Intrinsic Superconductivity in graphene ?

Electron-phonon mechanisms
(Raman scattering)

Pure Coulomb mechanisms

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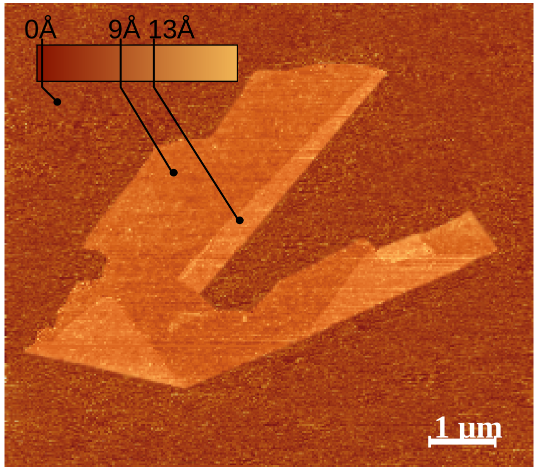
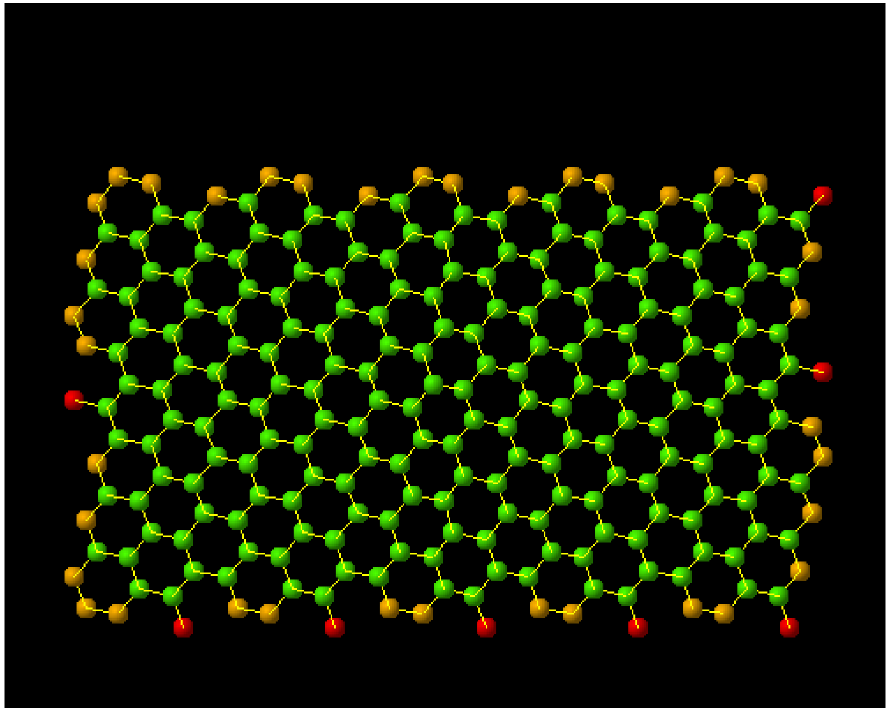


Novoselov et al, Science 306, 666 (2004)



cond-mat/0608543
cond-mat/0608515
cond-mat/0607343
Phys. Rev. B 73, 245426 (2006)
cond-mat/0604106
Phys. Rev. B 73, 241403 (2006)
Phys. Rev. B 73, 195411 (2006)
Phys. Rev. B 73, 214418 (2006)
Phys. Rev. B 73, 125411 (2006)
Phys. Rev. Lett. 96, 036801 (2006)
Phys. Rev. B 72, 174406 (2005)
Annals of Physics 321, 1559 (2006).

“The theoretical oriented scientist cannot be envied, because nature, i.e. the experiment, is a relentless and not very friendly judge of his work. In the best case scenario it only says ”maybe“ to a theory, but never ”yes“ and in most cases ”no“. If an experiment agrees with theory it means ”perhaps“ for the latter. If it does not agree it means ”no“. Almost any theory will experience a ”no“ at one point in time — most theories very soon after they have been developed.”

— **Albert Einstein**, Theoretische Bemerkungen zur Supraleitung der Metalle [Einstein, 1922].

A Personal Note

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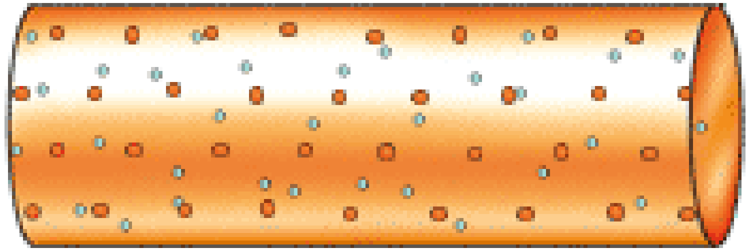
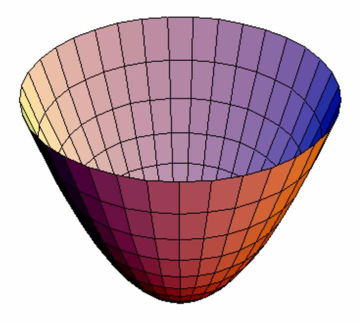
Free non-relativistic electrons in a box

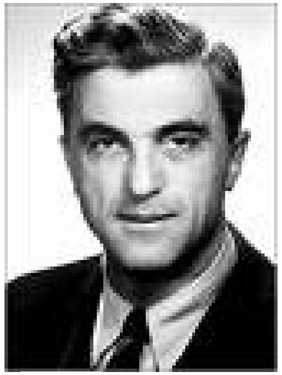


$$H = \frac{p^2}{2m}$$

Ignorance is dumped here !



LATTICE



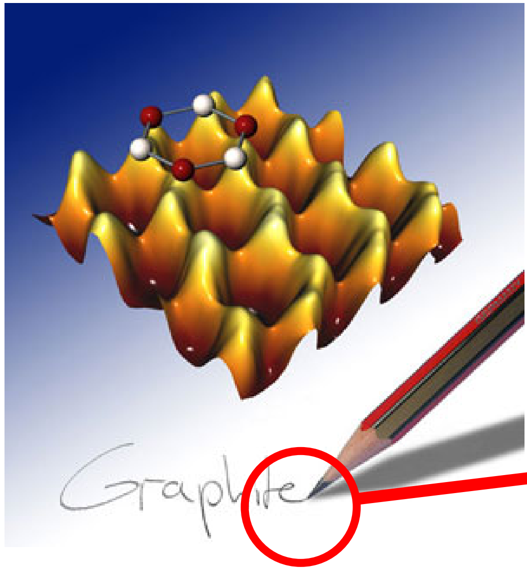
Felix Bloch

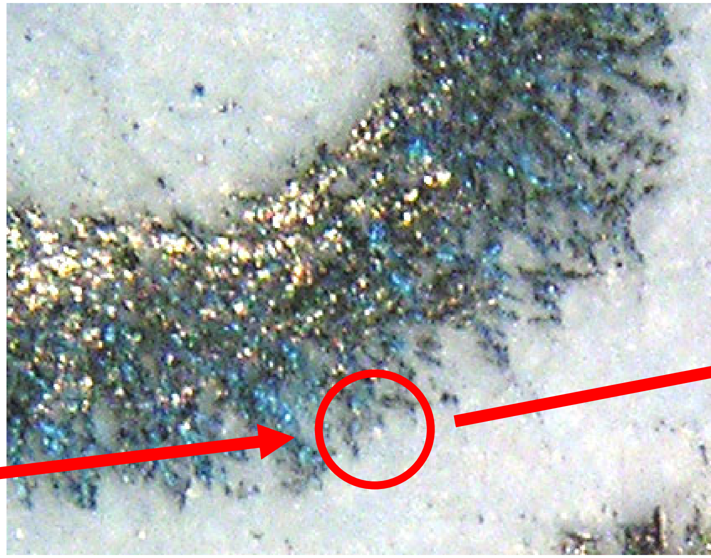
INTERACTIONS

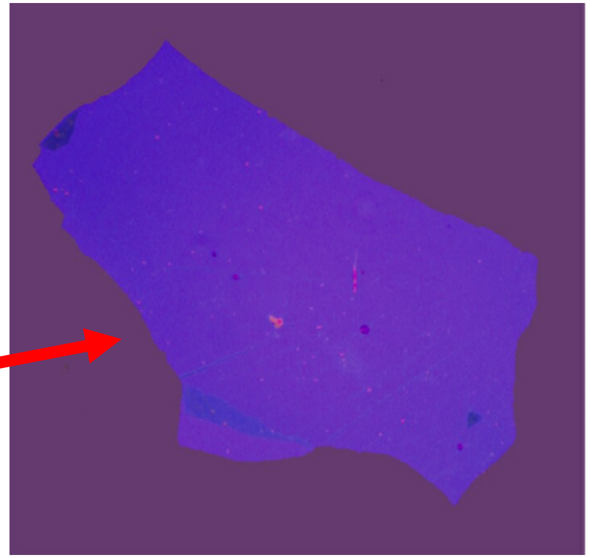
Lev Landau

Paul Drude

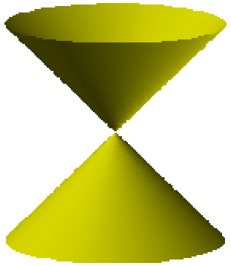
Arnold Sommerfeld







Novoselov et al, Science 306, 666 (2004)



$$H = vp \begin{pmatrix} 0 & e^{i\varphi(p)} \\ e^{-i\varphi(p)} & 0 \end{pmatrix}$$

Paul
Dirac

Free relativisitic electrons
in a box

DISORDER
INTERACTIONS

Ignorance is dumped here !

What would be the effective theory for superconductivity in graphene ?

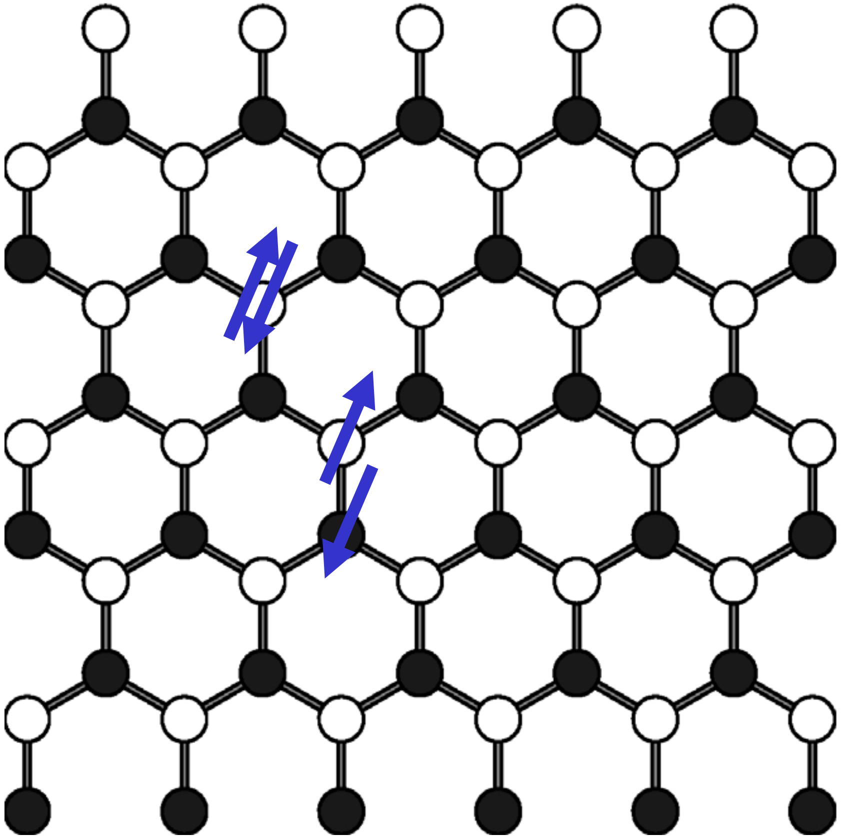
$$H^{g}_{ef} = -t\sum_{\sigma}\sum_{\langle ij\rangle} a^{\dagger}_{i\sigma}b_{j\sigma} + h.c. - \mu\sum_{i\sigma}\left[a^{\dagger}_{i\sigma}a_{i\sigma} + b^{\dagger}_{i\sigma}b_{i\sigma}\right]$$
$$+\frac{g_0}{2}\sum_{i\sigma}\left[a^{\dagger}_{i\sigma}a_{i\sigma}a^{\dagger}_{i-\sigma}a_{i-\sigma} + b^{\dagger}_{i\sigma}b_{i\sigma}b^{\dagger}_{i-\sigma}b_{i-\sigma}\right]$$
$$+g_1\sum_{\langle ij\rangle}\sum_{\sigma\sigma'} a^{\dagger}_{i\sigma}a_{i\sigma}b^{\dagger}_{j\sigma'}b_{j\sigma'}\,,$$

$$\Delta_{1,ij} = \langle a_{i\downarrow}b_{j\uparrow} - a_{i\uparrow}b_{j\downarrow}\rangle$$

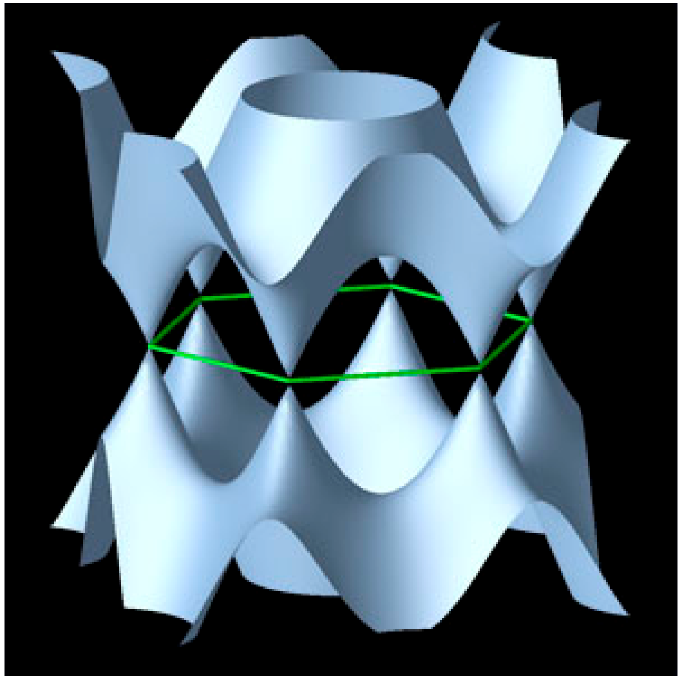
s-wave

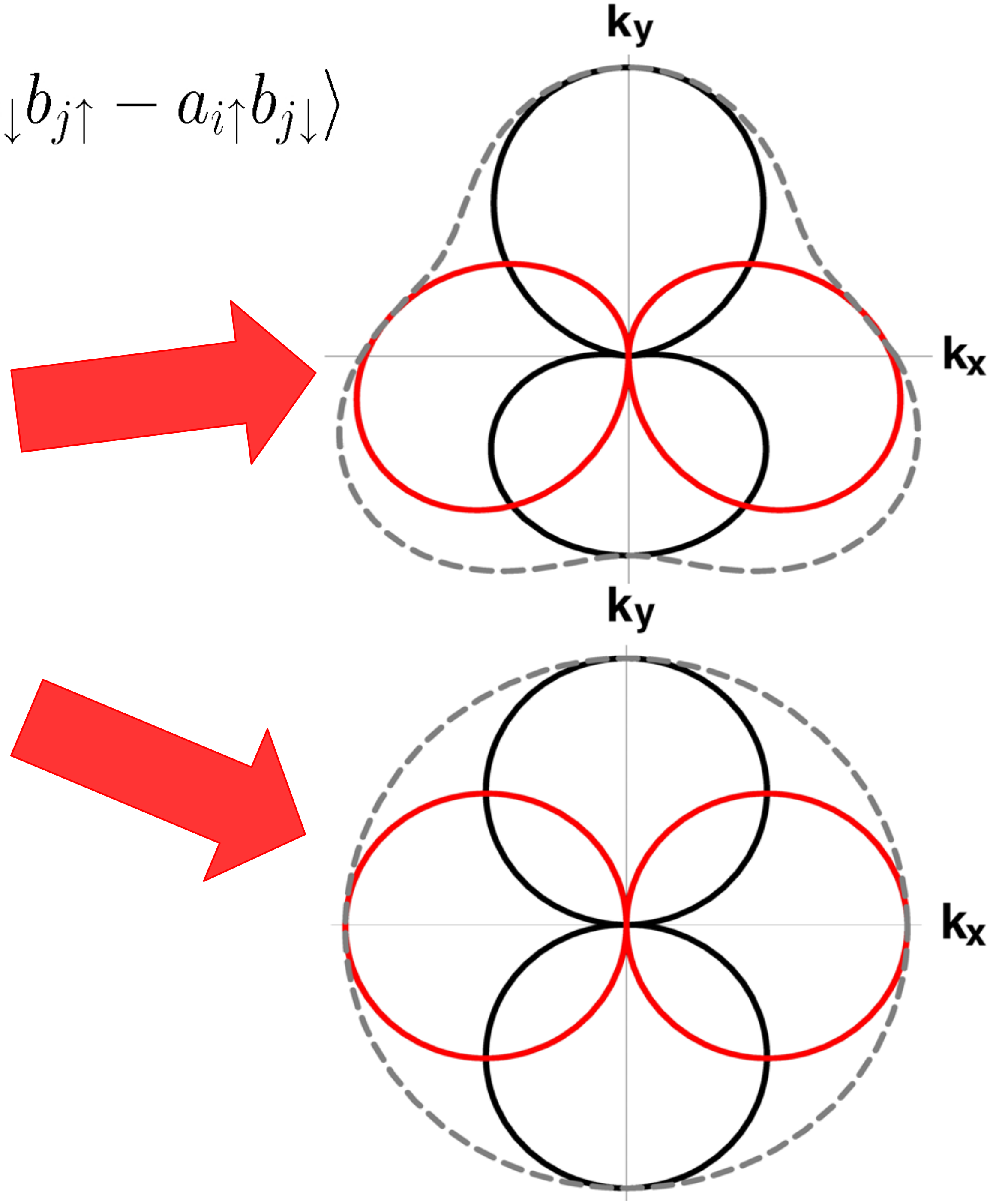
p+ip wave

$$\Psi = \psi_{S=0}\otimes\psi_{L=1}\otimes\psi_{A-B}$$



$$\Delta_{1,ij} = \langle a_{i\downarrow} b_{j\uparrow} - a_{i\uparrow} b_{j\downarrow} \rangle$$





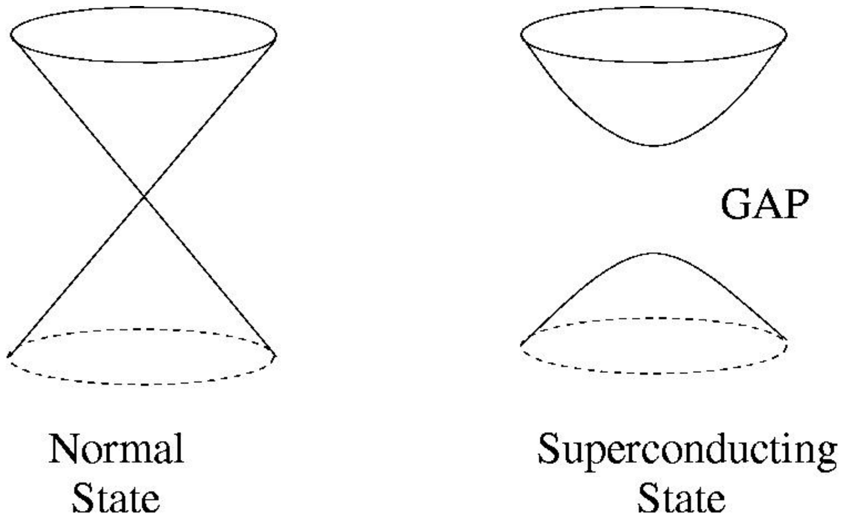
Dirac Fermion pairing

AHCN, Phys. Rev. Lett. 86, 4382 (2001)
B.Uchoa, AHCN, G. Cabrera, Phys.Rev.B 69, 144512 (2004)
B.Uchoa, G. Cabrera, AHCN, Phys.Rev.B 71, 184509 (2005)

$$H_D = \sum_{i,\mathbf{k},\sigma} \Psi^{\dagger}_{i,\sigma}(\mathbf{k})\left[v_F k_{\perp,i}\boldsymbol{\sigma}^z + v_0 k_{\parallel,i}\boldsymbol{\sigma}^x\right]\Psi_{i,\sigma}(\mathbf{k})$$

$$H_P = \sum_{\mathbf{k},a,b} [\sigma^y_{a,b}\Delta_s \psi^{\dagger}_{a,\uparrow}(\mathbf{k})\psi^{\dagger}_{b,\downarrow}(-\mathbf{k}) + \mathrm{H.c.}]$$

$$\Delta_s = -g\sum_{\mathbf{k},a,b}\sigma^y_{a,b}\langle\psi_{a,\uparrow}(\mathbf{k})\psi_{b,\downarrow}(-\mathbf{k})\rangle$$



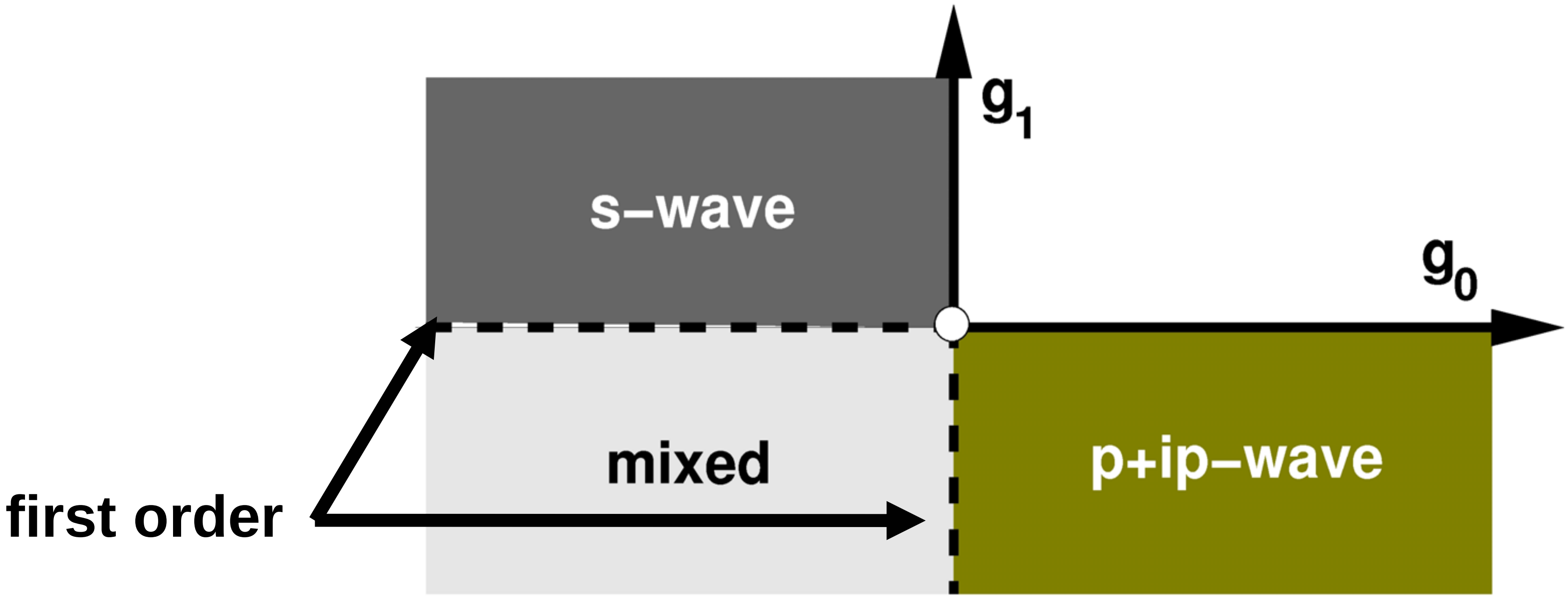
$$|\Delta_s(T,g)| = 2T\cosh^{-1}[\cosh[2\pi v_F v_0/(Tg_c)] \times e^{-(2\pi v_F v_0/Tg)}]$$

and $g_c = 4\pi^{3/2}\sqrt{v_F v_0}/\Lambda$. At $T = 0$ we have

$$|\Delta_s(0,g)| = 4\pi v_F v_0(1/g_c - 1/g),$$

The quantum theory is critical

$$\frac{T_c}{\mu} = \frac{2\gamma}{\pi} e^{\Lambda(1-g_c/g_0)\mu^{-1}-1}$$



Electron-phonon coupling

$$\begin{aligned}\mathcal{H}_{E-P} &= \frac{\partial t_0}{\partial l}\sum_{\mathbf{k},\mathbf{q}} c^\dagger_{A\mathbf{k}} c_{B\mathbf{k}+\mathbf{q}} \left\{ x_{A\mathbf{q}}\left[1-\frac{e^{i(\mathbf{k}+\mathbf{q})\mathbf{a}}}{2}-\frac{e^{i(\mathbf{k}+\mathbf{q})\mathbf{b}}}{2}\right] - x_{B\mathbf{q}}\left[1-\frac{e^{i\mathbf{k}\mathbf{a}}}{2}-\frac{e^{i\mathbf{k}\mathbf{b}}}{2}\right] + \right.\\ &+ \left. y_{A\mathbf{q}}\left[\frac{\sqrt{3}}{2}e^{i(\mathbf{k}+\mathbf{q})\mathbf{a}}-\frac{\sqrt{3}}{2}e^{i(\mathbf{k}+\mathbf{q})\mathbf{b}}\right] - y_{B\mathbf{q}}\left[\frac{\sqrt{3}}{2}e^{i\mathbf{k}\mathbf{a}}-\frac{\sqrt{3}}{2}e^{i\mathbf{k}\mathbf{b}}\right]\right\} + \text{h.c.}\end{aligned}$$

$$\frac{\partial t_0}{\partial l} = \alpha \approx 1.4\,\text{eV}\text{Å}^{-1}$$

$$\begin{pmatrix} x_{A\mathbf{q}} \\ y_{A\mathbf{q}} \\ x_{B\mathbf{q}} \\ y_{B\mathbf{q}} \end{pmatrix} \equiv \frac{1}{\sqrt{2M_C\omega_{\mathbf{q}}}}\left(b^\dagger_{\mathbf{q}} + b_{-\mathbf{q}}\right)\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix}$$

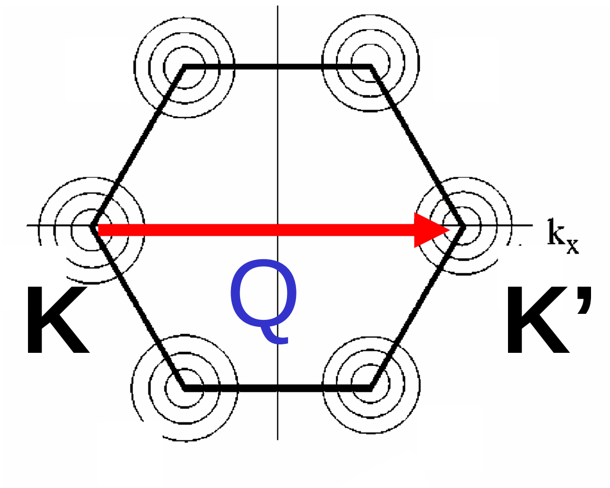
$$\lambda = \left(\frac{dt}{dl}\right)^2 \left\langle x^2 \right\rangle \frac{N(E_F)}{\omega_0}$$

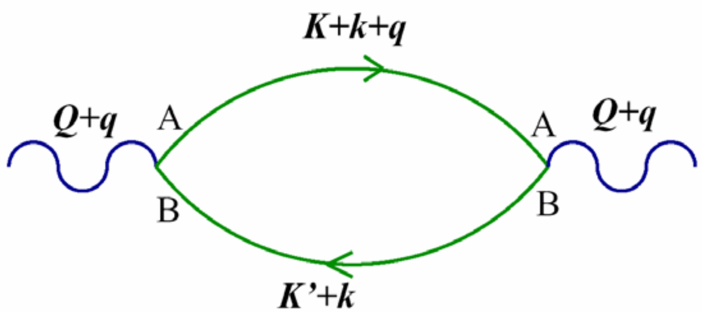
$$N_{2D}(E_F) \propto \frac{1}{W}$$

Weak coupling

$$N_g(E_F) \propto \frac{E_F}{v_F^2/a^2} \propto \frac{E_F}{W^2}$$

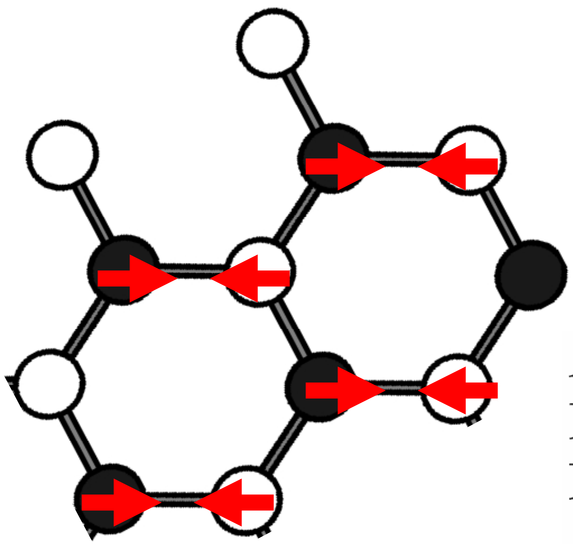
$$\mathcal{H}_{\mathbf{Q}} \equiv \frac{3\alpha}{2} \sum_{\mathbf{k}} c^{\dagger}_{A\mathbf{K}} c_{B\mathbf{K}'+\mathbf{k}} \left(x_{A\mathbf{Q}} - x_{B\mathbf{Q}} + \right.$$
$$\left. + \; i y_{A\mathbf{Q}} + i y_{B\mathbf{Q}} \right) + \text{h.c.} \, ,$$





$$\delta\omega_{\mathbf{Q}} = -\frac{9\sqrt{3}}{2\pi} \left(\frac{\partial t_0}{\partial l} \right)^2 \frac{k_{\mathrm{F}} a}{M_C \omega_{\mathbf{Q}} t_0}$$

$$\delta\omega_{\mathbf{Q}}(\mathrm{eV}) \approx -2.92 \times 10^{-10} \sqrt{n(\mathrm{cm}^{-2})}$$

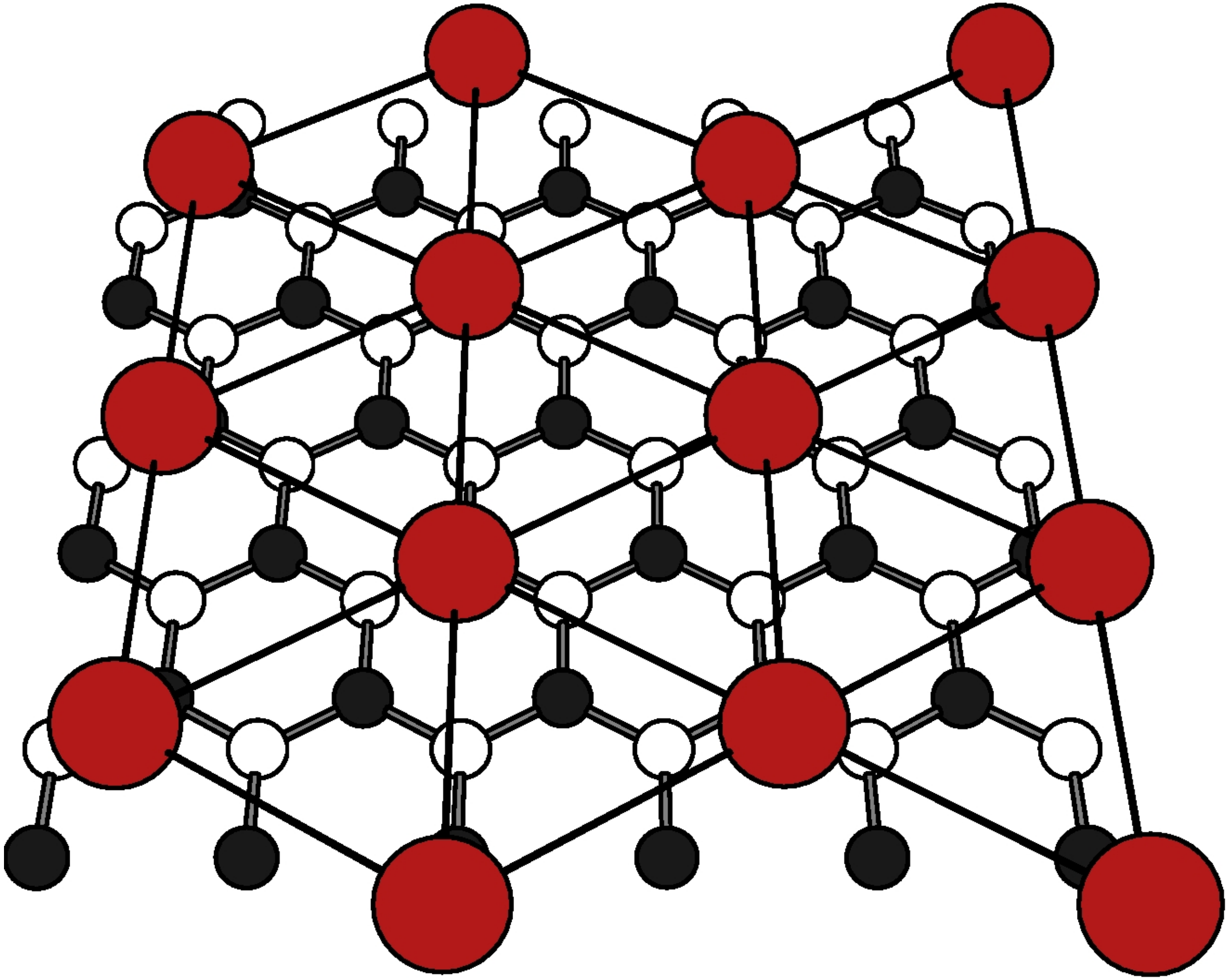


A. C. Ferrari, J. C. Meyer, V. Scardaci, C. Casiraghi, M. Lazzeri, F. Mauri, S. Piscanec, D. Jiang, K. S. Novoselov, S. Roth, et al., cond-mat/0606284.

A. Gupta, G. Chen, P. Joshi, S. Tadigadapa, and P. C. Eklund, cond-mat/0606593.

D. Graf, F. Molitor, K. Ensslin, C. Stampfer, A. Jungen, C. Hierold, and L. Wirtz, cond-mat/0607562.

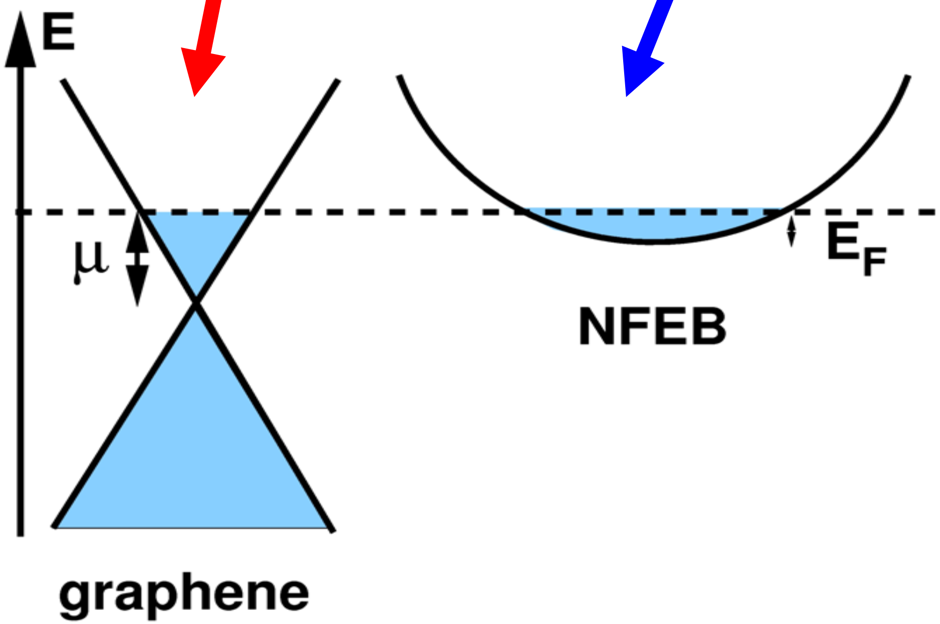
A proposal

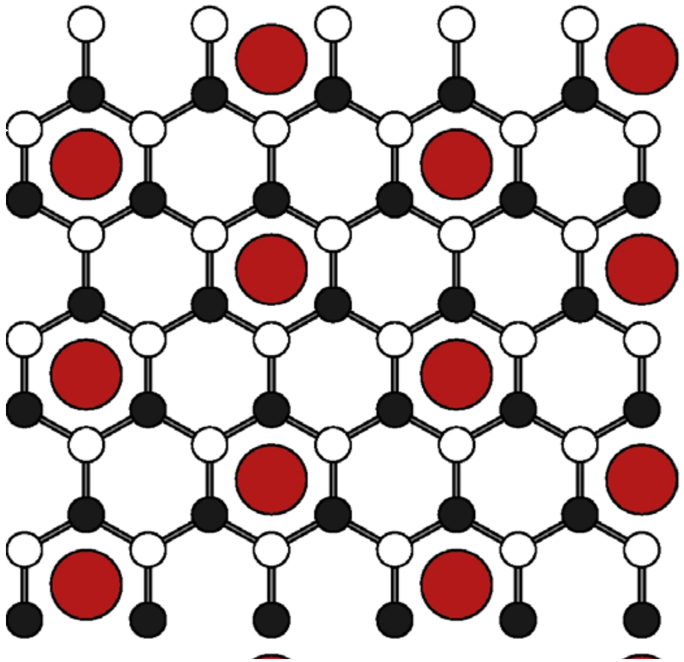


The idea: start with the non-interacting problem

$$H_t = -t \sum_{\langle ij \rangle} \sum_{\sigma=\uparrow\downarrow} (a^\dagger_{i,\sigma} b_{j,\sigma} + h.c.) - \mu_0 \sum_i \hat{n}_{g,i}$$

$$H_{NFEB} = \sum_{\sigma,\mathbf{k}} \epsilon_\mathbf{k} f^\dagger_{\mathbf{k}\sigma} f_{\mathbf{k}\sigma}$$

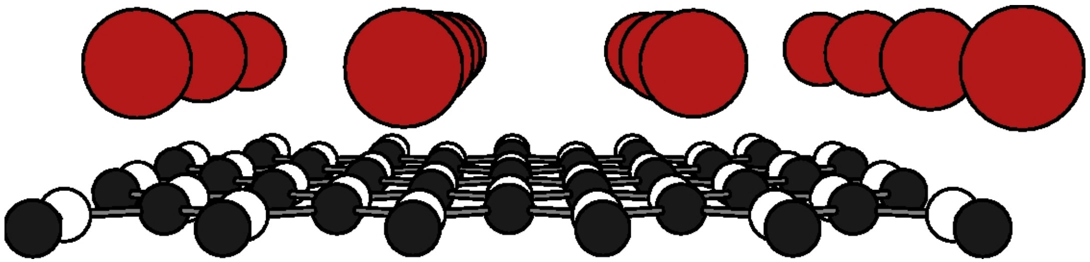




And add Coulomb interactions

$$H_I = \sum_{ij} V(\mathbf{x}_i - \mathbf{x}_j)\hat{n}_f(\mathbf{x}_i)\hat{n}_f(\mathbf{x}_j) + \sum_{ij} V(\mathbf{R}_i - \mathbf{R}_j)\hat{n}_{g,i}\hat{n}_{g,j}$$
$$+2\sum_{ij} V(\mathbf{x}_i - \mathbf{R}_j)\hat{n}_f(\mathbf{x}_i)\hat{n}_{g,j}\,.$$

plane distance d



All interactions are repulsive !

Graphene Effective Interaction (RPA)

$$V^{g}_{ef,\mathbf{k}}(\omega) \;=\; \frac{V_{0,\mathbf{k}}}{\epsilon_T(\mathbf{k},\omega)}\left[1-(V_{\mathbf{k}}-V_{d,\mathbf{k}})\chi^0_m(\mathbf{k},\omega)\right]$$

The interaction can be strongly attractive close to the zeros of the dielectric function

$$\epsilon_T[\mathbf{q},\Omega_p(q)] \;=\; 0$$

plasmon

where electrons resonate with plasmons !

Fröhlich, H.
J. Phys. C **1**, 544-548 (1968)

Dielectric function

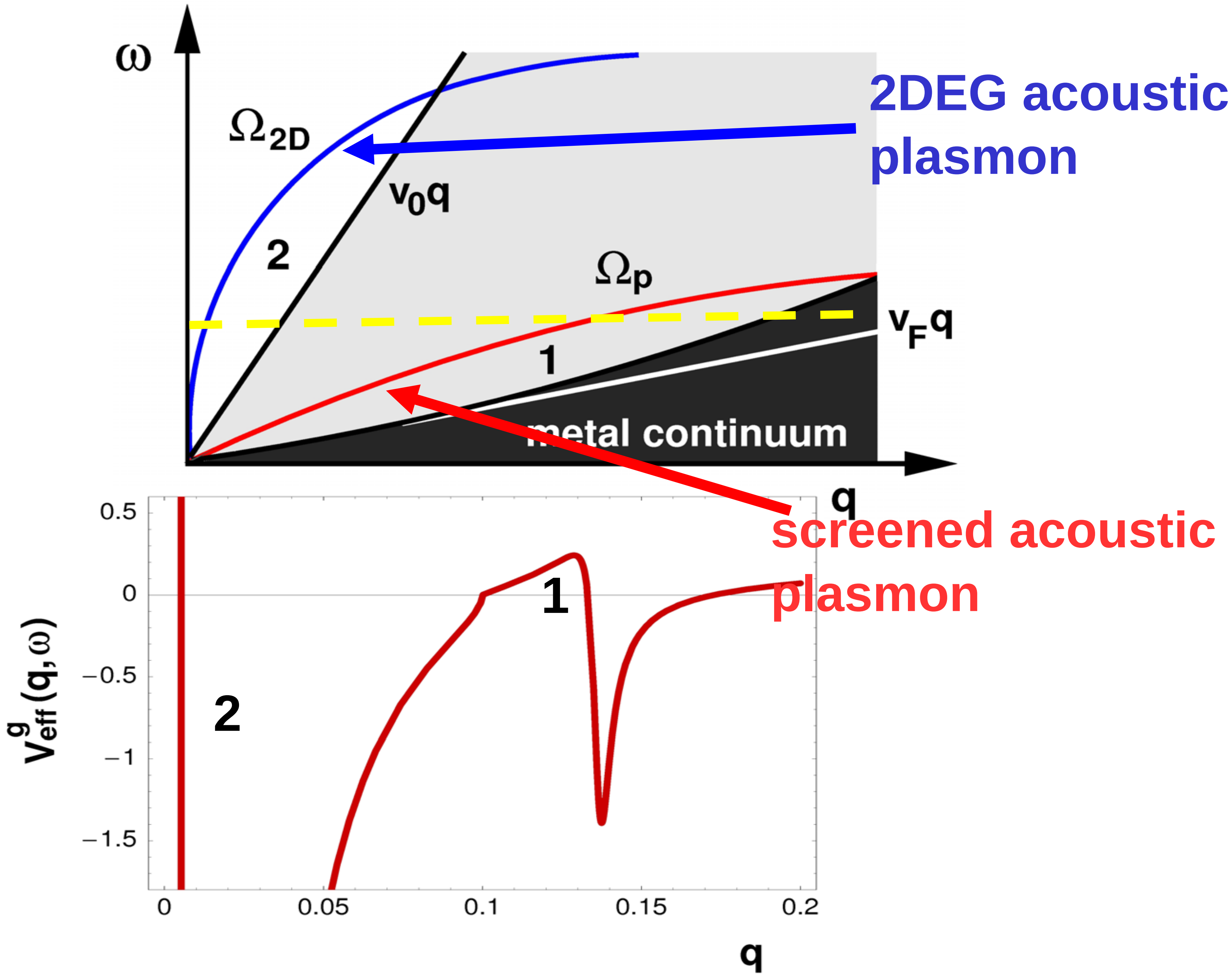
$$\epsilon_T(\mathbf{k},\omega) \;=\; 1 - V_{0,k}\left[\Pi_g^0(\mathbf{k},\omega) + \Pi_m^0(\mathbf{k},\omega)\right] + \left[V_{0,\mathbf{k}}^2 - V_{d,\mathbf{k}}^2\right]\chi_m^0(\mathbf{k},\omega)\chi_g^0(\mathbf{k},\omega)$$

where $V_{d,q} = \frac{2\pi e^2}{\epsilon_0 q}\,\mathrm{e}^{-qd}$ **2D Coulomb interaction**

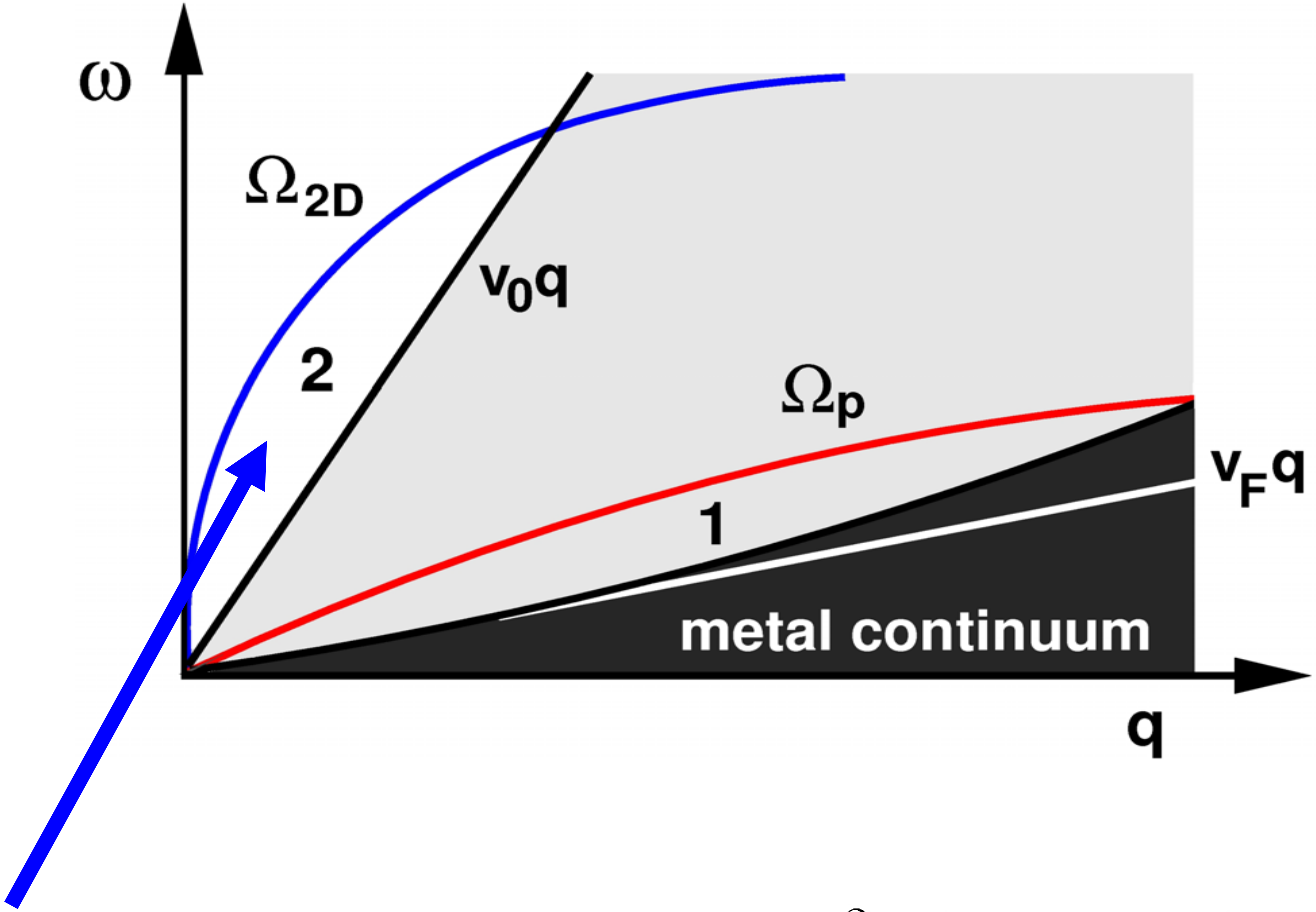
$$\Pi_g^0(q,\omega) \;=\; -(2\mu/\pi v_0^2)\left[1 - \omega/\sqrt{\omega^2 - v_0^2 q^2}\right]$$

$$V_{0,q}\Pi_m^0(q,\omega) = \Omega_m^2/\omega^2$$

$$\Omega_m(q) = e\sqrt{2E_F q/\epsilon_0}$$



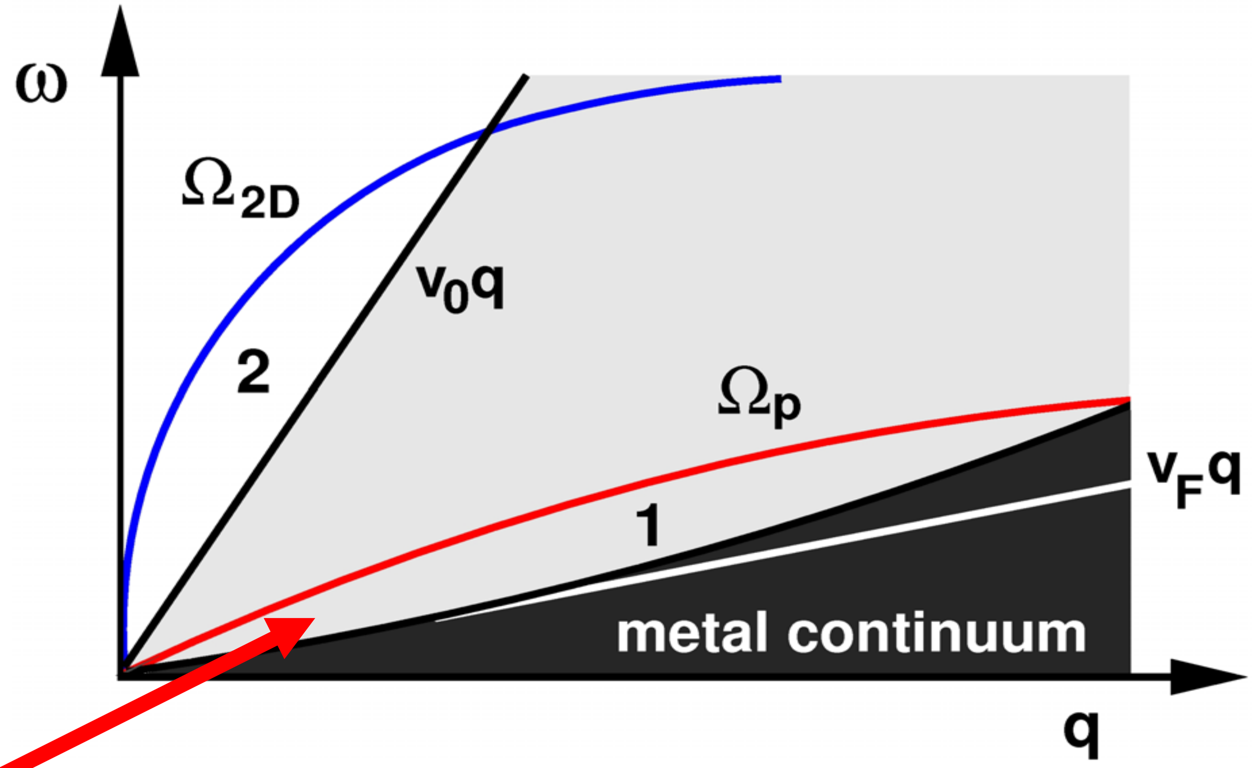
ω
Ω2D
v0q
2
Ωp
vFq
1
metal continuum
q
2DEG acoustic plasmon
screened acoustic plasmon
0.5
0
−0.5
−1
−1.5
2
1
0
0.05
0.1
0.15
0.2
q
V^g_eff(q,ω)



$$V^g_{ef}(q,\omega \gg v_0 q) \;=\; V_q \left[\frac{\Omega^2_{2D}(q)}{\omega^2 - \Omega^2_{2D}(q)} + 1 \right]$$

$$\Omega_{2D}(q) \;=\; e\sqrt{2(E_F+\mu)q/\epsilon_0}$$

2DEG plasmon



$$V^g_{ef}(q, \omega \ll v_0 q) = \frac{V_q}{\epsilon_g(q,0)}\left[\frac{\Omega_p^2(q)}{\omega^2 - \Omega_p^2(q)} + 1\right]$$

$$\Omega_p(q) \to \sqrt{E_F^*/(2\mu)}\, v_0 q$$

screened acoustic plasmon

The screened mode will exist if it is not overdamped by the electronic particle-hole continuum:

$$E_F \ll \frac{\epsilon_0 v_0}{4\alpha_g d} \sim 0.9 \text{ eV}$$

If *x* is the fraction of electrons per metallic atom that migrates to graphene in the compound, $\mathbf{C_m A_n}$

this condition is equivalent to

$$x \ll 0.12(m/n)$$

Superconductivity is favored for a sufficiently diluted coverage of the metal (large n) !

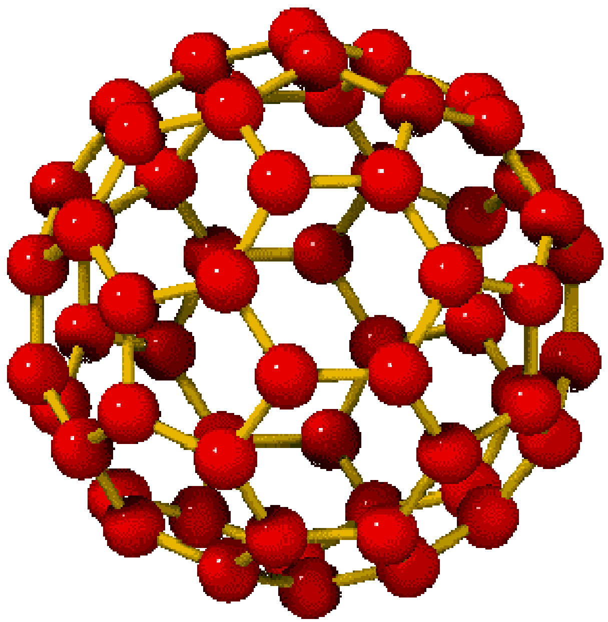
Carbon Superconductivity

Fullerenes

C_{60} is an insulator but

$C_{60}K_3$ is a superconductor (18 K)

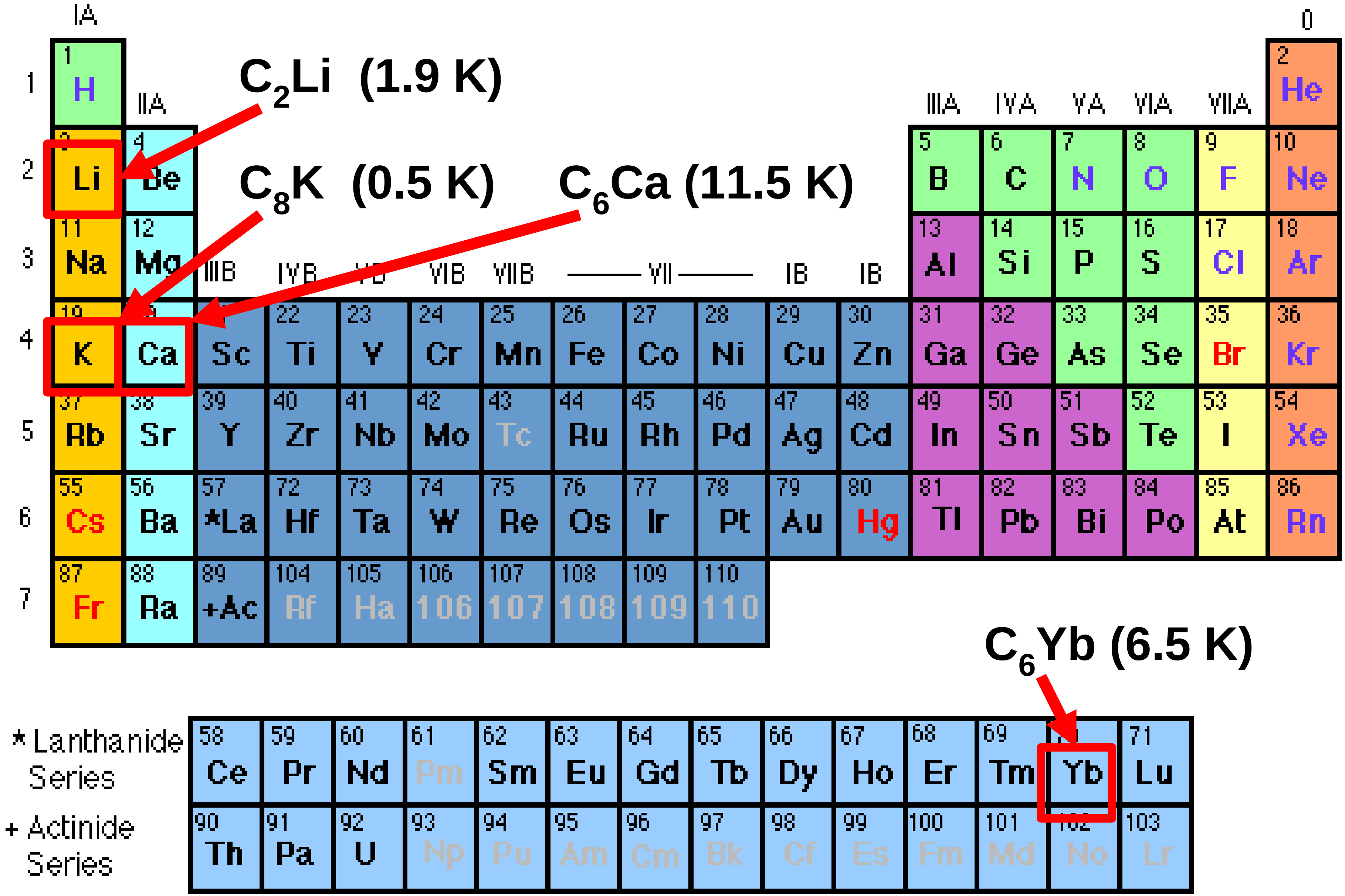
Hebard ***et al.*****, Nature 352, 223 (1991)**



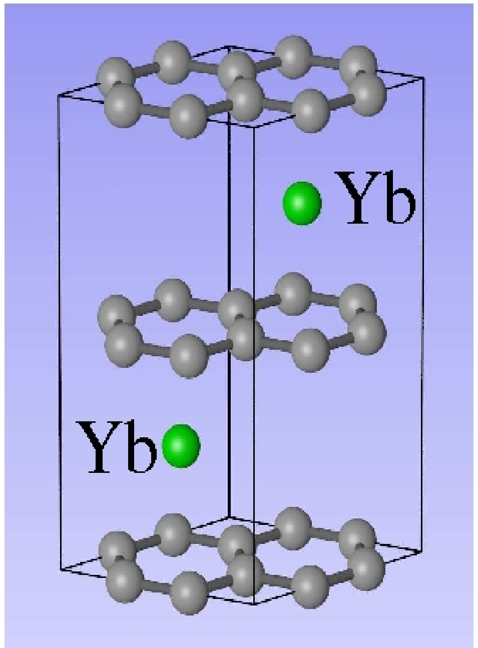
C_{60}buckyball

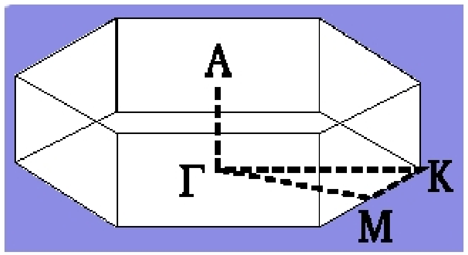
but $C_{60}K_4$ is an insulator indicating that superconductivity happens with metal dilution

Intercalated Graphite

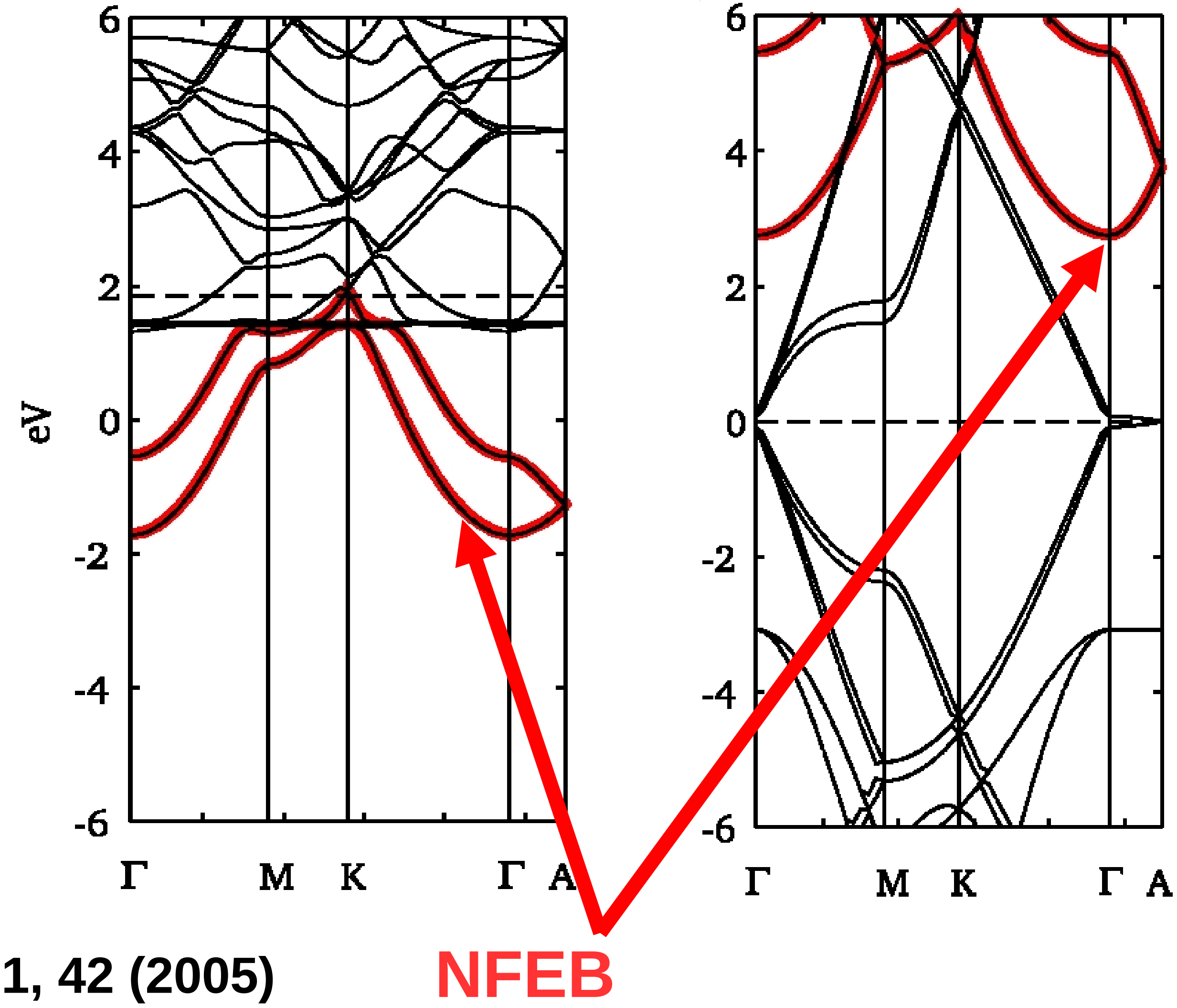


Graphite intercalated compounds (GIC)



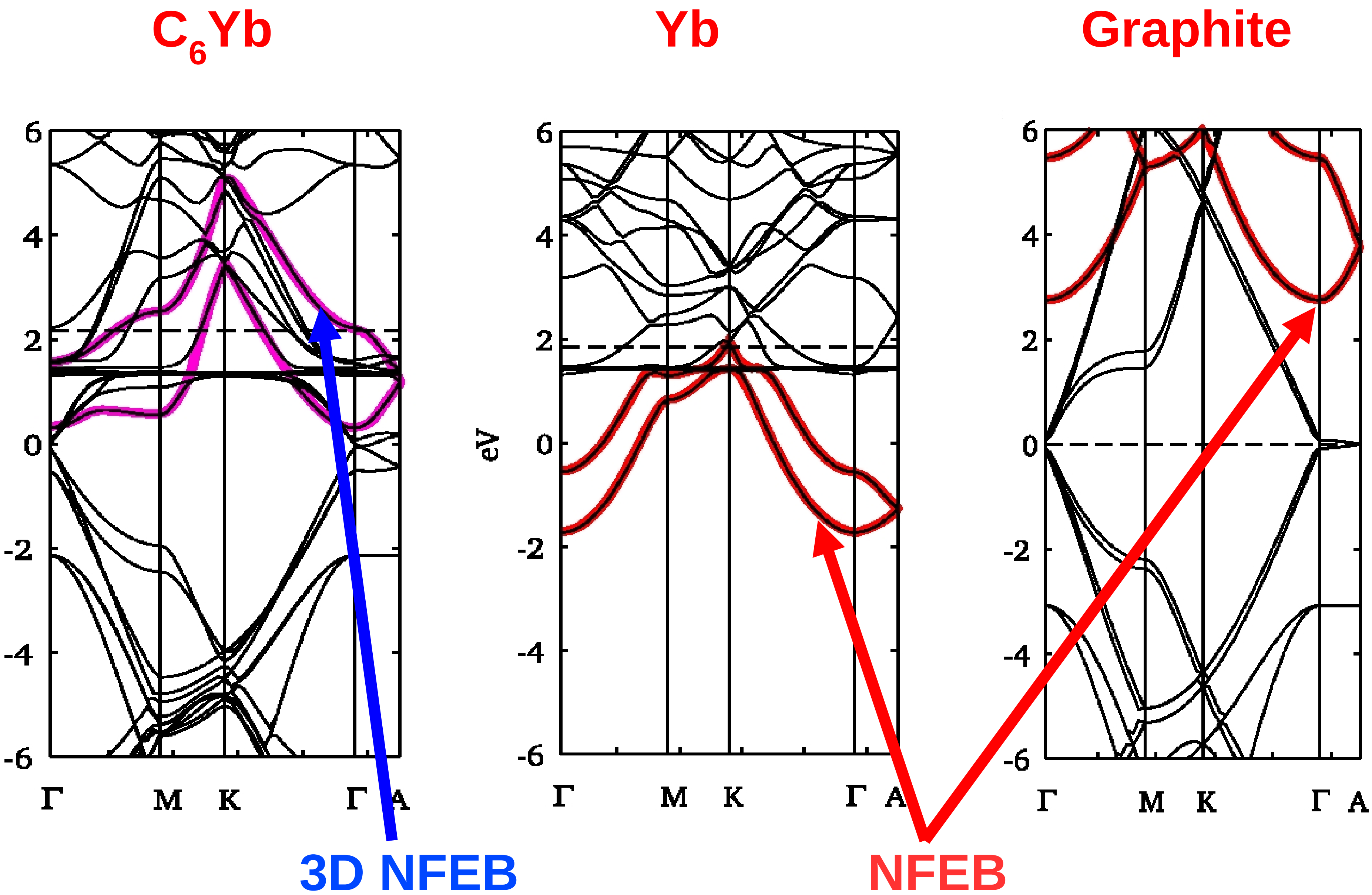


Yb **Graphite**

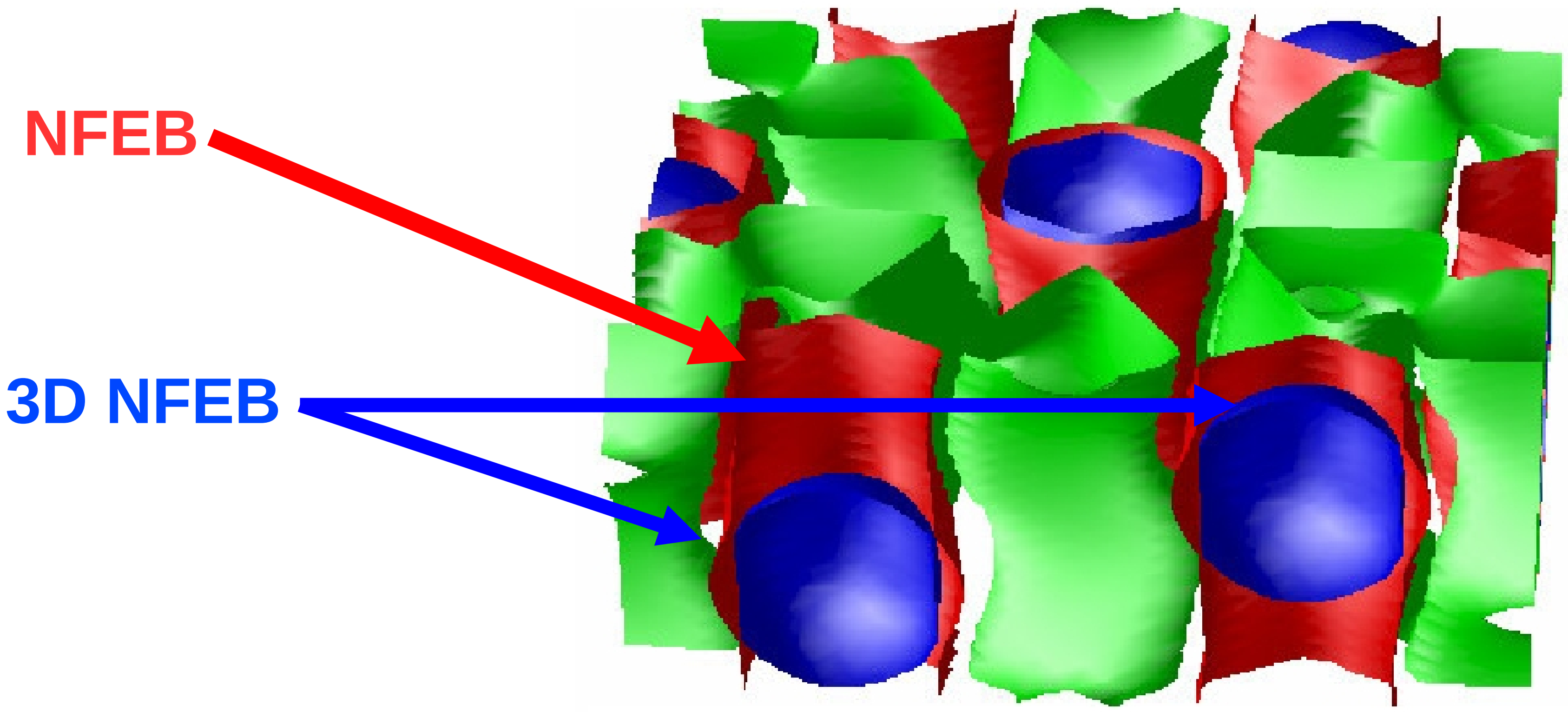


Czanyi *et al.*, **Nat. Phys. 1, 42 (2005)**

Graphite intercalated compounds (GIC)



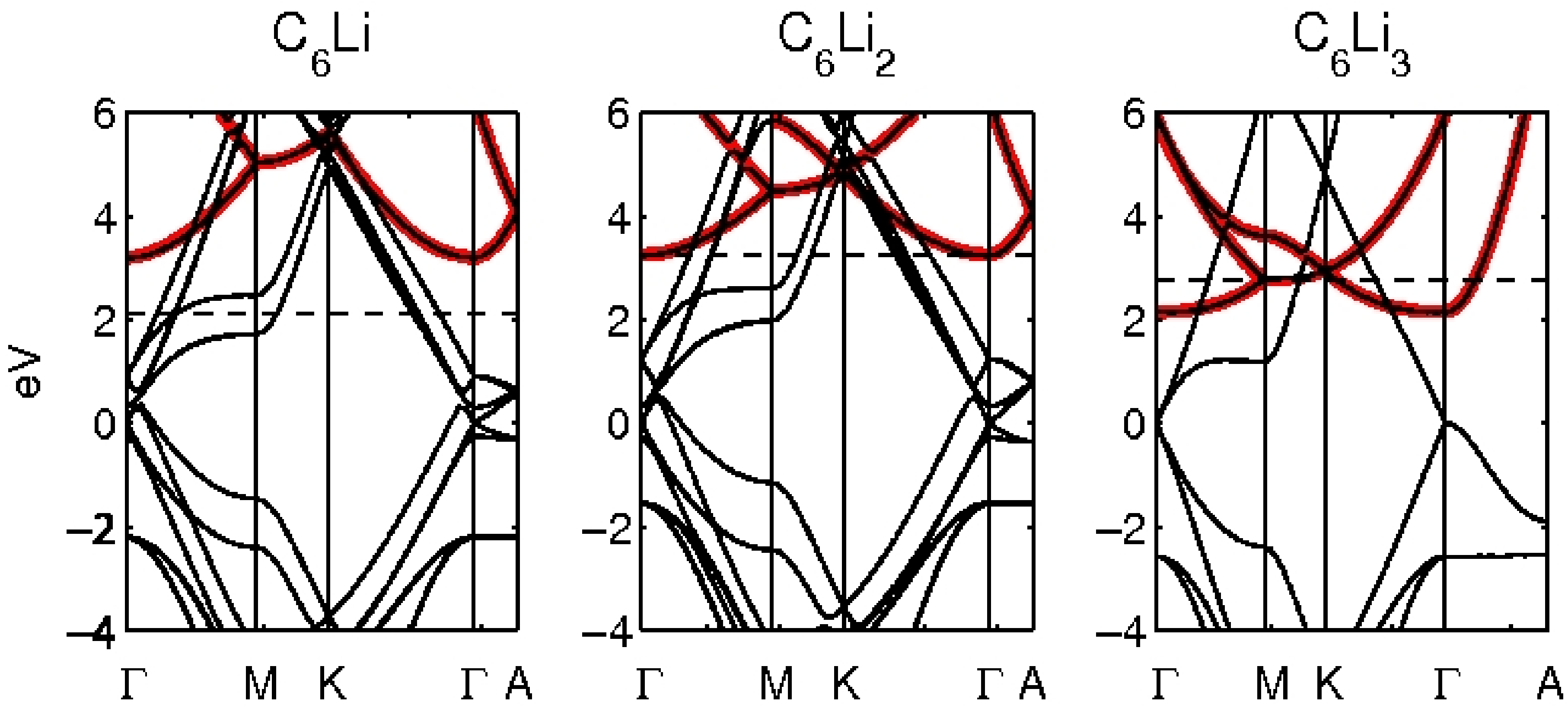
Graphite intercalated compounds (GIC)



Fermi surface

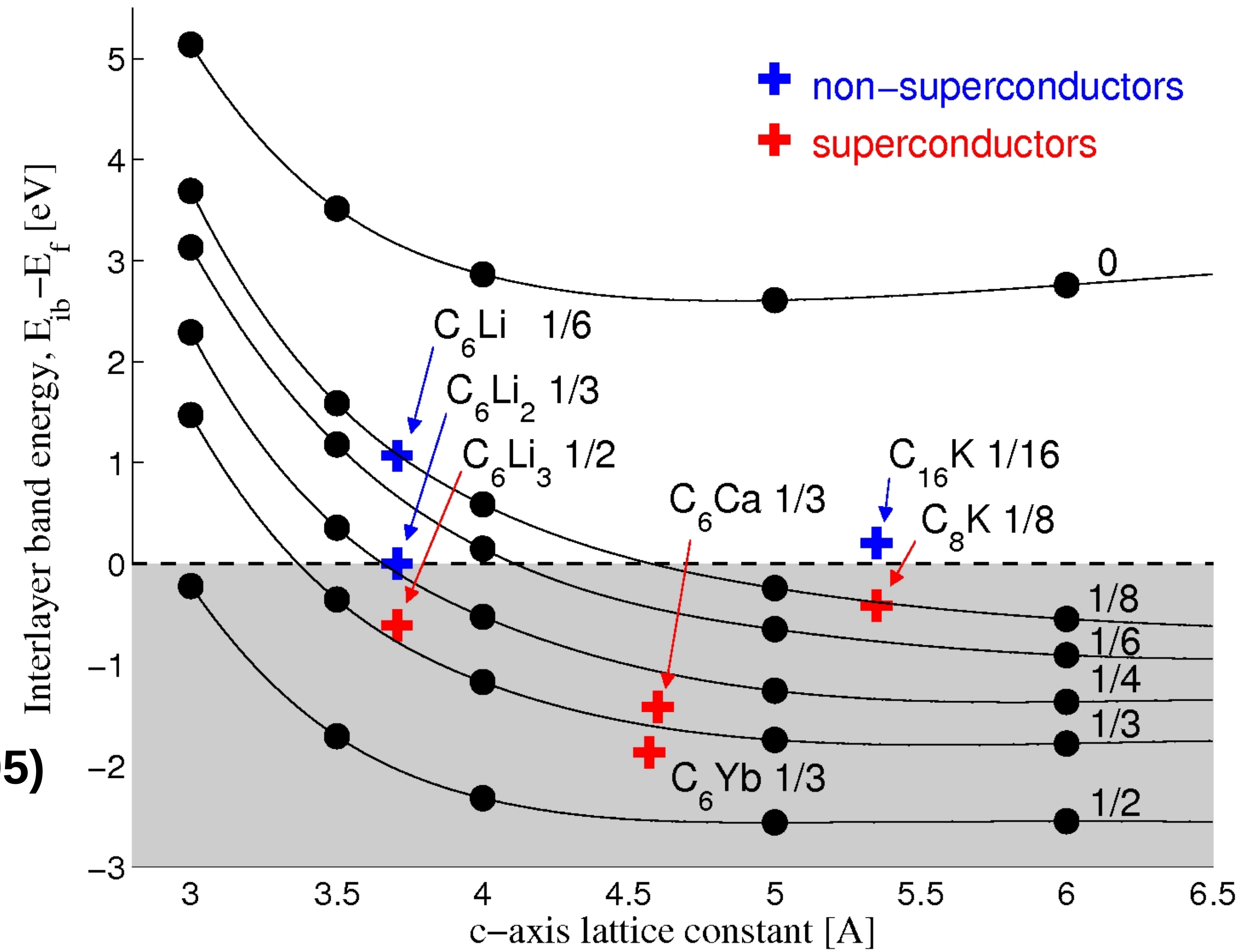
Mazin *et al.*, **cond-mat/0606404**

Graphite intercalated compounds (GIC)



NFEB FS?	**No**	**No**	**Yes**
SC?	**No**	**No**	**Yes**

Graphite intercalated compounds (GIC)



Czanyi *et al.*, Nat. Phys. 1, 42 (2005)

2 band model model superconductivity?

Conclusions

We showed that superconductivity is possible in graphene using a purely electronic mechanism

A reliable estimation of T_c requires the inclusion of retardation in the interaction (strong coupling theory)

Graphene has has a new superconducting phase with p+ip wave pairing in the singlet channel.