Decoherence in superconducting nanocircuits

Elisabetta Paladino

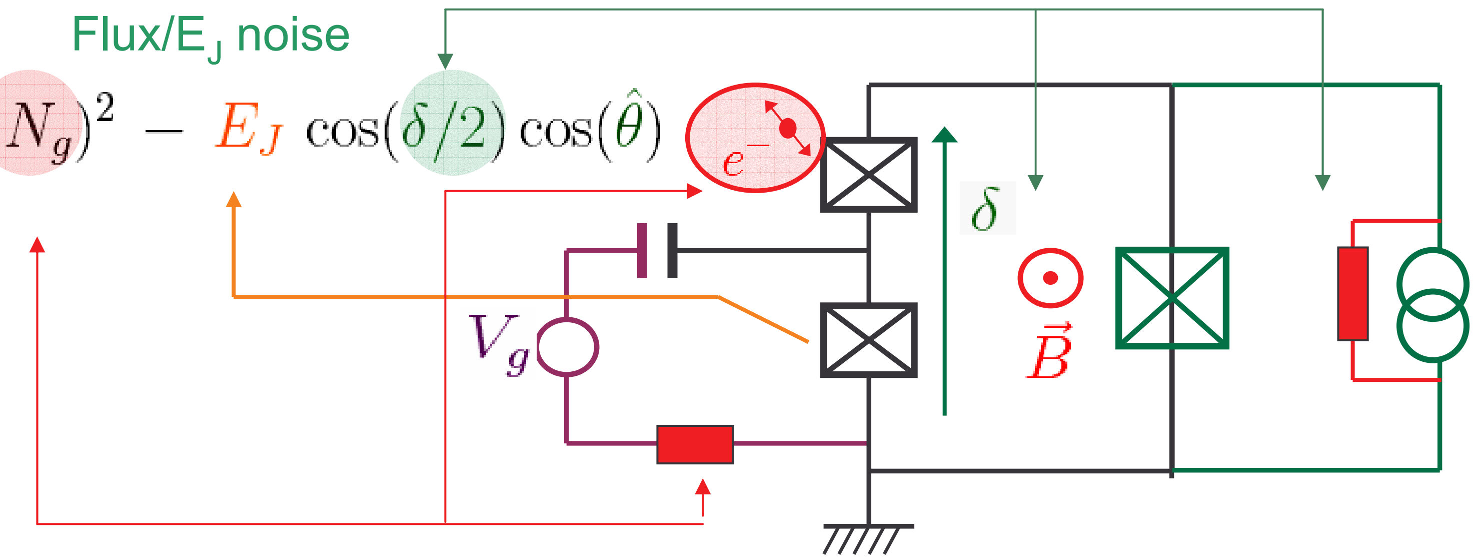
CNR INFM MATIS & SMT - DMFCI Catania

A. D'Arrigo, A. Mastellone, G. Falci

Noise sources in SC qubits

Flux/E_J noise

$$\mathcal{H} = E_C(\hat{N} - N_g)^2 - E_J \cos(\delta/2)\cos(\hat{\theta})$$



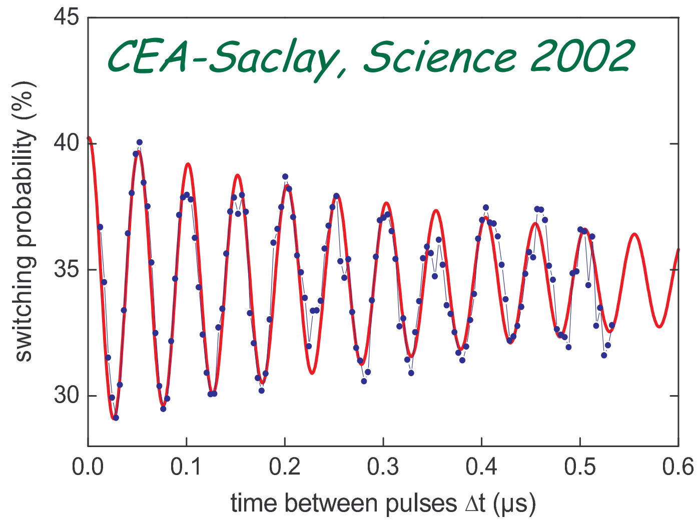
▬ = Z(ω) linear noise

(e^-) = S(ω) charge noise

Two-port design: Quantronium, Saclay 2002
(courtesy D. Esteve)

charge noise (1/f)

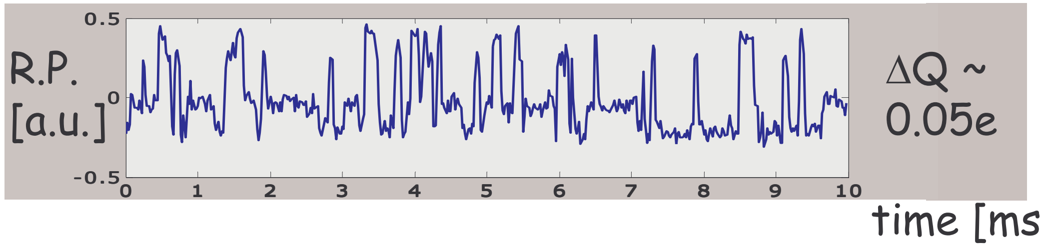
↔ switching impurities close to the device
Paladino, Faoro, Falci, Fazio, PRL 2002



- ➢**THE** problem in high Q charge based qubits
- ➢Affects two-qubit operations in spin-qubits

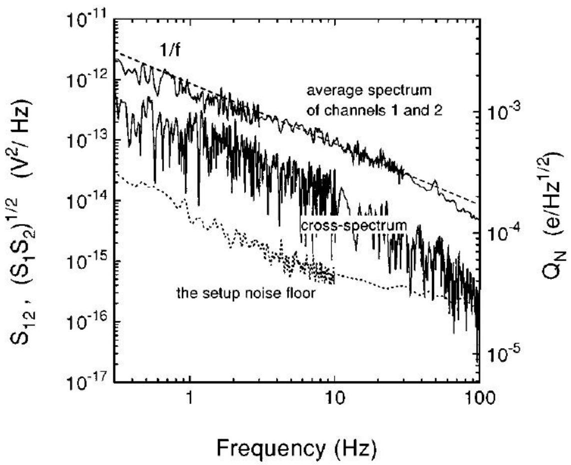
Noise characterization

- Noise due to *charged bistable impurities → RTN* Courtesy T. Duty, Chalmers '04



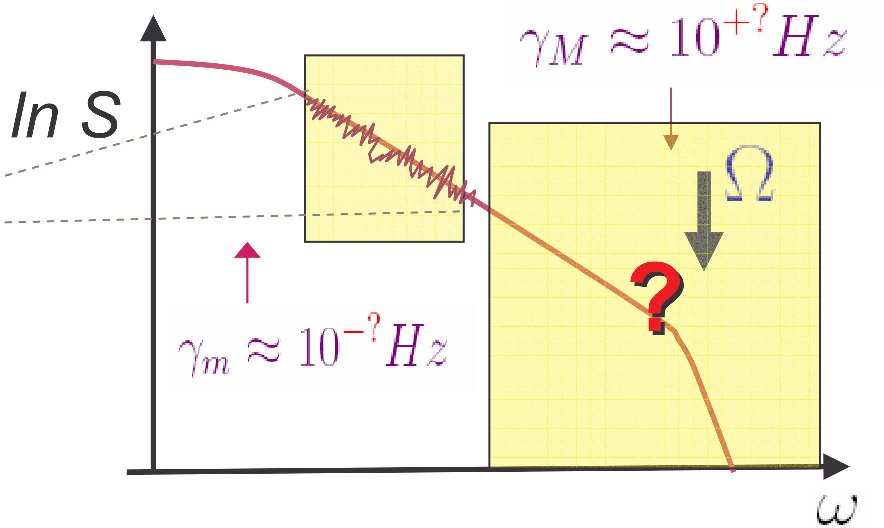
- Distribution of *charged bistable impurities* with switching rates $P(\gamma) = C/\gamma$ leads to **1/f noise**

non gaussian, non markovian



Experiments in SET: *Zorin et al. PRB (1996)*

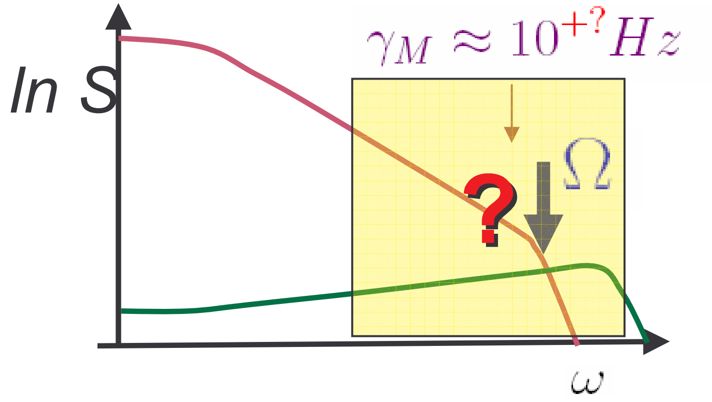
1/f noise measurements:
uncertainty on all high-frequency properties, eg. γ_M *vs* Ω

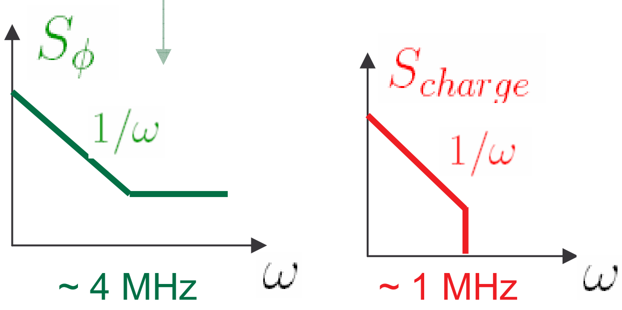


1/f noise measurements:

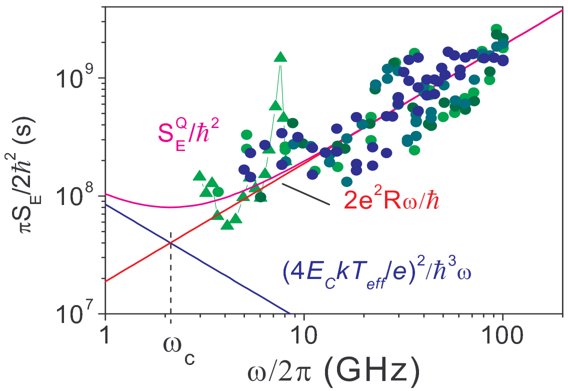
uncertainty on all high-frequency properties, eg. γ_M vs Ω

white noise, ohmic noise????





Saclay qubit *Ithier et al., subm PRB* **04**



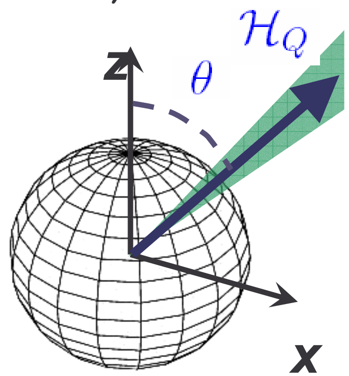
Charge qubit exps: *Astafiev et al., PRL 04*

- Variety of experimental features <u>material & device dependent</u>
- Slow noise components make unstable the calibration of the device (env. with memory) $\rightarrow$ signal decay **strongly depends on protocol**

Environment 1 : understanding the **nature** of noise sources

Works by groups in: Argonne, Basel, Catania, Karlsruhe, Landau Inst., Princeton, Saclay, ,Stony Brook …

$$\mathcal{H} = -\frac{\varepsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x - \frac{1}{2}\sigma_z \hat{X} + \mathcal{H}_R$$



Coupling with continous variables (bosons)

$$\hat{X} = \sum_\alpha \lambda_\alpha (a_\alpha^\dagger + a_\alpha) \qquad \mathcal{H}_R = \sum_\alpha \omega_\alpha a_\alpha^\dagger a_\alpha$$

- electromagnetic environment, sum of many microscopic variables

Coupling with impurities or switching noise sources

$$\hat{X} = \sum_\alpha v_\alpha d_\alpha^\dagger d_\alpha$$

intrinsic discrete nature

- eg. Fano impurities, Kondo like traps, defects in Junction oxides, flux noise

Goal: more and more accurate microscopic (semi-phenomenological) **characterization of the nature** of the environment

Solid-state coherent nanodevices **as detectors**

Environment 2 : handling the **effects** of noise sources

Solid-state coherent nanodevices **as processors**

Devices may have substantial coupling to the environment
but often they work in regimes of **limited sensitivity to its details**

Why ?

- **In practice:**

  Limited control of protocols $\longrightarrow$ *details* of the environment blurred
  (e.g. when inhomogeneous broadening dominates)
  Interest in protocols which effectively decouple the environment

- **In principle:** typical situation for Quantum Information

  several qubits nearly decoupled from the environment
  short time dynamics ~ 10000 operation
  time-dependent control

Goal: reasonable approximations including systematically **only relevant information** about noise, focusing on the **effects** of the environment on the controlled dynamics rather than on the specific nature of the noise sources.

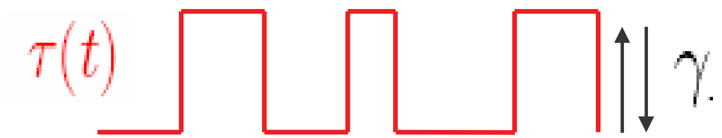
Applications to multiqubit devices

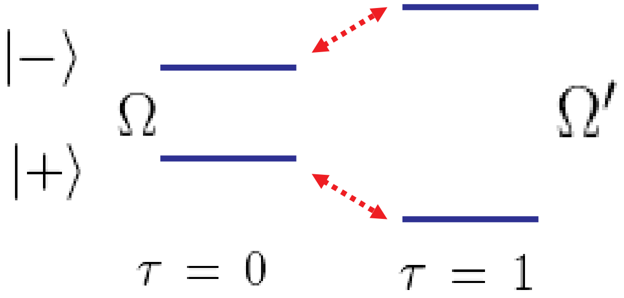
Main effects of the enviroment

Nanoscience

Paladino et al., Proc. MS+S2004

$$\mathcal{H} = -\frac{\epsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x - \frac{1}{2} v\,\tau(t)\,\sigma_z$$





visibility of the induced splitting

$$g = \frac{\Omega' - \Omega}{\gamma}$$

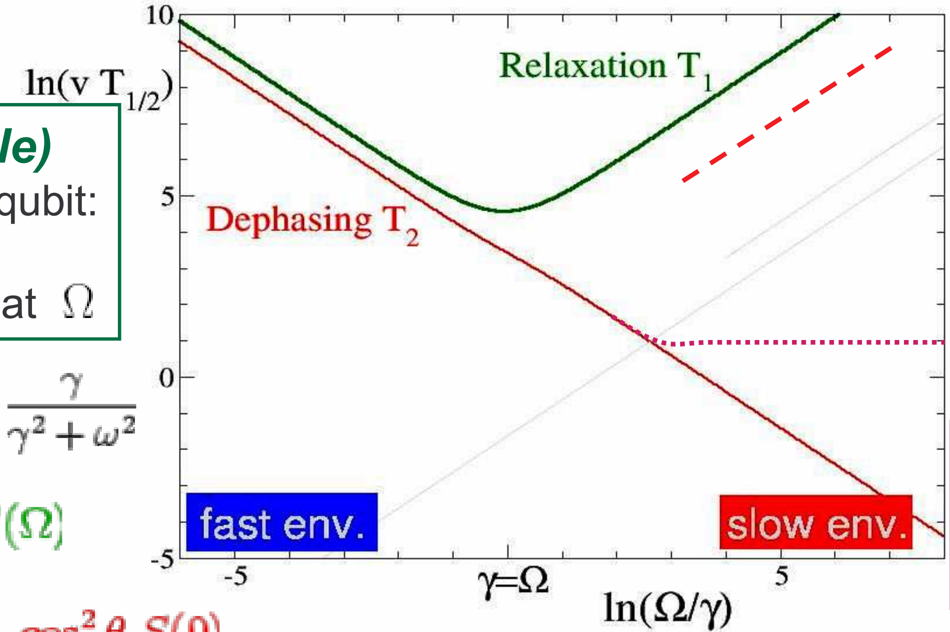
$$T_2 \propto 1/\gamma$$

g<1 (golden rule)
Nearly decoupled qubit:
Exponential decay
Single narrow line at Ω

$$S(\omega) = \frac{v^2}{2}(1 - \overline{\delta p}^2)\,\frac{\gamma}{\gamma^2 + \omega^2}$$

$$T_1^{-1} = \frac{1}{2}\sin^2\theta\, S(\Omega)$$

$$T_2^{-1} = \frac{1}{2}T_1^{-1} + \frac{1}{2}\cos^2\theta\, S(0)$$



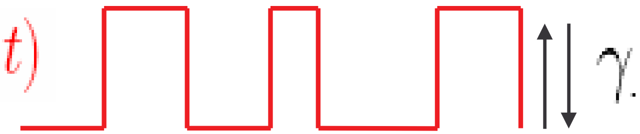
g>1: 2 narrow lines at Ω
and Ω'
linewidth $\gamma \rightarrow 0$
Beats → bad qubit
Good detector

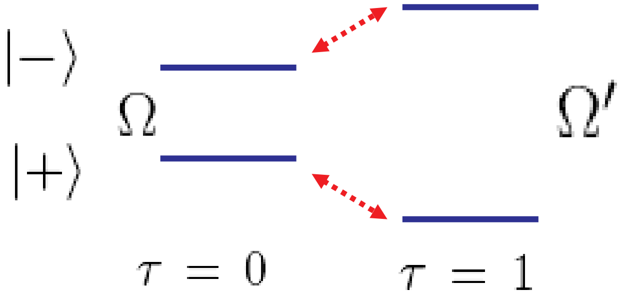
Inhomogeneous broadening
non-exponential decay
linewidth v

Main effects of the enviroment

Paladino et al., Proc. MS+S2004

$$\mathcal{H} = -\frac{\epsilon}{2}\,\sigma_z - \frac{\Delta}{2}\,\sigma_x - \frac{1}{2}\,v\,\tau(t)\,\sigma_z$$

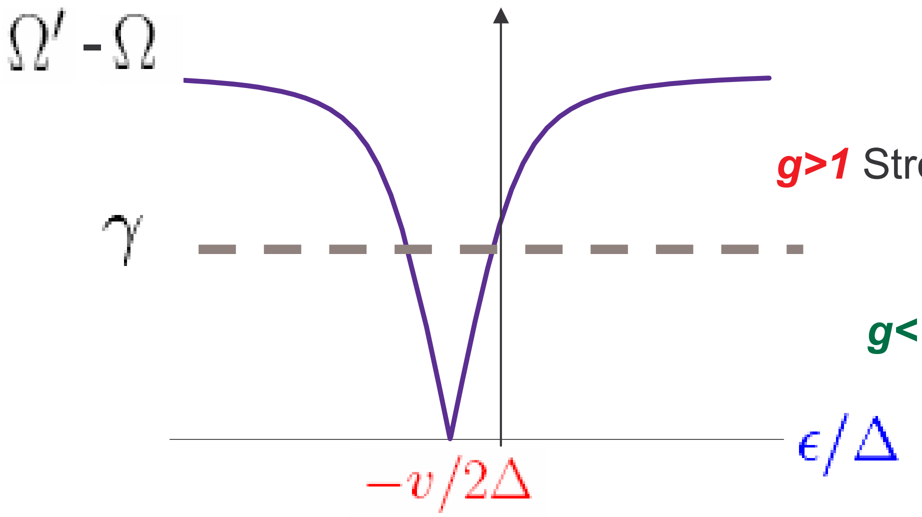




visibility of the induced splitting

$$g = \frac{\Omega' - \Omega}{\gamma}$$

sensitivity depends on the operating point

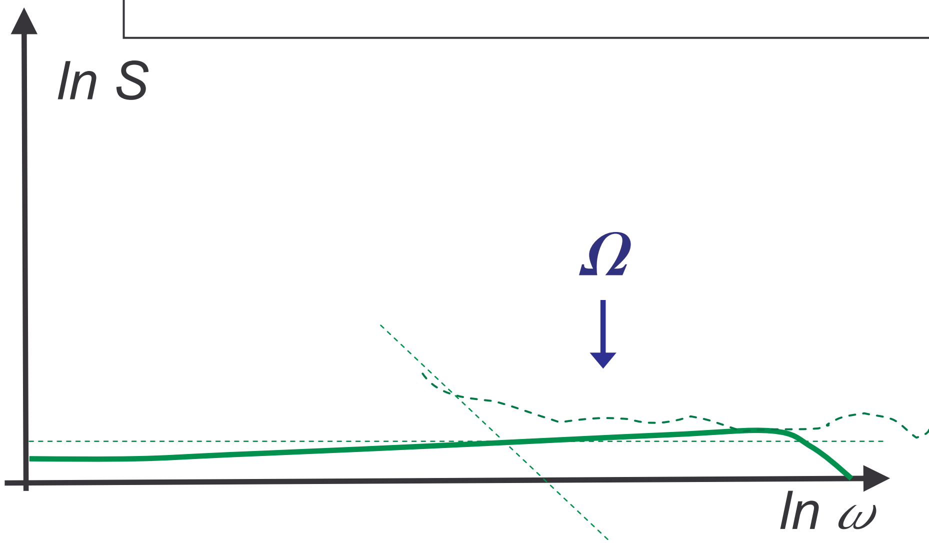


g>1 Strongly coupled

g<1 Nearly decoupled

Three classes of noise : I

Classification: according to the **effect** rather than to the nature of noise

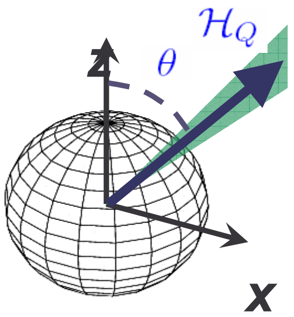


Quantum noise

1. **Spontaneuos decay** + …
2. Depends on S(ω), not on details
3. Weakly coupled **and** short-time correlated
4. **Markovian Master Equation**
5. Electromagnetic fluctuations, fast impurities, meter off, crosstalk, …

$$\mathcal{H} = -\frac{1}{2}\vec{\Omega}\cdot\vec{\sigma} - \frac{1}{2}\sigma_z\hat{E} + \mathcal{H}_E$$

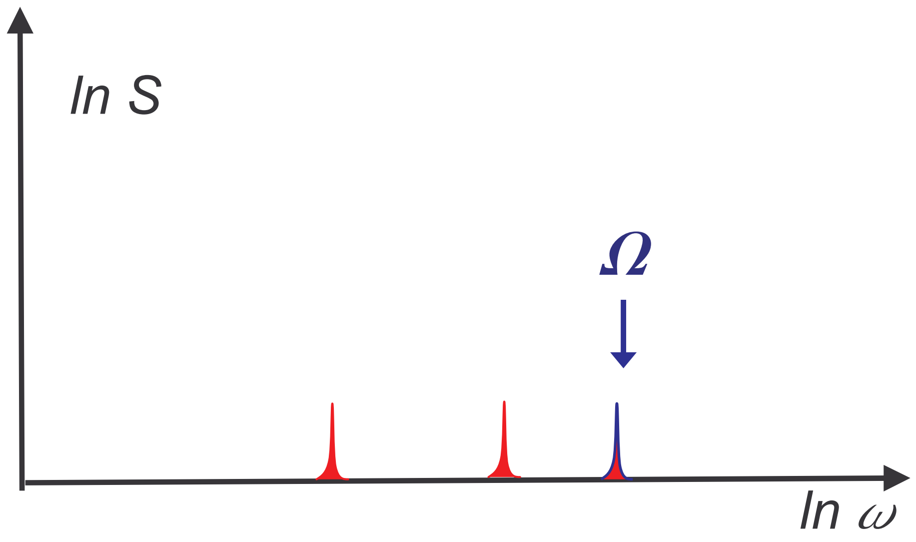
Master Equation $\quad \partial_t\,\rho^Q(t) = \hat{\mathcal{L}}[\,\rho^Q(0)\,] \qquad \rho^Q(t) = Tr_f\,[\,(W^{Q+f}(t))\,] = \mathcal{E}_t[\,\rho^Q(0)\,]$



Relaxation $\quad T_1^{-1} = \frac{1}{2}\sin^2\theta\; S(\Omega)$

Dephasing $\quad T_2^{-1} = \frac{1}{2}T_1^{-1} + \frac{1}{2}\cos^2\theta\; S(0)$

Three classes of noise: II



Strongly coupled noise

1. Uncontrolled "chemical shift" + …
2. Sensitive to details of protocol. Needed information beyond S(ω)
3. Possibly sample specific
4. **Enlarge Hilbert space of the system** including few environmental degrees of freedom

Enlarging system's Hilbert space

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E. Paladino et.al. PRL 2002

Quantum model

Bauernschmitt & Nazarov (1993)

$$\mathcal{H} = \underbrace{-\frac{1}{2}\vec{\Omega}\vec{\sigma} - \frac{v}{2}\sigma_z d^\dagger d + \varepsilon_c d^\dagger d}_{\text{System}} + \sum_k [T_k c_k^\dagger d + T_k^* d^\dagger c_k] + \sum_k \varepsilon_k c_k^\dagger c_k$$

(Fano impurity: $\varepsilon_c d^\dagger d + \sum_k [T_k c_k^\dagger d + T_k^* d^\dagger c_k] + \sum_k \varepsilon_k c_k^\dagger c_k$)

Fano impurity included in the system

"strong coupling" technique $\longrightarrow$ theory from adiabatic to quantum noise

Enlarging system's Hilbert space

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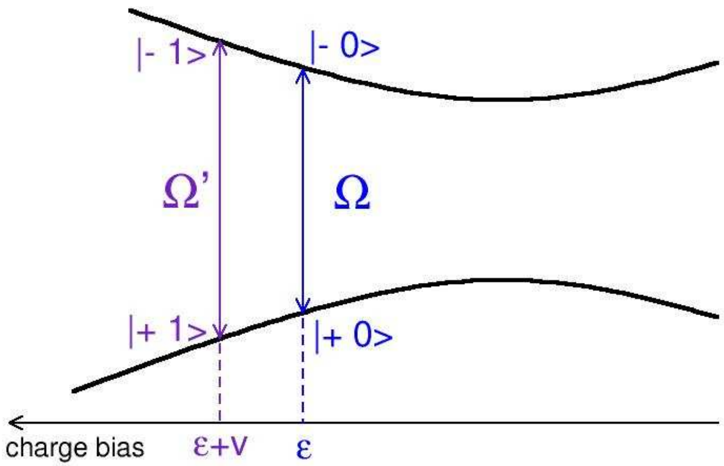
$$\mathcal{H} = \underbrace{-\frac{1}{2}\vec{\Omega}\,\vec{\sigma} - \frac{v}{2}\sigma_z\, d^\dagger d + \varepsilon_c\, d^\dagger d}_{\text{System}} + \overbrace{\varepsilon_c\, d^\dagger d + \sum_k [T_k\, c_k^\dagger d + T_k^*\, d^\dagger c_k] + \sum_k \varepsilon_k\, c_k^\dagger c_k}^{\text{Fano impurity}}$$

System



4 state system

$$|\,qubit\rangle \otimes |\sim incoher.\, impurity\rangle$$

Master eq. for 4×4 reduced density matrix

$$\partial_t\, \rho^{Q+S}(t) = \hat{\mathcal{L}}_{Q+S}[\rho^{Q+S}(0)]$$

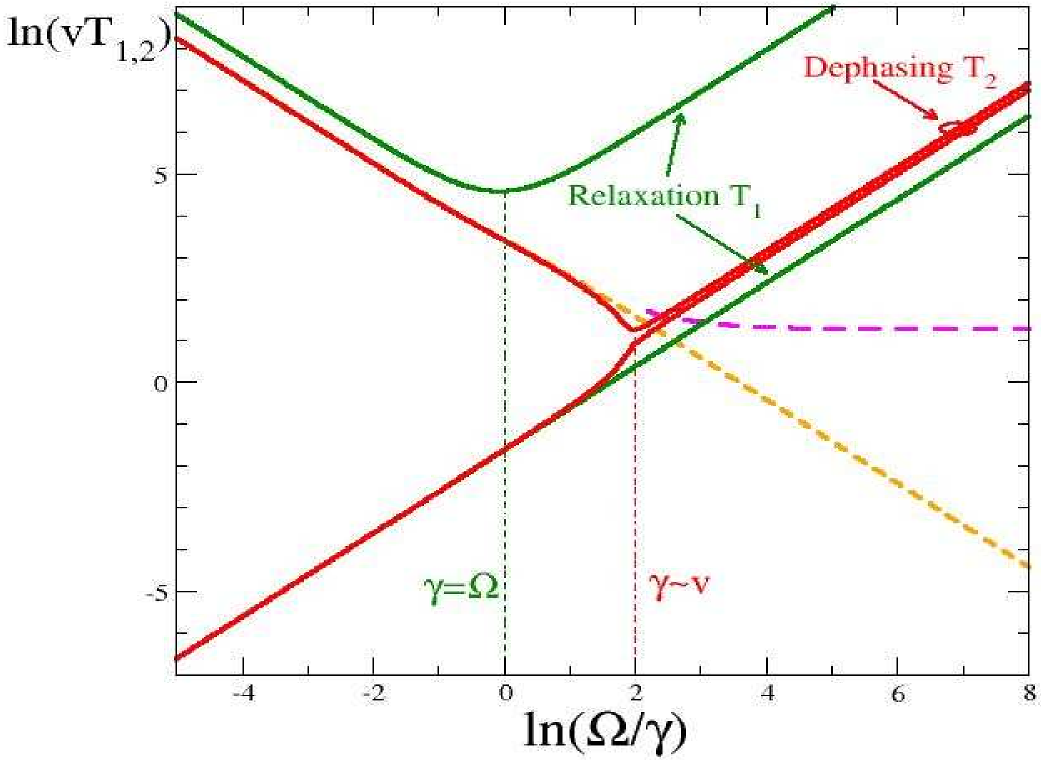
Trace over the impurity to get the qubit density matrix

$$\rho^Q(t) = Tr_S\,[\rho^{Q+S}(t)]$$

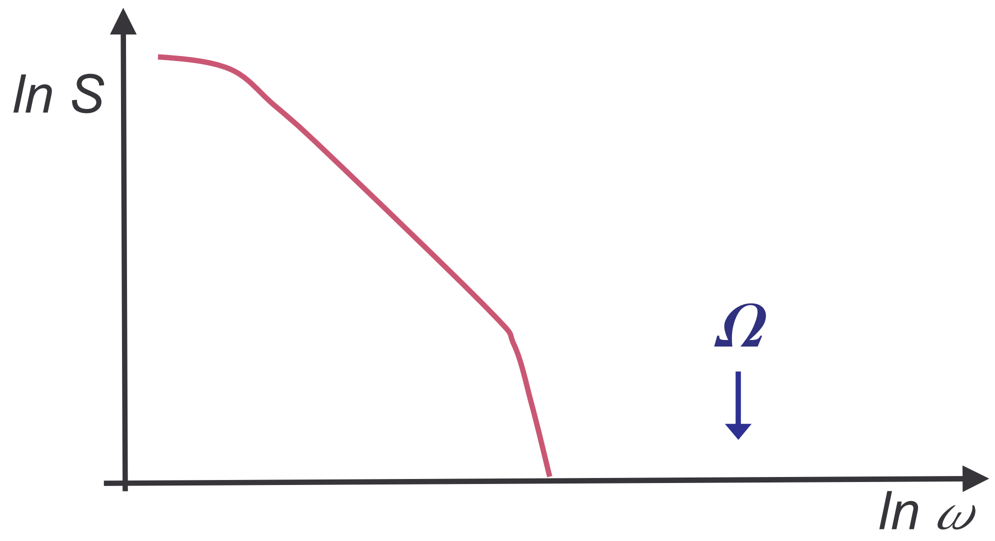
E. Paladino et. al. Adv.Sol.St.Phys. 2003;
Cf. also Marquard et al. PRB 2002

Enlarging system's Hilbert space

& exact diagonalisation of the 8 × 8 Redfield tensor



Three classes of noise: III



Adiabatic noise

1. Inhomogeneuos broadening + …
2. Sensitive to details of protocol. Beyond S(ω) ?
3. Long time-correlated
4. **Adiabatic approximation**
5. Low-frequency part of 1/f noise.

Adiabatic noise (Cooper-pair–box based nanodevices)

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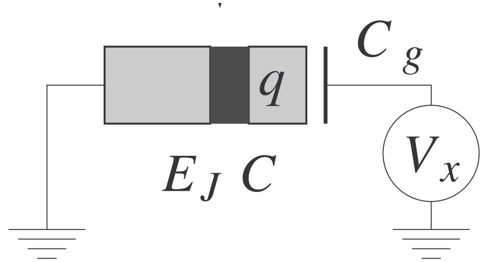
$$\mathcal{H}_{BOX} = E_C\,(\hat{q} - q_x)^2 - E_J \cos\hat{\varphi} \qquad [\hat{\varphi}, \hat{q}] = i$$

Cooper pair box



charge fluctuactions $\quad q_x \longrightarrow q_x + \delta q_x$

splittings $\quad \Omega(q_x, X) = \Omega_0(q_x) + \delta\Omega(q_x, X)$

Environment as a classical stochastic drive

$$\mathcal{H} = \mathcal{H}_{BOX} + q\,X(t) \qquad X(t) \to -2E_c\,\delta q_x(t)$$

Adiabatic noise (Cooper-pair–box based nanodevices)

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$$\mathcal{H} = \mathcal{H}_{BOX} + q\,X(t)$$ Environment as a classical stochastic drive

$$\gamma_M \ll \Omega \longrightarrow t \ll T_1$$ Adiabatic approximation

$$|\psi\, t\rangle = \sum_m |m(X_t)\rangle\langle m(X_0)|\psi\, 0\rangle\, \mathrm{e}^{-i\int_0^t ds\, E_m[X(s)]}$$

Average over realizations

$$\rho(t) = \int \mathcal{D}X(t)\, P[X(t)]\, \rho(t|X(s))$$

$$\rho(t) = \int \mathcal{D}X(t)\, P[X(t)] \sum_{mn} \underbrace{\hat{R}_{mn}(X_0, X_t)}_{transverse}\, \underbrace{\mathrm{e}^{-i\int_0^t ds\, \Omega_{mn}(s)}}_{longitudinal}$$

$$P[X(s)] = F[X(s)]\; p[X(s)]$$

Filter (protocol)

Joint probability distribution

$$p[X(s)] = \lim_{n\to\infty} p_{n+1}(X_n t; \ldots; X_1 t_1; X_0 0)$$

Adiabatic noise: inhomogeneous broadening

Falci, D'Arrigo, Mastellone, Paladino, PRL 2005, & cond-mat/0407484

Phase fluctuations accumulate in time $\longrightarrow$ **retain** longitudinal noise $\rho_{mn}(t) = \rho_{mn}(0)\, e^{-i\,\Phi_{mn}(t)}$

$$-i\,\Phi_{mn}(t) = \ln \int \mathcal{D}X(t)\, P[X(t)]\, e^{-i\int_0^t ds \Omega_{mn}[X(s)]}$$

Static Path Approximation (SPA) $\quad -i\,\Phi_{mn}(t) = \ln \int dX_0\, P[X_0]\, e^{-i\,\Omega_{mn}(X_0)\,t}$

Large N_{fl} central limit theorem $\rightarrow$ $p(X_0)$ gaussian distributed

Variance from number of switching impurities during $t_{meas} = 1/\gamma^*$

$$\sigma_X^2 = \frac{\langle\langle v^2 \rangle\rangle}{4} N_{fl} = 16 E_C\, A \ln \frac{\gamma_M}{\gamma^*} \longrightarrow \int_{\gamma^*}^{\gamma_M} \frac{d\omega}{\pi} S(\omega) \qquad S(\omega) \rightarrow 1/\omega$$

for 1/f noise

Adiabatic noise: inhomogeneous broadening

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Falci, D'Arrigo, Mastellone, Paladino, PRL 2005, & cond-mat/0407484

Static Path Approximation (SPA)

$$-i\,\Phi_{mn}(t) \;=\; \ln \int dX_0\, P[X_0]\, e^{-i\,\Omega_{mn}(X_0)\,t}$$

Steepest descent ~ quadratic expansion (*lowest eigenstates*)

$$\Omega(q_x, X) \;\approx\; \Omega_0(q_x) + c(q_x)\, X + \frac{1}{2}\, s^2(q_x)\, \frac{X^2}{\Omega_0(q_x)}$$

$$-i\,\Phi_{01}(t) \;=\; -i\,\Omega_0 - \frac{1}{2}\ln\!\left(1 + i\, s^2 \frac{\sigma_X^2}{\Omega_0} t\right) - \frac{1}{2}\,\frac{(c\sigma_X t/\Omega_0)^2}{1 + i\, s^2\, \sigma_X^2\, t\,/\Omega_0^2}$$

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low-frequency noise characterization in:

1) Time domain: short-time dynamics
dephasing time

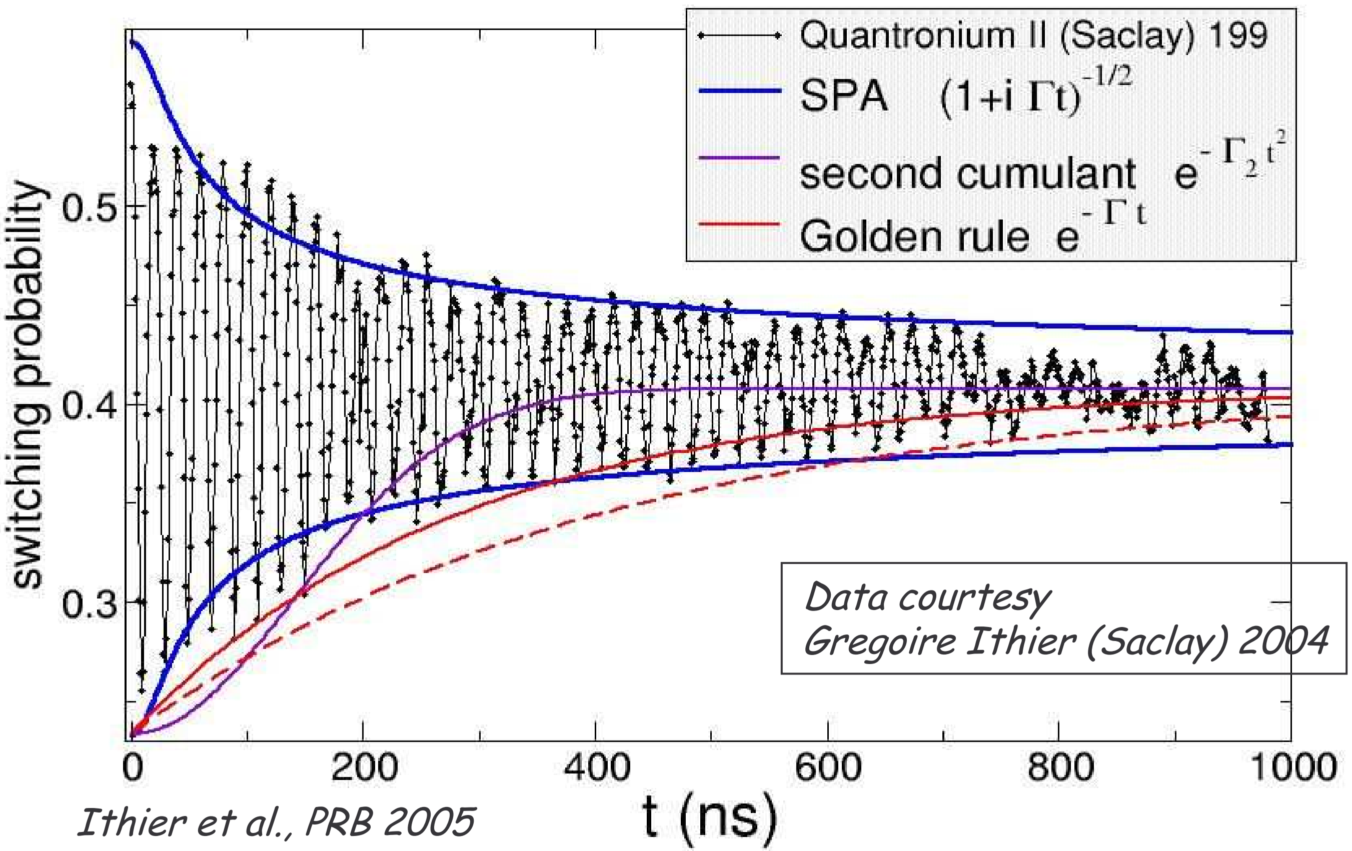
2) Frequency domain

Adiabatic noise in the Quantronium

Falci, D'Arrigo, Mastellone, Paladino, PRL 2005, & cond-mat/0407484

***Cf. Diagrammatic approach**: Shnirman-Makhlin, PRL 2004*
***Cf. Ornstein-Uhlenbeck processes**: Rabenstein-Averin 2004*

$$\rho_{10}(t) = \rho_{10}(0)\, e^{-i\Omega_0 t} \left(1 + i\,\frac{\sigma_X^2\, t}{\Delta}\right)^{-\frac{1}{2}} \propto t^{-1/2}$$



Ithier et al., PRB 2005

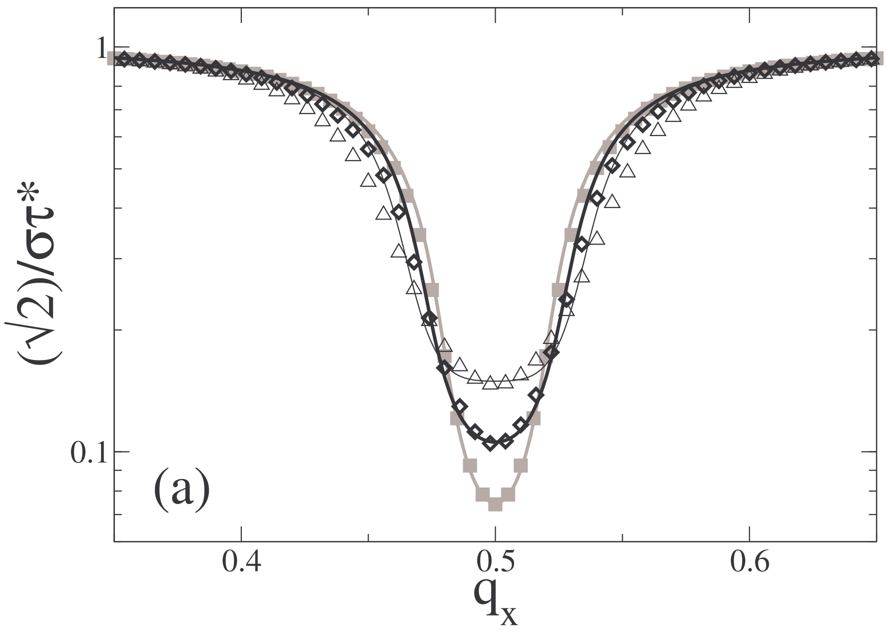
For simple protocols relevant effects due to **static impurities** (inhom. broadening)

Low frequency noise characterization: T_2^*

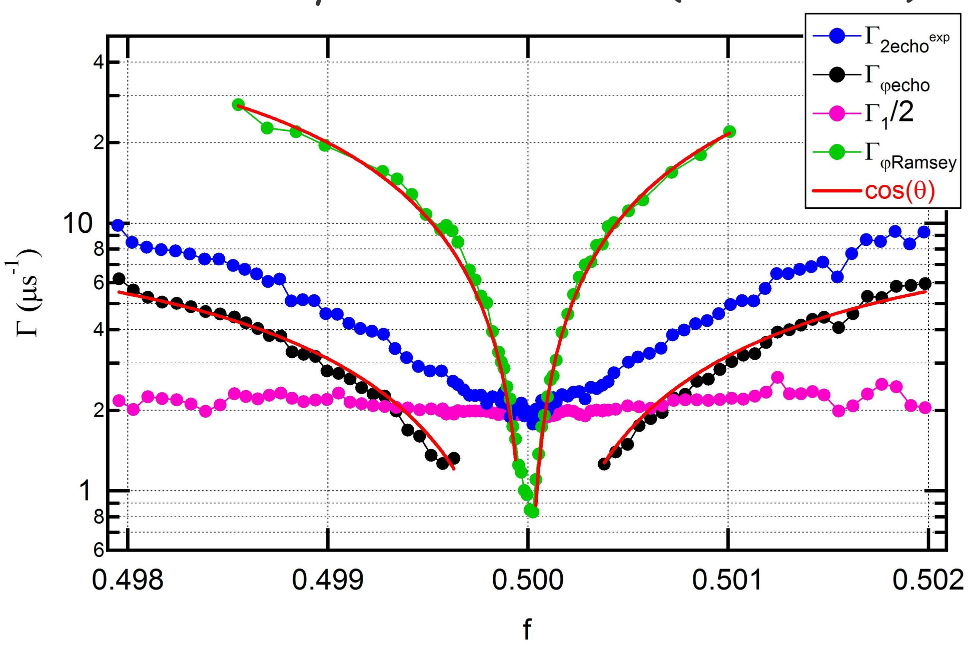
D'Arrigo, Falci, Mastellone, Paladino, proc. MS+S2006

Extract the dephasing time from $\mathrm{Im}[\Phi_{01}(T_2^*)] = -1$

Charge qubit



courtesy of J. Nakamura (MS+S2006)



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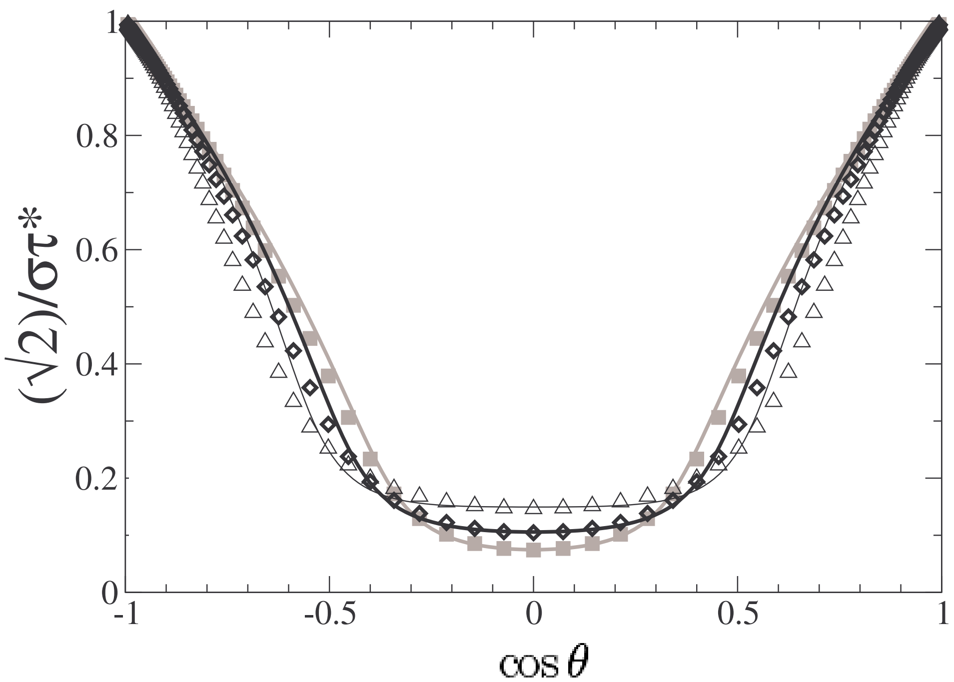
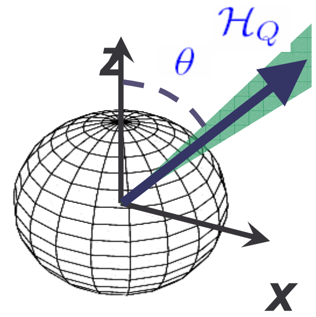


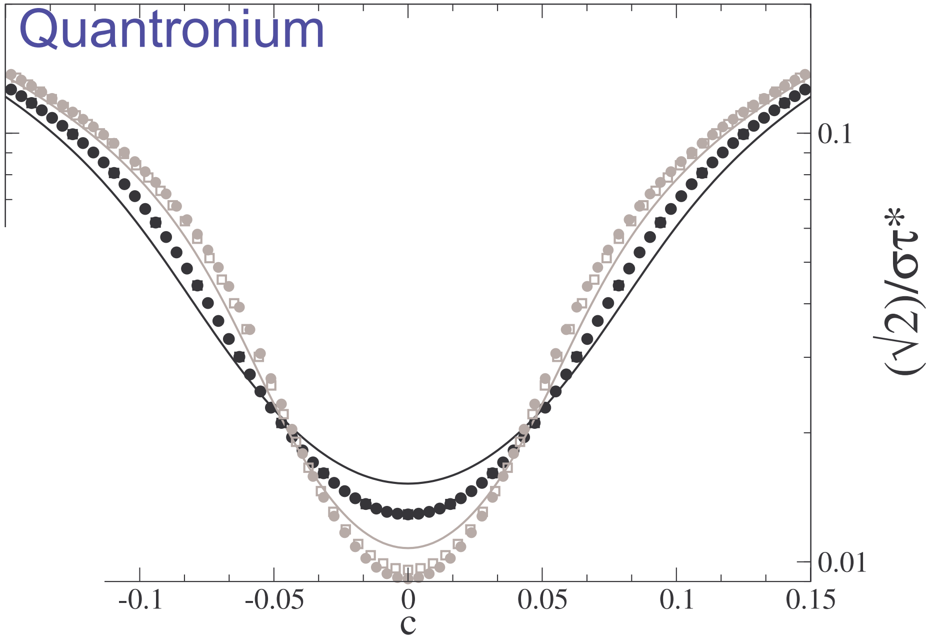


$c = 1$ $\quad T_2^{*-1} \approx \frac{\sigma_X |c|}{\sqrt{2}}$

$c = 0$ $\quad T_2^{*-1} \approx \frac{\sigma_X^2}{\sqrt{2}\Omega_0}$

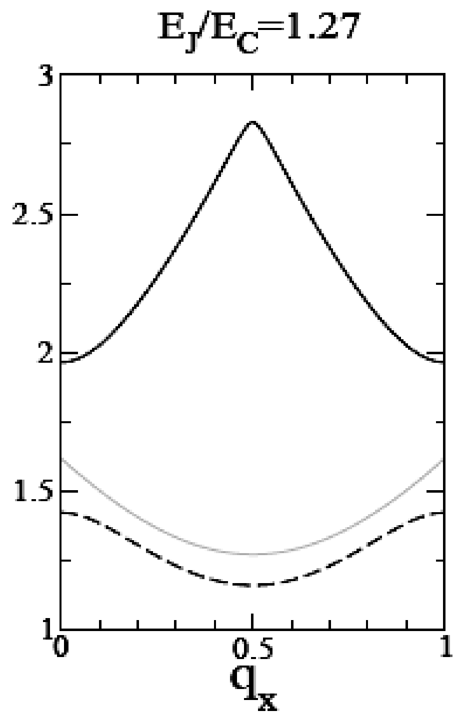
Master equation $\quad T_2^{*-1} \approx c^2 S(0) \quad$ where $\quad \sigma_X^2 \leftrightarrow S(0)$

Low frequency noise characterization: T_2^*



T_2^* vs $c(q_x)$: low-frequency noise characterization

BUT charge fluctuations → leakage from Hilbert space spanned by the lowest eigenstates at the optimal point



$c(q_x) = (q_{11} - q_{00})$ ↔ charge sensitivity of the device

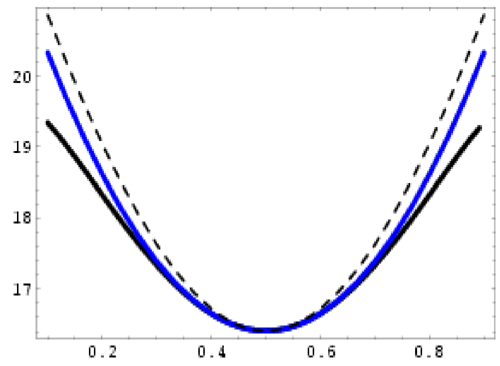
$$s^2(q_x) = 2\{2q_{10}^2 - \sum_{m\geq 2}[\frac{q_{1m}^2\Omega_0}{E_m - E_1} - \frac{q_{0m}^2\Omega_0}{E_m - E_0}]\}$$

↔ quantum capacitance CPB $C_Q^k \equiv (C_g/e)^2 \partial^2 E_k/\partial q_x^2$

$$\Omega(q_x, X) \approx \Omega_0(q_x) + c(q_x)\,X + \frac{1}{2}s^2(q_x)\,\frac{X^2}{\Omega_0(q_x)}$$

Low-frequency noise characterization: Lineshapes

$$\rho_{10}(t) = \rho_{10}(0)\, e^{-\Gamma t} \int dX\, p(X)\, e^{-i\int_0^t ds\, \Omega[X(s)]}$$



Quadratic splitting about the optimal point

$$\Omega(X) = \Omega_0 + a\,X + \frac{X^2}{2\Delta}$$

Close to the optimal point: sharp asymmetric lineshape

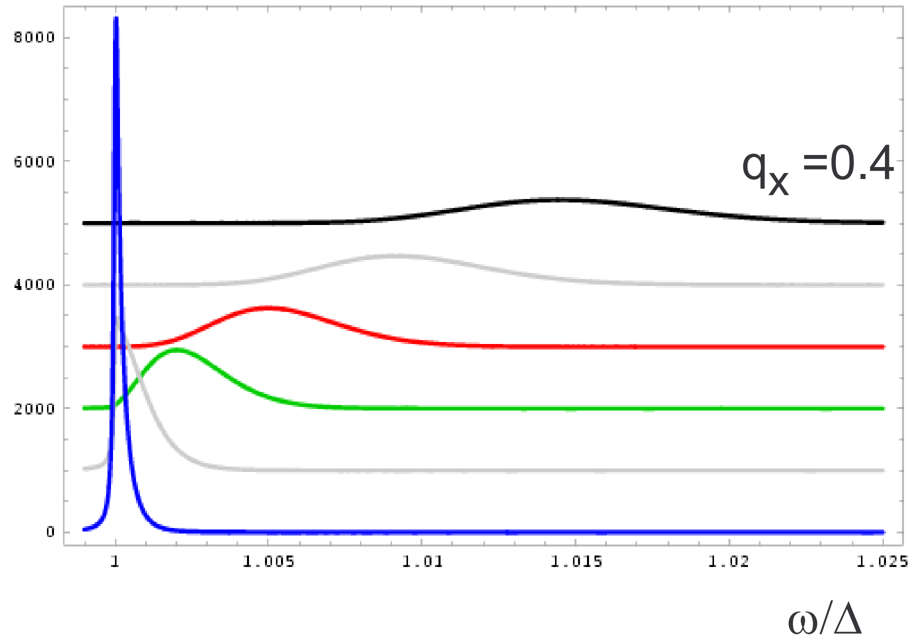
$$|a| < \sqrt{2\sigma_X/\Delta} \qquad a = 2E_C(q_x - 1/2)/E_J.$$

$$\tilde{\rho}_{10}(\omega) \approx \rho_{10}(0)\, 2\sqrt{2}\pi\,\Delta\, \frac{\Theta(\omega-\Delta)}{\sqrt{\omega-\Delta}}\, p(0)$$

Regularized by exponential tail $\Gamma = \frac{1}{2T_1}$

$$\tilde{\rho}_{10}(\omega) = \rho_{10}(0)\, 2\frac{\sqrt{2\pi}\Delta}{z\,\sigma_X}\, e^{-\frac{(z\Delta)^2}{2\sigma_X^2}} \left[1 + \mathrm{Erf}\left(\frac{iz\Delta}{\sqrt{2}\sigma_X}\right)\right]$$

$$z = \sqrt{\frac{2}{\Delta}(\omega - \Delta + i\Gamma)}$$



insensitive to details of fluctuations

D'Arrigo, Falci, Mastellone, Paladino, proc. MS+S2006

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Low-frequency noise characterization: Lineshapes

D'Arrigo, Falci, Mastellone, Paladino, proc. MS+S2006

$$\rho_{10}(t) = \rho_{10}(0)\, e^{-\Gamma t} \int dX\, p(X)\, e^{-i\int_0^t ds\,\Omega[X(s)]}$$

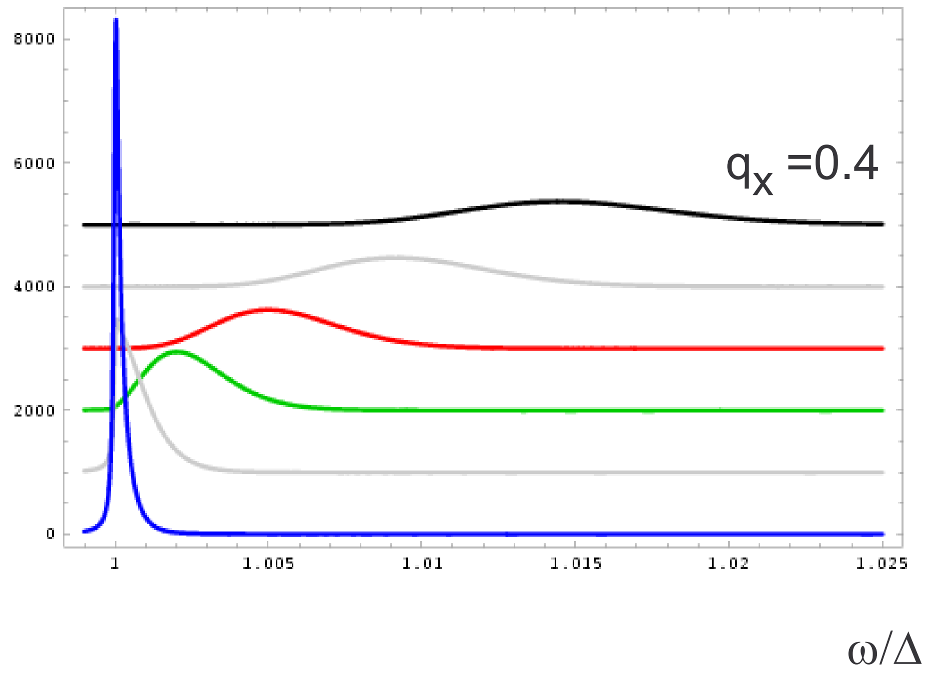
$$\Omega(X) = \Omega_0 + a\,X + \frac{X^2}{2\Delta}$$

$$a = 2E_C(q_x - 1/2)/E_J.$$

Non optimal points: $|a| > \sqrt{2\sigma_X/\Delta}$

broadened line peaked at $\omega = \Omega_0$

containing information on $p(X)$



$$\tilde{\rho}_{10}(\omega) \approx \rho_{10}(0)\,\frac{\sqrt{2\pi}\,\Delta}{\sqrt{\omega-\Delta}}\, p(|a|\Delta - \sqrt{2(\omega-\Delta)}) \quad \omega > \Delta$$

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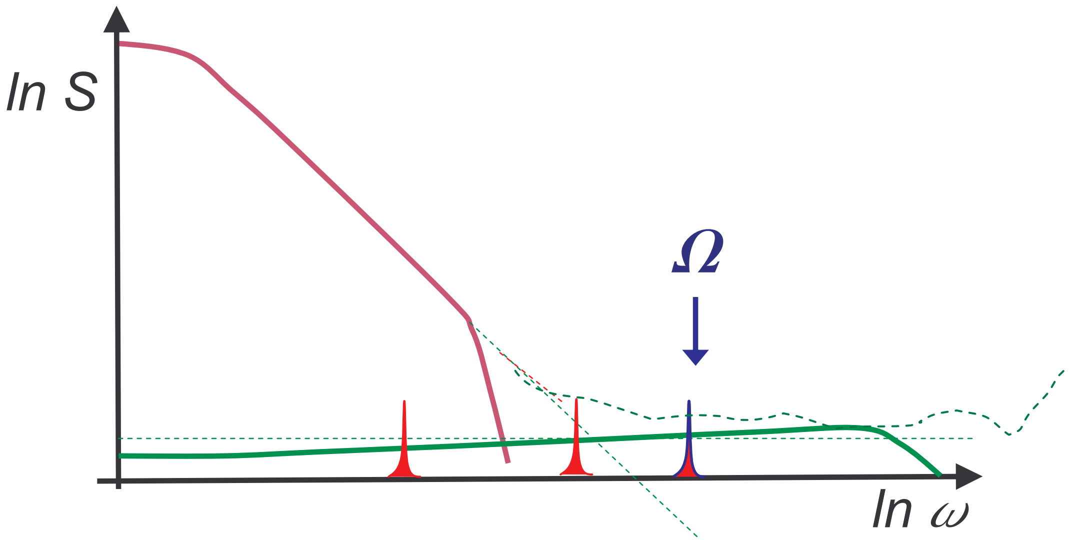
Three classes of noise

Cf. Falci, et al., PRL 2005
Astafev et al PRL. 2004



Adiabatic noise

1. Inhomogeneuos broadening + …
2. **Adiabatic approximation**
3. Low-frequency part of 1/f

Strongly coupled noise

1. Uncontrolled chemical shift + …
2. **Enlarge Hilbert space of the system**
3. Not weakly coupled impurities

Quantum noise

1. Spontaneuos decay + …
2. **Markovian Master equation**

Classification according to the **effect** rather than to the nature of noise

Each class has its **specific approximation scheme** which does not work (or it is impractical) for other classes of noise

Multistage elimination

$$\mathcal{H} = -\frac{\varepsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x - \frac{1}{2}\sigma_z\hat{X} + \mathcal{H}_R$$

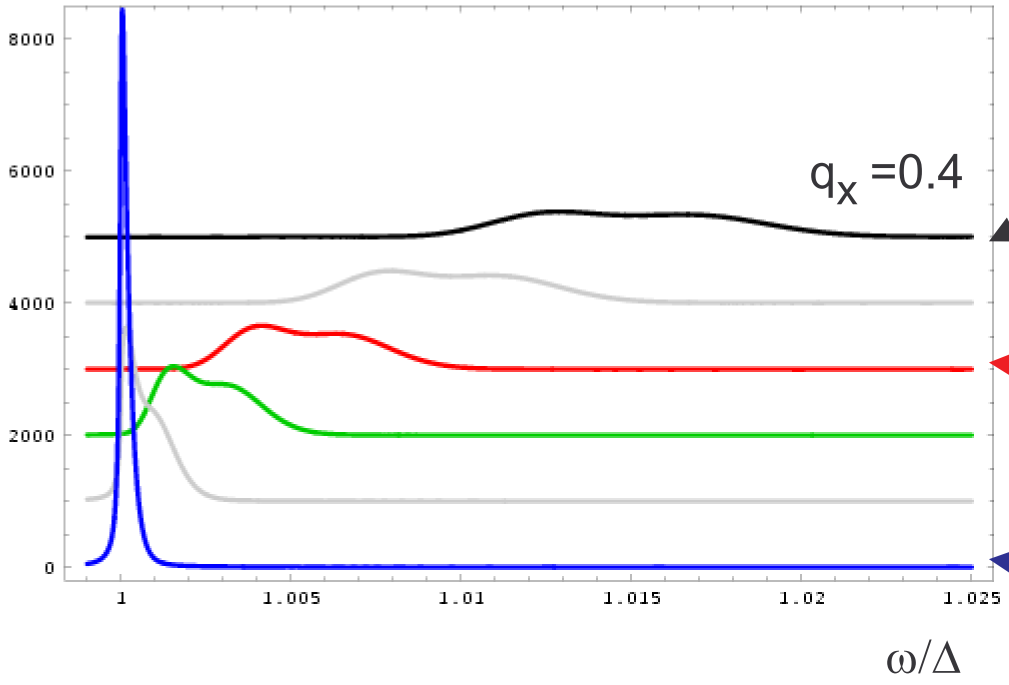
Split $$-\frac{1}{2}\sigma_z\hat{X}_S + \mathcal{H}_{RS} - \frac{1}{2}\sigma_z X(t) - \frac{1}{2}\sigma_z\hat{X}_f + \mathcal{H}_{Rf}$$

In general

$$\rho^Q(t) = Tr_S\{\int \mathcal{D}X(t)\,P[X(t)]\,Tr_f\,[W^{Q+S+f}(t|X(t))]\}$$

Adiabatic + one extra fluctuator

$$\rho^Q(t) = e^{-\Gamma t}\, Tr_S\{ \int dX\, p(X)\, \rho^{Q+S}(t|X(t)) \}$$



Features washed out by Inhomogeneuos broadening away from optimal point

(Asymmetric) features of the extra fluctuator appear close to the optimal point

Does not modify the lineshape at optimal point

Extra strongly coupled fluctuators

- limit flexible control
- Pose reliability problems in networks

Paladino, Mastellone, D'Arrigo, Falci, cond-mat/0407484
c.f. Non gaussian effects in *Bergli, Galperin, Altshuler PRB 2006*

Systematic path-integral

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Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

1. n-joint probability of the sum of many independent stochastic processes via product of the n-point generating functionals of each variables

2. An individual fluctuator is markovian $\longrightarrow$ n-point generating functional via (2-point) conditional probability

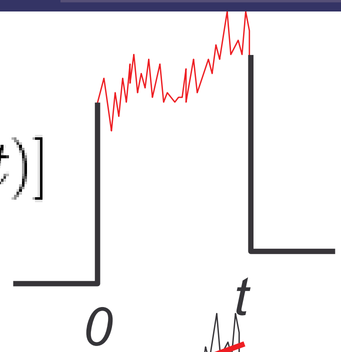
3. If needed gaussian approximation via retaining second cumulant

4. Systematic approach ~ derivative expansion

Systematic path-integral

Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

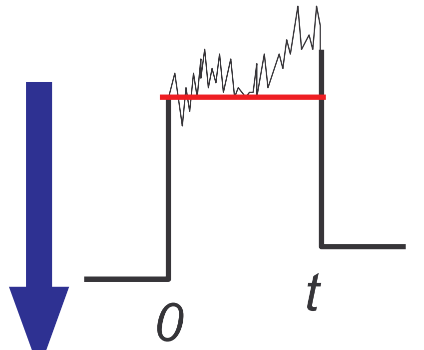
$$P[X_t, t\,;\, X_{t_{M-1}}, t_{M-1}\,;\, \cdots\,;\, X_{t_1}, t_1\,;\, X_0, 0] \xrightarrow{\Delta t \to 0} P[X(t)]$$

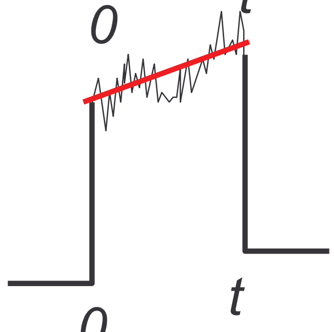


Static path $P[X_0, 0]$

1-st correction $P[X_t, t\,;\, X_0, 0]$

2-nd correction $P[X_t, t\,;\, X_{t_1}, t_1\,;\, X_0, 0]$





More and more accurate sampling of the process

"Derivative" expansion

First correction: adiabatic noise during evolution in FID or Ramsey

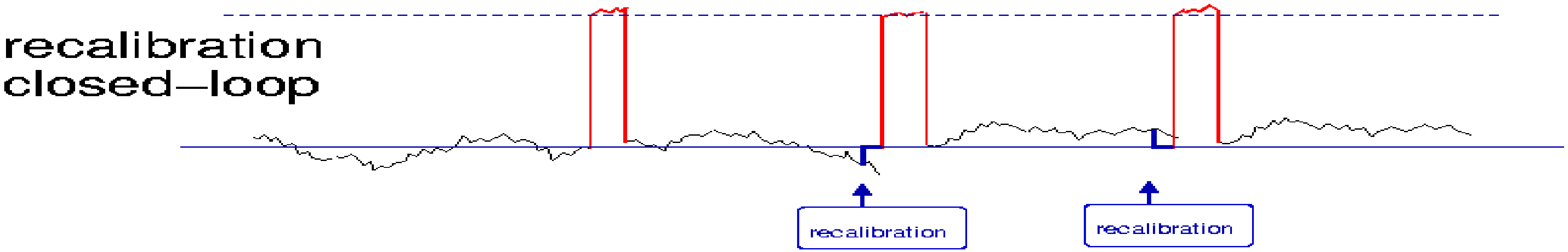
$$i\Phi(t) = \frac{1}{2}\ln\left[1 + i\frac{\sigma_X^2[1-\pi(t)]t}{\Omega}\right] + \frac{1}{2}\ln\left[1 + i\frac{\sigma_X^2\pi(t)\,t}{3\,\Omega}\right] \qquad \pi(t) = \frac{1}{2\,\sigma^2}\int_0^\infty \frac{d\omega}{\pi} S(\omega)\,(1-\cos\omega t)$$

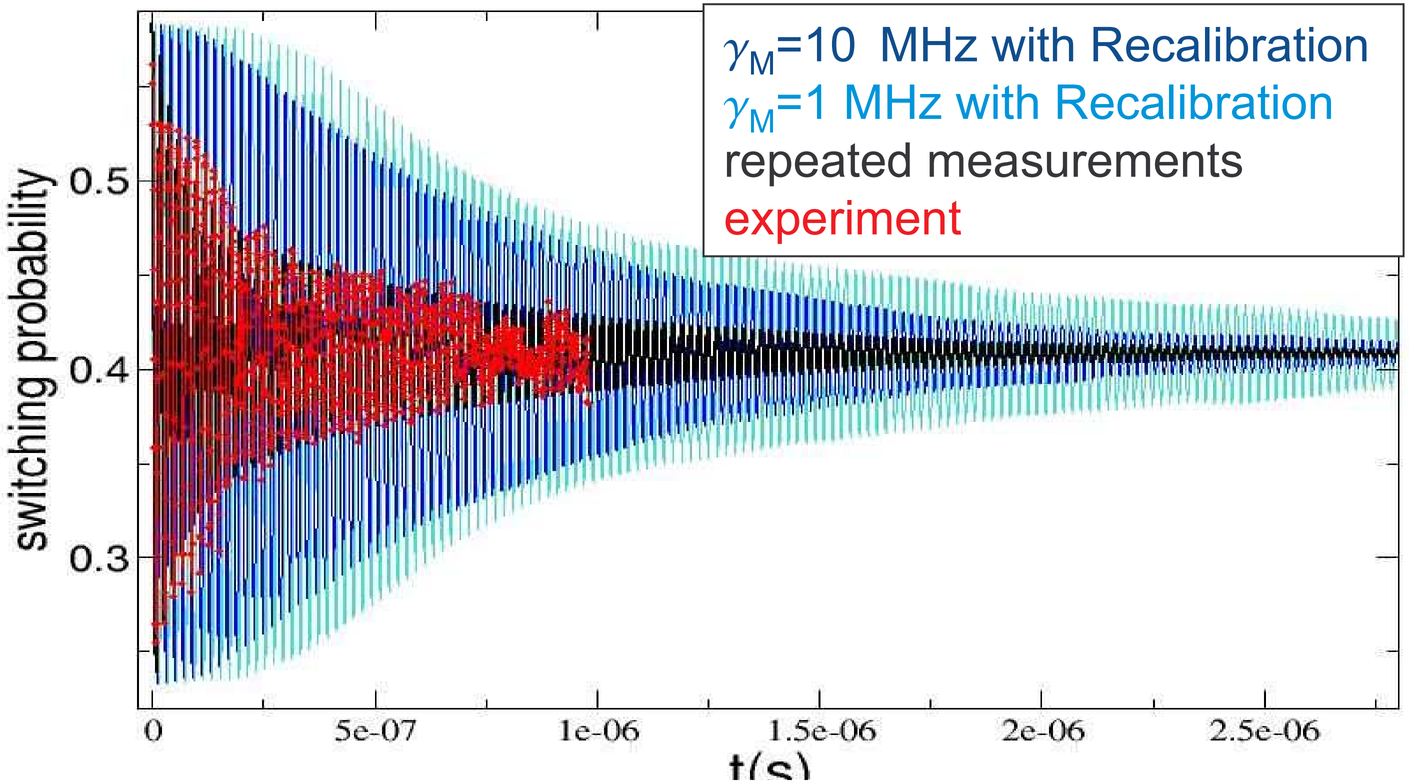
First correction: adiabatic noise during evolution with Feedback and Echo

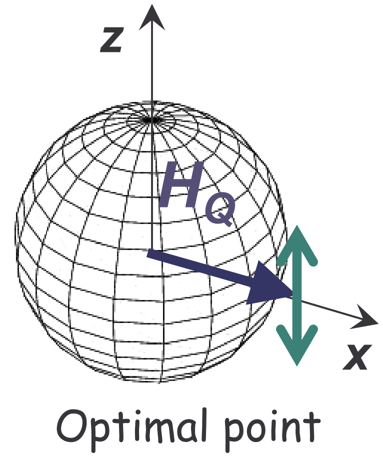
$$\Gamma_F(t) \approx -\frac{1}{4}\ln\left[1 + \left(\frac{4s^2\sigma^2 t}{3\Omega}\pi_2(t)\right)^2\right] \qquad \Gamma_1(t) \approx -\frac{1}{4}\ln\left[1 + \left(\frac{2s^2\sigma^2 t}{2\Omega}\pi_2(t)\right)^2\right]$$

$$\pi_2(t) = \pi(t)\,[1-\pi(t)]$$

Feedback control and nongaussian effects



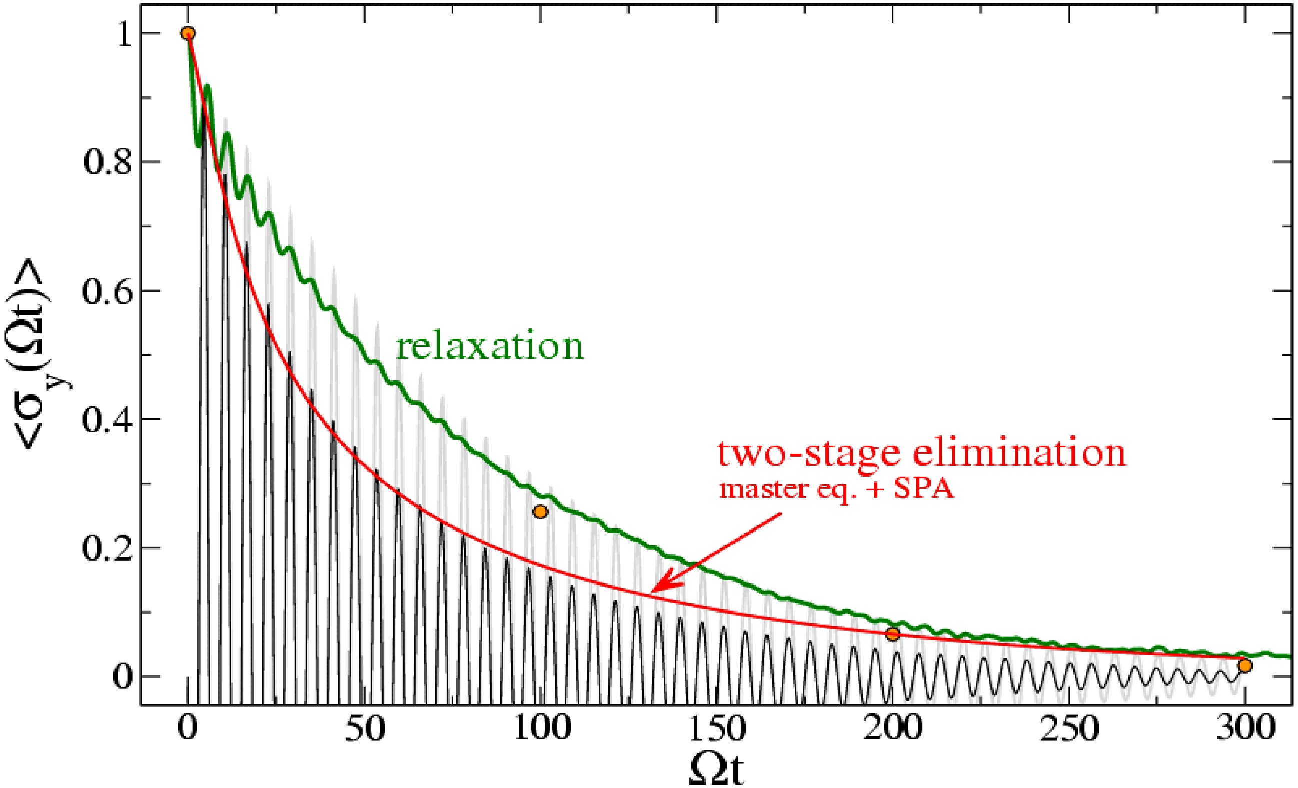




Slower decay, starts as $\approx \exp(-\Gamma^3 t^3)$ for discrete environments

Protocol more sensitive to details of the environment

1/f adiabatic + 1/f quantum



Environment of slow + fast impurities, $\gamma_M = 10\,\Delta$

No separation of time scales

$$T_1 \approx T_2$$

Theory: two-stage elimination

$$\hat{X} \rightarrow X + \hat{X} \qquad \rho^Q(t) = \int dX\, p(X)\, \rho^Q_f(t|X)$$

$$\langle \sigma_y(t) \rangle = \langle \sigma_y(0) \rangle\, \mathrm{e}^{-t/(2T_1)} \left(1 + [i\Delta + T_1^{-1}]\, \frac{\sigma_X^2 t}{\Delta^2} \right)^{-1/2}$$

Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

Applications to more complex systems

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Quantum bits coupled to resonators

- Selection rules between entangled doublet $\longrightarrow$ adiabatic approximation is exact
- System tunable to a working point where fluctuations are

$$\Omega(X) = \Omega_0 + \alpha X^2 + \beta X^4$$

extra protection against fluctuations due to interaction.

G. Falci, et al., Proc. MS+S2004.

Driven multistate systems

- Adiabatic fluctuations may induce level crossings
- Limitations in the implementation of quantum optics protocols as STIRAP

Acknowledgements: IST-SQUBIT2, EuroSQUIP,