Quantum Transport through Coulomb-Blockade Systems

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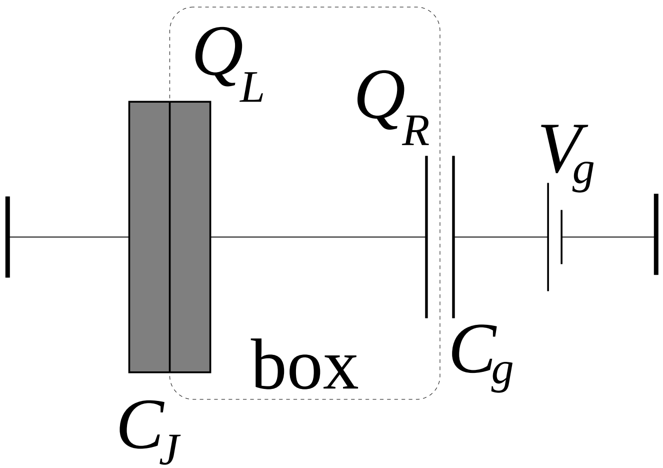
Overview

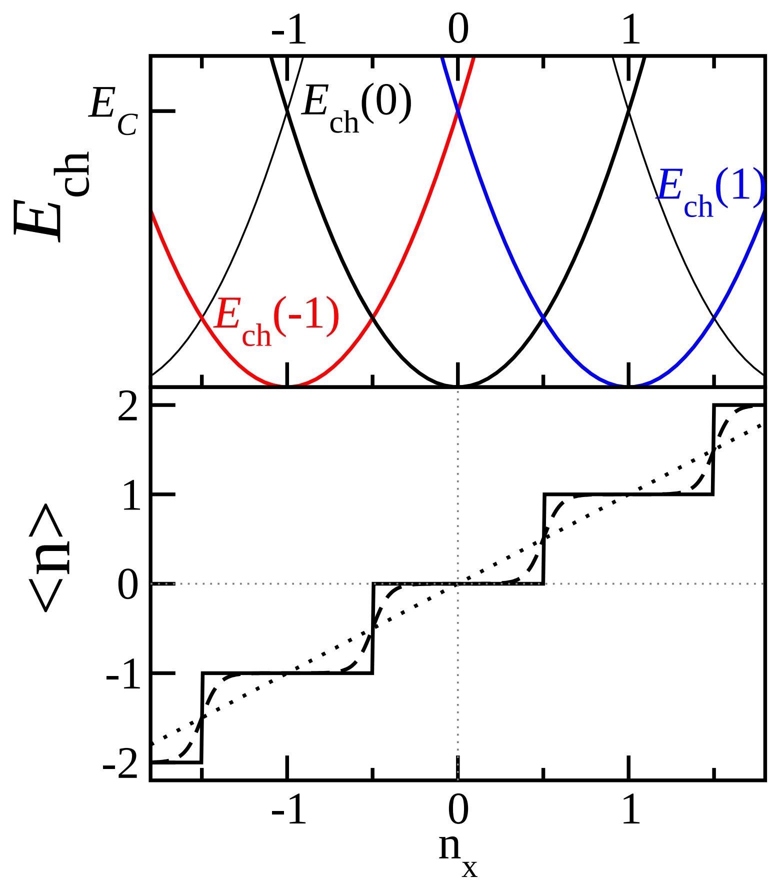
- Motivation
  - Single-electron box/transistor
  - Coupled single-electron devices
- Model and Technique
  - Real-time diagrammatics
- Thermoelectric transport
  - Thermal and electrical conductance
  - Quantum fluctuation effects on thermopower
- Multi-island systems
  - Diagrammatics for complex systems
  - New tunneling processes

Single-Electron Box

Gate attracts charge to island.

Tunnel barrier → quantized charge





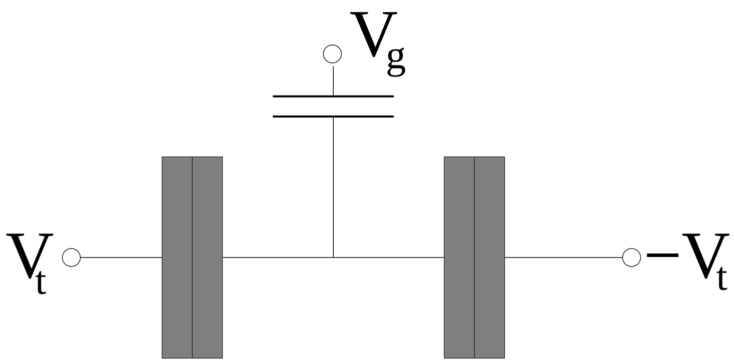
Quantitatively:

$$
\begin{aligned}
E_{\text{ch}} &= \frac{Q_L^2}{2C_J} + \frac{Q_R^2}{2C_g} + Q_R V_g \\
&= \frac{e^2}{2C_\Sigma}(n - n_x)^2 + \text{const.},
\end{aligned}
$$

Coulomb staircase

Single-Electron Transistor

Two contacts → transport



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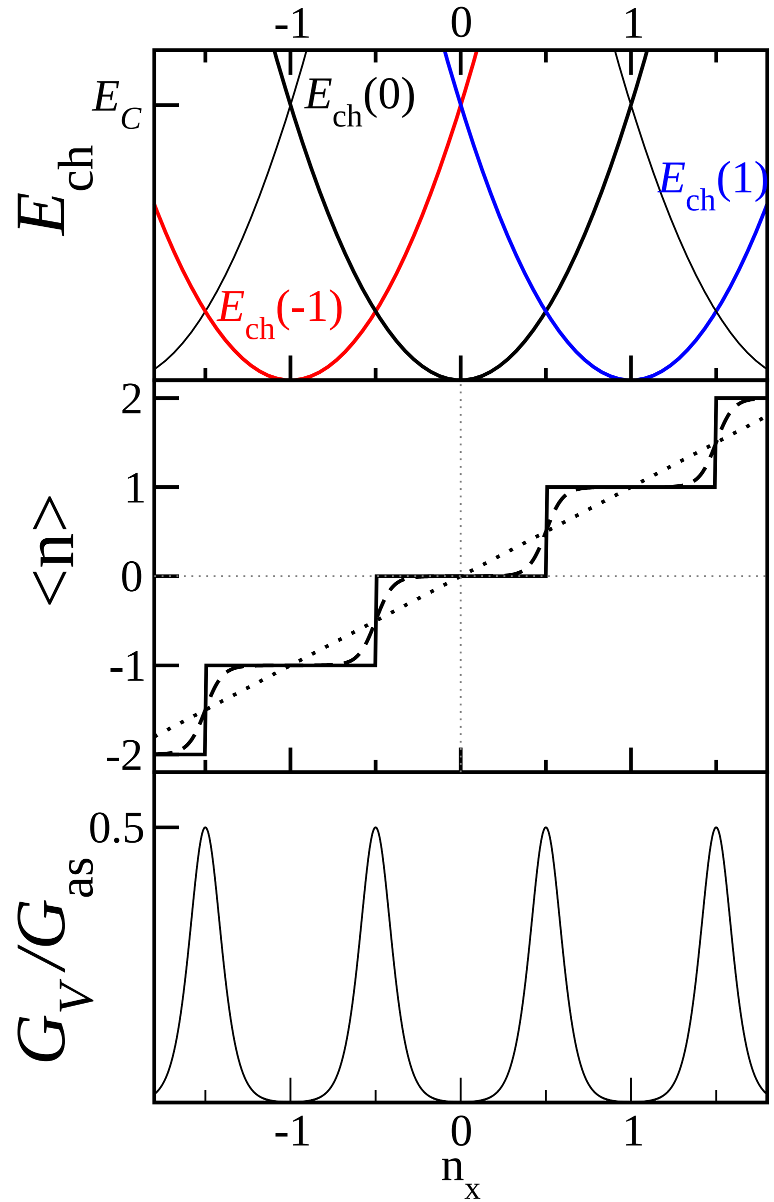
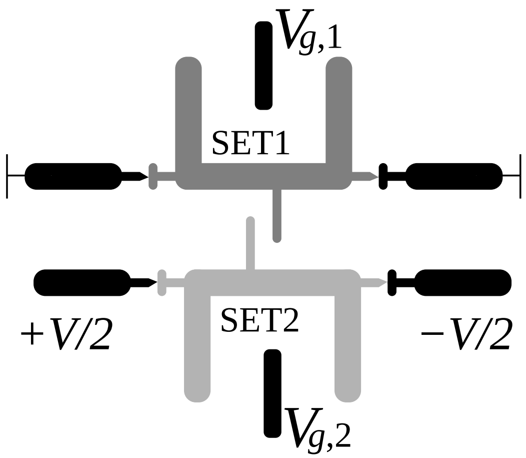
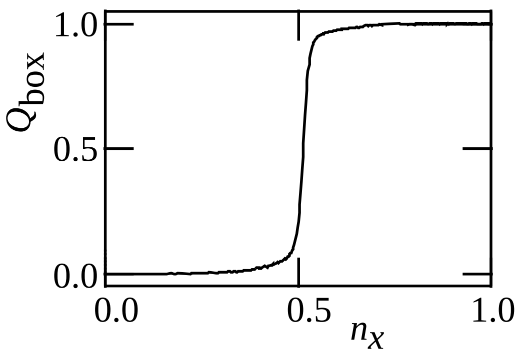


Coulomb oscillations

Simple Coupled Device



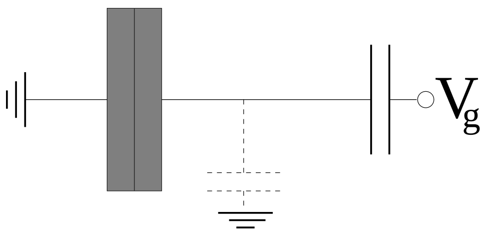
Transistor measures
box charge



one step of Coulomb staircase

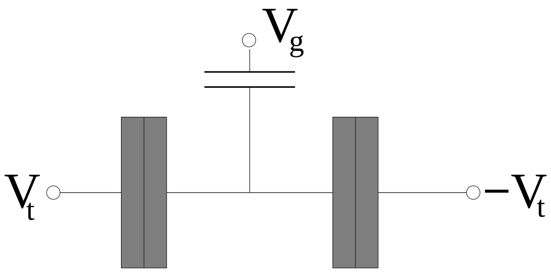
(Lehnert et al. PRL '03,
Schäfer et al. Physica E '03)

Box:



charge on C_c (sawtooth)
input for transistor

Transistor:



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Real-time diagrammatics for an SET

(Schoeller and Schön, PRB '94)

Hamiltonian: $H = H_L + H_R + H_I + H_{\mathrm{ch}} + H_T = H_0 + H_T$

charge degrees of freedom separated from fermionic degrees:

charging energy

$$H_{\mathrm{ch}} = \frac{e^2}{2C}(\hat{N} - n_x)^2$$

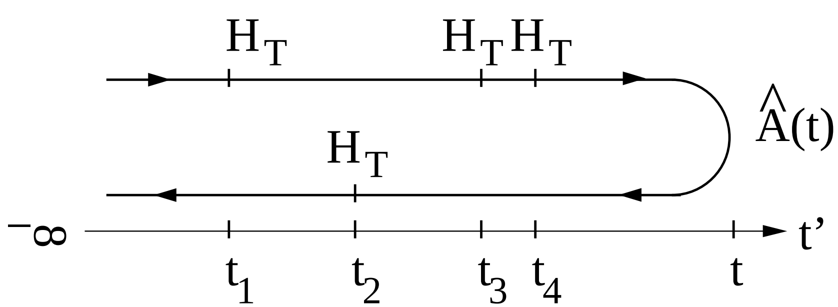
tunneling

$$H_T = \sum_{r=R,L} \sum_{kln} \left(T_{kl}^{rn} a_{krn}^{\dagger} c_{ln} e^{-i\varphi} + \mathrm{h.c.} \right)$$

Time evolution of e.g. density matrix of charge governed by propagator $\prod$:

$$\prod_{n_2',n_2}^{n_1',n_1} = \underbrace{\mathrm{Trace}}_{\text{fermionic d.o.f's}} \left[\langle n_2' | \tilde{T} \exp\left(-i \int_t^{t_0} dt' H_T(t')_I \right) | n_2 \rangle \langle n_1 | T \exp\left(-i \int_{t_0}^{t} dt' H_T(t')_I \right) | n_1' \rangle \right]$$

$\Rightarrow$ **Keldysh contour**



Dyson-equation

Integrating out reservoirs/ contracting tunnel vertices $\Rightarrow$
each contraction $\Leftrightarrow$ golden-rule rate:

$\alpha^{r\pm}(\omega) = \int dE \alpha_0^r\, f_r^{\pm}(E+\omega) f^{\mp}(E) = \pm \alpha_0^r \frac{\omega - \mu_r}{e^{\pm\beta(\omega-\mu_r)}-1}$ with $\alpha_0^r = \frac{R_K}{4\pi^2 R_r}$.

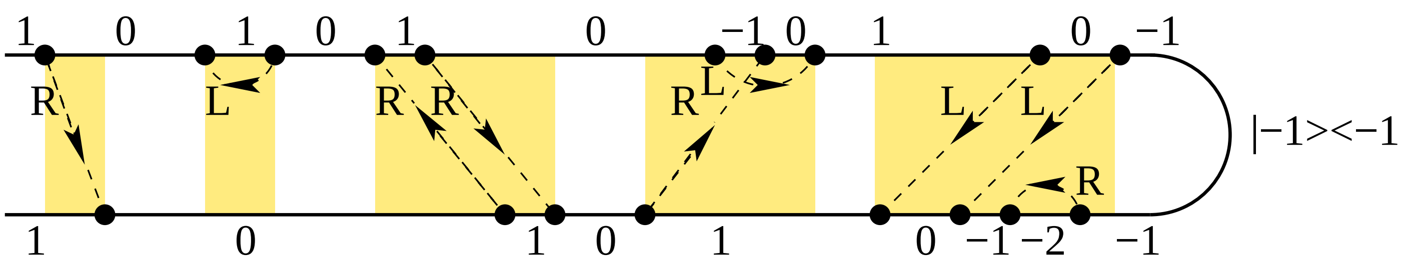
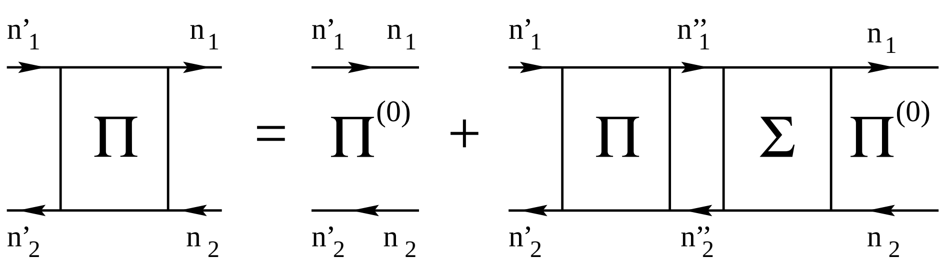


diagram with sequential, cotunneling and 3rd order processes

Write full propagator $\prod$ as **Dyson equation**:



$$\prod \;=\; \prod^{(0)} + \prod \sum \prod^{(0)}$$

with free propagator (w/o tunneling) $\prod^{(0)}$

to calculate:

self-energy $\sum$

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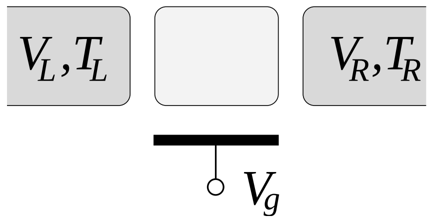
Electrical and thermal conductance

$$G_V = G_{\text{as}} \int d\omega \frac{\beta\omega/2}{\sinh \beta\omega} A(\omega) \; ; \quad G_T = -G_{\text{as}} \frac{k_B}{e} \int d\omega \frac{(\beta\omega/2)^2}{\sinh \beta\omega} A(\omega)$$



$$g_V = \frac{G_V}{G_{\text{as}}} \quad ; \quad g_T = -\frac{e}{k_B} \frac{G_T}{G_{\text{as}}}$$

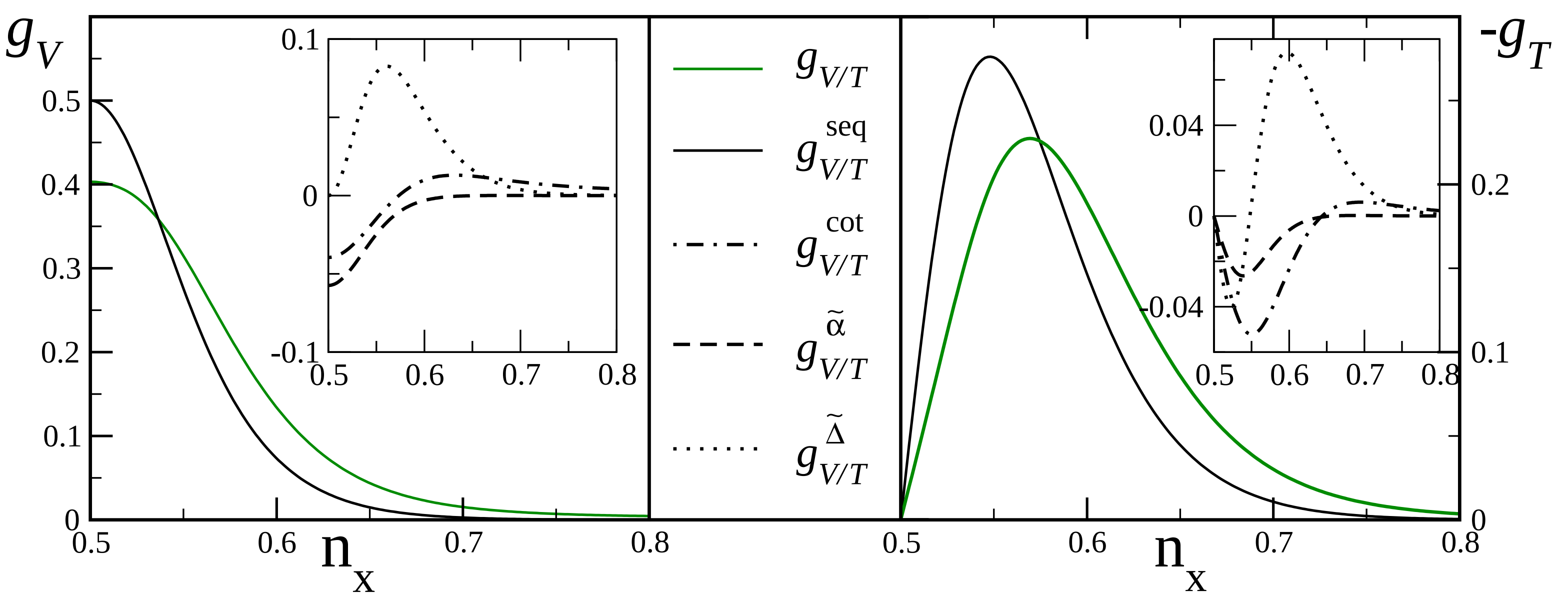
Thermoelectric transport:

$$I = G_V V + G_T \, \delta T$$

$$A(\omega) = [C^<(\omega) - C^>(\omega)]/(2\pi i)$$

perturbative expansion to 2nd order in coupling α_0

$$g_{V/T} = g_{V/T}^{\text{seq}} + g_{V/T}^{\tilde{\alpha}} + g_{V/T}^{\tilde{\Delta}} + g_{V/T}^{\text{cot}}$$

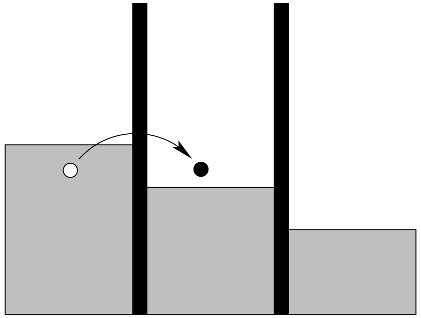


Sequential tunneling

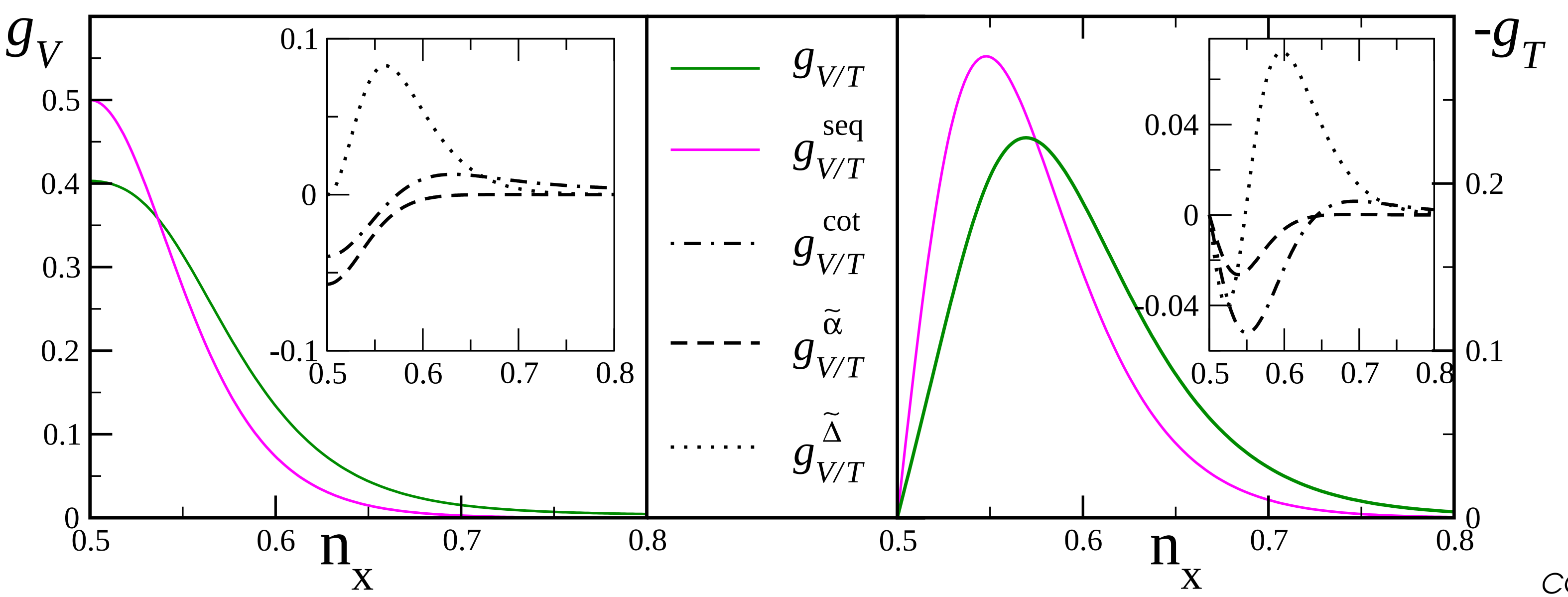
$$g_{V/T} = g_{V/T}^{\text{seq}} + g_{V/T}^{\tilde{\alpha}} + g_{V/T}^{\tilde{\Delta}} + g_{V/T}^{\text{cot}}$$

- sequential tunneling:

$$g_{V/T}^{\text{seq}} = \kappa_0 \frac{\beta\Delta_0/2}{\sinh \beta\Delta_0} \quad \text{with} \quad \kappa_0 = \begin{cases} 1 & : \; V \\ \beta\Delta_0/2 & : \; T \end{cases}$$



Resonances around degeneracy points $\Delta_n = 0$.



Cotunneling

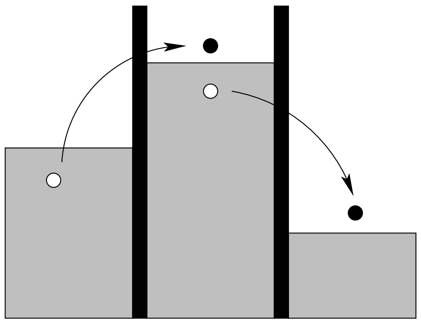
$$g_{V/T} = g_{V/T}^{\text{seq}} + g_{V/T}^{\tilde{\alpha}} + g_{V/T}^{\tilde{\Delta}} + g_{V/T}^{\text{cot}}$$

- standard cotunneling:

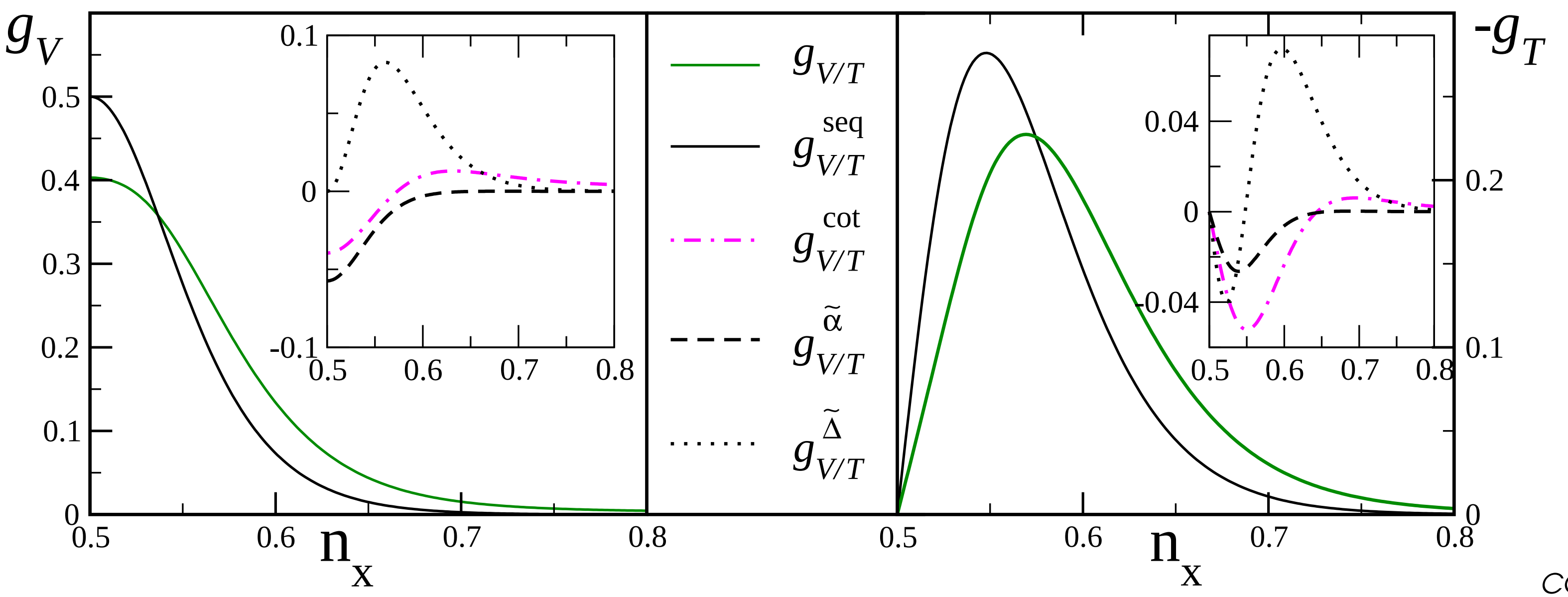
$$g_V^{\text{cot}} = \alpha_0 \frac{2\pi^2}{3}(k_B T)^2 \left(\frac{1}{\Delta_0} - \frac{1}{\Delta_{-1}}\right)^2$$

$$g_T^{\text{cot}} = \alpha_0 \frac{8\pi^4}{15}(k_B T)^3 \left(\frac{1}{\Delta_0} - \frac{1}{\Delta_{-1}}\right)^2 \left(\frac{1}{\Delta_0} + \frac{1}{\Delta_{-1}}\right)$$

dominant away from resonance $|\Delta_n| \gg k_B T$.



virtual occupation of unfavourable charged state.



Cotunneling

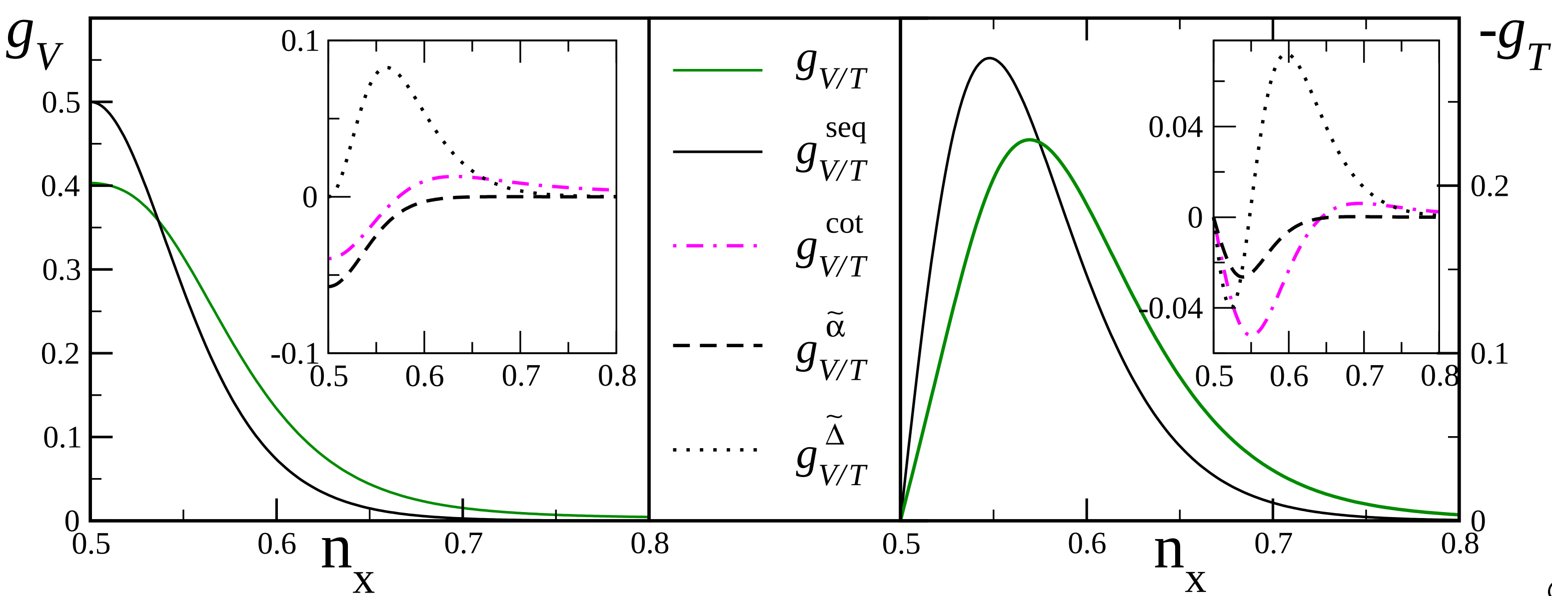
$$g_{V/T} = g^{\text{seq}}_{V/T} + g^{\tilde{\alpha}}_{V/T} + g^{\tilde{\Delta}}_{V/T} + g^{\text{cot}}_{V/T}$$

- standard cotunneling:

$$g^{\text{cot}}_{V} = \alpha_0 \frac{2\pi^2}{3} (k_B T)^2 \left(\frac{1}{\Delta_0} - \frac{1}{\Delta_{-1}} \right)^2$$

$$g^{\text{cot}}_{T} = \alpha_0 \frac{8\pi^4}{15} (k_B T)^3 \left(\frac{1}{\Delta_0} - \frac{1}{\Delta_{-1}} \right)^2 \left(\frac{1}{\Delta_0} + \frac{1}{\Delta_{-1}} \right) \qquad \kappa_n = \begin{cases} 1 & : \quad V \\ \beta\Delta_n/2 & : \quad T \end{cases}$$

$$g^{\text{cot}}_{V/T} = \kappa_{-1}\Delta_{-1}\partial^2\phi_{-1} + \kappa_0\Delta_0\partial^2\phi_0 + \frac{\kappa_0 + \kappa_{-1}}{2} \cdot \frac{\phi_0 - \phi_{-1} + \Delta_{-1}\partial\phi_{-1} - \Delta_0\partial\phi_0}{E_C}$$



Renormalized sequential tunneling

$$g_{V/T} = g^{\text{seq}}_{V/T} + g^{\tilde{\alpha}}_{V/T} + g^{\tilde{\Delta}}_{V/T} + g^{\text{cot}}_{V/T}$$

$$\kappa_0 = \begin{cases} 1 & : \quad V \\ \beta\Delta_0/2 & : \quad T \end{cases}$$

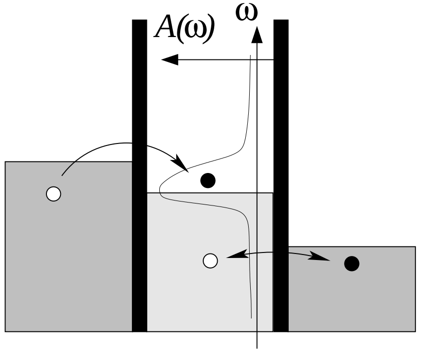
- Renormalization of

coupling:

$$g^{\tilde{\alpha}}_{V/T} = \kappa_0 \frac{\beta\Delta_0/2}{\sinh \beta\Delta_0} \left[\partial \left(2\phi_0 + \phi_{-1} + \phi_1 \right) + \frac{\phi_{-1} - \phi_1}{E_C} \right]$$

energy gap:

$$g^{\tilde{\Delta}}_{V/T} = \frac{\partial}{\partial \Delta_0} \left[\kappa_0 \frac{\beta\Delta_0/2}{\sinh \beta\Delta_0} \right] \left(2\phi_0 - \phi_{-1} - \phi_1 \right)$$



sequential tunneling but spectral density $A(\omega)$ broadened and shifted.

$\Downarrow$

renormalized parameters

for coupling: $\tilde{\alpha}$

charging energy gap: $\tilde{\Delta}_n$

Renormalization by quantum fluctuations

$$g_{V/T}^{\tilde{\alpha}} = \kappa_0 \frac{\beta\Delta_0/2}{\sinh\beta\Delta_0}\left[\partial\left(2\phi_0+\phi_{-1}+\phi_1\right)+\frac{\phi_{-1}-\phi_1}{E_C}\right]; \quad g_{V/T}^{\tilde{\Delta}} = \frac{\partial}{\partial\Delta_0}\left[\kappa_0\frac{\beta\Delta_0/2}{\sinh\beta\Delta_0}\right]\left(2\phi_0-\phi_{-1}-\phi_1\right)$$

Quantum fluctuations $\Rightarrow$ **renormalization** of system parameters

$$G(\alpha_0,\Delta_0) = G^{\text{seq}}(\tilde{\alpha},\tilde{\Delta}) + \text{cot. terms}$$

expand: $$G^{\text{seq}}(\tilde{\alpha},\tilde{\Delta}) = \tilde{\alpha}\,\frac{\partial G^{\text{seq}}(\alpha_0,\Delta_0)}{\partial\alpha_0} + \left(\tilde{\Delta}-\Delta_0\right)\frac{\partial G^{\text{seq}}(\alpha_0,\Delta_0)}{\partial\Delta_0}$$

---

renormalization of parameters (perturbative in α_0):

$$\frac{\tilde{\alpha}}{\alpha_0} = 1-2\alpha_0\left\{-1+\ln\left(\frac{\beta E_C}{\pi}\right)-\partial_{\Delta_0}\left[\Delta_0\,\mathrm{Re}\,\Psi\left(i\frac{\beta\Delta_0}{2\pi}\right)\right]\right\}$$

$$\frac{\tilde{\Delta}}{\Delta_0} = 1-2\alpha_0\left[1+\ln\left(\frac{\beta E_C}{\pi}\right)-\mathrm{Re}\,\Psi\left(i\frac{\beta\Delta_0}{2\pi}\right)\right]$$

$\tilde{\alpha}$ and $\tilde{\Delta}$ **decrease logarithmically** by renormalization!
(for lowering temperature and increasing coupling α_0)

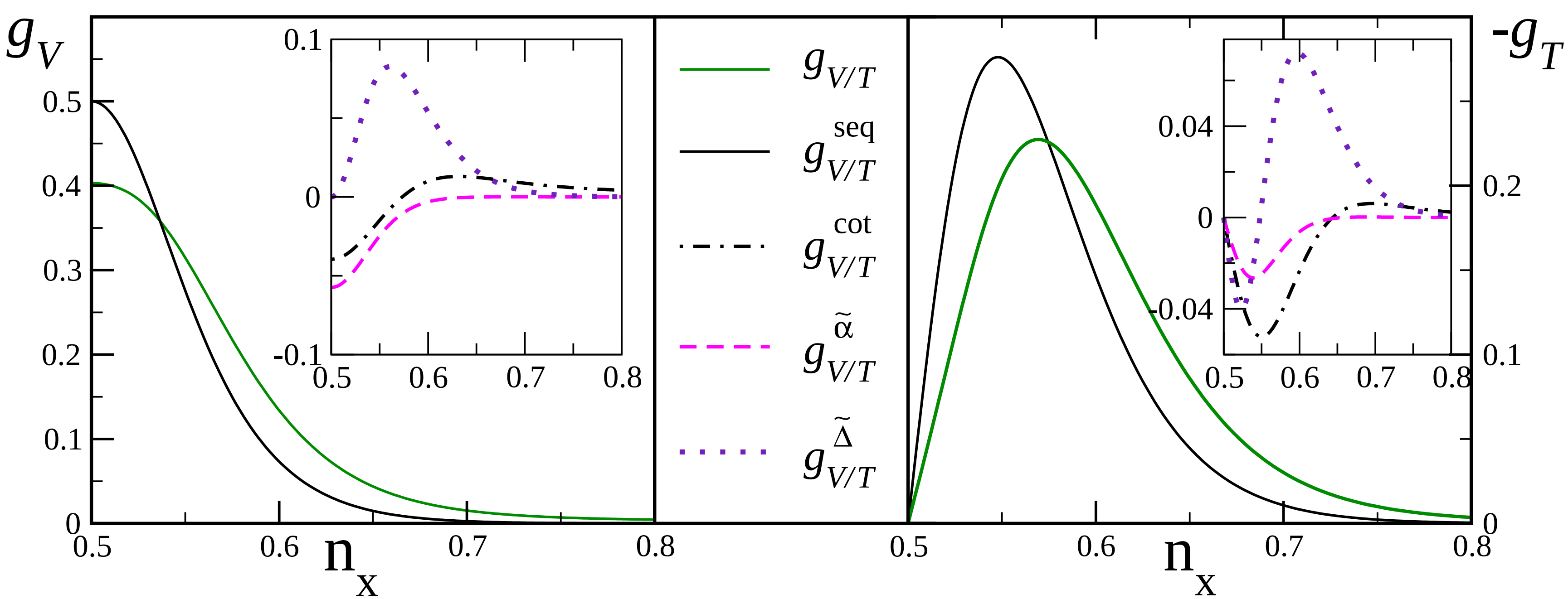
$\Leftrightarrow$ many-channel **Kondo-physics**

Renormalization effects on $G_{V/T}$

$$G(\alpha_0, \Delta_0) = G^{\text{seq}}(\tilde{\alpha}, \tilde{\Delta}) + \text{cot. terms}$$

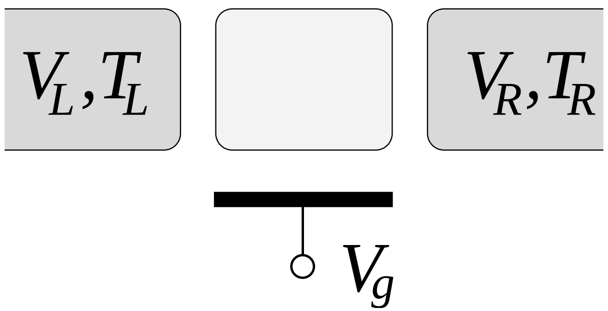
$\tilde{\alpha}$ and $\tilde{\Delta}$ decrease logarithmically by renormalization:

- $\tilde{\alpha} \searrow \quad \longrightarrow$ peak structure reduced by quantum fluctuations.
- $\tilde{\Delta} \searrow \quad \longrightarrow$ closer to resonance; peak broadened by quantum fluct.



(logarithmic reduction of maximum electrical conductance (König et al. PRL '97)
experimentally observed by Joyez et al. PRL '97)

Thermopower



Thermoelectric transport:

$$I = G_V V + G_T\,\delta T$$

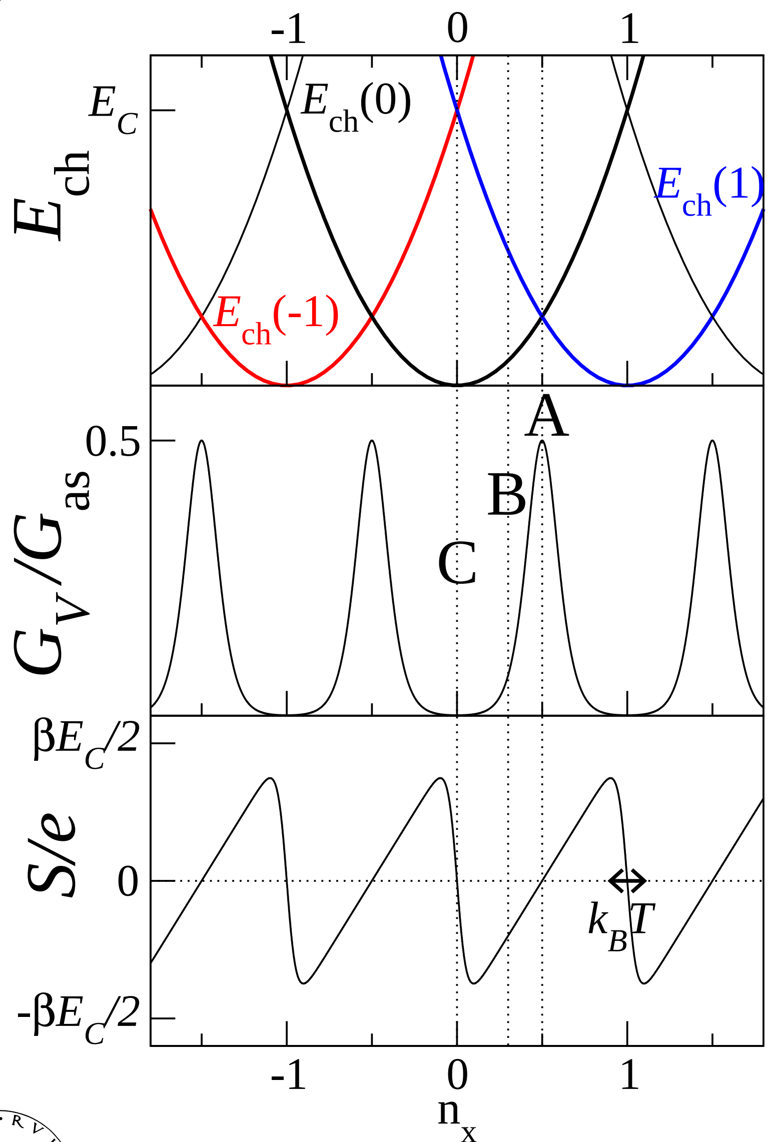
Thermopower:

$$\boxed{S = -\lim_{\delta T \to 0} \frac{V}{\delta T}\bigg|_{I=0} = \frac{G_T}{G_V}}$$

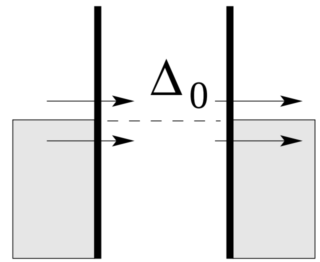
S measures average energy:

$$S = -\frac{\langle \varepsilon \rangle}{eT}.$$

Charging energy gaps determine S



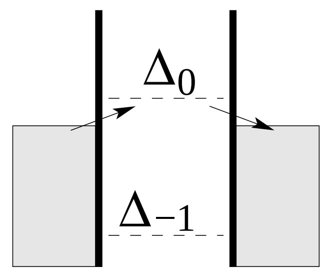
$$\Delta_n = E_{\mathrm{ch}}(n+1) - E_{\mathrm{ch}}(n)$$
$$= E_C[1 + 2(n - n_x)]$$



A) at resonance:

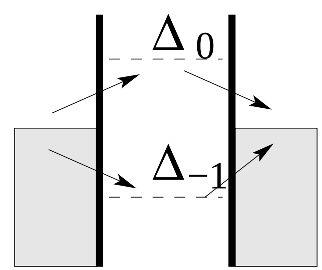
- peak in G_V
- $S \propto \langle \varepsilon \rangle = 0$

  $\varepsilon \gtrless E_F$ cancels



B) sequential

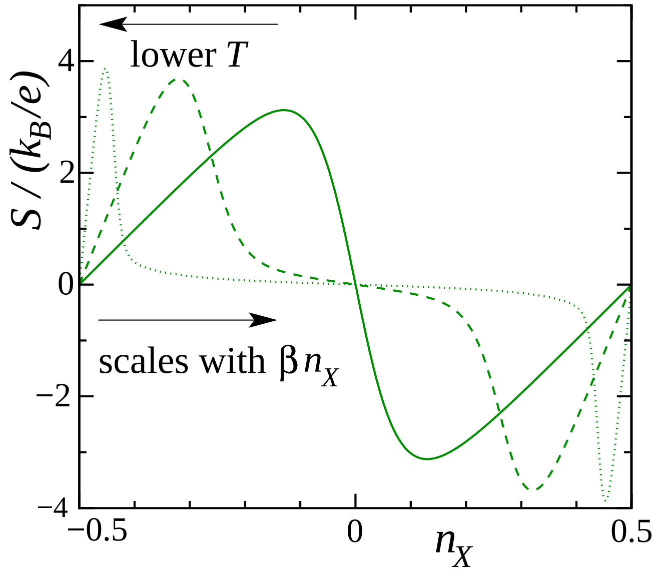
- G_V decays off resonance
- $S \propto \langle \varepsilon \rangle \propto \Delta_0 \propto n_x$

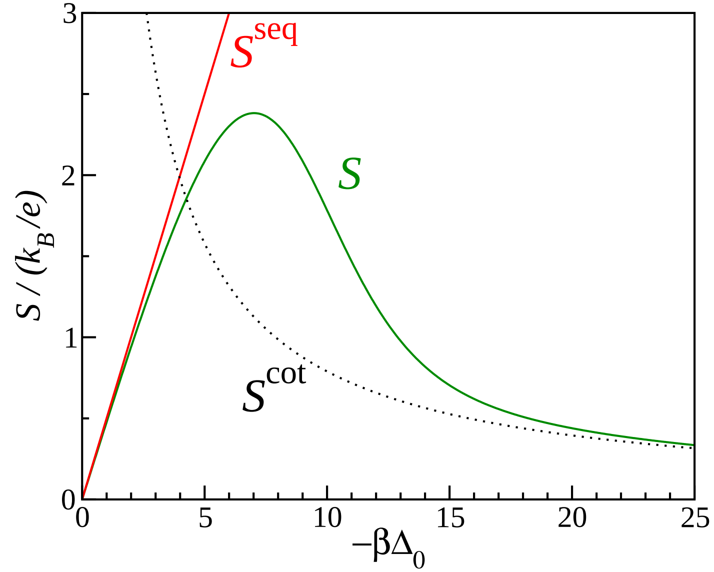


C) $n_x = 0 \Leftrightarrow \Delta_{-1} = -\Delta_0$

- two levels add for G_V
- two levels cancel for S

Sequential and cotunneling only





(Turek and Matveev, PRB '02)

$$-SeT = \langle \varepsilon \rangle = \frac{g_V^{\text{seq}}\Delta_0/2 + g_V^{\text{cot}}(k_B T)^2/\Delta_0}{g_V^{\text{seq}} + g_V^{\text{cot}}}$$

'universal low-T behavior'

- $S^{\text{seq}} = -\Delta_0/(2eT) \propto n_x \to$ sawtooth
- sawtooth suppressed by cotunneling

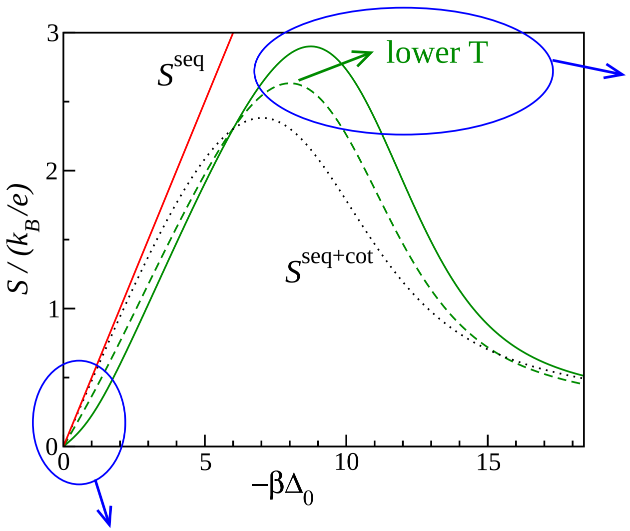
$$S^{\text{cot}} = -\frac{k_B}{e}\frac{4\pi^2}{5}\frac{1}{\beta\Delta_0}$$

$$S^{\text{seq+cot}} = S(\beta\Delta_0)$$

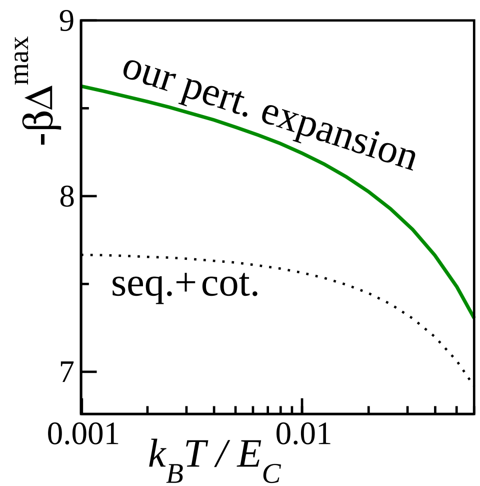
How do quantum fluctuations change this picture?

Renormalization effects on thermopower

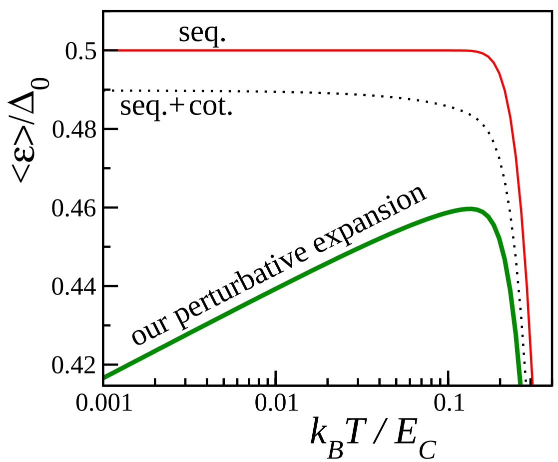
Low T properties governed by renormalization:



- Maximum of S



- charging-energy gap



$$-SeT = \langle \varepsilon \rangle = \frac{g_V^{\mathrm{seq}} \Delta_0/2 + g_V^{\mathrm{cot}} (k_B T)^2/\Delta_0}{g_V^{\mathrm{seq}} + g_V^{\mathrm{cot}}}$$

crossover from g_V^{seq} to g_V^{cot} $\Rightarrow$ maximum position

system closer to resonance $\Rightarrow$ crossover for larger Δ_{max}

reduced charging-energy gap

$\Rightarrow$ smaller $\langle \varepsilon \rangle$ (measures $\tilde{\Delta}$)

$\Rightarrow$ Further support for renormalization picture!

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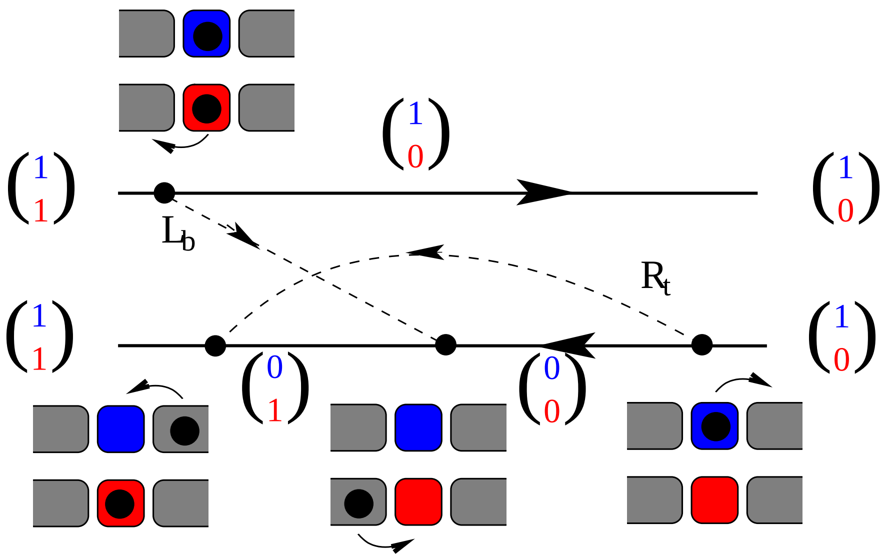
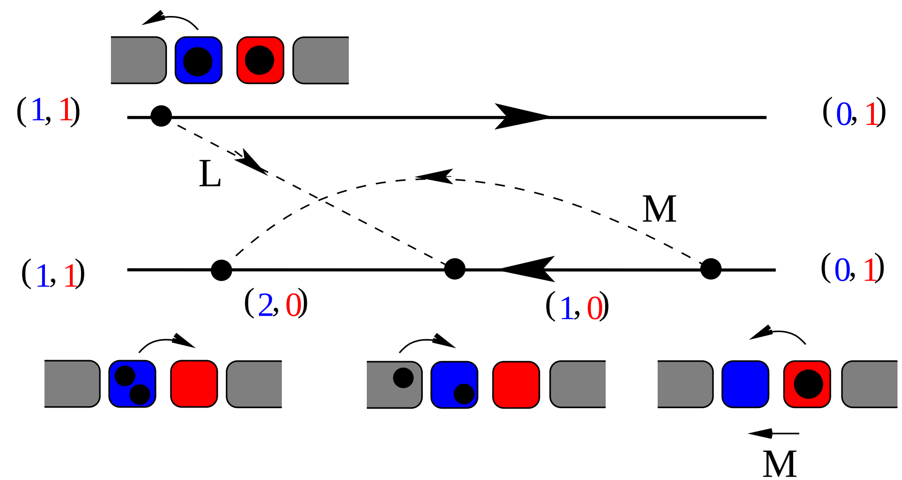
Multi-island geometries:

parallel setup: series setup:



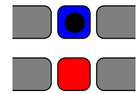


Trivial changes allow application to different setups !

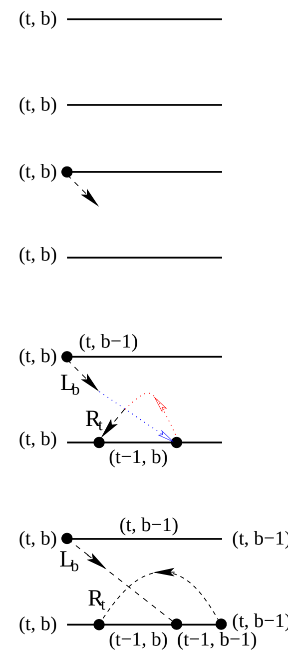
(no changes in calculation of diagrams)

Algorithm for multi-island systems

e.g. parallel setup:

- **charge states**: (t, b) $\quad (1, 0)$ 
- **Electrostatics** :

$$E_{ch}(t, b) = E_t(t - t_x)^2 + E_b(b - b_x)^2 + E_{\mathrm{coupl.}}(t - t_x)(b - b_x)$$



- **generate all diagrams:**

  - start from any charge state
  - choose vertex positions
  - choose tunnel junctions and directions of lines
  - connect vertices
  - change charge states

- **Calculate value of diagram**, contributing to $\Sigma_{(t,b)\to(t,b-1)}$

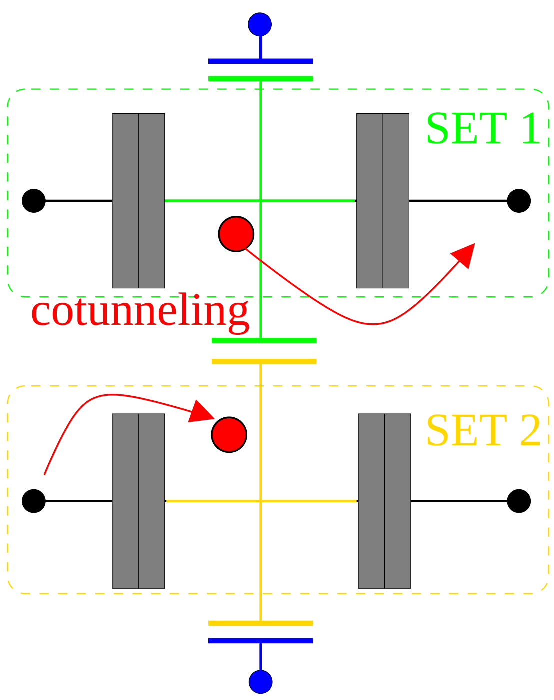
simple rules but plenty of diagrams (in 2nd order 2^{11} per charge state)

$\Rightarrow$ **Automatically generate and calculate all diagrams !**

New processes in coupled SETs

-before:

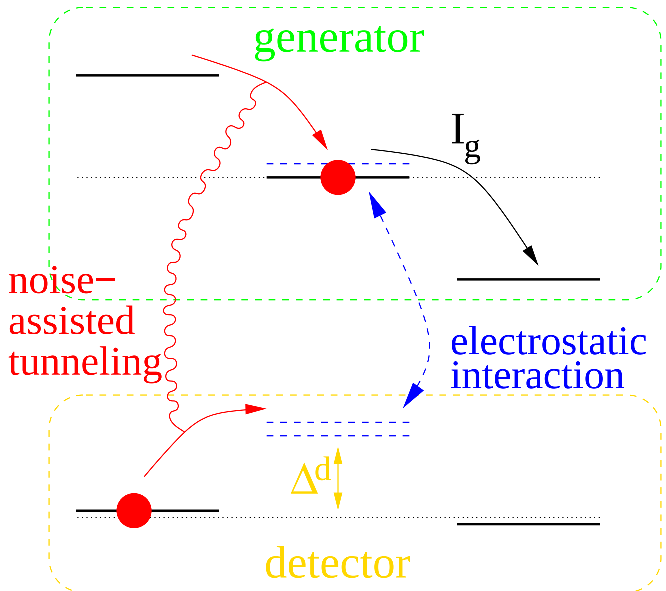
- study building blocks separately
- link blocks together:
  e.g., **average** charge of one SET
  $\rightarrow$ input for other SET



full treatment:

- **complete 2nd order** theory
- **quantum fluctuations**
- **backaction** of SET on box
- **new** class of processes:
  double-island cotunneling with energy exchange
  $\Rightarrow$ noise $\leftrightarrow$ $P(E)$ theory
  (SET1 noisy environment for SET2)

Noise assisted tunneling



limiting cases:

– **small** driving

detector-cotunneling independent of I_g

– **strong** driving of generator

$P(E) \propto S_g^Q / E^2 \;\Leftarrow$ generator noise

$$\Gamma_{01}^d = \Gamma(\Delta^d) = \alpha_0 \int dE \frac{E}{1 - e^{-\beta E}} P(-\Delta^d - E)$$

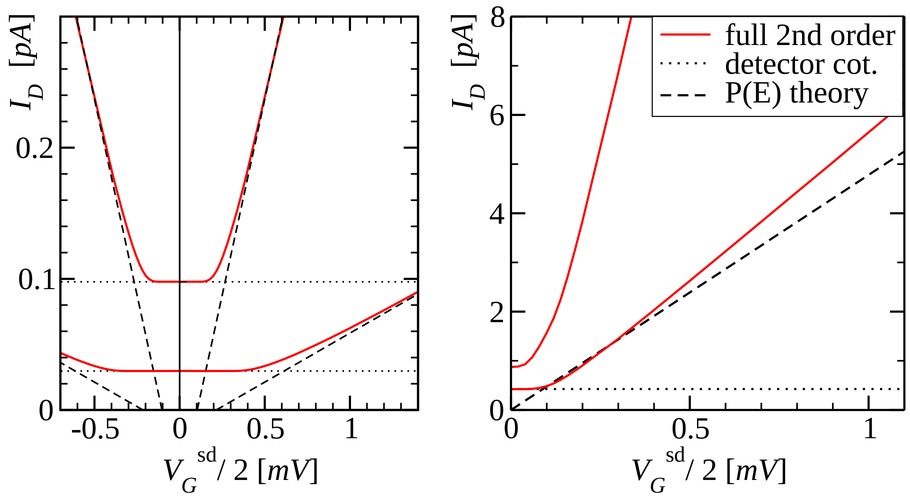
noise-assisted tunneling $\propto |I_g|$
(instead of exponential suppression).

$$P(n_g = 0) \approx \tfrac{1}{2} \approx P(n_g = 1)$$

$$P(n_d = 0) \approx 1$$

$$P(n_d = 1) \neq 0$$

by noise-assisted tunneling



Conclusions

- **Higher-order tunneling effects beside cotunneling**
- Thermoelectric properties of an SET
  - **Quantum fluctuations renormalize** system parameters
  - Electrical and thermal conductance renormalized similarly
  - Thermopower measures average energy $\Rightarrow$ logarithmic (Kondo-like) reduction of charging-energy gap
- **General scheme** to analyze **multi-island** systems
  - **All 2nd order diagrams computed automatically**
  - Detailed study of mutual influence of coupled SETs possible, **backaction** and **quantum fluctuations**
  - **New tunneling processes exchange energy between islands**

B. Kubala, G. Johansson, and J. König, Phys. Rev. B **73**, 165316 (2006).

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