Decoherence by controlled spin baths

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Motivation

Decoherence → Paradigm models

- Harmonic oscillators
- Spin baths

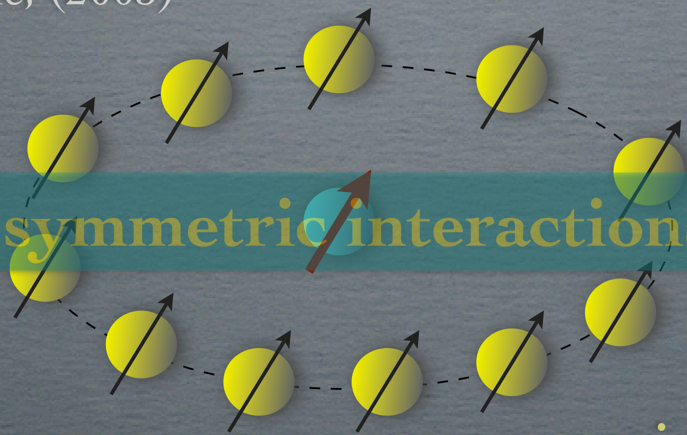
Engineered Quantum Baths

Spin baths

N.V. Prokof'ev and P.C.E. Stamp,
Rep. Prog. Phys. **63**, 669 (2000)

indipendent spins

W.H. Zurek (1982)
F.M. Cucchietti, J.P Paz, and W.H. Zurek, (2005)
L. Tessieri and J. Wilkie, (2003)

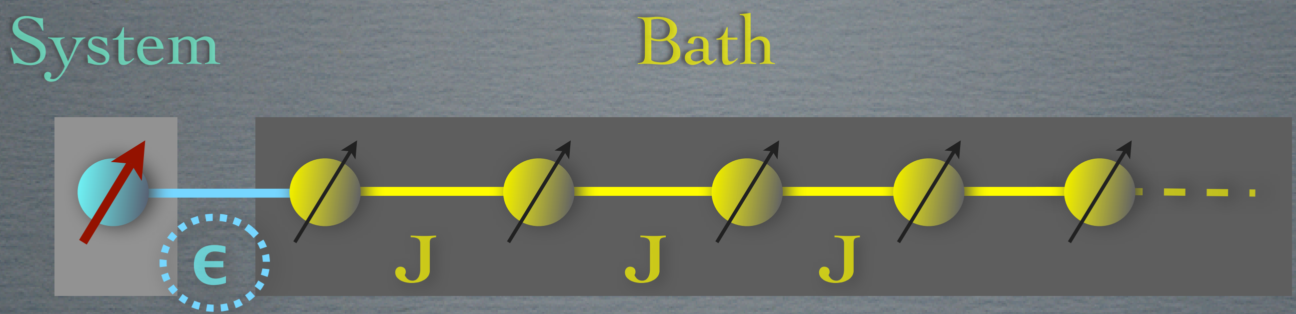


Highly symmetric interactions and baths

interacting spins

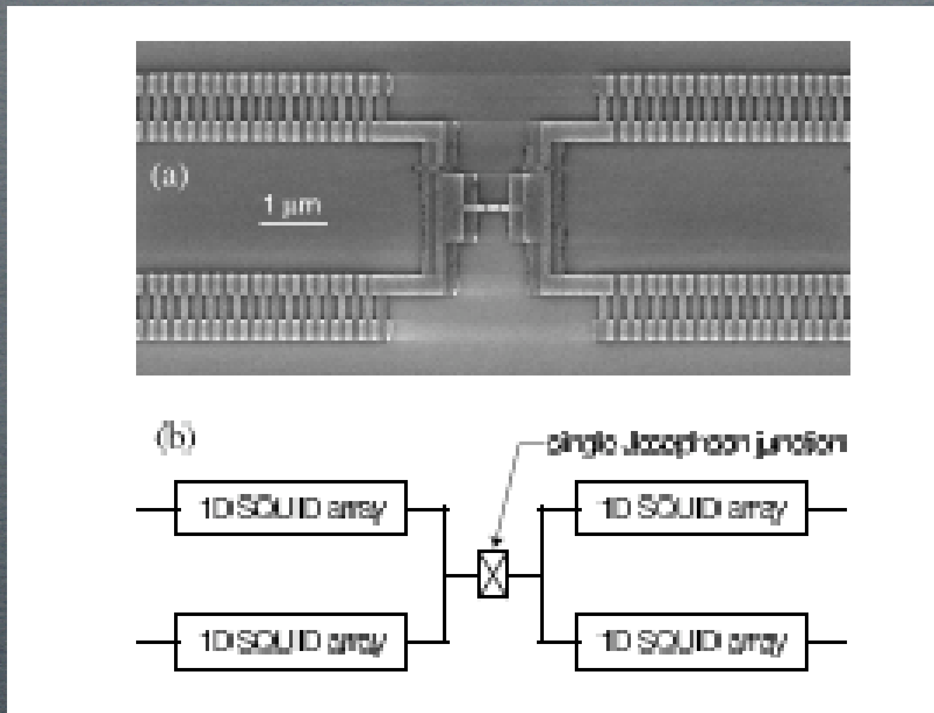
D.V. Khveshchenko, (2003)
C.M. Dawson *et al.*, (2005)
S. Paganelli, F. de Pasquale, and S.M. Giampaolo, (2002)
H.T. Quan *et al* (2006).
F.M. Cucchietti, S. Fernandez-Vidal, and J.P. Paz, (2006)

The setup



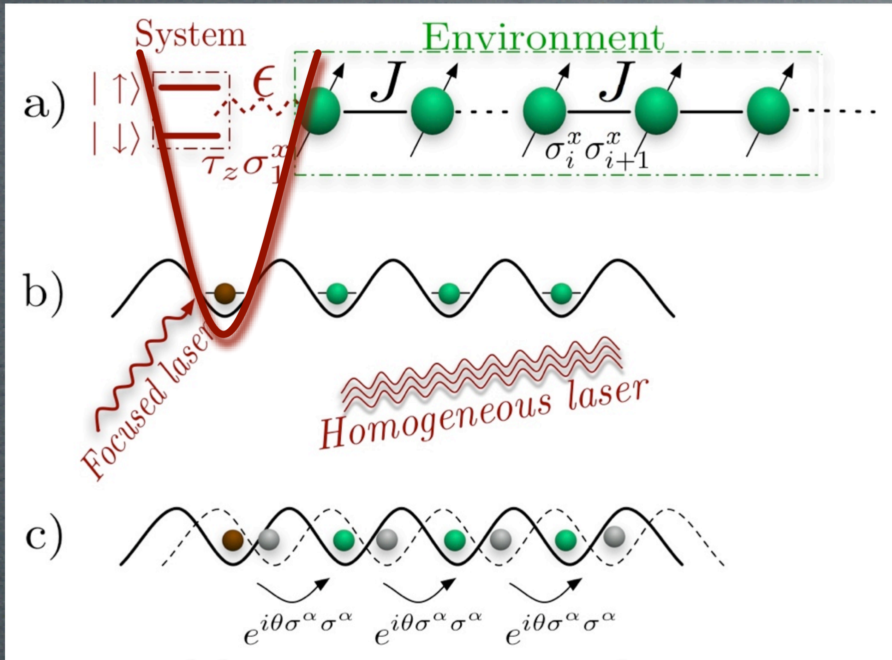
A two-level system(the quantum system) is coupled to a single spin of a one-dimensional spin-1/2 chain (the environment).

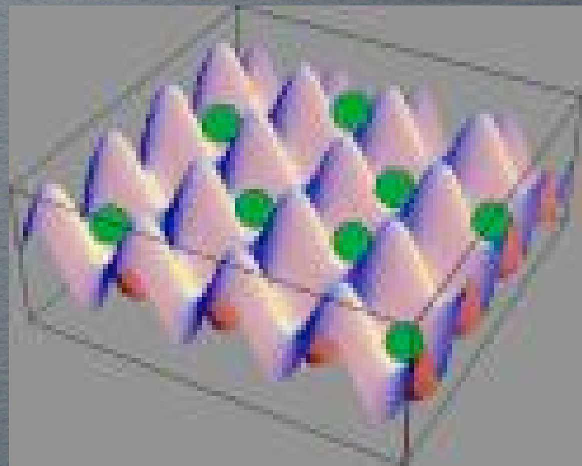
Physical realizations - JJAs

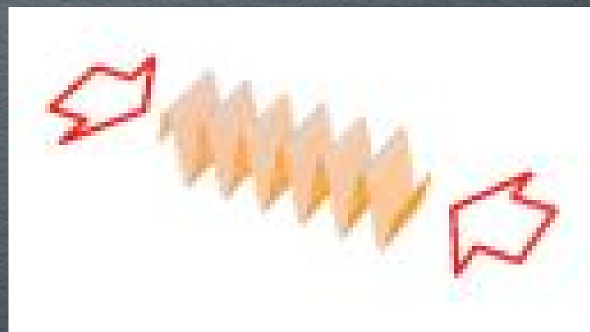


D. Haviland's group

Physical realizations - optical lattices







Optical lattices as simulators of interacting spin systems

E. Janè *et al* (2003)
D. Jaksch *et al* (1999)
O. Mandel *et al* (2003)

The Model

$$\mathcal{H} = \mathcal{H}_{TL} + \mathcal{H}_E + \mathcal{H}_{IN}$$

$$\mathcal{H}_{TL} = \omega_1 |1\rangle\langle 1|$$

$$\mathcal{H}_{TL} = -\epsilon |1\rangle\langle 1| \sigma_1^z$$

Pure dephasing

Unruh (1995)
Palma, Suominen, and Ekert, (1996)

$$\rho(t) = \begin{pmatrix} \rho_{00}(0) & \rho_{01}(t) \\ \rho_{10}(t) & \rho_{11}(0) \end{pmatrix}$$

$$\rho_{10}(t) = \rho_{10}(0) D(t)$$

$$D(t) = \langle e^{i\mathcal{H}t} e^{-i(\mathcal{H}_{TL}+\mathcal{H}_E)t} \rangle$$

Loschmidt echo

$$\mathcal{L}(t) = |D(t)|^2$$

MODELS FOR THE ENVIRONMENT

$$\mathcal{H}_E$$

Ising chain in a transverse field

$$H = -\frac{J}{2}\sum_{i=1}^{N}(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y - h\sum_{i=1}^{N}\sigma_i^z$$

$$\langle\sigma_x\rangle \neq 0$$

$\gamma \neq 0$ Ising universality class

$\gamma = 0$ XY universality class

1

$$\lambda = \frac{h}{J}$$

Exact solution - free fermions

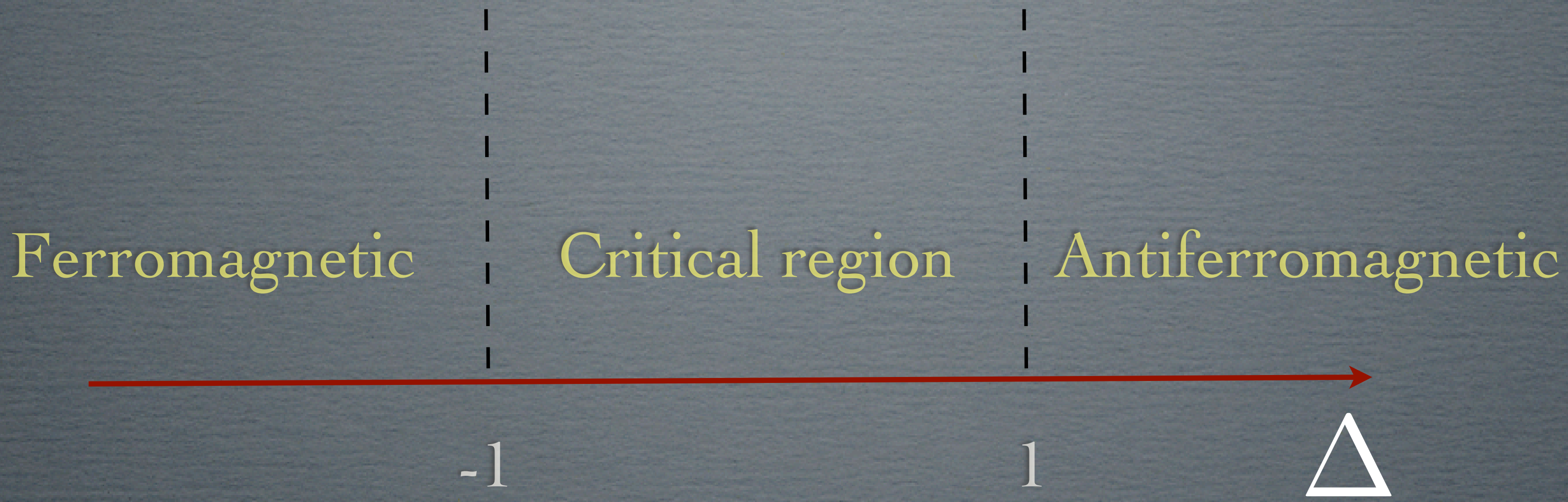
Pfeuty '70

Heisenberg Model

$$H = \sum_i J \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \, \sigma_i^z \sigma_{i+1}^z \right)$$

Heisenberg Model

Constant couplings



Ising chain environment

$$\mathcal{L}(t) = \det\left(1 - \mathbf{r} + \mathbf{r}e^{i\mathbf{C}t}\right)$$

$$\mathbf{C} = \begin{pmatrix} A & B \\ -B & -A \end{pmatrix}$$

$$r_{i,j} = \langle \Psi_i^\dagger \Psi_j \rangle$$

$$B_{j,k} = -J\gamma\left(\delta_{k,j+1} - \delta_{j,k+1}\right)$$

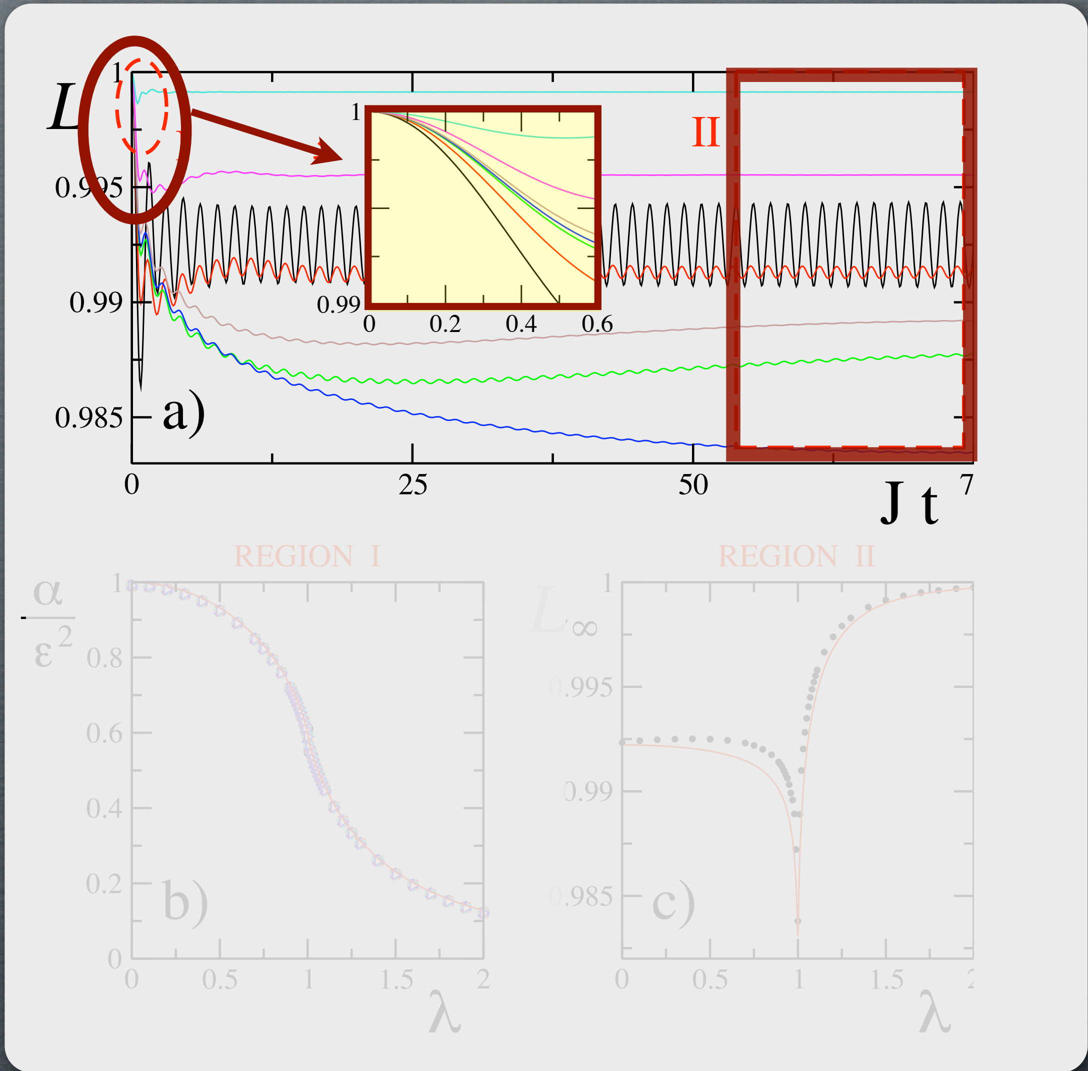
$$A_{j,k} = -J(\delta_{k,j+1} + \delta_{j,k+1}) - 2(\lambda + \epsilon_j)\delta_{j,k}$$

Ising chain environment

$$\mathcal{L}_I(t) \sim e^{-\alpha t^2}$$

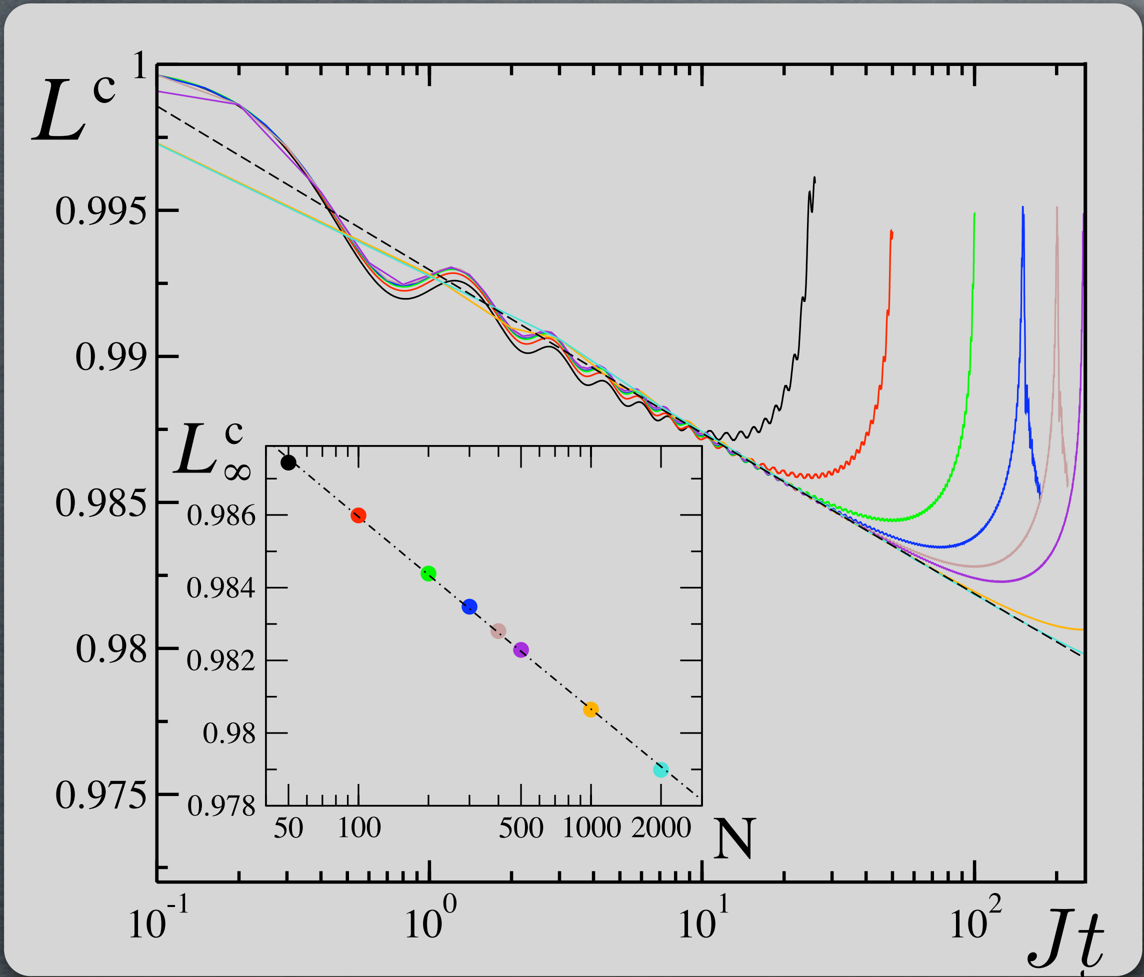
$\lambda < 1$ oscillations

$\lambda > 1$ constant



Ising chain environment

$$\lambda = \lambda_c$$

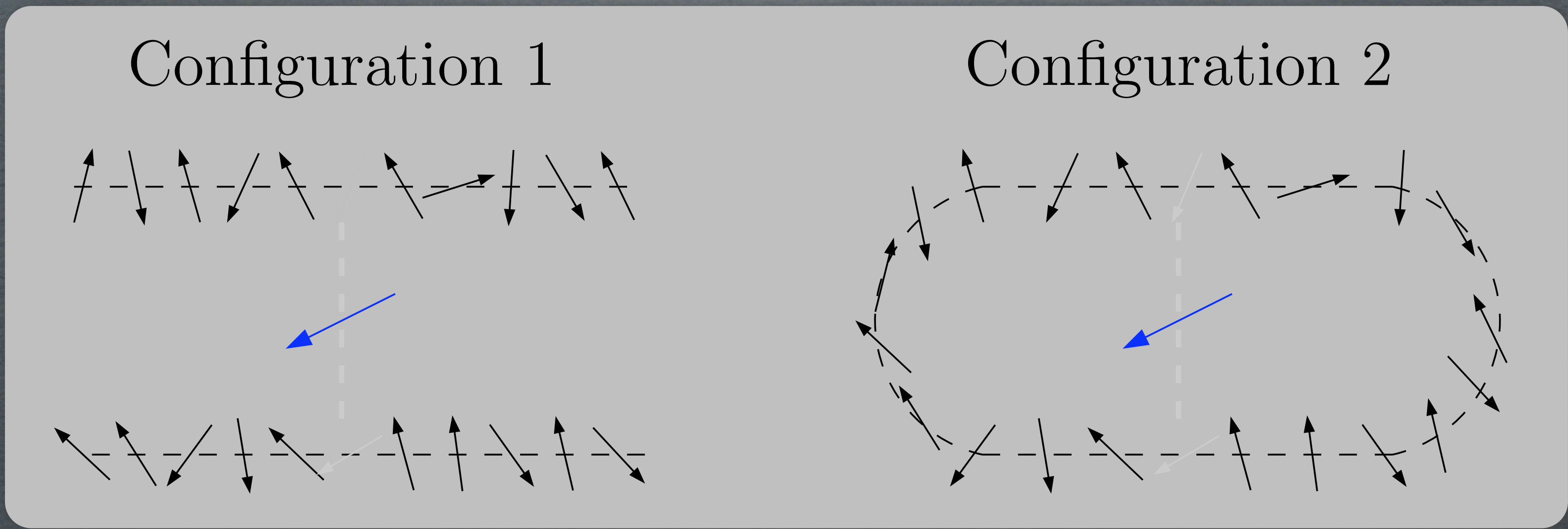


$$\mathcal{L}^c(t) = \frac{c_0}{(1 + c_1 \ln t)}$$

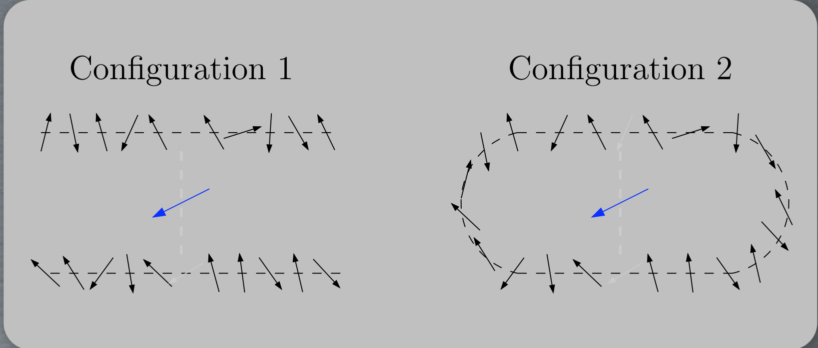
$$\mathcal{L}^c_\infty = \frac{l_\infty}{1 + \beta \ln N}$$

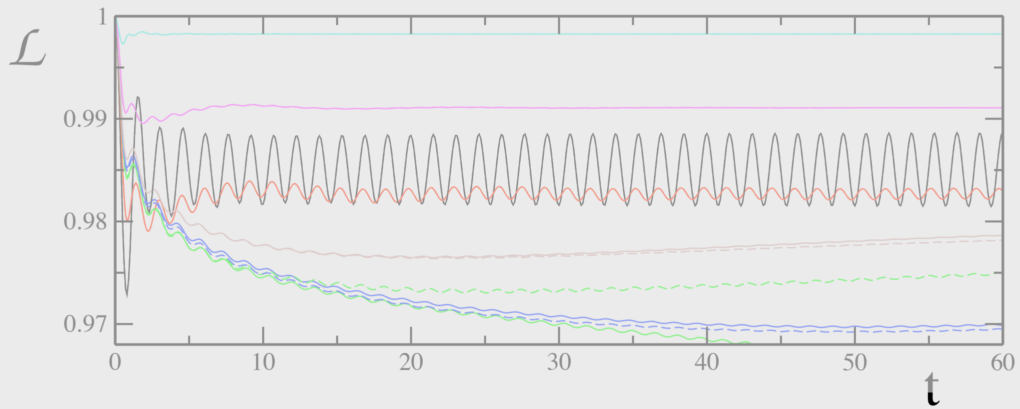
Ising chain - correlations in the bath

If the chain is not at the critical point, the decay of the coherences is independent of the number of spins in the bath



Ising chain - correlations in the bath

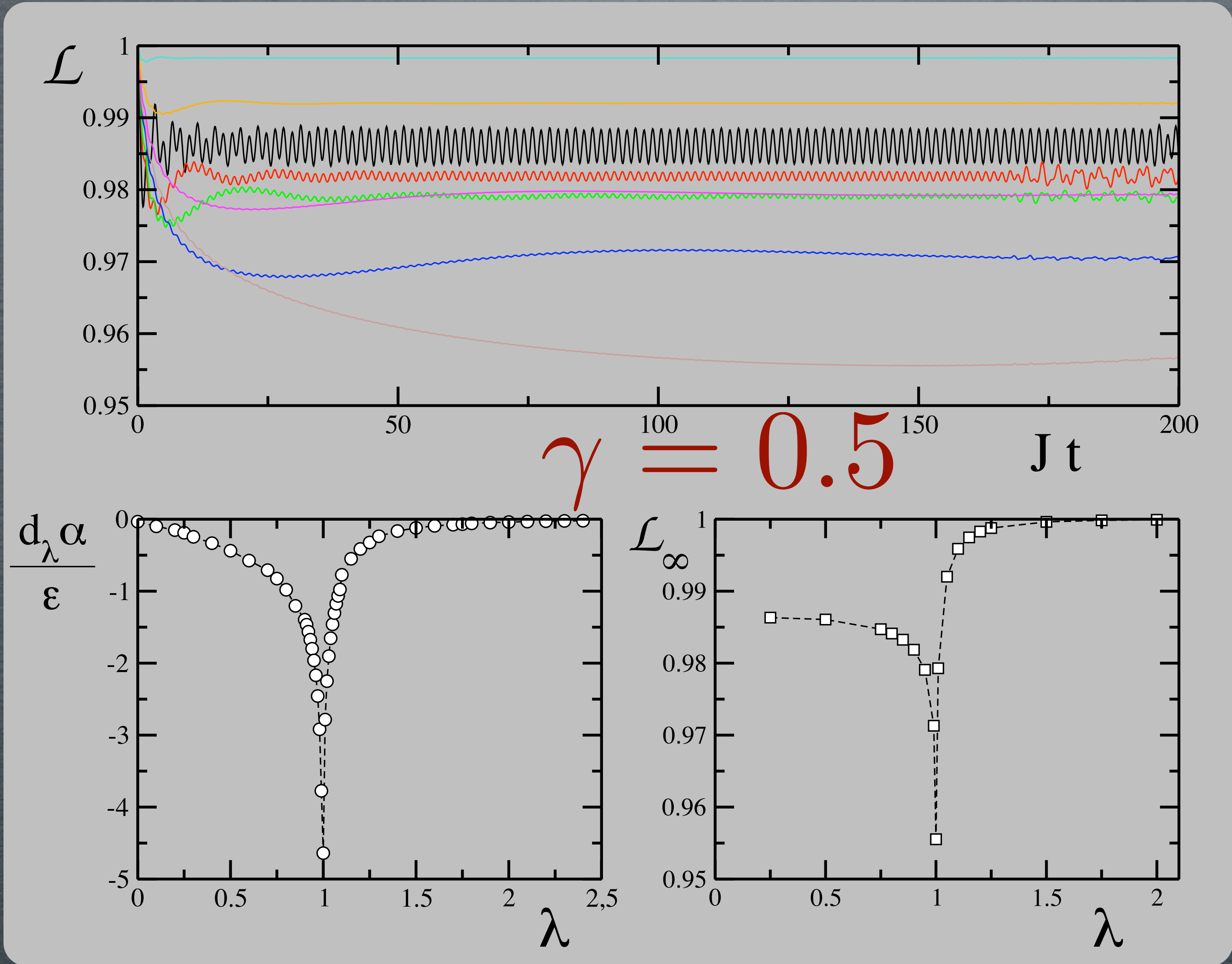




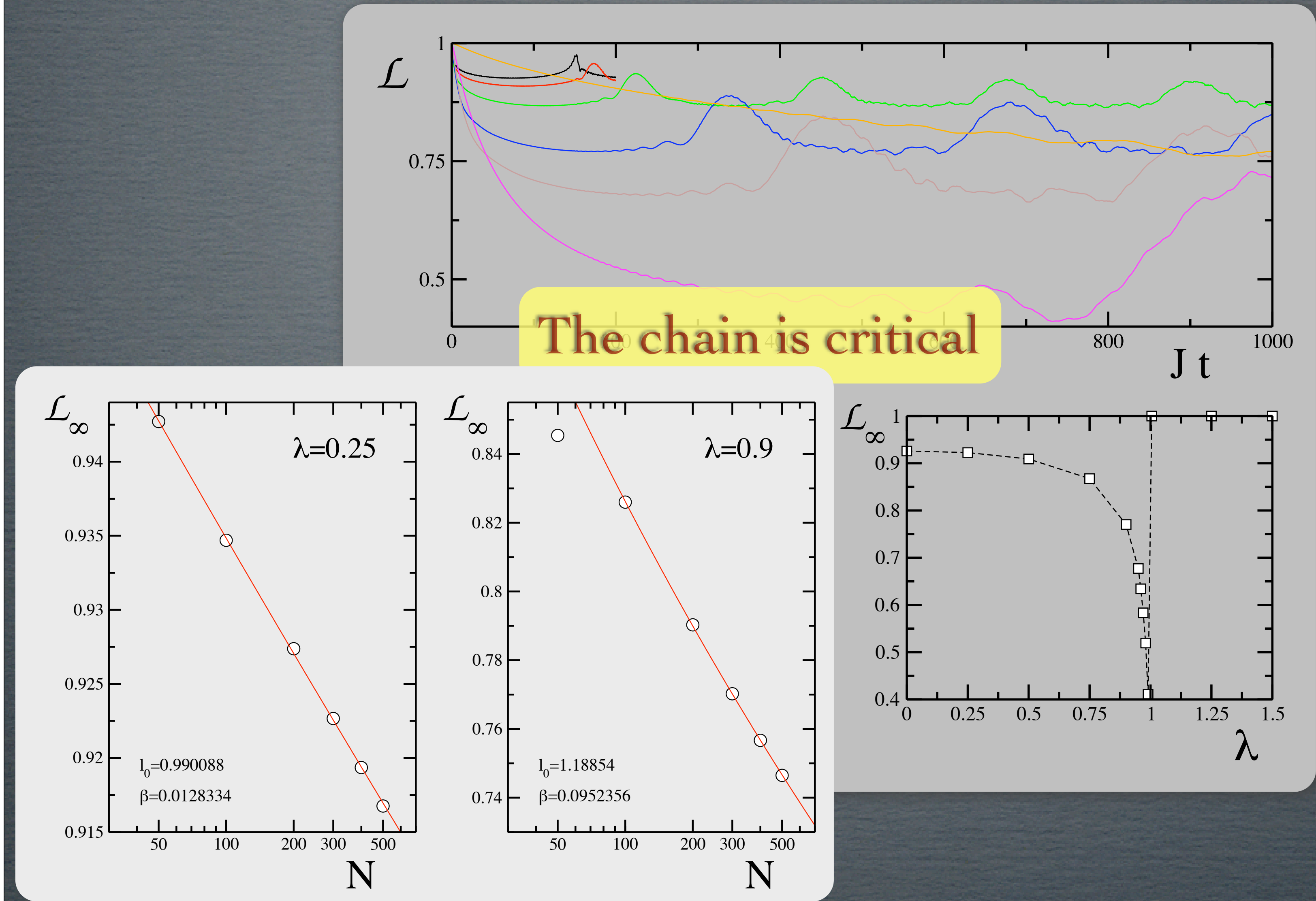
$$\lambda \neq \lambda_c$$

Ising chain environment - Universality

$\gamma \neq 0$



XY chain environment



Decoherence
vs
interaction & entanglement in the bath

see the suggestion of
C. M. Dawson, A.P. Hines, R.H. McKenzie, and G.J. Milburn G.J. (2005).

How to measure entanglement?

- Entanglement between two spins in the network

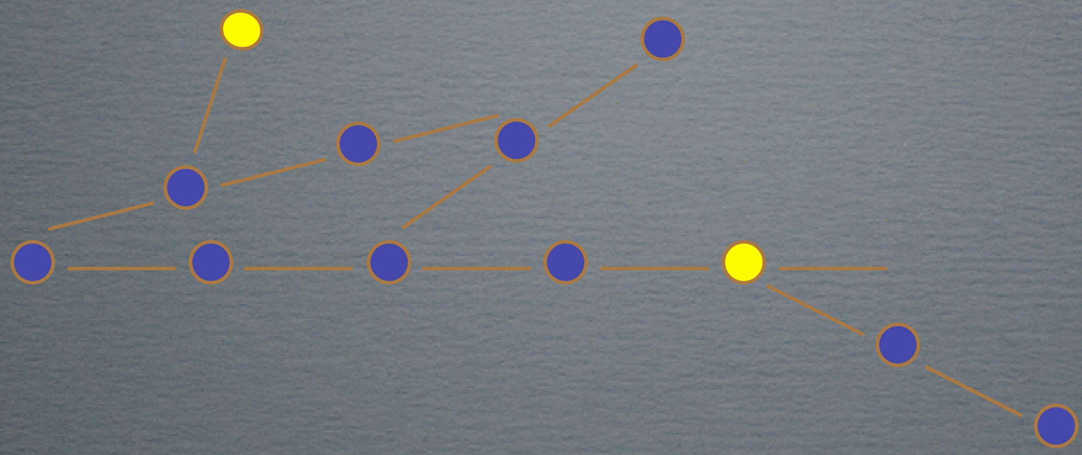
- Multipartite entanglement

- Block entropy

- Localizable entanglement

- ...

Bipartite entanglement



The state of the two selected spins is mixed

Measure of mixed state entanglement for two spin-1/2 states

- Separable state

$$| \alpha\rangle =| 00\rangle$$

- "NOT"

$$| \beta\rangle =| 11\rangle$$

$$\langle\beta | \alpha\rangle = 0$$

-Entangled state state

$$| \alpha\rangle = \frac{1}{\sqrt{2}}(| 00\rangle+ | 11\rangle)$$

- "NOT"

$$| \beta\rangle = \frac{1}{\sqrt{2}}(| 00\rangle+ | 11\rangle)$$

$$\langle\beta | \alpha\rangle = 1$$

Concurrence between spins at sites i and j

- construct

$$R = \rho(i,j)\tilde{\rho}(i,j)$$

where

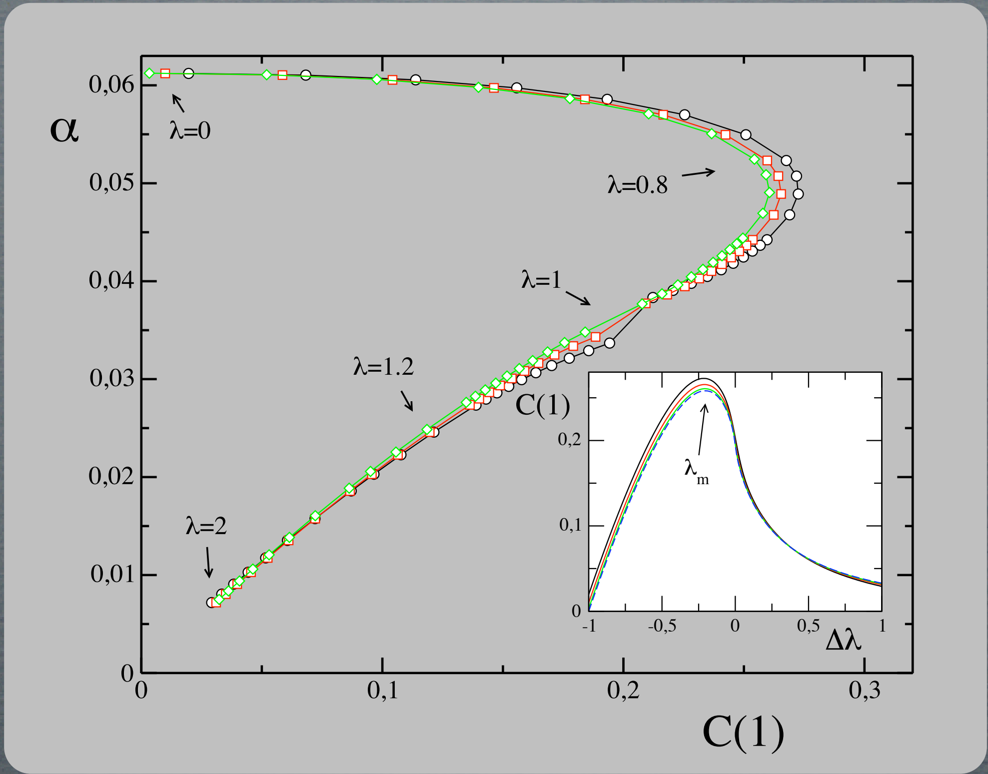
$$\tilde{\rho} \doteq \sigma^y \otimes \sigma^y \rho^* \sigma^y \otimes \sigma^y$$

- the concurrence is defined as

$$C(i,j) = \max\{0, \lambda_1(i,j) - \lambda_2(i,j) - \lambda_(i,j) - \lambda_4(i,j)\}$$

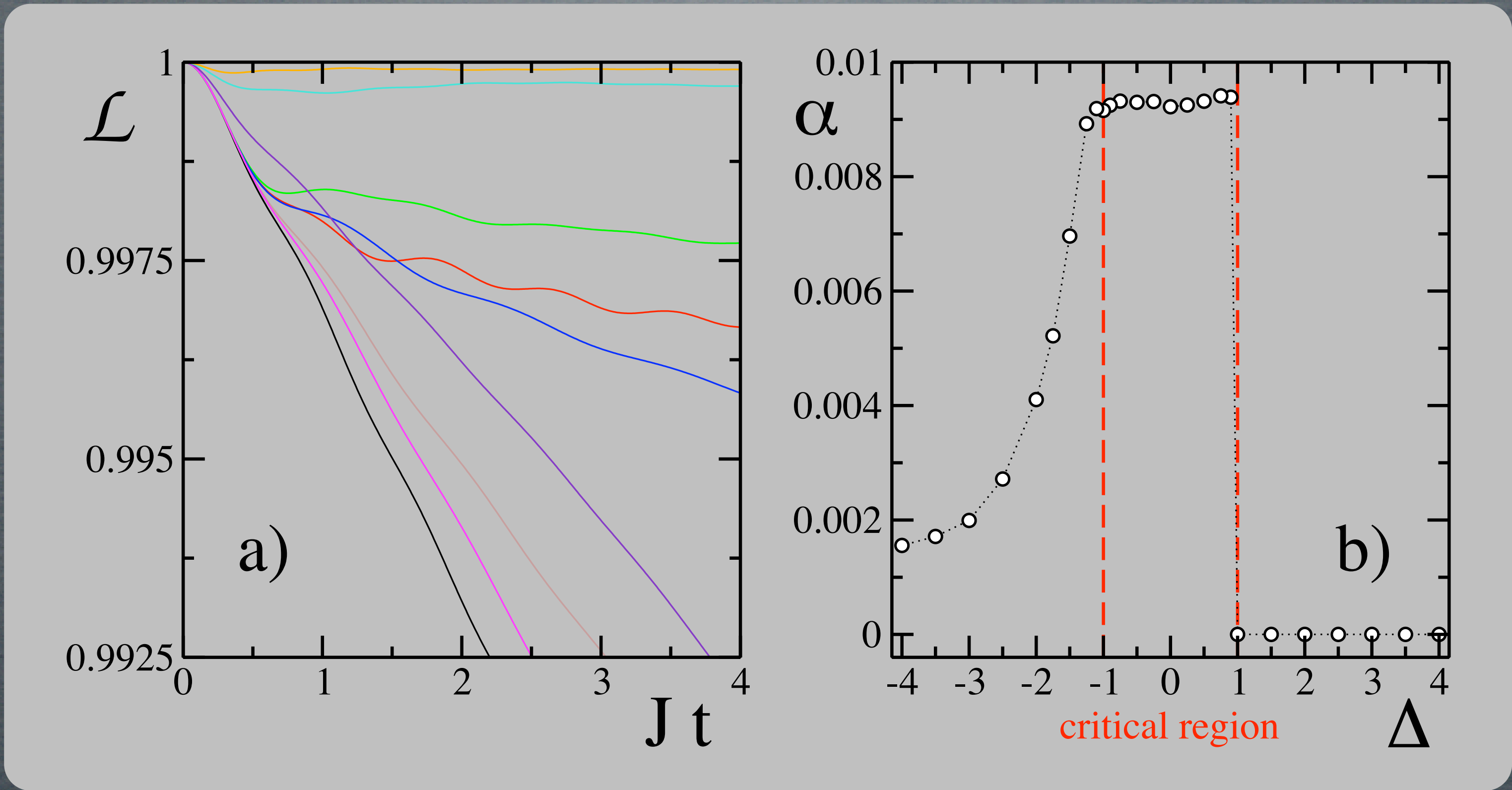
where λs are the eigenvalues of R in ascending order

Decoherence vs entanglement in the bath

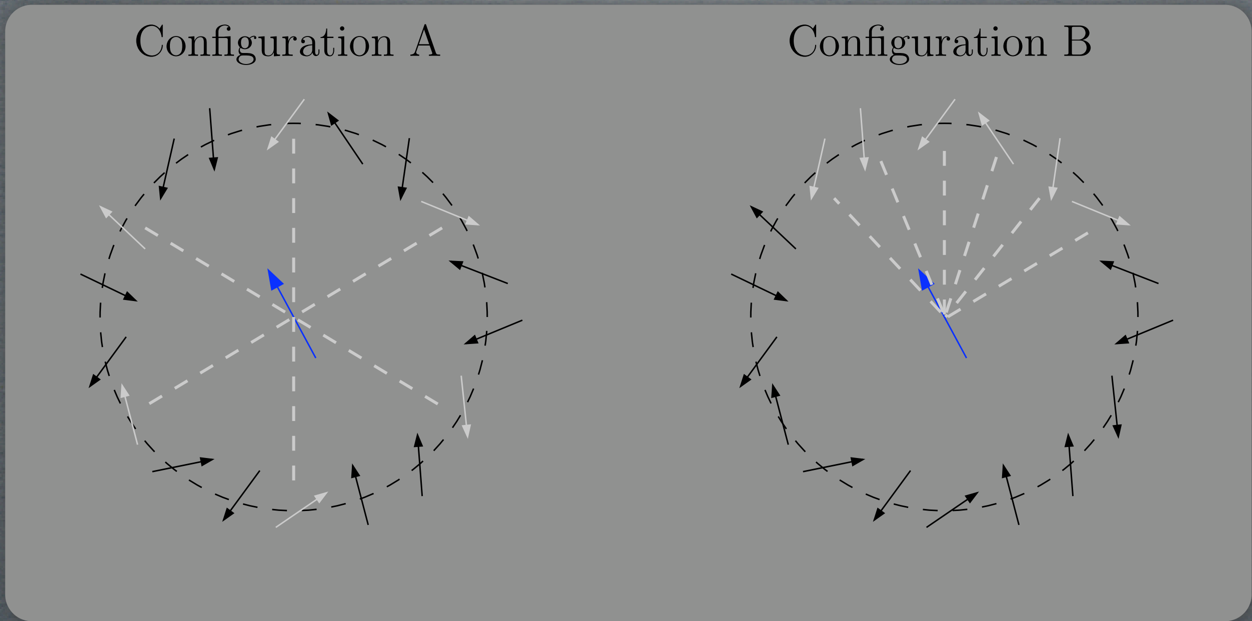


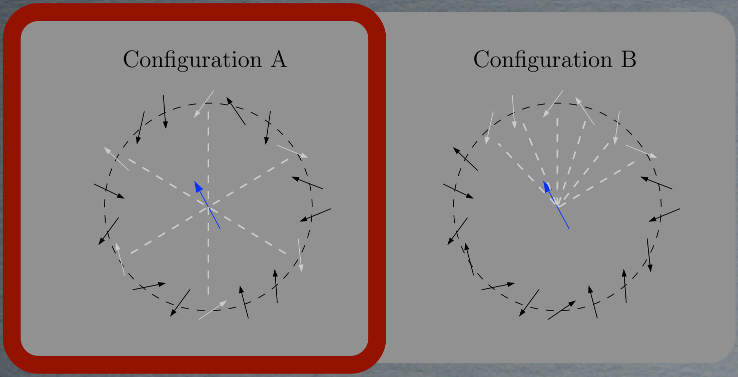
the suggestion of C.M. Dawson *et al.* (2005)

Heisenberg chain environment - tDMRG

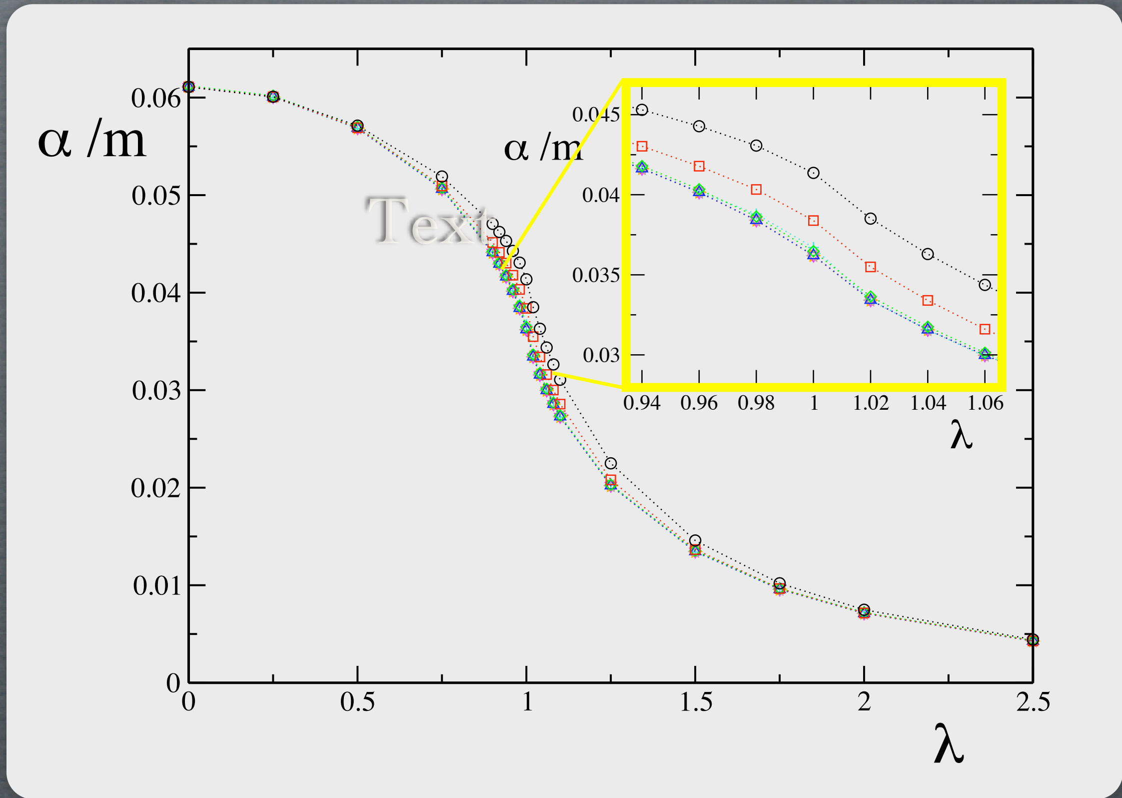


Coupling to m spins in the environment

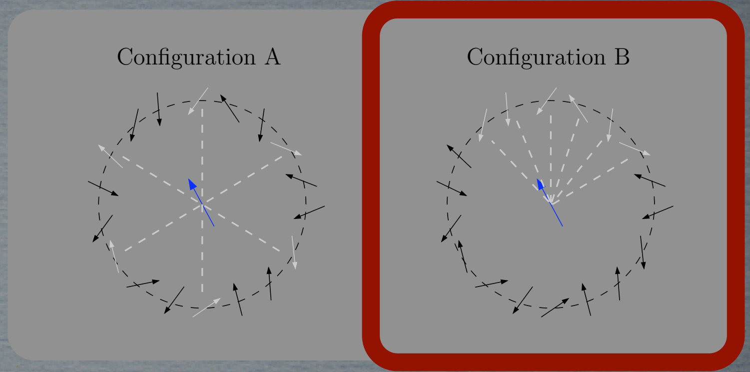


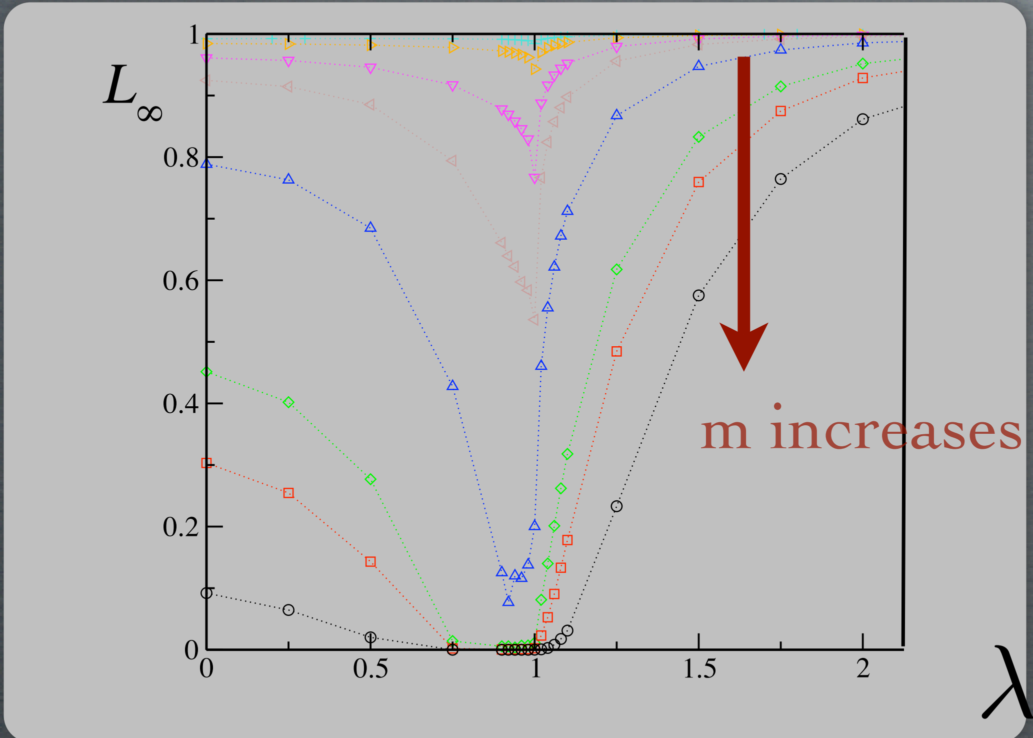


Weakly dependent on m



Strong dependence on the number of couplings





Conclusions

- ✔ Optical lattices to simulate quantum baths
- ✔ Exact solution for an interacting baths
- ✔ Numerical t-DMRG for Heisenberg bath