Testing Theories:

Finding Functional Fixed Points for Pinned Manifolds

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Support from NSF, ANR

MPI-PKS Workshop "Dynamics and Relaxation in Complex Quantum and Classical Systems and Nanostructures"

3 August, 2006

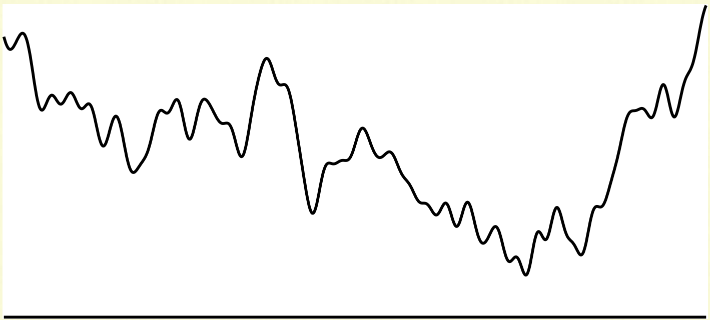
Organization

- Reverse historical approach.
- “Experimental” talk.
- See cond-mat/0606160.
- [Reminded of ancient Greek theater festivals.]

This is a glass talk, so we need this diagram

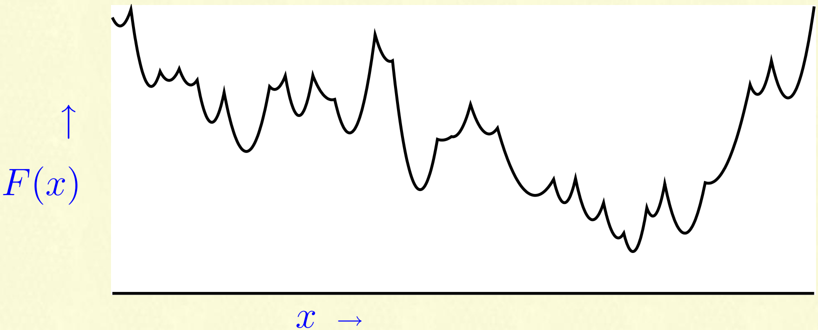
$\uparrow$

$F(\vec{x})$



$\vec{x} \;\rightarrow$

However, we will mostly see this



How Computer Scientists Taught Physicists to Be Lazy

Physicists want: low E, long t, large λ behavior of complex, heterogeneous systems, e.g., random magnets, superconductors with dirt.

- The ground state (or even partition fn. Z) can often be computed *very quickly, even when the physical system has **many** local minima and **extremely** slow dynamics.*
- This speed can be exploited in models with quenched disorder
  - to *precisely* study phase transitions
  - to study the effects of perturbations
  - to answer qualitative questions (e.g., # of states)
- Warning: some reasonable physical systems have no known fast algorithms for all cases. These correspond to NP-hard problems.

To study materials, learn computer science

Rather informally:

- A **decision problem** is one for which one replies yes/no for a given input.
- The set **P** consists of decision problems that can be solved in time bounded by a polynomial N^k in the problem length N. “Tractable”.
- The set **NP** (“nondeterministic polynomial”) consists of decision problems for which “yes” answers can be *verified* in time polynomial in N.

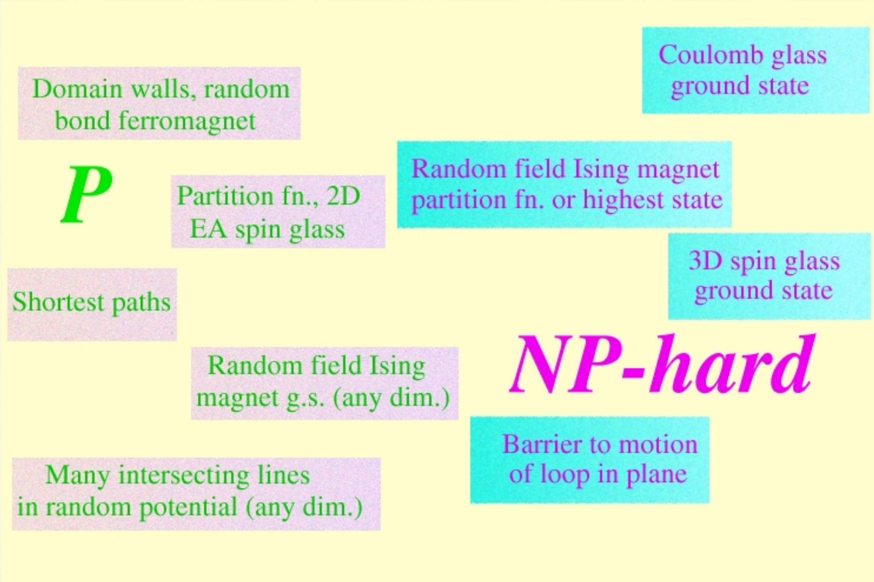
P and NP

Example decision problem instance:

> Can you find a train itinerary from Trieste to Dresden that takes less than 15 hours?
> [Shortest path problem is in P.]

$P \subset NP$, but we don't know if $P = NP$.

Which problems are tractable?



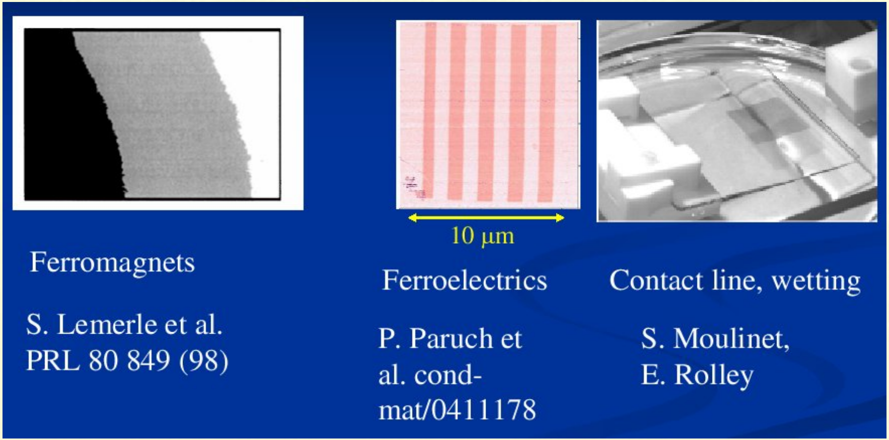
How accurate for P?

AS EXACT AS YOUR INPUT:

* The algorithms expand the configuration space.
* The "rough landscape" is smooth and downhill* in this space.
* At the "bottom", translate back to a physical solution,
. . . which is *guaranteed to be the **exact** g.s.*

- The combinatorial math and particular rep'ns are often unfamiliar to physicists.
- But we are used to imaginary time for QM, e.g.

The Cover to the Program



[Collection “courtesy” of T. Giamarchi]

Inspiration

- Statics of surfaces pinned by disorder
  - Domain walls in random magnets, contact lines on a rough surface, vortex lines in superconductor, electron world lines in a space AND time dependent potential, periodic scalar fields, e.g., vortex-free superconductors.
- "Simplest" finite-d glassy phases (?)
  - Elastic, no plastic rearrangements.
  - At low T, disorder is irrelevant . . .
    - Theme of dramatic tension: elasticity v. disorder
- Characterize by roughness, $w \sim L^{\zeta}$, energy fluctuations $\sim L^{\theta}$. Statics are preliminary to
  - barriers to equilibration
  - dynamics (creep or sliding) in disordered background.

Plot Summary

The effective long wavelength pinning potential for $d < 4$ interface is **universal** (depends on symmetries of pinning potential).

$\Rightarrow$Find fixed points for force-force correlation functions $\Delta(u)$.
$\Rightarrow$Quantitatively confirm shape of $\Delta(u)$.

- First evidence for cusp at zero u (20 yrs)
- "Chaos" (sensitivity to disorder)
- Universal amplitudes.

Production Crew

P. Le Doussal, K. Wiese, AAM, and 100 1GHz processors.
$\Rightarrow$C++ code to find **exact ground state** for discrete interfaces $u(x)$ in dimensions $d = 1, 2, 3, 4, \ldots$ with

- User-defined lattices.
- Choice of disorder correlations, corresponding to
  - Random field (RB): $\langle [U(u', x') - U(u, x)]^2 \rangle = |u - u'| \delta(x - x')$
  - Random bond (RF): $\langle [U(u', x') - U(u, x)]^2 \rangle = e^{-|u-u'|} \delta(x - x')$
  - Periodic pinning (RP): $\langle [U(u', x') - U(u, x)]^2 \rangle = \sin[\frac{2\pi(u-u')}{P}] \delta(x - x')$
- Add in a moving harmonic well to the disorder [P. Le Doussal].

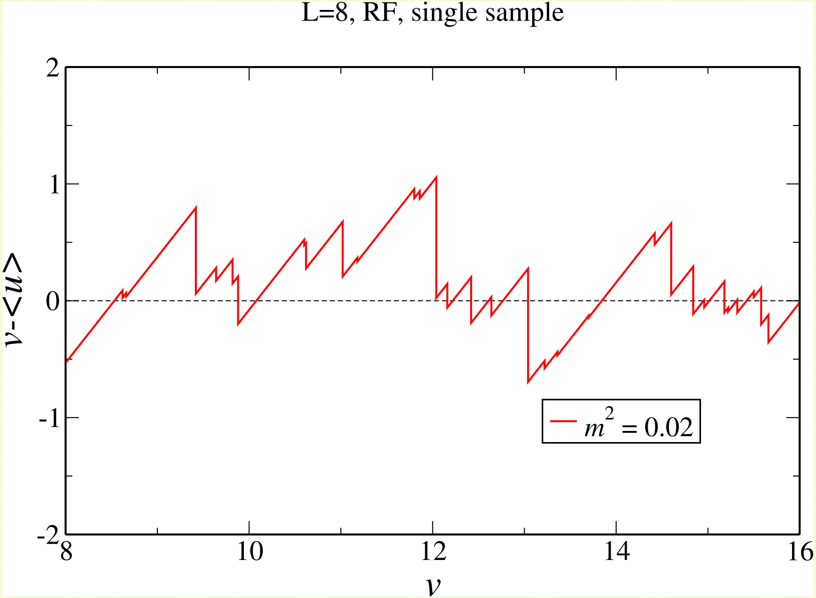
$$U_{\text{harmonic}}[u(x)] = \frac{m^2}{2}(u - v)^2$$

Simulation uses rolling disorder and can incrementally find $v \rightarrow v + \delta v$.

The Play

Act 1: Random field pinning, $D = 2{+}1$ interface, $m^2 = 0.1$, $L{\times}W = 20 \times 20$, $\delta v = 0.04$, 100 steps.
Act 2: Same interface, but $m^2 = 0.01$
Act 3: Back to scene 1, but highlights: avalanches/droplets.
Act 4: The shocking events from scene 2.

Critics: quantify? context?



Theory - Functional Renormalization Group

FRG seems to be a controlled verifiable approach to manifolds in a disordered potential.

- Below $d = 4$, ∞ number of relevant operators and metastability.
- Writing $\langle [V_\ell(u,\vec{x}) - V_\ell(0,\vec{0})]^2 \rangle = -2R_\ell(u)\delta(\vec{x})$, D. S. Fisher (1986) derived flow equations, using $\Delta(u) = -R''(u)$,

$$\frac{d\Delta(u)}{d\ell} = (\epsilon - 4\zeta)\Delta(u) + \zeta u \Delta'(u) + \frac{1}{2}\left[\Delta''(u)\right]^2 - \Delta''(u)\Delta''(0)$$

- *Non-analytic fixed points:* $\Delta(u)$, force-force correlations, have a **cusp** at $u = 0$.

Relevance

$R(u)$ and its derivatives $\Rightarrow$ the physical picture of pinned interfaces:

- Fisher, Narayan, Balents; Balents, Bouchaud, Mezard (1986-1996): sequence of scalloped potentials [singularity in $R(u)$] due to hopping between metastable states, suggestive connections to Burgers equation.
- Le Doussal, recently: scallops derived from harmonic well + disorder; precise connection to Burgers equation.
- Fixed points for flow of $R(u)$ gives exponent ζ for roughness, etc.
- Finite drive, changing disorder [”chaos”], and temperature round out the singularity at different scales [zero pinning force $\Delta'''(0)$].

Measured correlations vs. 1-loop predictions

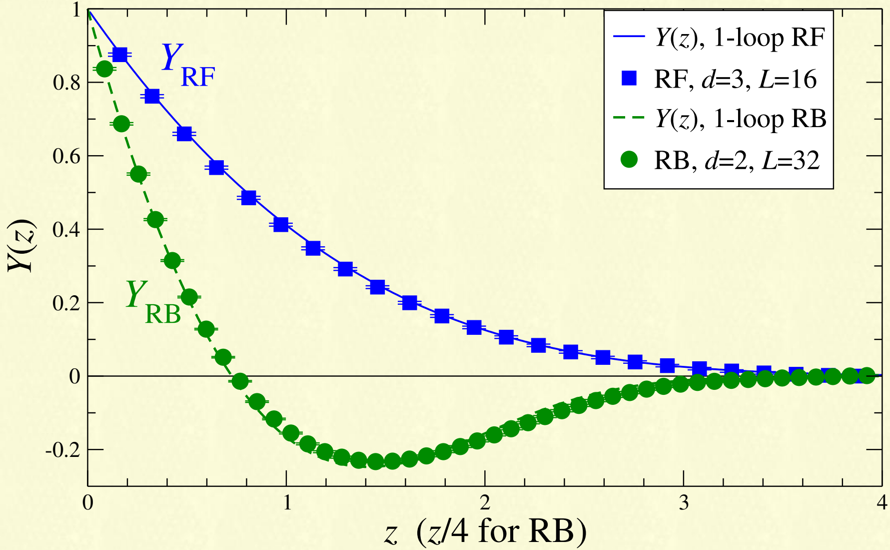
- Compute fixed point: large enough L, small enough m, so that

$$\tilde{\Delta}[m(v-v')^{\zeta}] = m^{\epsilon-4\zeta-d}\overline{[v'-\langle u(v')\rangle][v-\langle u(v)\rangle]}$$

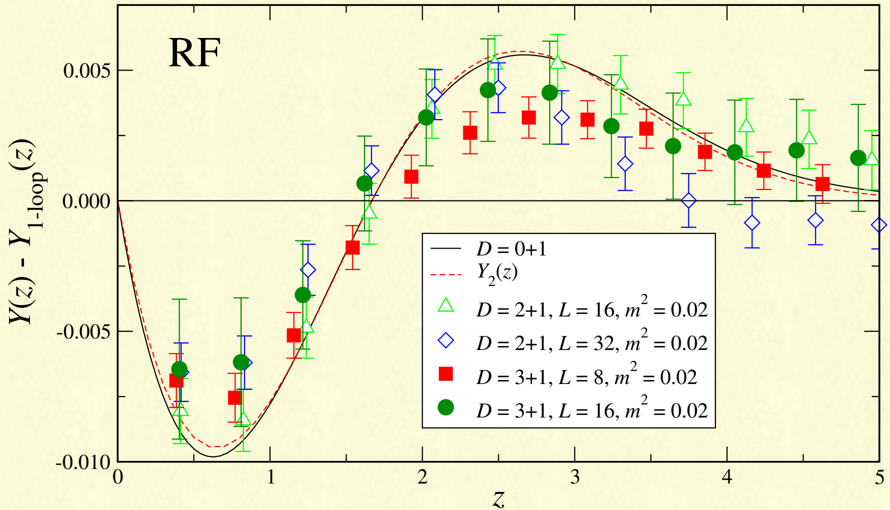
is converged.

- Rescale to $Y(u) = \tilde{\Delta}(u)/\tilde{\Delta}(0)$ and scale $z = um^{\zeta}$ to get $\int Y = 1$ (RF), $\int Y^2 = 1$(RB).

Measured correlations vs. 1-loop predictions

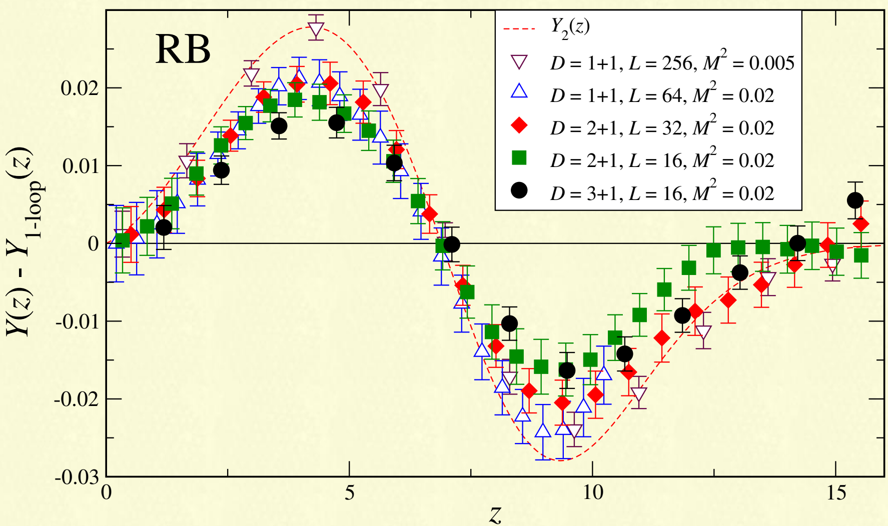


Residuals, RF



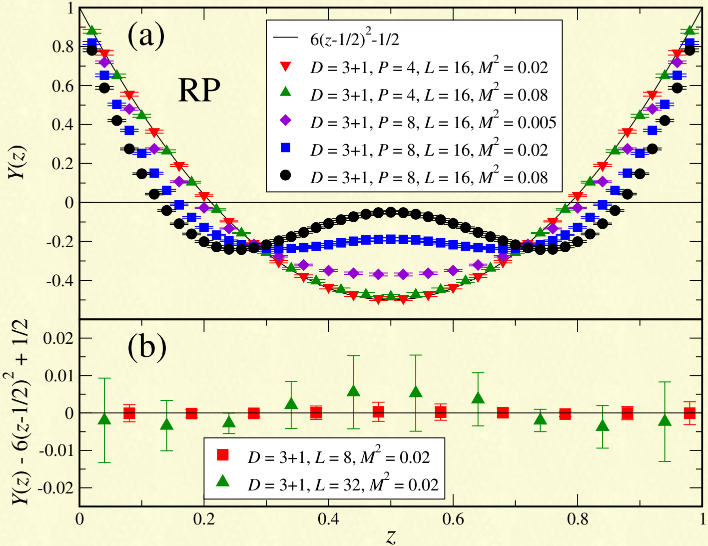
Where *one form* of the *2*-loop prediction is $Y(z) = Y_1(z) + (4 - d)Y_2(z)$

Residuals, RB



Where *one form* of the 2-loop prediction is $Y(z) = Y_1(z) + (4-d)Y_2(z)$

RP: crossover from RB to RP



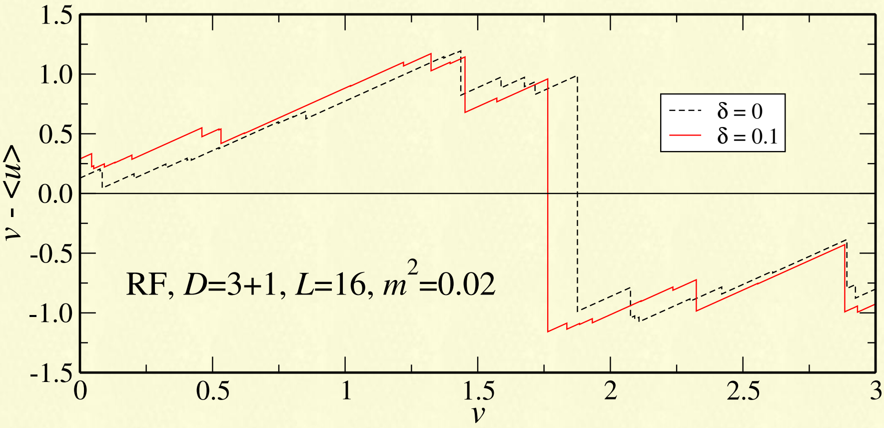
General prediction: $Y(z)$ is a parabola with zero mean (i.e., $6(z-\frac{1}{2})^2-\frac{1}{2}$).

"Chaos" (sensitivity to disorder)

Recent predictions by P. Le Doussal [PRL **96**, 235702 (2006)] for correlations

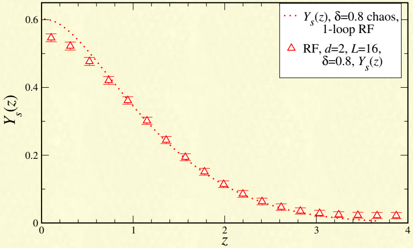
$$\Delta_{12}(y) = \langle [v + y - u_1(v + y)][v - u_2(v)] \rangle$$

between samples with disorders U_1 and U_2, with difference measured by δ.
We can check this - shapes of curves (1 adjustable parameter).

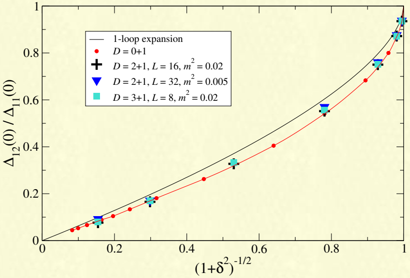


Chaos

Normalized $\Delta_{12}(y)$, fixed perturbation δ



$\Delta_{12}(0)/\Delta_{11}(0)$, varying δ [parameter free ratio]



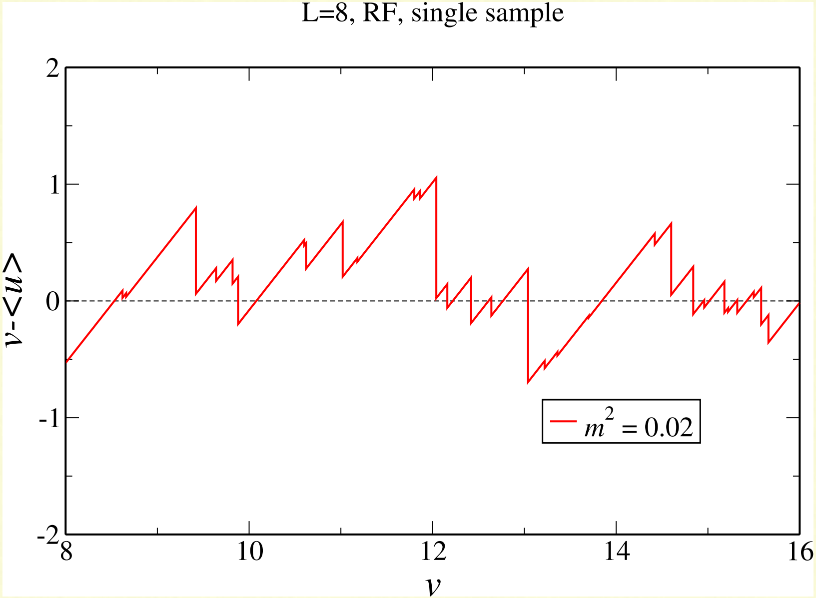
Functional Burgers Equation

$d = 0$: particle in a single $V(u)$ given by a random walk + $\frac{m^2}{2}(u-v)^2$. *Exact correspondence* between $v - \langle u \rangle$ and velocity in Burgers equation, given $t \to m^{-2}$, $V \to v - \langle u \rangle$, $\nu \to t$: jumps in $\langle u \rangle$ are shocks in 1D decaying Burgers equation.

$$\partial_t V + V \partial_x V = \nu \partial_x^2 V$$

Functional equation: formally similar.
Consequences:
In a single sample, see coalescence of jumps as decrease m^2.

Sequence of m^2 in a single sample



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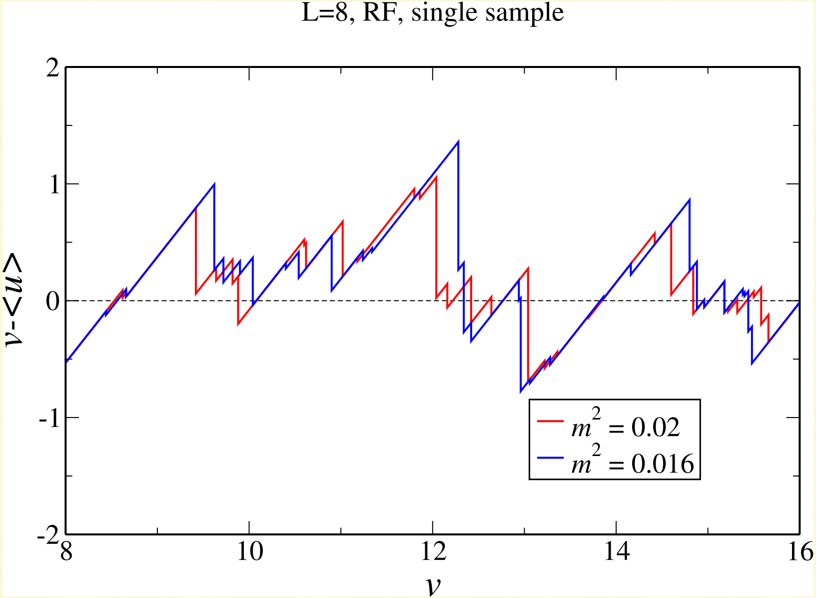

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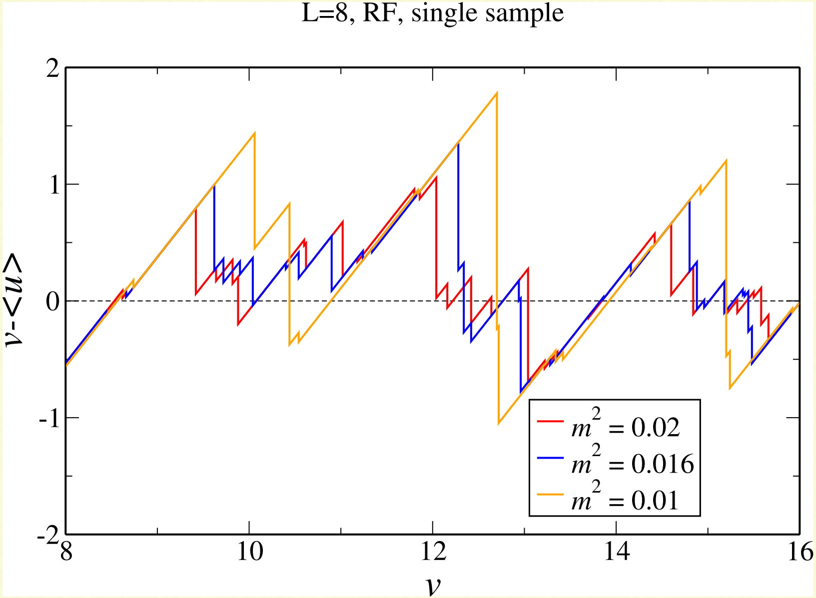

Highlights & Sequels

- Can precisely study disordered systems.
- Confirmed prediction for nonanalytic form for pinned manifolds: ***linear cusps*** in *force-force correlator* $\Delta(u)$ [20 years ago].
- One-loop calculation appears to be unreasonably good, but not the full story for RF, RB; RP shows expected exact parabola.
- Supports exponent values, validates approach, physical picture.
- Functional decaying Burgers eqn. for $v - u(x)$.