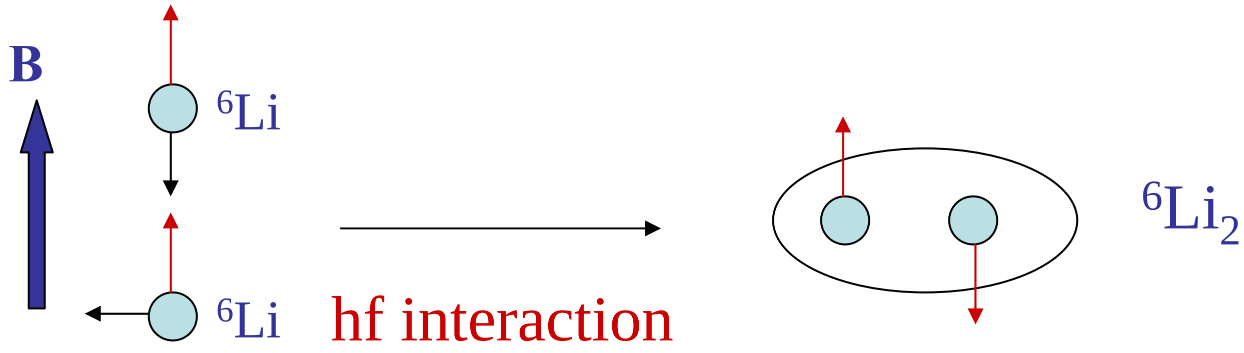


FIG. 3. Dependence of atom loss on inverse sweep rate. The field is ramped linearly from high to low field. The solid circles represent the average of 6–10 measurements and the error bars are the standard deviations of these measurements. The solid line is an exponential fit, giving a decay constant of 1.3 ms/G.

Feshbach Resonance driven by sweeping magnetic field



Hamiltonian: $H = H_0 + V$

$$H_0 = \sum_{\mathbf{p}} \left[\left(\varepsilon_{\mathbf{p}} - \mu - h(t) \right) \left(a^{\dagger}_{\mathbf{p}} a_{\mathbf{p}} + b^{\dagger}_{\mathbf{p}} b_{\mathbf{p}} \right) \right] + \sum_{\mathbf{q}} \left(E_{\mathbf{q}} - 2\mu \right) c^{\dagger}_{\mathbf{q}} c_{\mathbf{q}}$$

$$h(t) = \dot{h} t$$

$$V = \frac{g}{\sqrt{V}} \sum_{\mathbf{p},\mathbf{q}} \left(a_{\mathbf{p}} b_{\mathbf{q}} c^{\dagger}_{\mathbf{p}+\mathbf{q}} + h.c. \right); \qquad g \sim \varepsilon_{hf} \sqrt{a_m^3}$$

Suggestions of LZ probability for the molecule production

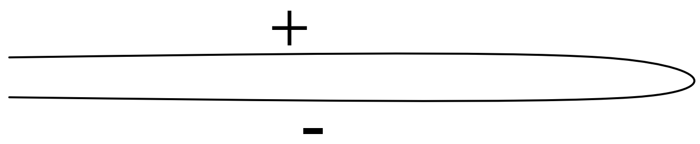
$$\frac{N_{mol}(t)}{N_a(0)} = \alpha P_{LZ}(t,\gamma) \qquad \gamma^2 = \frac{g^2 n}{\hbar^2 \dot{\Omega}} \sim \frac{\varepsilon_{hf}^2}{\hbar^2 \dot{\Omega}} n a_m^3$$

$\alpha = ?$ - combinatorial factor. $\alpha = 1/2$

Perturbation theory for molecular production

B. Dobrescu and VP, Phys. Lett. A **350**, 154 (2006)

Keldysh technique for time-dependent field



Green functions

$$\mathcal{A}_{\alpha\beta}(\mathbf{p};t,t') = -i\left\langle T_c\left(a_\alpha(\mathbf{p},t)a_\beta^\dagger(\mathbf{p},t')\right)\right\rangle$$

$$\mathcal{B}_{\alpha\beta}(\mathbf{p};t,t') = -i\left\langle T_c\left(b_\alpha(\mathbf{p},t)b_\beta^\dagger(\mathbf{p},t')\right)\right\rangle$$

$$C_{\alpha\beta}(\mathbf{p};t,t') = -i\left\langle T_c\left(c_\alpha(\mathbf{p},t)c_\beta^\dagger(\mathbf{p},t')\right)\right\rangle$$

$$\alpha,\beta = \pm$$

Number of molecules

$$N_m(t) = i\sum_{\mathbf{p}\in(\text{Fermi sphere})} C_{+,-}(\mathbf{p};t,t)$$

Interaction representation

$$\mathcal{A}^{(0)}_{+,-}(\mathbf{p},t,t') = i\theta(\varepsilon_F - \varepsilon_{\mathbf{p}})\exp\left[\frac{i}{\hbar}\int_t^{t'}\left(\varepsilon_{\mathbf{p}} - \mu - h(t)\right)dt\right]$$

$$\mathcal{A}^{(0)}_{-,+}(\mathbf{p},t,t') = -i\theta(\varepsilon_{\mathbf{p}} - \varepsilon_F)\exp\left[\frac{i}{\hbar}\int_t^{t'}\left(\varepsilon_{\mathbf{p}} - \mu - h(t)\right)dt\right]$$

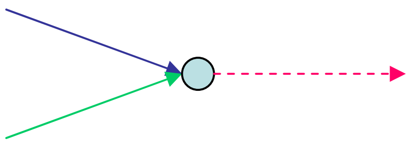
$$\mathcal{B}_{\alpha,\beta}(\mathbf{p},t,t') = \mathcal{A}_{\alpha,\beta}(\mathbf{p},t,t')$$

$$\mathcal{C}^{(0)}_{+,-}(\mathbf{p},t,t') = 0;\; \mathcal{C}^{(0)}_{-,+}(\mathbf{p},t,t') = \exp\left[\frac{i}{\hbar}\left(\varepsilon_c(\mathbf{p}) - 2\mu\right)(t'-t)\right]$$

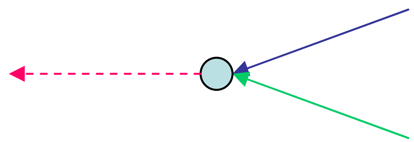
$$\mathcal{G}_{+,+}(\mathbf{p},t,t') = \theta(t-t')\mathcal{G}_{-,+}(\mathbf{p},t,t') + \theta(t'-t)\mathcal{G}_{+,-}(\mathbf{p},t,t')$$

$$\mathcal{G}_{-,-}(\mathbf{p},t,t') = \theta(t-t')\mathcal{G}_{+,-}(\mathbf{p},t,t') + \theta(t'-t)\mathcal{G}_{-,+}(\mathbf{p},t,t')$$

Vertices:

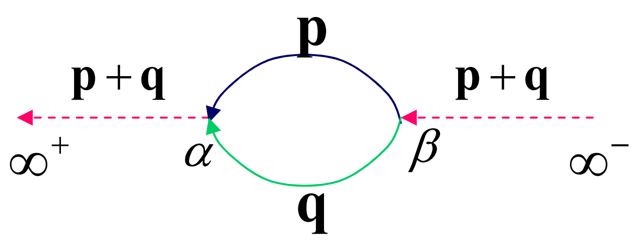


$$\frac{g}{\sqrt{V}}$$

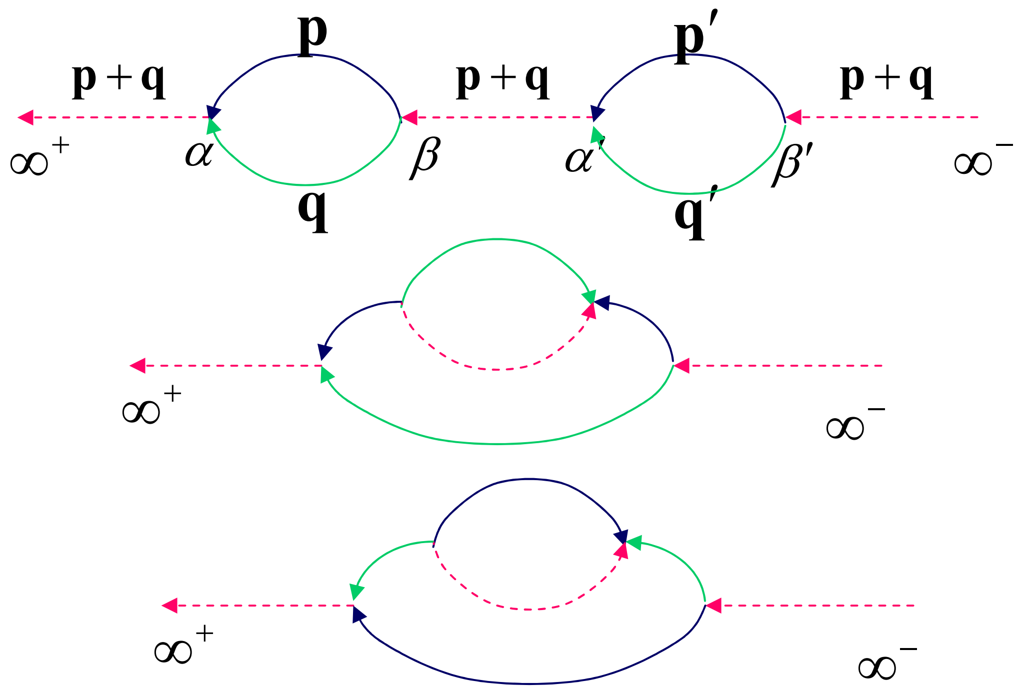


Graphs

Second order:



Fourth order:



Results

$$\frac{N_m(t=\infty)}{N_a(t=-\infty)} = \Gamma - \frac{88}{105}\Gamma^2 + 0\left(\Gamma^4\right)$$

$$\Gamma = \frac{g^2 n_a}{\hbar^2 \dot{\Omega}} \qquad \Omega = \frac{g_S \mu_B B}{\hbar} = \frac{h(t)}{\hbar}$$

In first two orders of perturbation theory $\frac{N_m(t=\infty)}{N_a(t=-\infty)} = f(\Gamma)$

Compare to effective LZ theory: $\frac{N_m(t=\infty)}{N_a(t=-\infty)} = \Gamma - \frac{1}{2}\Gamma^2 + 0\left(\Gamma^4\right)$

Collective effects suppress the transition probability

During the transition process atoms can perform an exchange.

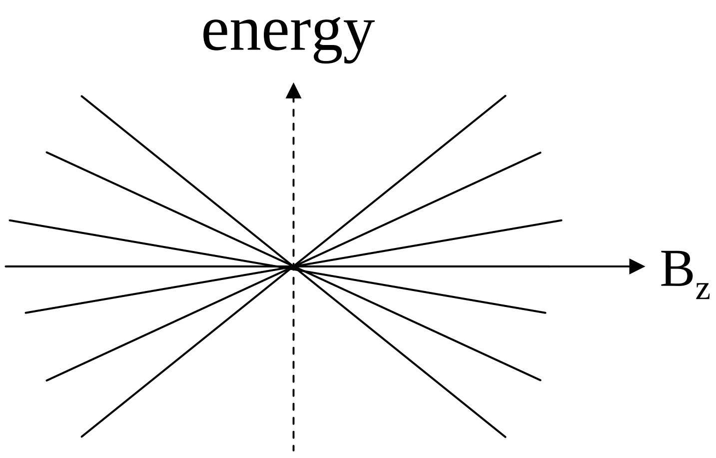
Renormalizability?

Conclusions

- LZ transition proceeds during time interval $\tau_{\mathrm{LZ}} = \Delta / \dot{\Omega}$
- Fast transverse noise produces transitions during the time interval $\tau_{acc} = \left(\dot{\Omega}\tau_n\right)^{-1}$. Condition $\tau_n \ll \tau_{acc}$ is assumed
- Transitions at any moment of time are due to the spectral component of noise which is in the resonance with the current frequency of the LZ system
- Classical noise tends to establish equal population of levels. Strong quantum noise also leads to the equal population on the timescale between τ_{acc} and τ_{sat} and equilibrates the LZ system on longer time scale.
- Transformation of the Fermi gas of cooled alkali atoms into molecules driven by sweeping magnetic field is a collective process which can not be described by the LZ formula. Collective effects suppress the transitions.
- The decoherence time due to longitudinal noise is $\tau_{dec} = \frac{1}{\langle \eta_z^2 \rangle \tau_n} \gg \tau_n$

What else was done

Transitions of spin S>1/2 in the sweeping magnetic field



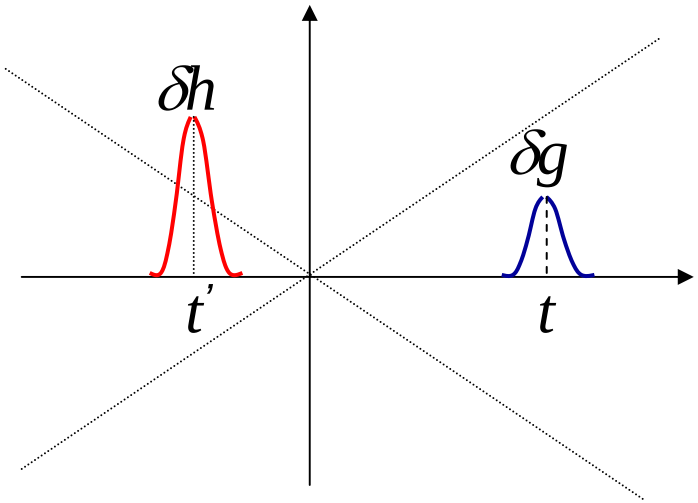
$$H_s = -\mathbf{bS}; \quad \mathbf{b} = g\mu_B \mathbf{B}$$

$$\mathbf{b} = \left(b_x, 0, \dot{b}_z t\right)$$

VLP and N.A. Sintsyn, Phys. Rev. B **69,** 104414 (2004)

Correlations in the noisy LZ transitions

Response to a weak pulse signal



$$\langle \delta g_\alpha(t) \rangle = \int_{-\infty}^{t} \langle g_\alpha(t) g_\beta(t') \rangle \delta h_\beta(t') dt'$$

$$\langle g_\alpha(t) g_\beta(t') \rangle = K_{\alpha\beta}(t,t')$$

Solution of the Bethe-Salpeter equations

VLP and S, Scheidl, Phys. Rev. B **70**, 014416 (2004)

Solution of the Bloch equations

$$\dot{g}_z = \frac{1}{2i}\left(\eta_+ g_- - \eta_- g_+\right);\ \dot{g}_\pm = \mp i\dot{\Omega} t g_\pm \pm i\eta_\pm g_z$$

Integral equation for g_z

$$\dot{g}_z = -\frac{1}{2}\int_{-\infty}^{t}\left[e^{\frac{i\dot{\Omega}(t^2-t'^2)}{2}}\eta_+(t)\eta_-(t') + c.c.\right]g_z(t')dt'$$

Complete initial decoherence

Averaging procedure: $\left\langle \eta_+(t)\eta_-(t')g_z(t')\right\rangle = \left\langle \eta_+(t)\eta_-(t')\right\rangle\left\langle g_z(t')\right\rangle$

Precision: $\tau_n / \tau_{acc} = \dot{\Omega}\tau^2$