Persistent currents in two dimension: New regimes induced by the interplay between electronic correlations and disorder

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Outline:

- Overview of the strongly correlated 2D electron-gas problem;
- Introduction of the numerical model: few interacting particles on a lattice
- Persistent current maps with disorder

References:

Z.Á. Németh and J.-L. Pichard, Eur. Phys. J. B **45**, 111 (2005)

J.-L. Pichard and Z.Á. Németh, J. Phys. IV France **131**, 155 (2005)

Quantum solid state physics

- Fermi: weakly-interacting quantum particles
- Wigner: strongly interacting particles

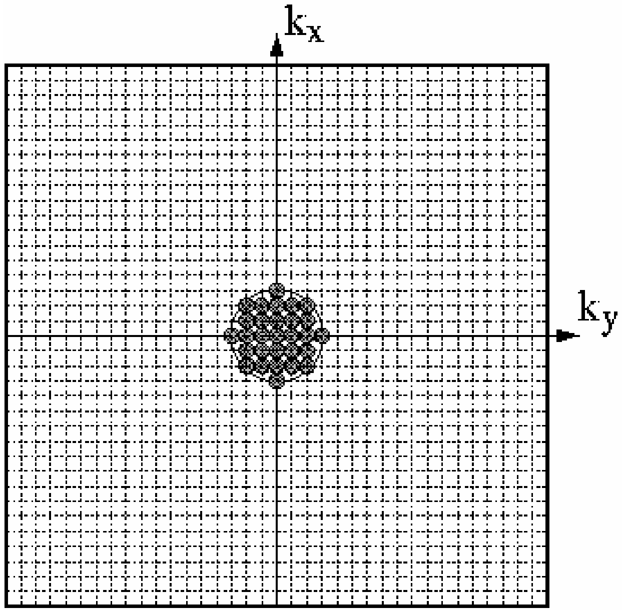
TWO LENGTH SCALES:

- average interparticle spacing a
- Bohr-radius a_B

Dimensionless scaling parameter: $r_s = a/a_B$

Weak interaction limit

- Electrons localized in *k*-space (Fermi liquid behavior).



high density limit, $r_s \to 0$

kinetic energy

$$E_0^{Fermi} = \frac{a_1}{r_s^2} + \frac{a_2}{r_s} + O(\ln r_s)\ [\times NRy]$$

exchange energy

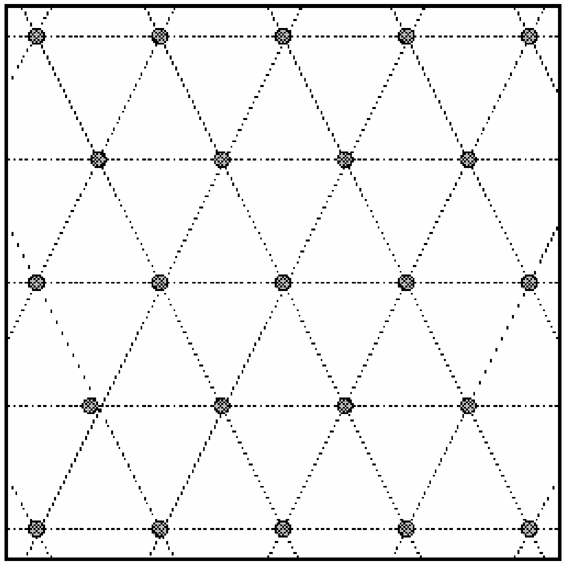
$a_1 = 2.0$

$a_2 = -1.6972$

M. Gell-Mann and K. A. Brueckner
Phys. Rev. **106**, 364 (1957)

Strong interaction limit

- Electrons localized in real space (Wigner crystal)



low density limit, $r_s \to \infty$

classical elctrostatic energy

$$E_0^{Wigner} = \frac{c_1}{r_s} + \frac{c_{3/2}}{r_s^{3/2}} + O\left(r_s^{-2}\right) \left[\times\ NRy\ \right]$$

quantum zero-point motion

$c_1 = -2.21$
$c_{3/2} = 1.63$

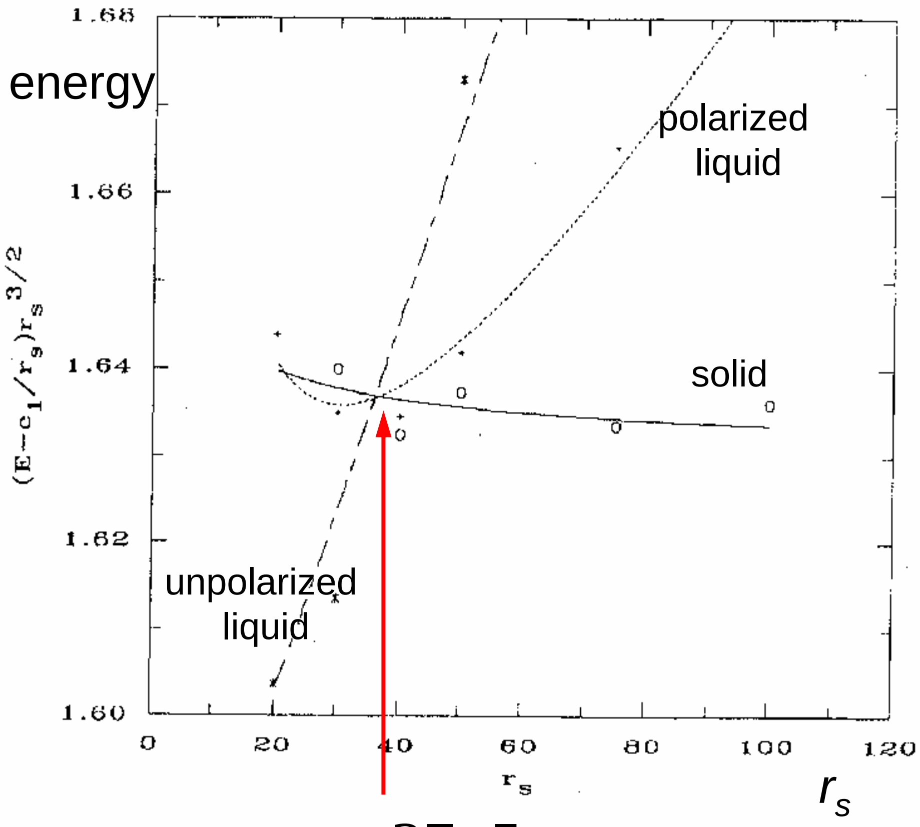
W. J. Carr Jr., *Phys. Rev.* **122**, 1437 (1961)

Quantum Monte-Carlo

- Fixed node Monte-Carlo method:

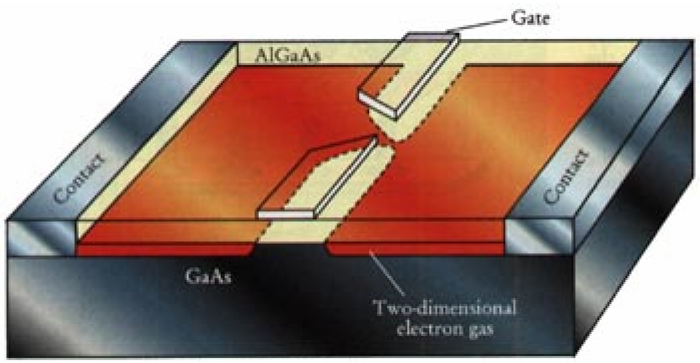
B. Tanatar and D. M. Ceperley
PRB **39**, 5005 (1989)

GS energy



$r_s = 37 \pm 5$

Semiconductor heterostructures

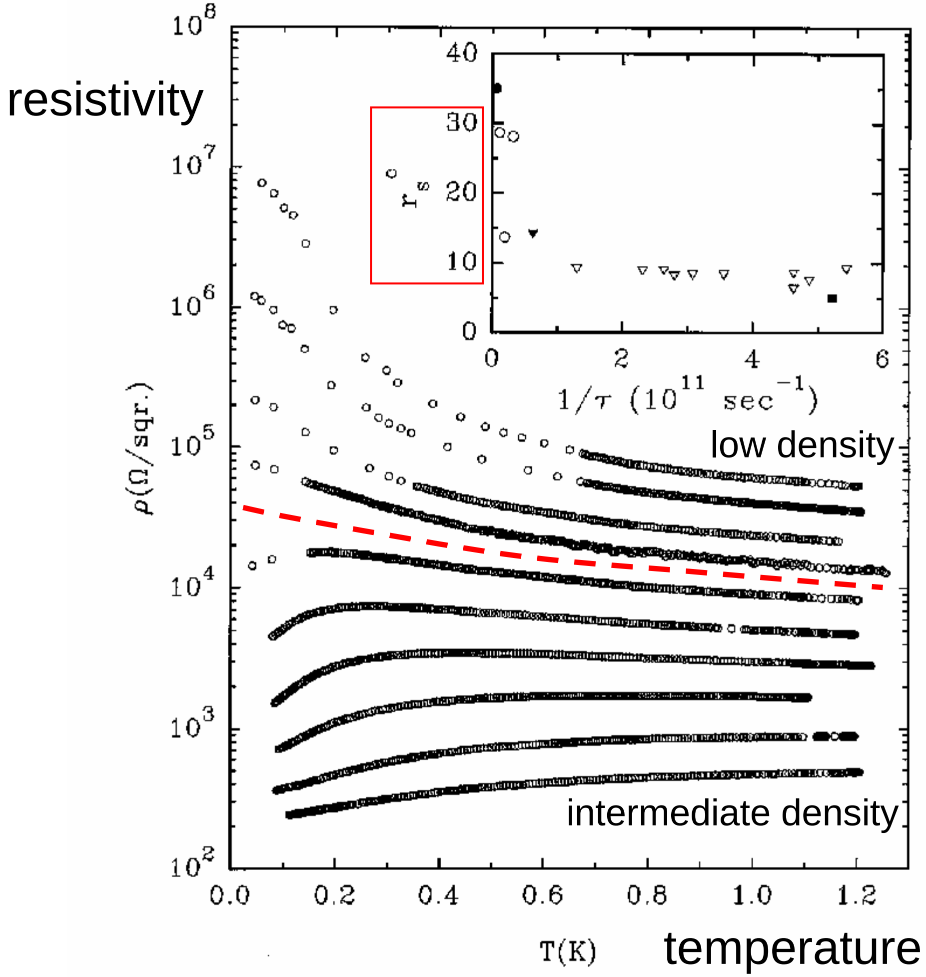


Example:
quantum point-contact

Since the '70s it is possible to fabricate 2D electron gas in semiconductor devices.

Electron density and r_s are varied through voltage gates.

Unexpected metallic behavior in 2D



In ultra-clean heterostructures:

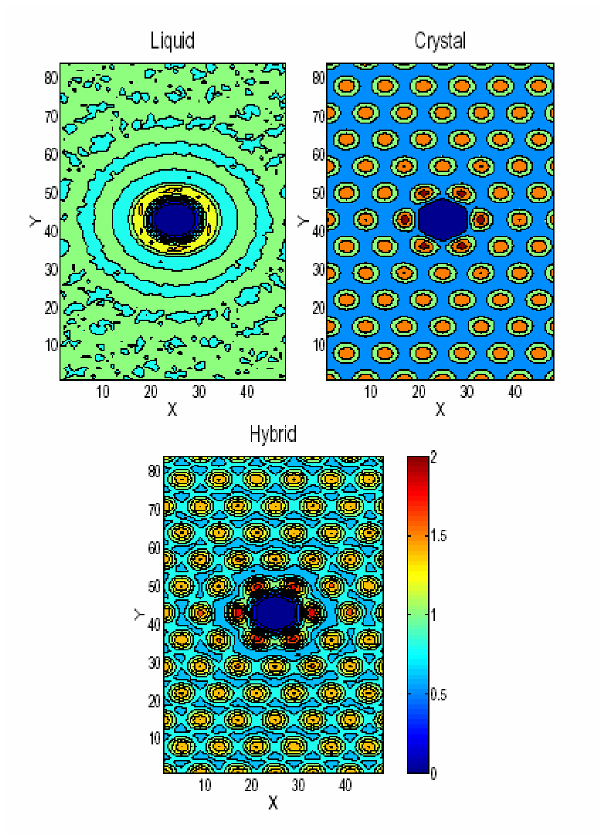
r_s can reach ≈ 40

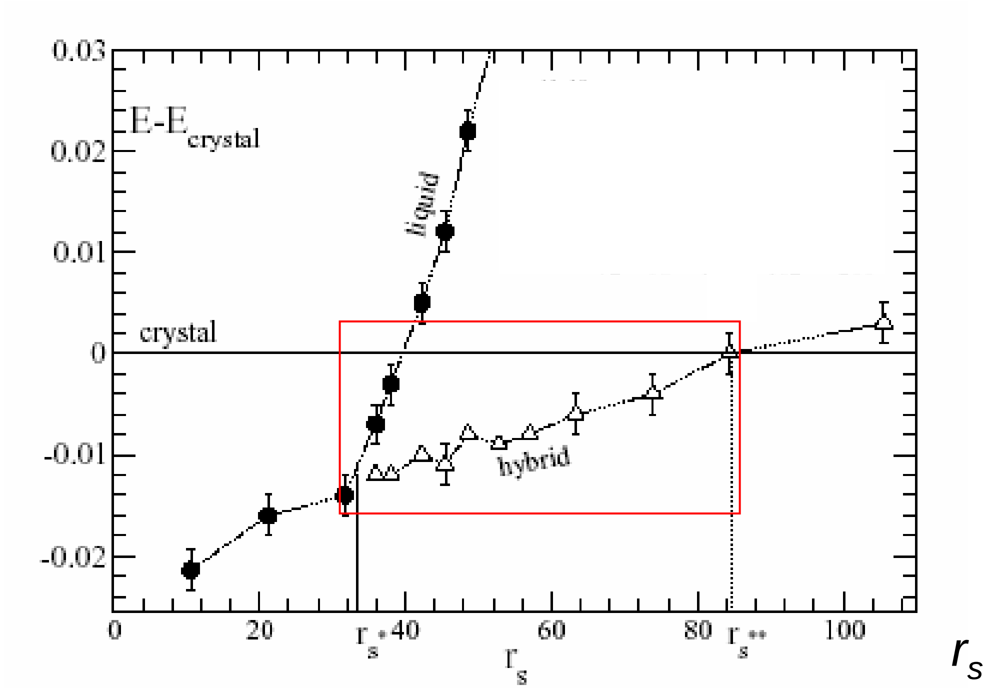
Observed metallic behavior at intermediate r_s

- S.V. Kravchenko *et al.*, *PRB* **50**, 8039 (1994)
- J. Yoon *et al.*, *PRL* **82**, 1744 (1999)

Hybrid phase in QMC

Density-density correlation function





Hybrid phase: nodal structure of

Slater determinants in the crystal potential:

mixed liquid-solid behavior

H. Falakshahi, X. Waintal: Phys. Rev. Lett. **94**, 046801 (2004)

Theory for intermediate r_s

Still mainly speculations...

- *Andreev-Lifshitz* « supersolid » state (relation with *He*-physics)

- Inhomogeneous phases, stripes and bubbles (*B. Spivak*)

Relation with the physics of high-T_c cuprates and with Hubbard model (high lattice filling, contact interaction).

Lattice model

N spinless fermions on ***L*** $\times$ ***L*** square lattice with periodic boundary conditions (lattice spacing ***s***):

$$\mathrm{H} = t \cdot \left(4N - \sum_{<i,j>} \hat{c}_i^{+} \hat{c}_j \right) + \frac{U}{2} \cdot \sum_{i \neq j} \frac{\hat{n}_i \hat{n}_j}{d_{ij}} + W \sum_j \varepsilon_j \hat{n}_j$$

discrete Laplacian

disorder

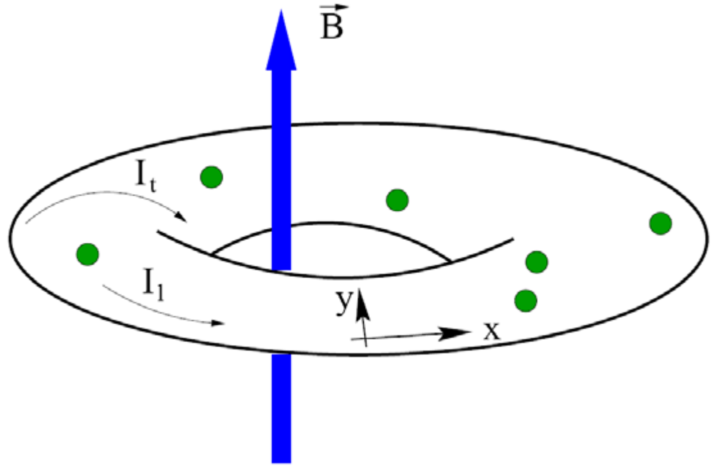
THE r_s AND r_l PARAMETERS:

$$r_l = \frac{UL}{t} \qquad r_s = \frac{a}{a_B} \rightarrow \frac{r_l}{2\sqrt{\pi N}}$$

the ***t*** and ***U*** parameters of this Hamiltonian:

$$U = \frac{e^2}{s} \qquad t = \frac{\hbar^2}{2ms^2}$$

Persistent current



Longitudinal and transverse currents

•local: $$j_j^{long}(\Phi) = 2\,\mathrm{Im}\left\langle \Psi_0 \middle| c_j^{+} c_{j+(1,0)} e^{2\pi i \Phi / L} \middle| \Psi_0 \right\rangle$$

•total: $$I_{long} = \frac{1}{L} \sum_j j_j^{long} \sim \Delta E_0(\Phi) = E_0(\Phi = 0) - E_0(\Phi)$$

Transverse current is analogous.

Persistent currents with disorder

- Effects of an infinitesimal disorder: new lattice perturbative regime

  – Ballistic motion
  – Coulomb Guided Stripes
  – Localization if the Wigner crystal

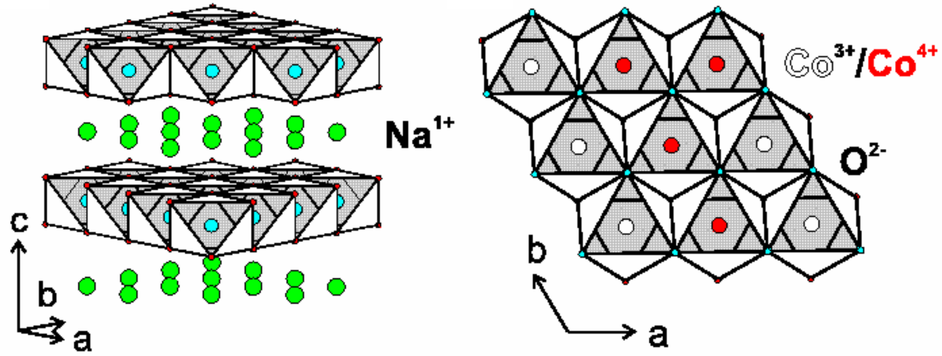
Strong interaction, lattice regimes

Can be relevant in real materials

e.g. Cobalt-oxides

(Na_xCoO_2)

- effective mass: $m^*/m = 175$
- relative dielectric constant: $\varepsilon_r = 20$
- lattice spacing s = 2.85 Å
- carrier density depends on Na^+ concentration



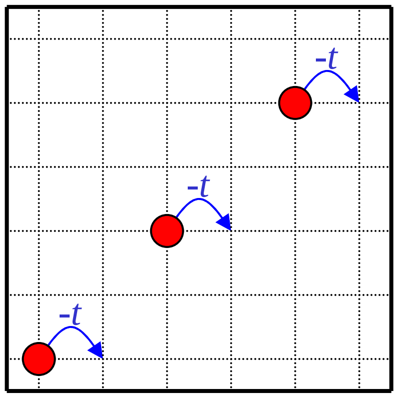
Lemmens *et al.*, cond-mat/0309186

Lattice perturbation theory when $t \to 0$

Example: the persistent current

perturbation

$$H = t \cdot \left(4N - \sum_{<i,j>} c_i^{+} c_j \right) + \frac{U}{2} \cdot \sum_{i \neq j} \frac{n_i n_j}{d_{ij}}$$



rigid hopping

$$E(\Phi = 0) = -2t_{eff} \left(\cos K_x + \cos K_y \right)$$

$$E\left(\Phi = \Phi_0 / 2 \right) = -2t_{eff} \left(\cos\left(K_x + \frac{N\pi}{L} \right) + \cos K_y \right)$$

$$I_{lattice} \sim \left| E(\Phi = 0) - E\left(\Phi = \Phi_0 / 2 \right) \right| \approx t_{eff} \frac{9\pi^2}{L^2}$$

where *teff* describes the rigid hopping of the three particle « molecule »

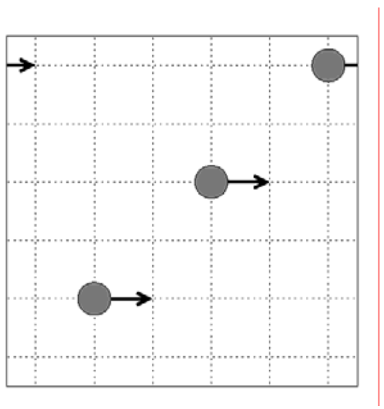
$$t_{eff} = \frac{1296}{49} \frac{t^3 L^6}{U^2 \pi^2}$$

Ballsitic motion (BWM)

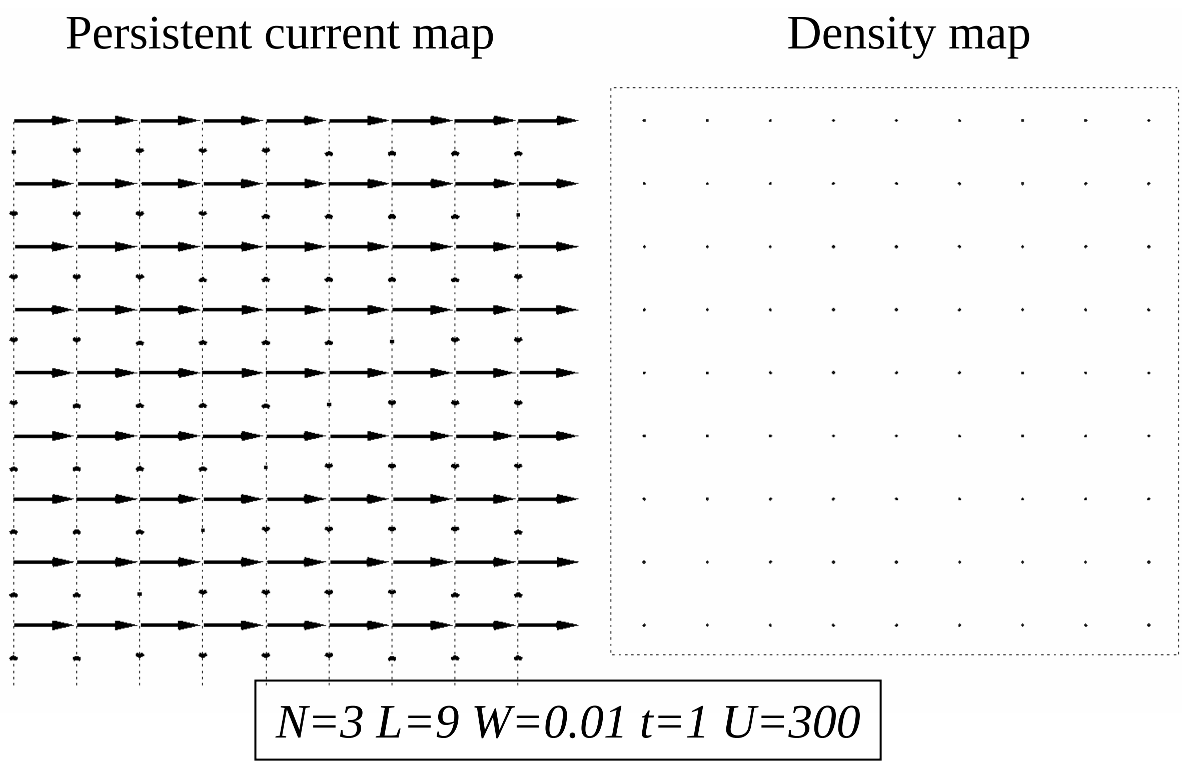
Effective Hamiltonian:

$$\mathrm{H}_{eff} = 4Nt - E_{Coul} - t_{eff} \sum_{\langle j,j' \rangle} C_j^+ C_{j'}$$

$$t_{eff} \sim \frac{t^N}{U^{N-1}} L^{3N-3}$$



$C_j^+ C_{j'}$



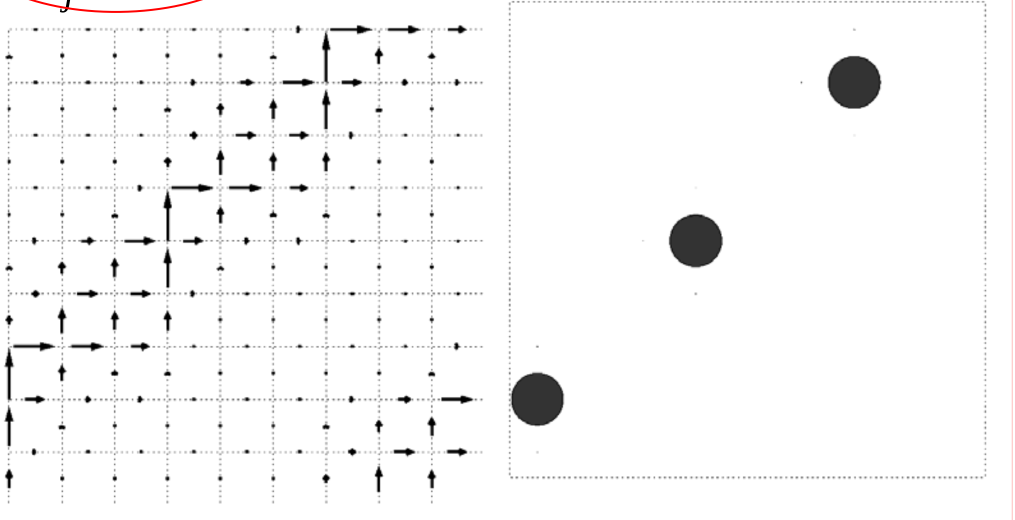
Coulomb Guided Stripes (CGS)

Disorder correction in the effective Hamiltonian:

$$\mathrm{H}_{eff} = 4Nt - E_{Coul} - t_{eff} \sum_{\langle j,j'\rangle} C_j^{+} C_{j'} + W \sum_j \varepsilon'_j \hat{N}_j$$

$$I_{long} \sim \Delta E_0(\Phi) \sim \frac{t_{eff}^{\;2L/3} \cos(2\pi\Phi)}{W^{2L/3-1}}$$

$$t_{eff} \sim \frac{t^N}{U^{N-1}} L^{3N-3}$$



current of a collective motion

N=3 L=9 W=1 t=1 U=1000

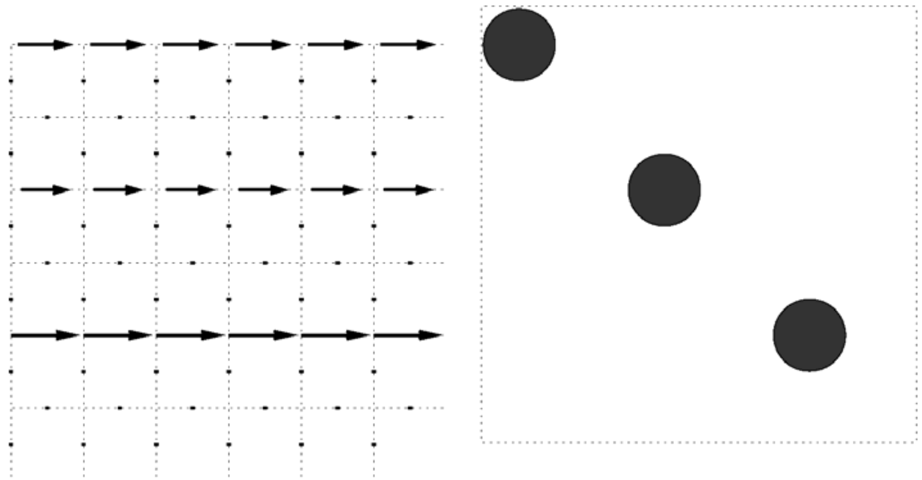
transverse current: $I_{trans} = I_{long}$

Localized Wigner Molceule (LWM)

Standard perturbation therory:

$$I_{long} \sim \Delta E_0(\Phi) \sim \frac{t^L L^{3L-3}\cos(2\pi\Phi)}{U^{L-1}}$$

current of independent particles

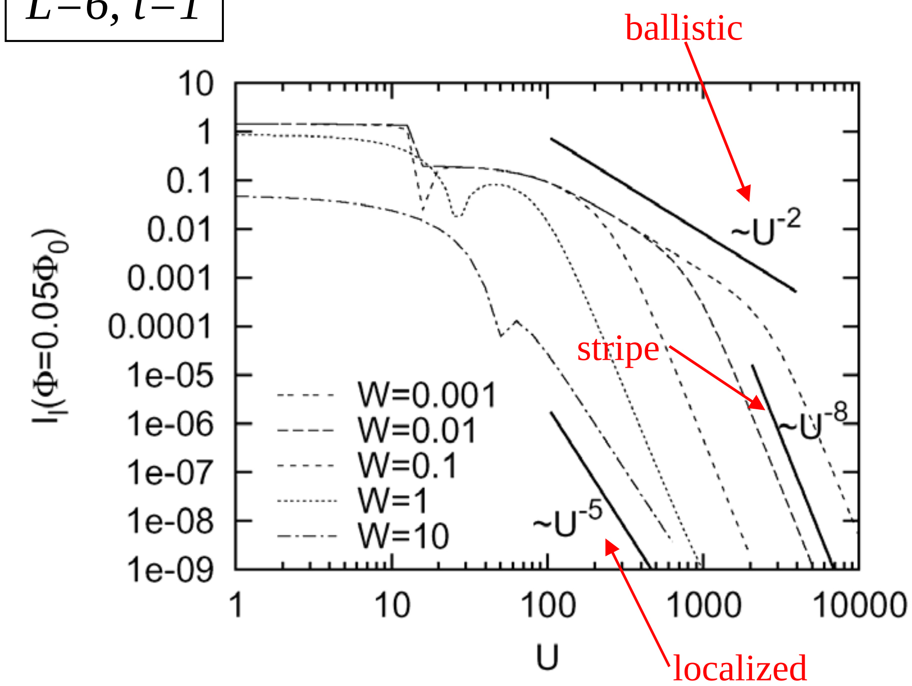


N=3 L=6 W=20 t=1 U=1000

transverse current: $I_{trans} = 0$

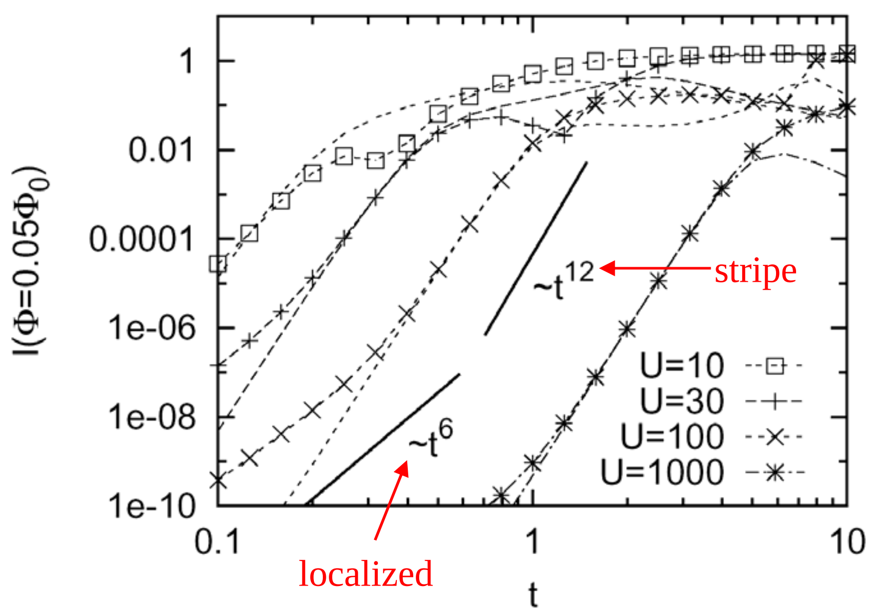
Numerical check of the different regimes

L=6, t=1



longitudinal

L=6, W=1



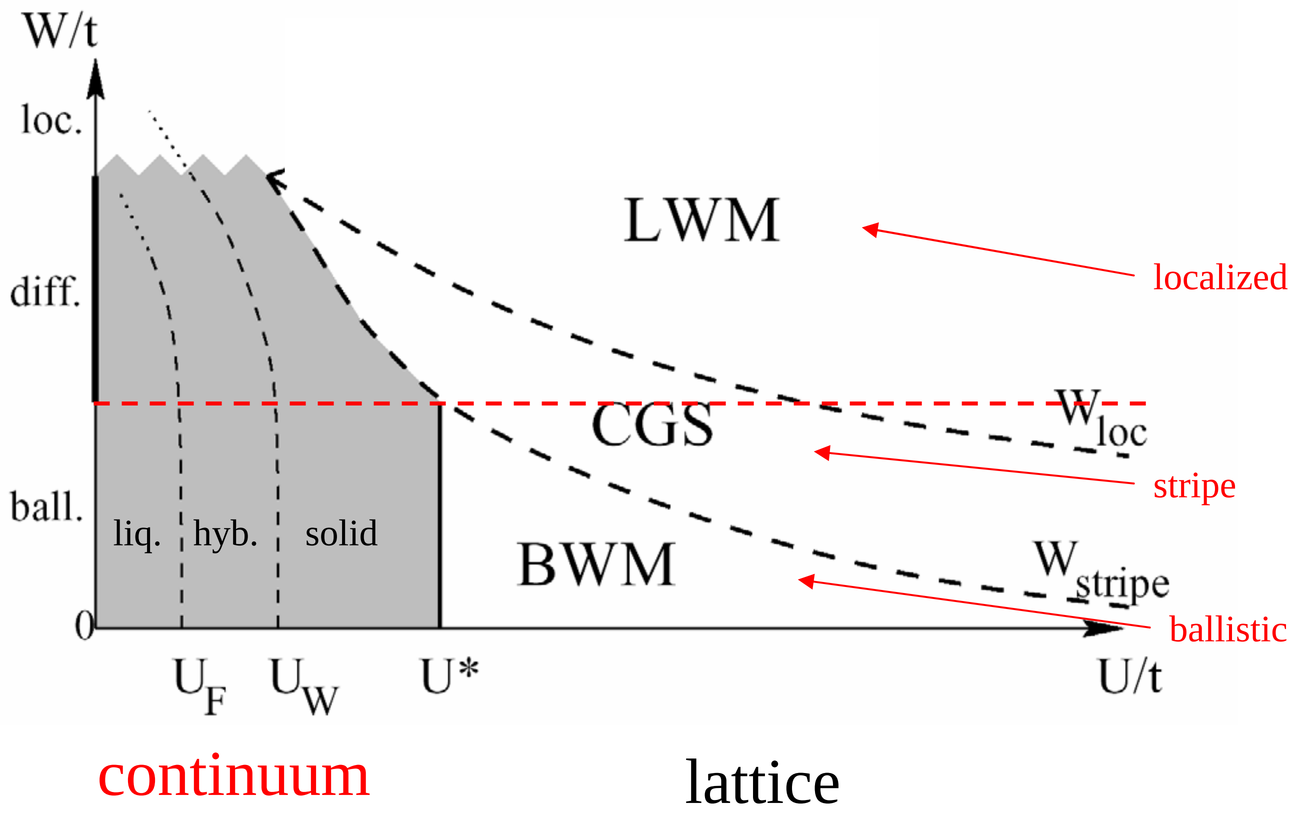
longitudinal & transverse

Phase diagram for weak disorder

Critical lines:

$$\frac{W_{stripe}}{t} \propto \frac{t^2}{U^2} L^6$$

$$\frac{W_{loc}}{t} \propto \frac{t^{1/2}}{U^{1/2}} L^{3/2}$$



In case of long-range interaction,
lattice models without disorder
exhibit a latice-continuum transition.

Continuum perturbation theory when $r_s \to \infty$

Zero point motion in the vibrating mode of the molecule

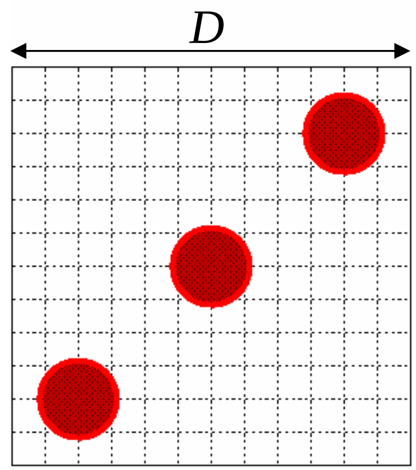
2nd order expansion around equilibrium

$$H \approx -\frac{\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2+\nabla_3^2\right)+E_{el}+\vec{\mathbf{X}}\hat{\mathbf{M}}\vec{\mathbf{X}} \Rightarrow$$

$$H \approx -\frac{\hbar^2}{2m}\sum_{\alpha=1}^{6}\frac{\partial^2}{\partial\chi_\alpha^2}+E_{el}+10B\left(\chi_3^2+\chi_4^2\right)+4B\left(\chi_5^2+\chi_6^2\right)$$

Longitudinal modes — Transverse modes

where $B=\frac{\sqrt{6}}{24}\frac{e^2\pi}{D^3}$



$$E_{\mathbf{K}=0}=E_{el}+\hbar\left(\omega_L+\omega_T\right)$$

$$\omega_L=\sqrt{\frac{20B}{m}} \qquad \omega_T=\sqrt{\frac{8B}{m}}$$

Limit for the zero point motion

electrostatic energy

$$F_N(L,U,t) = \frac{E_0(L,U,t) - E_0(L,U,t=0)}{E_0(L,U=0,t)}$$

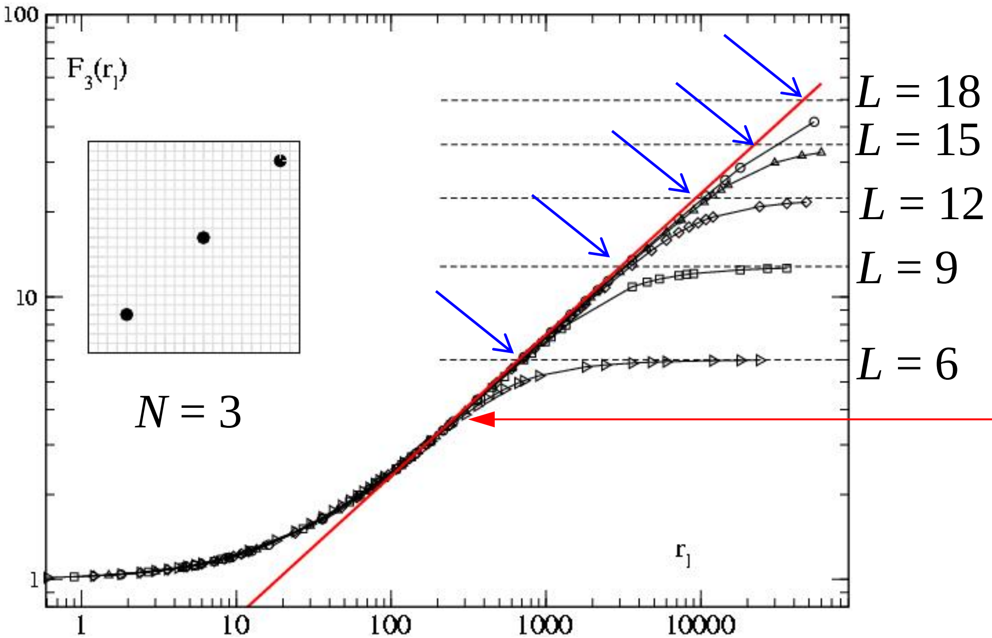
E_0: ground state energy

F_N: scaling function

Lattice behavior:

$$E_0 - E_{el} = 4Nt$$

Example:
Three spinless fermions on $L \times L$ lattice



$r_l = UL/t$

Harmonic vibration of the solid in the continuum:

$$F_3 = 0.2327\sqrt{r_l}$$

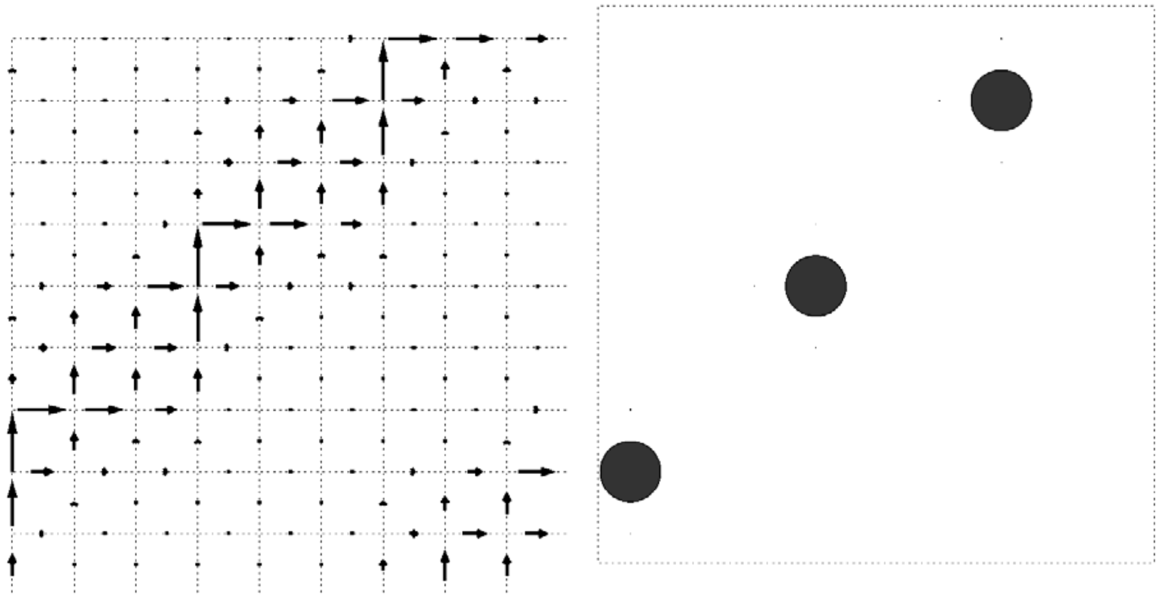
Persistent currents with disorder

Do we see similar thing with disorder?

- Effect of an intermediate disorder in the continuum limit:
  - Coulomb guided stripes
  - Level crossing and supersolid behavior

Coulomb guided stripe on a lattice

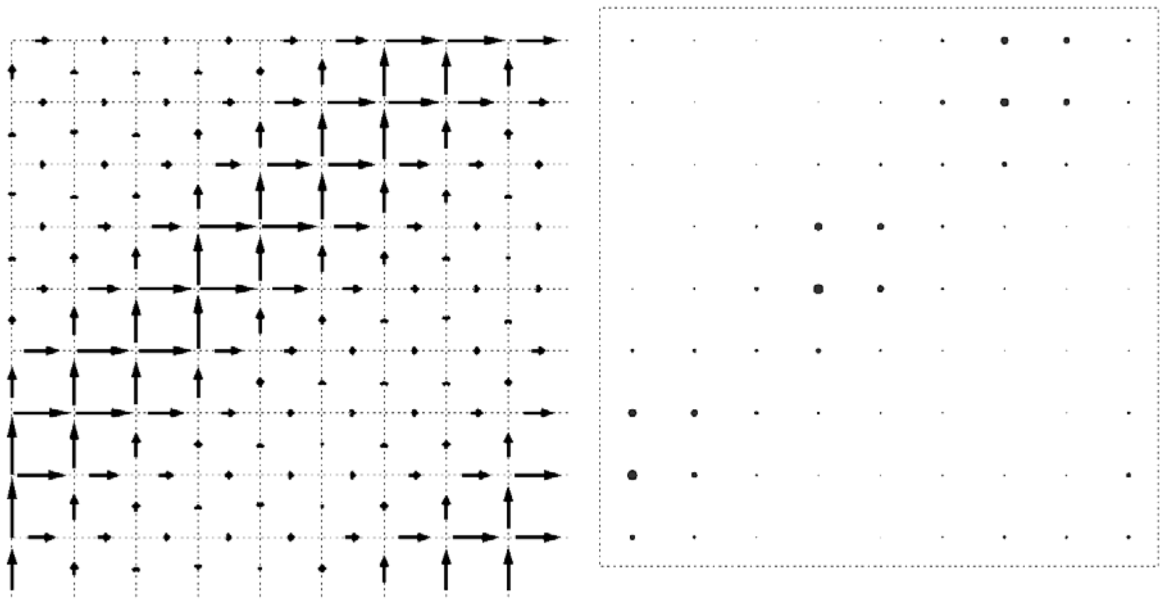
N=3 L=9 W=1 t=1 U=1000



Parallel density and current

Continuum version of the Coulomb guided stripe

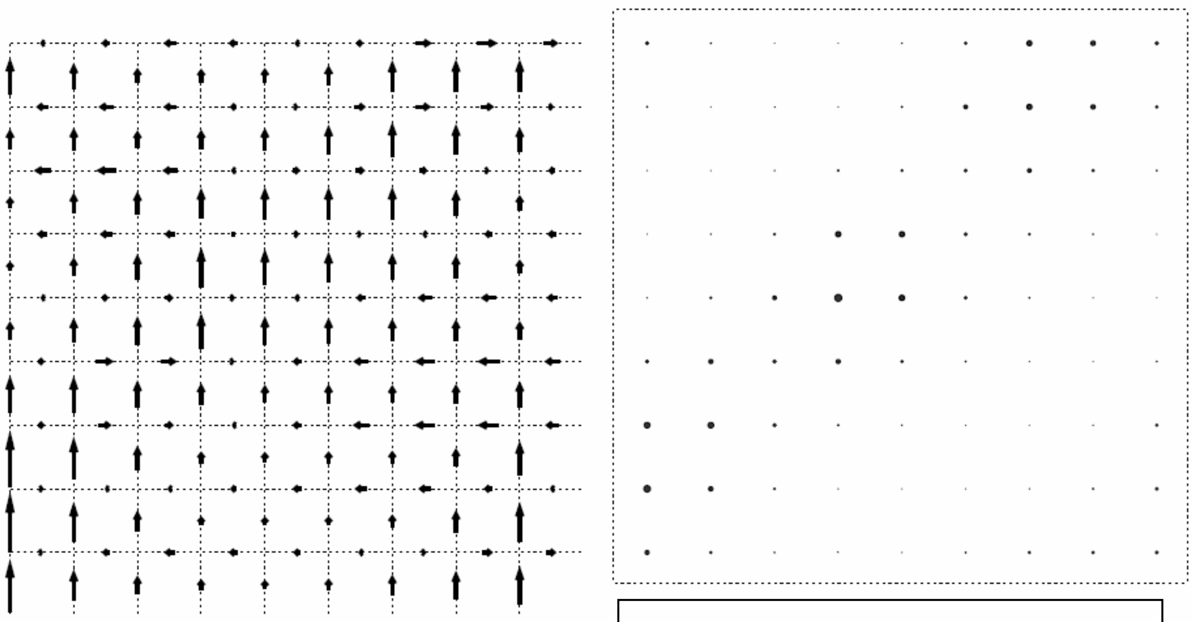
N=3 L=9 W=1 t=1 U=50



Crossover regime

Disconnected current

and density



N=3 L=9 W=1 t=1 U=15

N=3 L=9 W=1 t=1 U=7

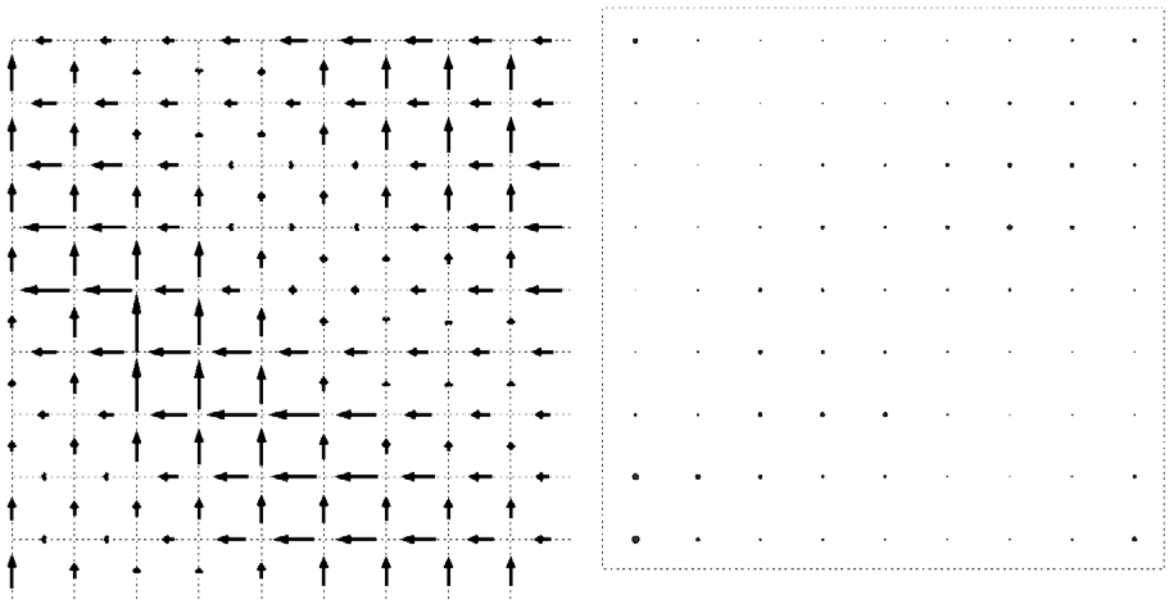
Legett’s rule:

1D motion diamagnetic

means even number of

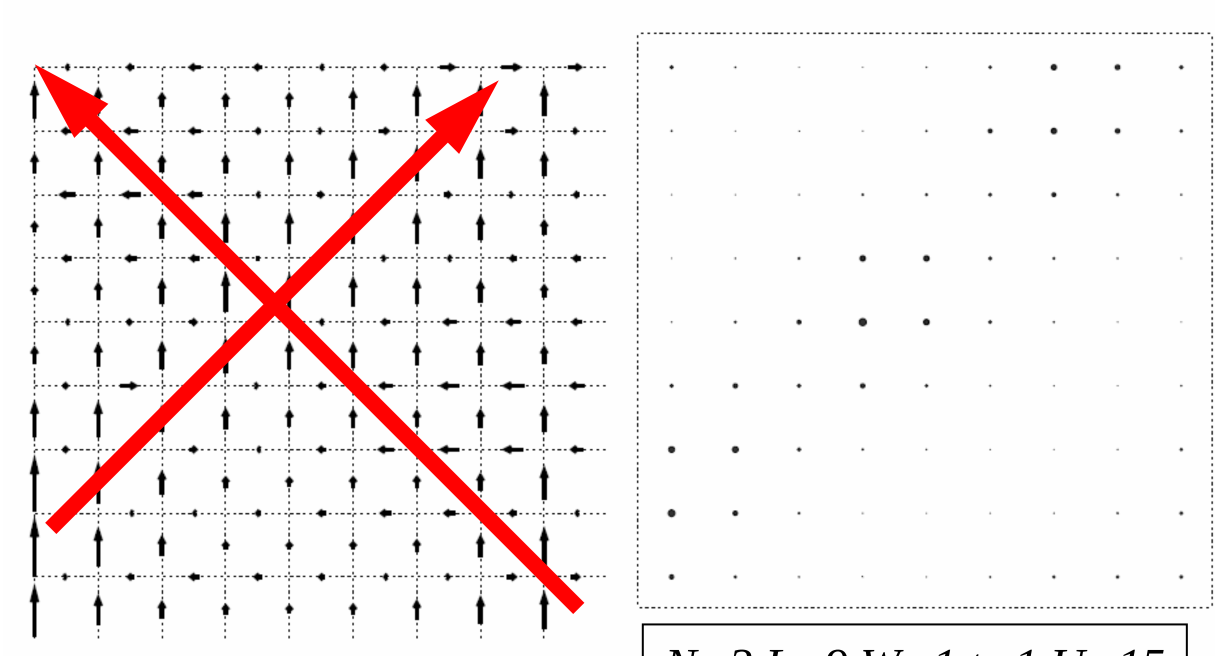
particles

Sign of supersolid



Crossover regime

Disconnected current and density



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Legett’s rule:

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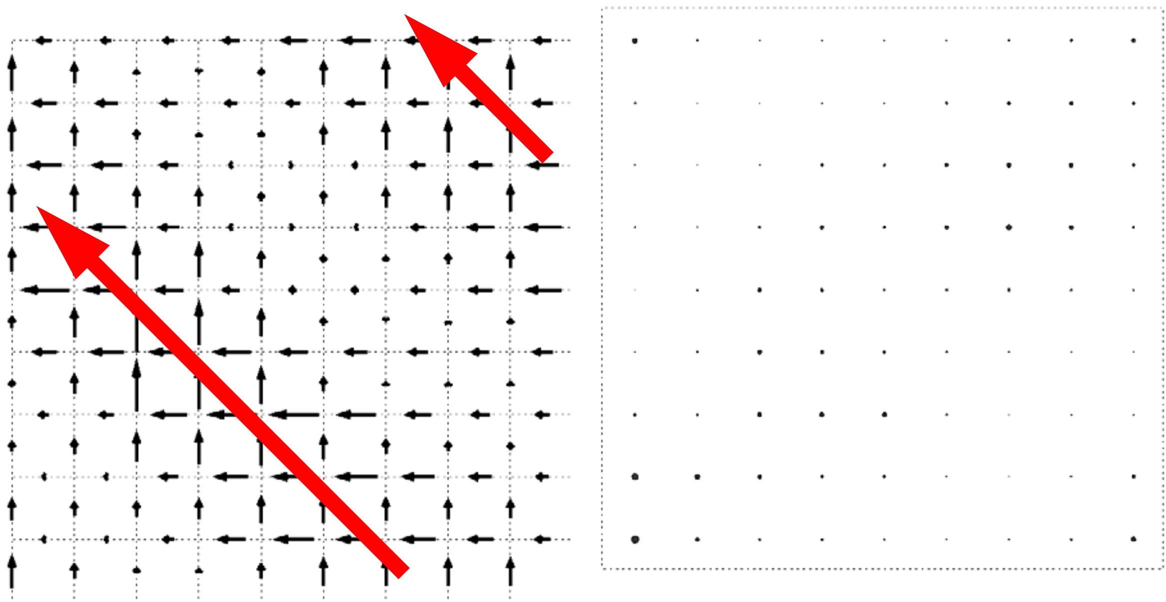
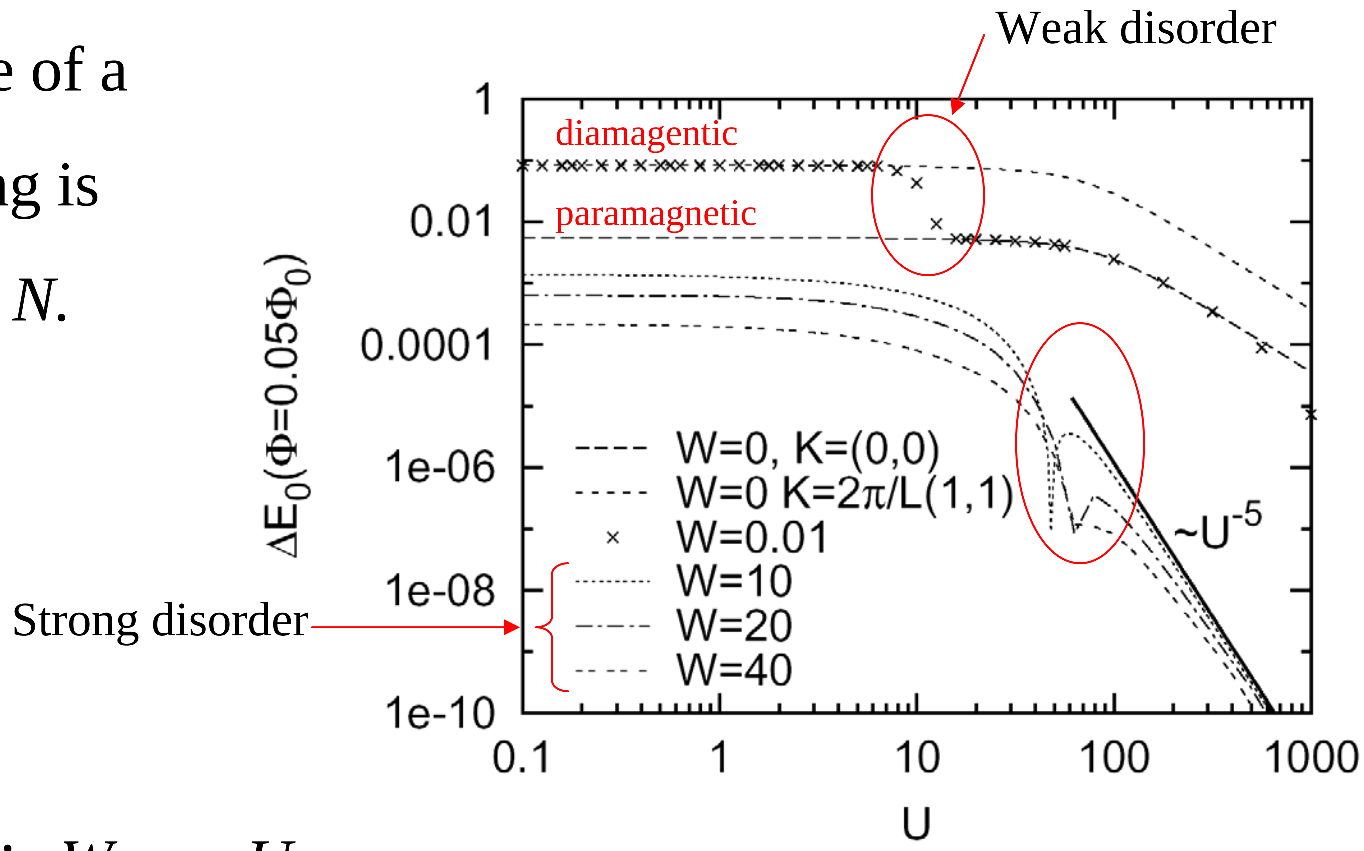


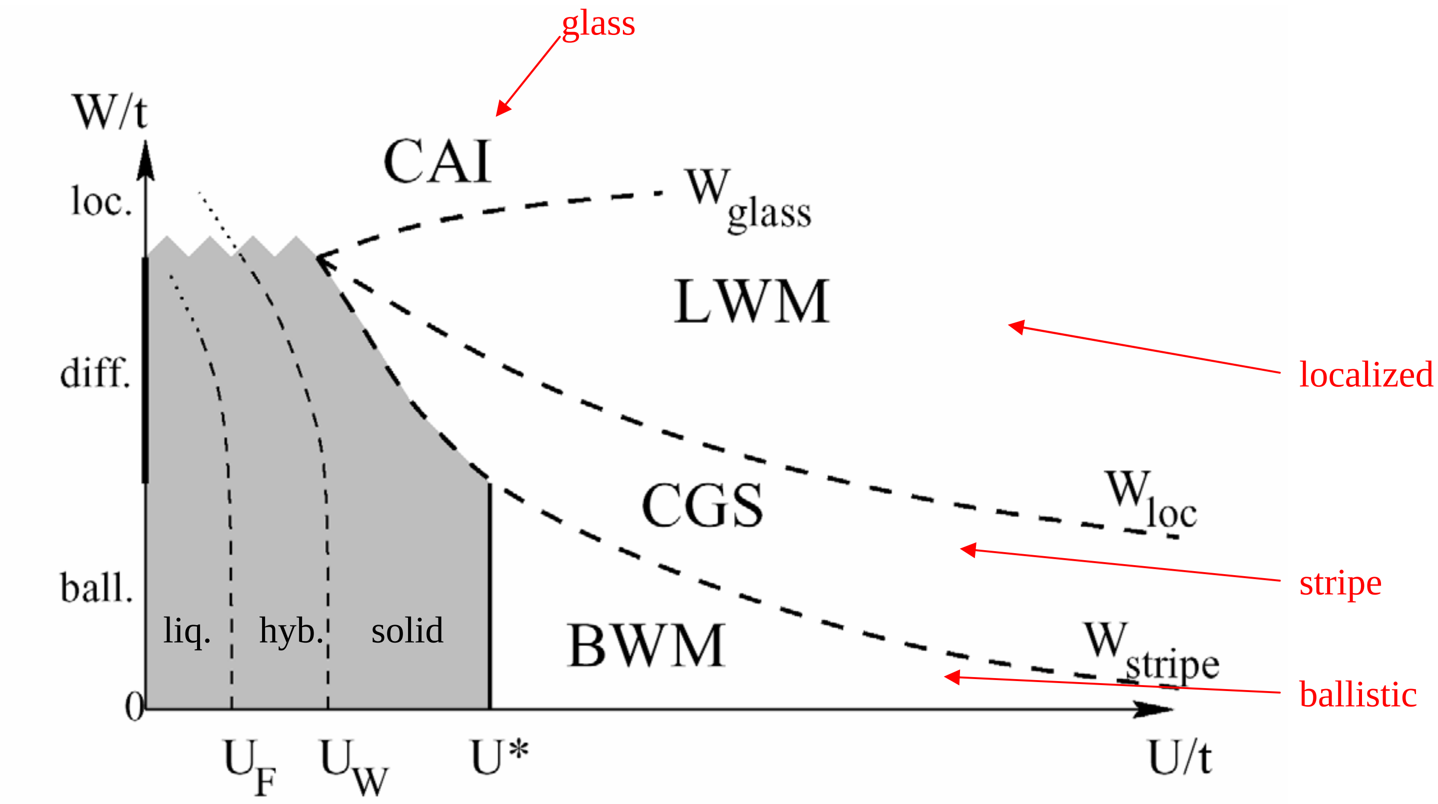
Level crossing & strong disorder

The presence of a level crossing is specific to *N*.



Crossover: $W_{glass} \sim U$

Phase diagram (*N*=3)



continuum

lattice

Conclusion

- In the presence of a weak disorder, we have identified for large U/t three lattice regimes, characterized by different power-law decays as a function of U, t and W, L, N.
- The physics of the continuum is also affected by disorder (signatures of the supersolid).