Dynamics, Synchronization and Inverse problem in Neural Networks with synaptic plasticity

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From Microscopic to collective dynamics in Neural Circuits
Dresden, 2016
Neural Network

\( Y(t) \) Electrical activity

Avalanches activity

- Delta (\( \delta \)): 0.5-4 Hz
- Infants, sleeping adults
- Spikes
- Epilepsy: petit mal

Graphs showing electrical activity and avalanche activity.
Outline

• LIF neurons with synaptic plasticity
• Heterogeneous Mean Field
• Partially synchronous and asynchronous regimes
• From collective activity to network structure
LIF Neuronal Model

\[ \dot{v} = a - v + I_{\text{syn}} \]

\[ v > v_{\text{th}} = 1 \begin{cases} \text{spike} \\ v = 0 \end{cases} \]

\[ \alpha > 1 \]

\[ \text{Spiking Regime} \]

Short term plasticity:
TUM model for excitatory neurons

\[ I^{(i)}_{\text{syn}}(t) = \tilde{g} \sum_{j \in \text{presyn.i}} y_j(t) \]

\[ S_j(t) = \sum_{n|t_n < t} \delta(t - t_n(j)) \]

\[ k_{\text{in}} = k \]

\[ k_{\text{out}} \]

\[ \dot{y}_j = -\frac{y_j}{\tau_{\text{in}}} + u x_j S_j \]

\[ \dot{x}_j = \frac{\tilde{z}_j}{\tau_r} - u x_j S_j \]

\[ x_j + y_j + z_j = 1 \]
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LIF Neuronal Model

Finite size network of $N$ neurons

\[
\dot{v}_i = a - v_i + \frac{g}{N} \sum_{j \neq i} g_{ij} y_j
\]

\[
\dot{y}_i = -\frac{y_i}{\tau_{in}} + ux_i S_i
\]

\[
\dot{x}_i = \frac{1 - x_i - y_i}{\tau_r} - ux_i S_i
\]

$\tau_{in} = 0.2$ ; $\tau_r = 26.6$

$u = 0.5$ ; $a = 1.3$

$g = 30$
Erdös–Renyi random Network

each link connected with probability $p$

large $N$ : $P_N(k) = G(Np, Np(1 - p))$

\[
Y(t) = \frac{1}{N} \sum_i y_i(t)
\]

\[
\tilde{k} = \frac{k}{N}
\]
Thermodynamic limit

Erdös–Renyi: \( P(\tilde{k}) = G(p, p(1-p)/N) \)

fluctuations \( \sigma_{\tilde{k}} \sim 1/\sqrt{N} \)

Dynamics \( (N \rightarrow \infty) \neq \) Dynamics (finite \( N \))

New Network construction

\( P(\tilde{k}) \) fixed

extract \( \tilde{k}_i \) from \( P(\tilde{k}) \)

&

assign randomly \( N\tilde{k}_i \) inputs

Gaussian \( P(\tilde{k}) : \langle \tilde{k} \rangle = 0.7 \) \( \sigma_{\tilde{k}} = 0.06 \)
Heterogeneous Mean Field

\[ \dot{v}_i = a - v_i + \frac{g}{N} \sum_j \epsilon_{ij} y_j \]

\[ \frac{1}{k_i} \sum_j g_{ij} y_j(t) \simeq \frac{1}{N} \sum_j y_j(t) = Y(t) \implies F_i(t) = \frac{g}{N} \sum_j g_{ij} y_j(t) \to g\tilde{k}_i Y(t) \]

\[ \dot{v}_k(t) = a - v_k(t) + g\tilde{k}Y(t) \]

\[ y_k(t) = -\frac{y_k(t)}{\tau_{in}} + u x_k(t) S_k(t) \]

\[ \dot{x}_k(t) = \frac{(1 - y_k(t) - x_k(t))}{\tau_r} - u x_k(t) S_k(t) \]

\[ Y(t) = \int_0^1 P(\tilde{k}) y_{\tilde{k}}(t) d\tilde{k} \]

Gaussian \( P(\tilde{k}) : \langle \tilde{k} \rangle = 0.7 \quad \sigma_{\tilde{k}} = 0.06 \)
Stability Analysis

Take \( Y(t) \) from HMF simulation

\( Y(t) \) periodic of period \( T \)

\[ \dot{v}_k(t) = a - v_k(t) + g_kY(t) \]

Obtain a map:

\[ t_{n+1}(\tilde{k}) = M_k t_n(\tilde{k}) \]
From partial synchrony to asynchronous phase: the role of degree disorder

\[ R = \left\langle \frac{1}{N} \left| \sum_{j=1}^{N} e^{i \phi_j(t)} \right| \right\rangle \]

\[ \phi_i(t, m) = 2\pi \frac{t - t_i(m)}{t_i(m+1) - t_i(m)} \]
HMF model is non-chaotic

\[ \lambda_{max} \sim \frac{1}{\sqrt{M}} \]
Global Inverse Problem: \[ Y(t) \rightarrow P(\tilde{k}) \]
Different setups: the disorder in neurons excitability

**in vitro experiments**
Robinette et al., Front. Neuroeng. (2011)

Model: disorder on \( a_i \) around \( a_c = 1 \)
Different setups: the inversion procedure

Uniform distribution $P(a)$ around threshold, All-to-All network

$$Y(t) = \int P(a)y_a(t) \, da$$
Conclusions

- Heterogeneous Mean Field reproduces finite size dynamics
- Rich dynamical phase
- Connectivity distribution from global signals

Collaborators:

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