

# Analytical Mesoscale Modeling of Aeolian Sand Transport

Marc Lämmel, Anne Meiwald, Klaus Kroy

# Analytical Mesoscale Modeling of Aeolian Megaripples

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# **Aeolian Sand Transport**

**turbulent streaks**

**= sand or dust?**

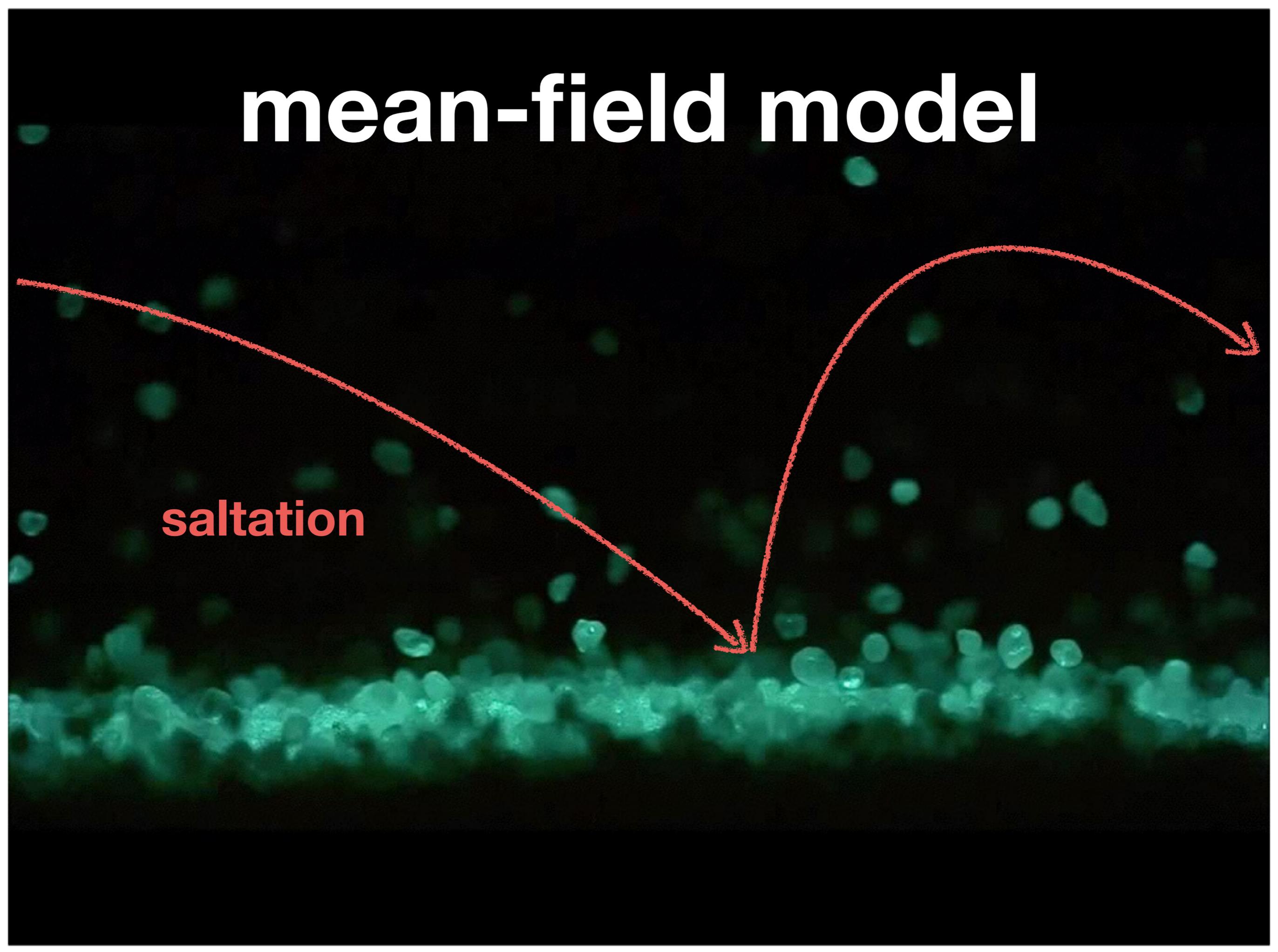
**= important or  
averaged out?**

**mesoscale phenomenon**

**potentially amenable  
to analytical modeling**

# mean-field model

saltation

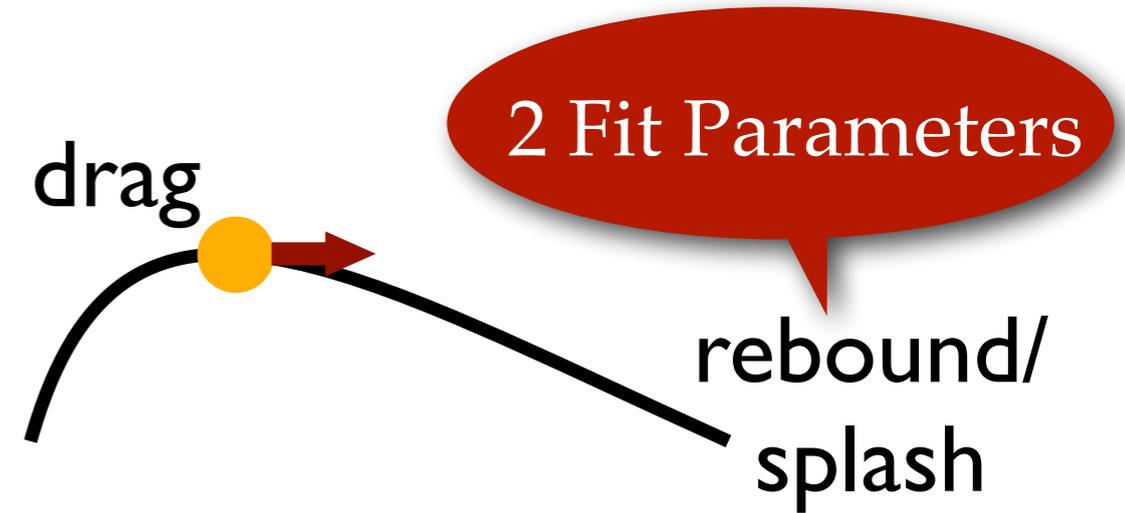


# mean-field model

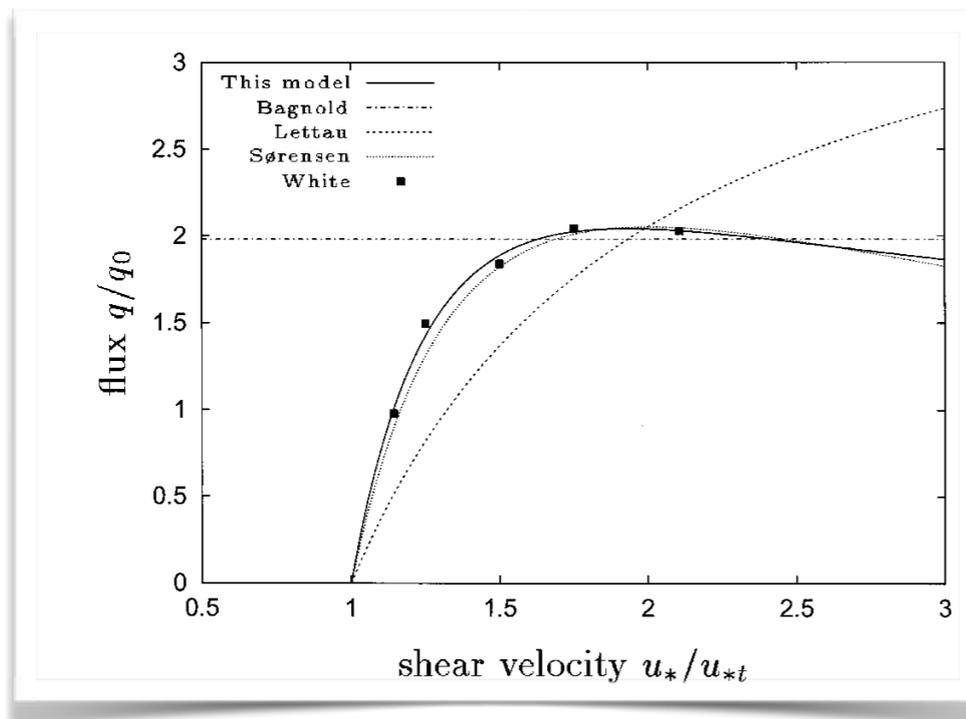
PHYSICAL REVIEW E, VOLUME 64, 031305 (2001)

## Continuum saltation model for sand dunes

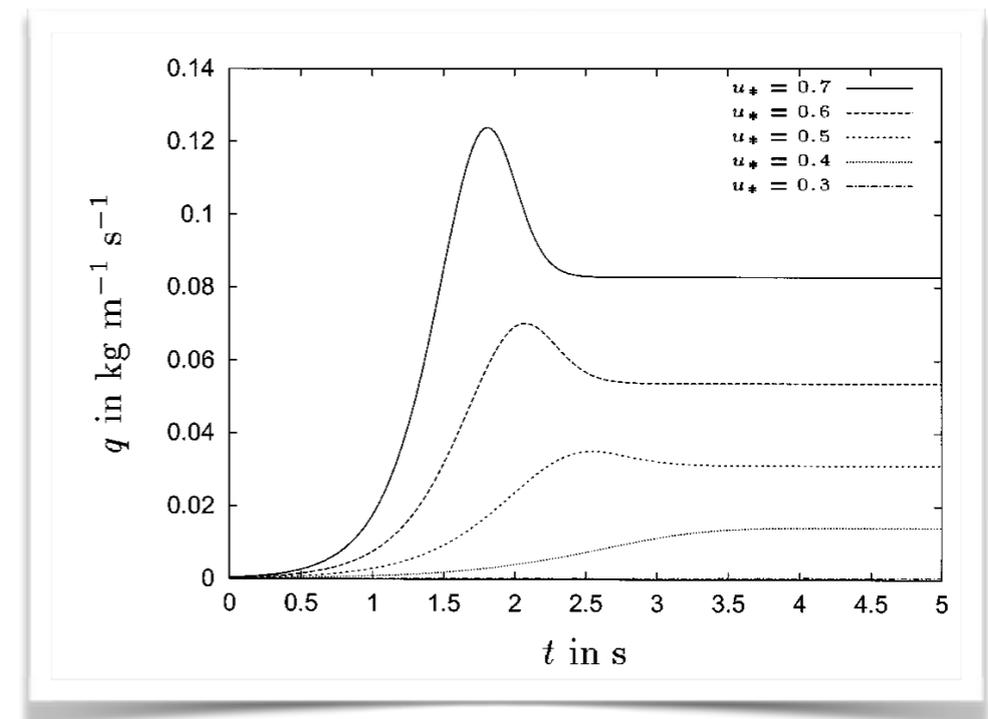
Gerd Sauermann,<sup>1,2</sup> Klaus Kroy,<sup>1,\*</sup> and Hans J. Herrmann<sup>1,2</sup>



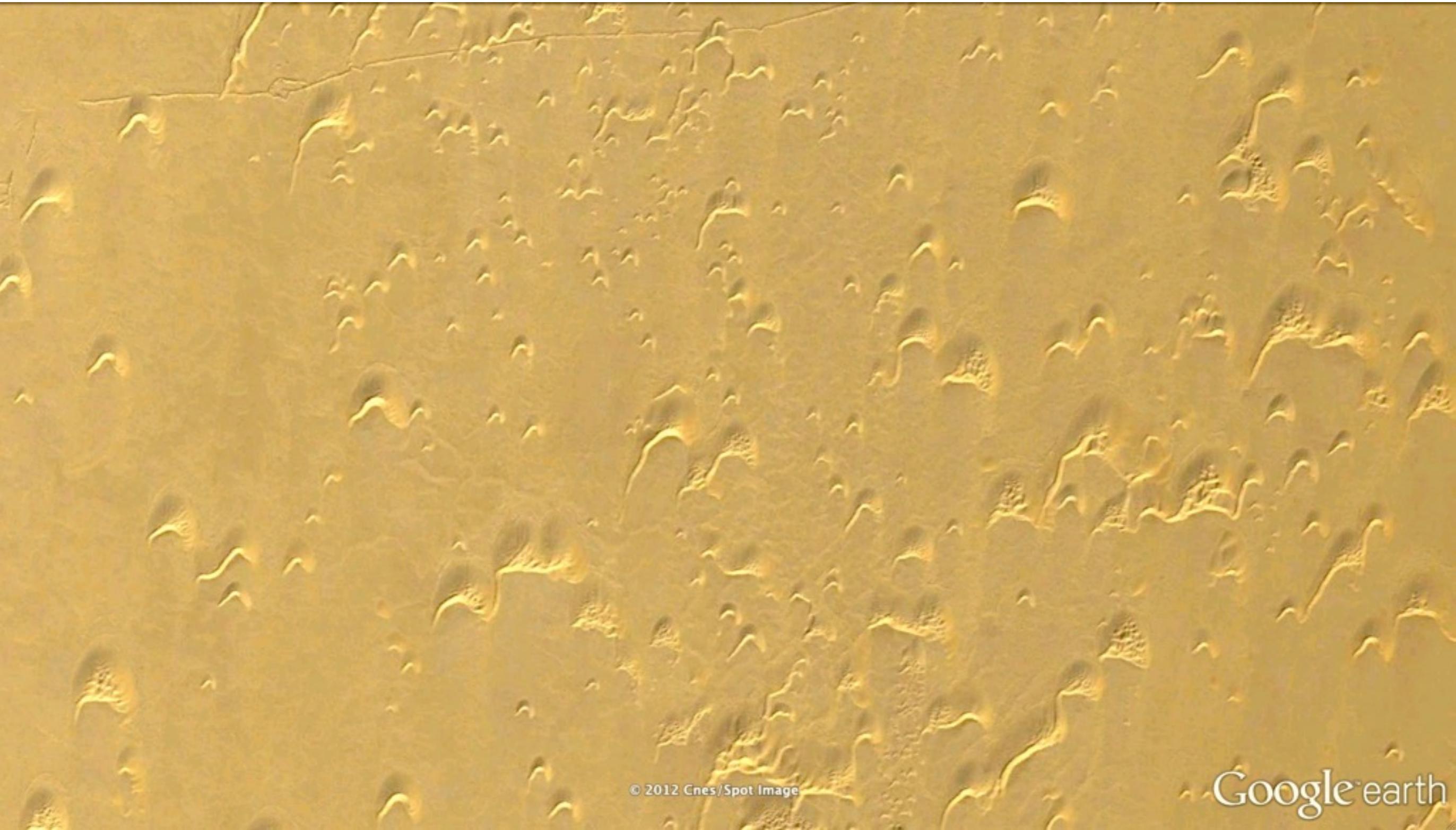
depth-averaged  
saturated flux/ $u_*^3$



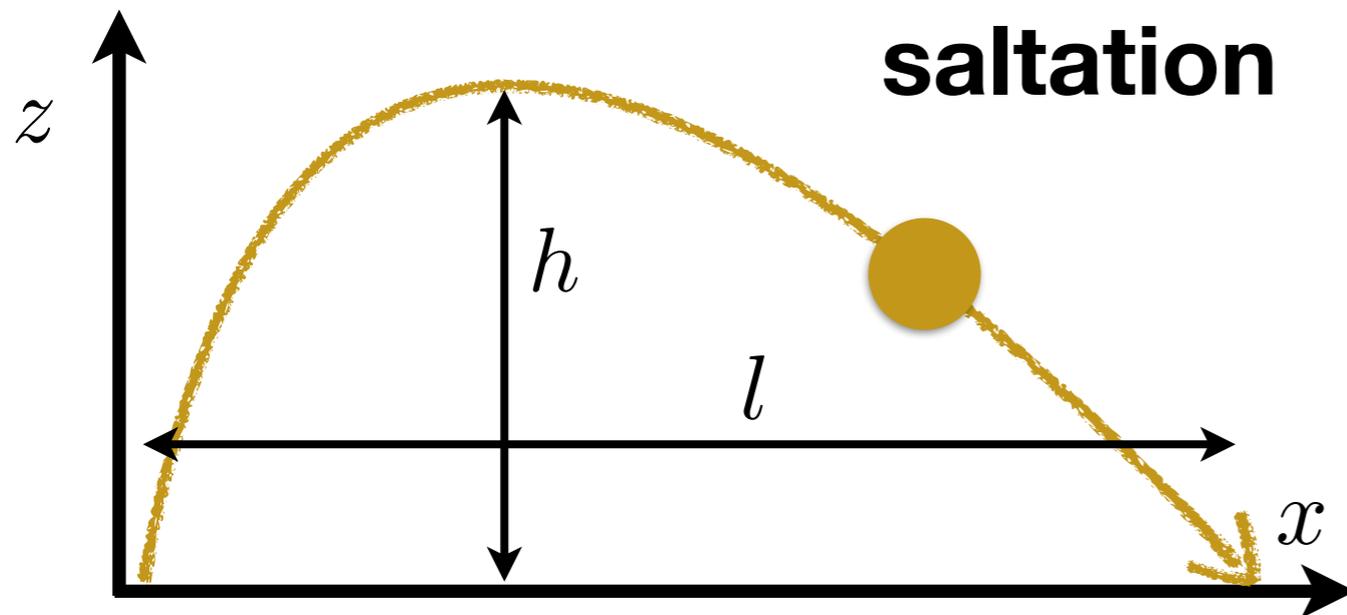
saturation transients  
(mesoscale)



# Aeolian Structure Formation



# Mesoscale Phenomena



**above focus  
extracts energy  
from wind**

**saltation**

**below focus  
screens bed  
from wind  
reptation**

*J. Fluid Mech.* (2004), vol. 510, pp. 47–70. © 2004 Cambridge University Press  
DOI: 10.1017/S0022112004009073 Printed in the United Kingdom

## **A two-species model of aeolian sand transport**

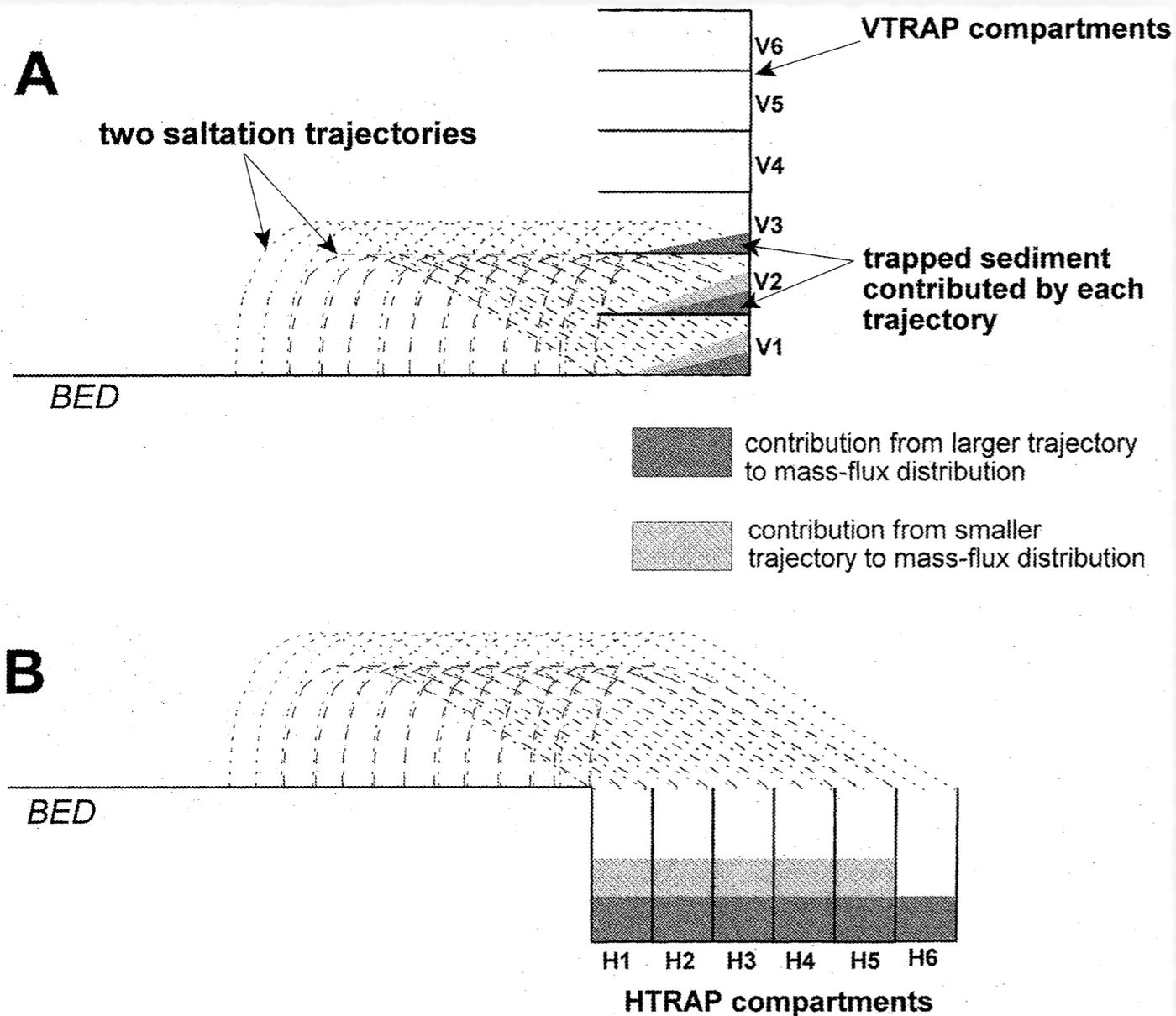
By BRUNO ANDREOTTI

**New Journal of Physics**

The open-access journal for physics

**A two-species continuum model for aeolian sand transport**

M Lämmel, D Rings and K Kroy<sup>1</sup>



- ## HEIGHT RESOLVED
- particle concentration
  - particle velocity
  - particle flux
  - hop length & height
  - wind speed

*J. Fluid Mech.* (1964), vol. 20, part 2, pp. 225–242  
 Printed in Great Britain

225

### Saltation of uniform grains in air

By P. R. OWEN

Department of Aeronautics, Imperial College, London

(Received 14 April 1964)

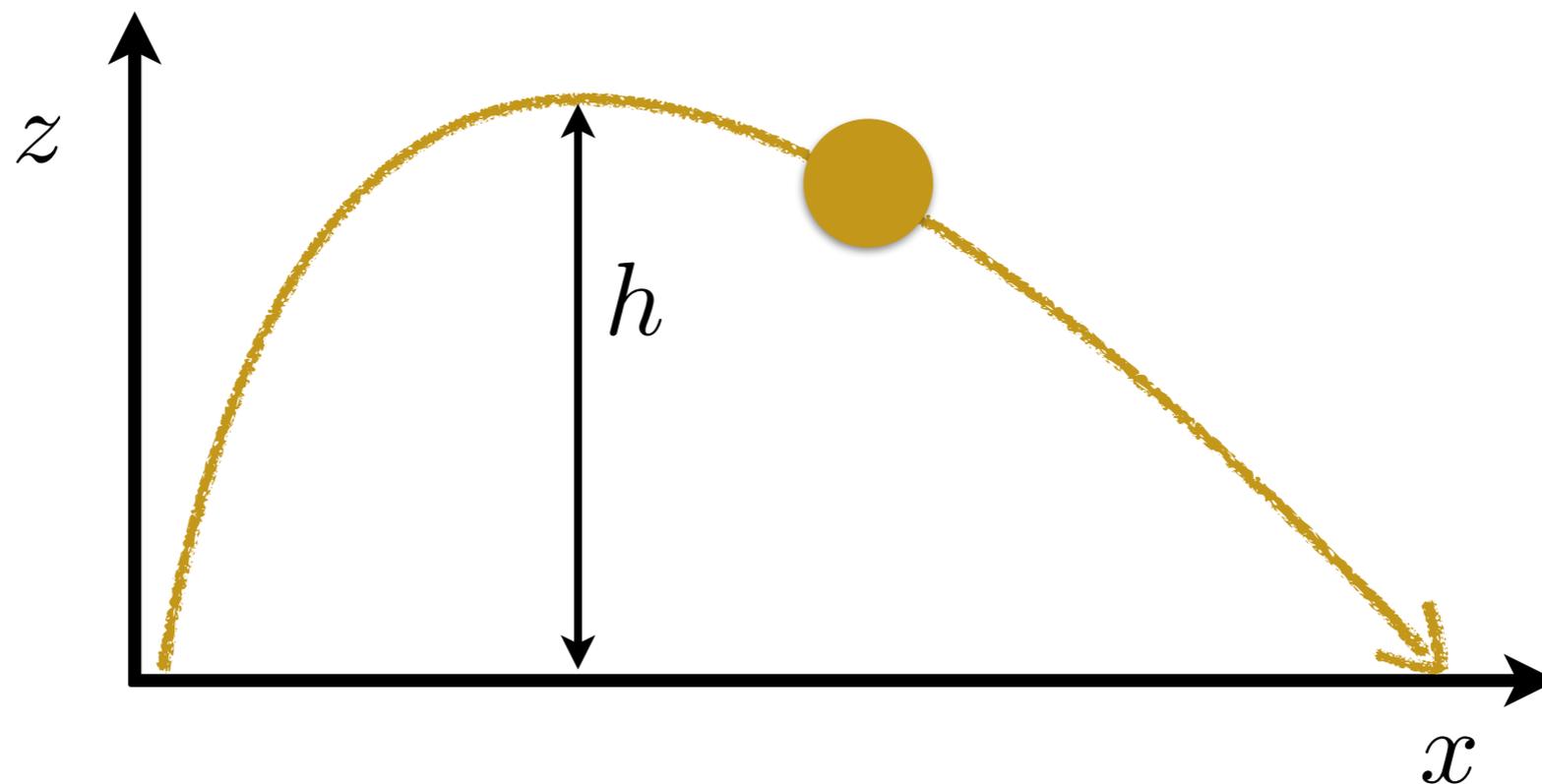


**DISTRIBUTIONS**  
*of grain trajectories*

# particle distribution

$$P(z, h)$$

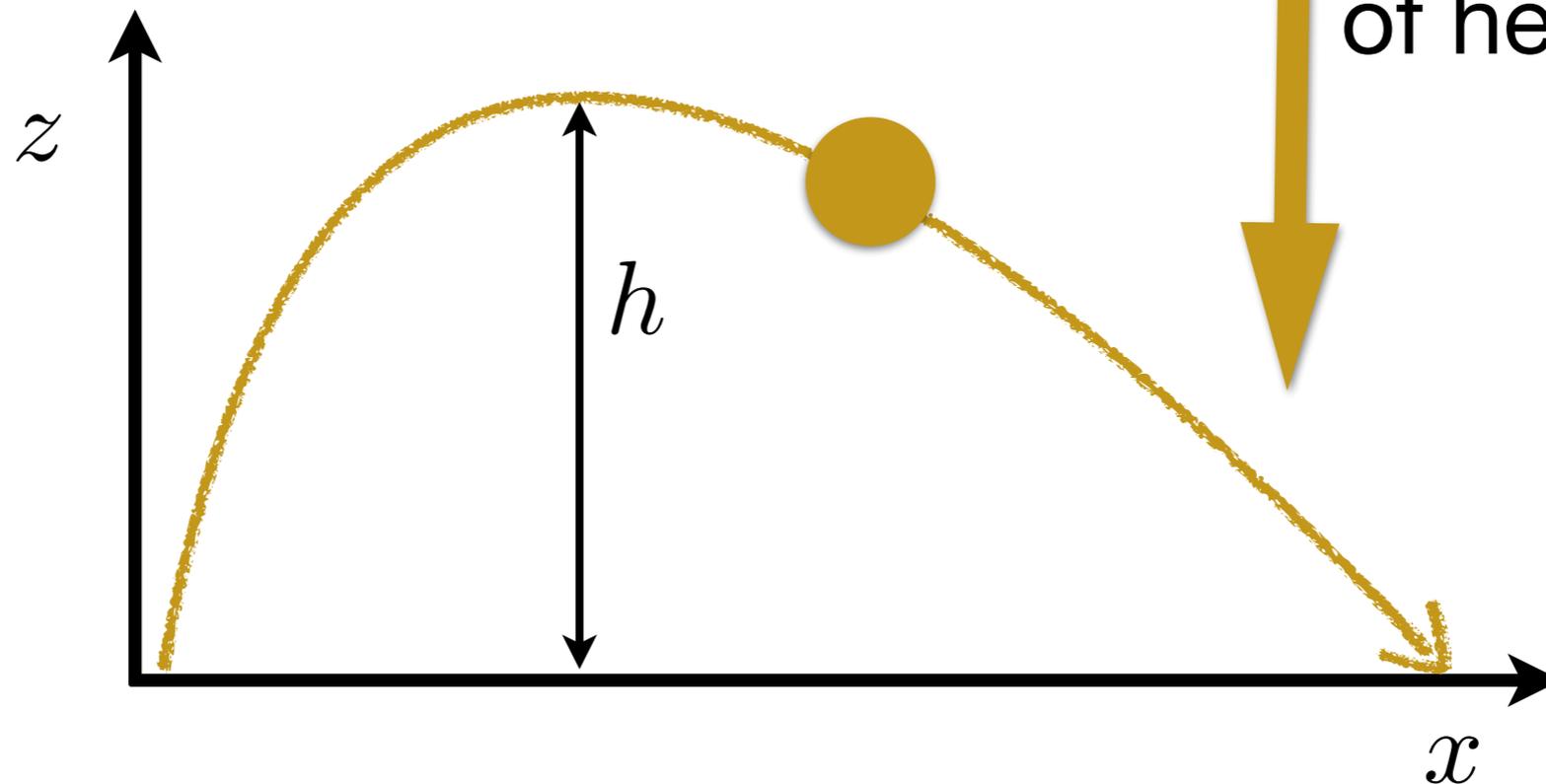
Prob to observe particle  
on trajectory of height  $h$  at  $z$



# particle distribution

$$P_{\bar{h}}(z, h) = P(z|h)P_{\bar{h}}(h)$$

wind  
2-spec

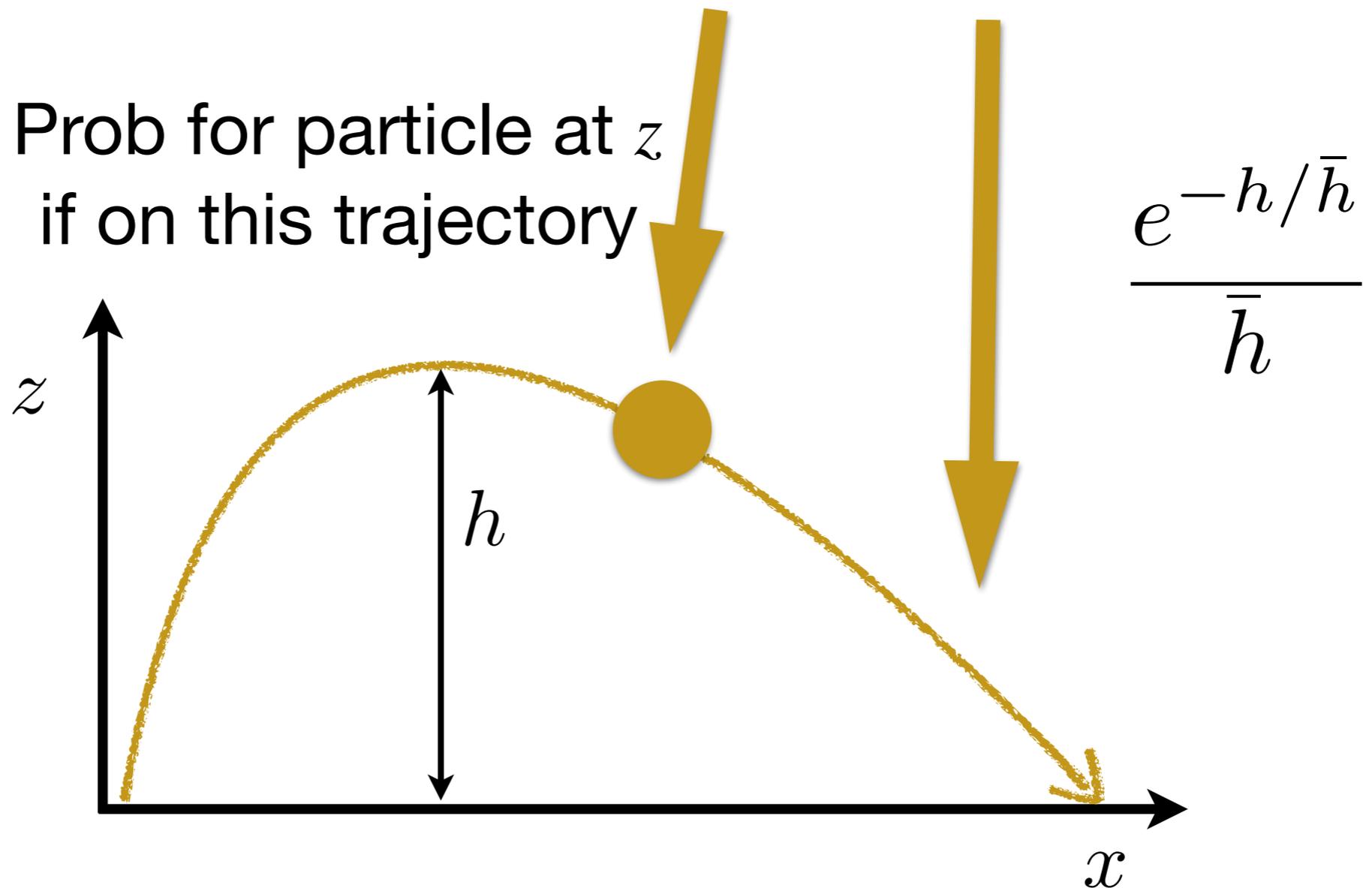


Prob for  
trajectory  
of height  $h$

# particle distribution

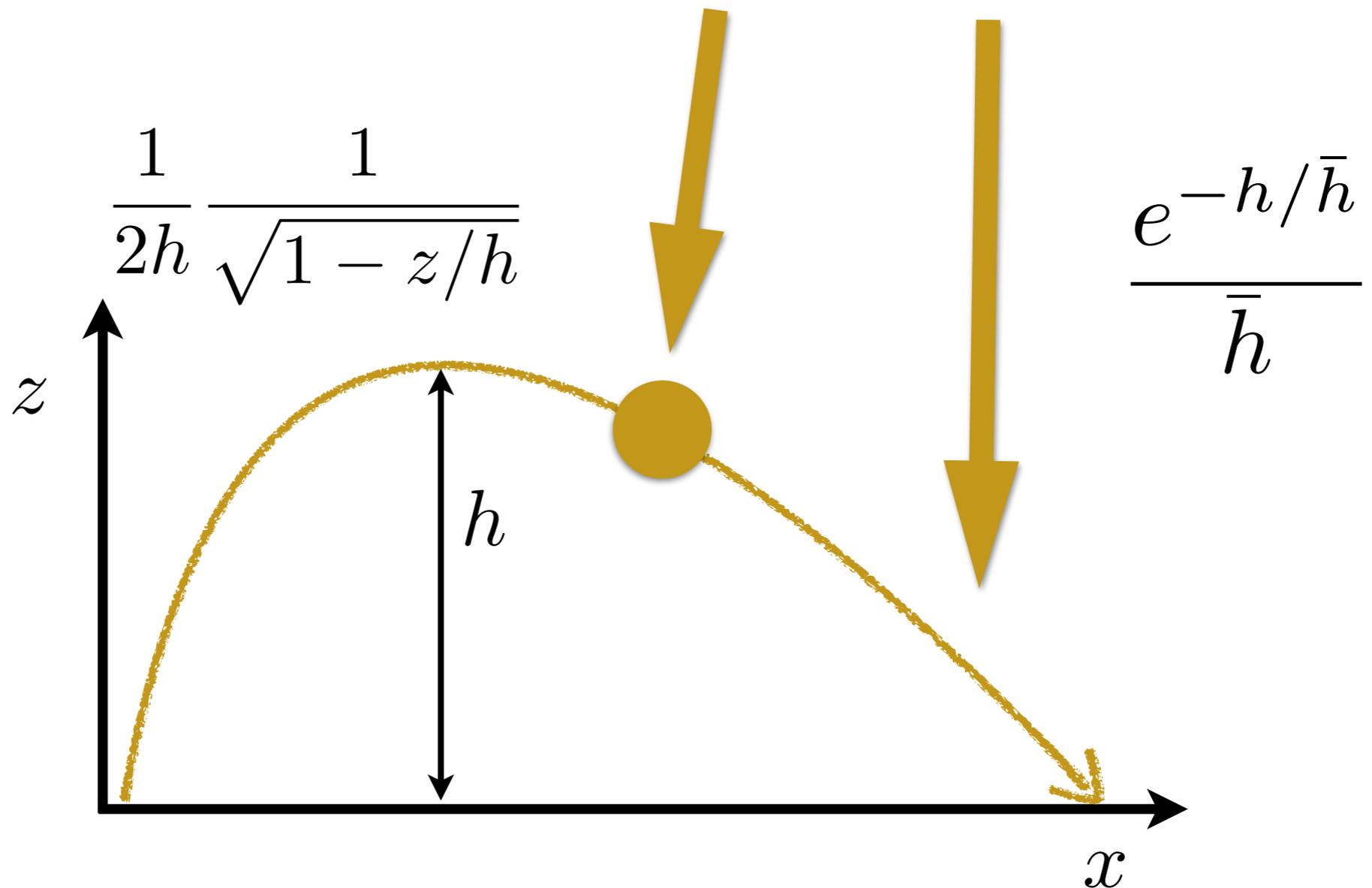
$$P_{\bar{h}}(z, h) = P(z|h)P_{\bar{h}}(h)$$

Prob for particle at  $z$   
if on this trajectory



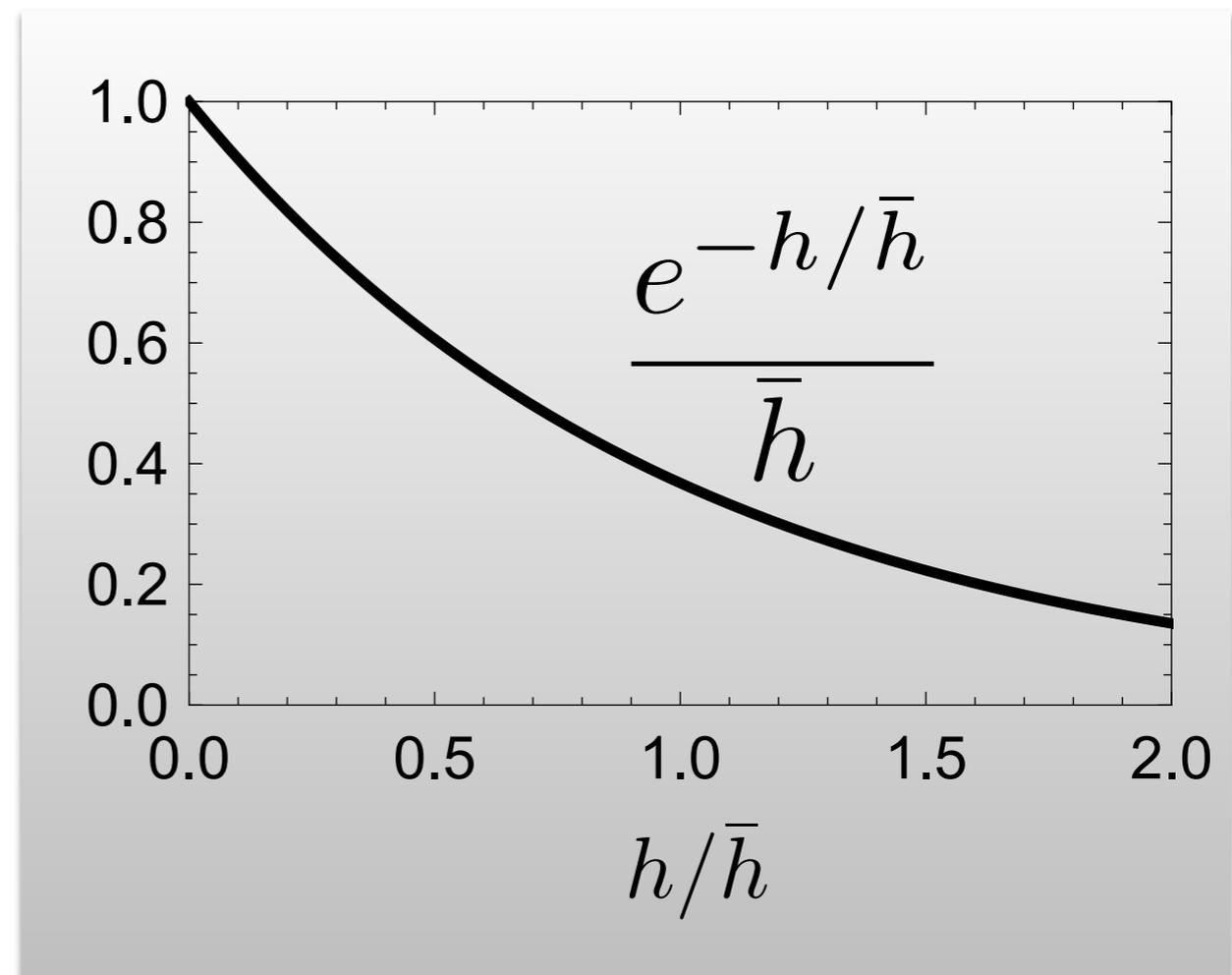
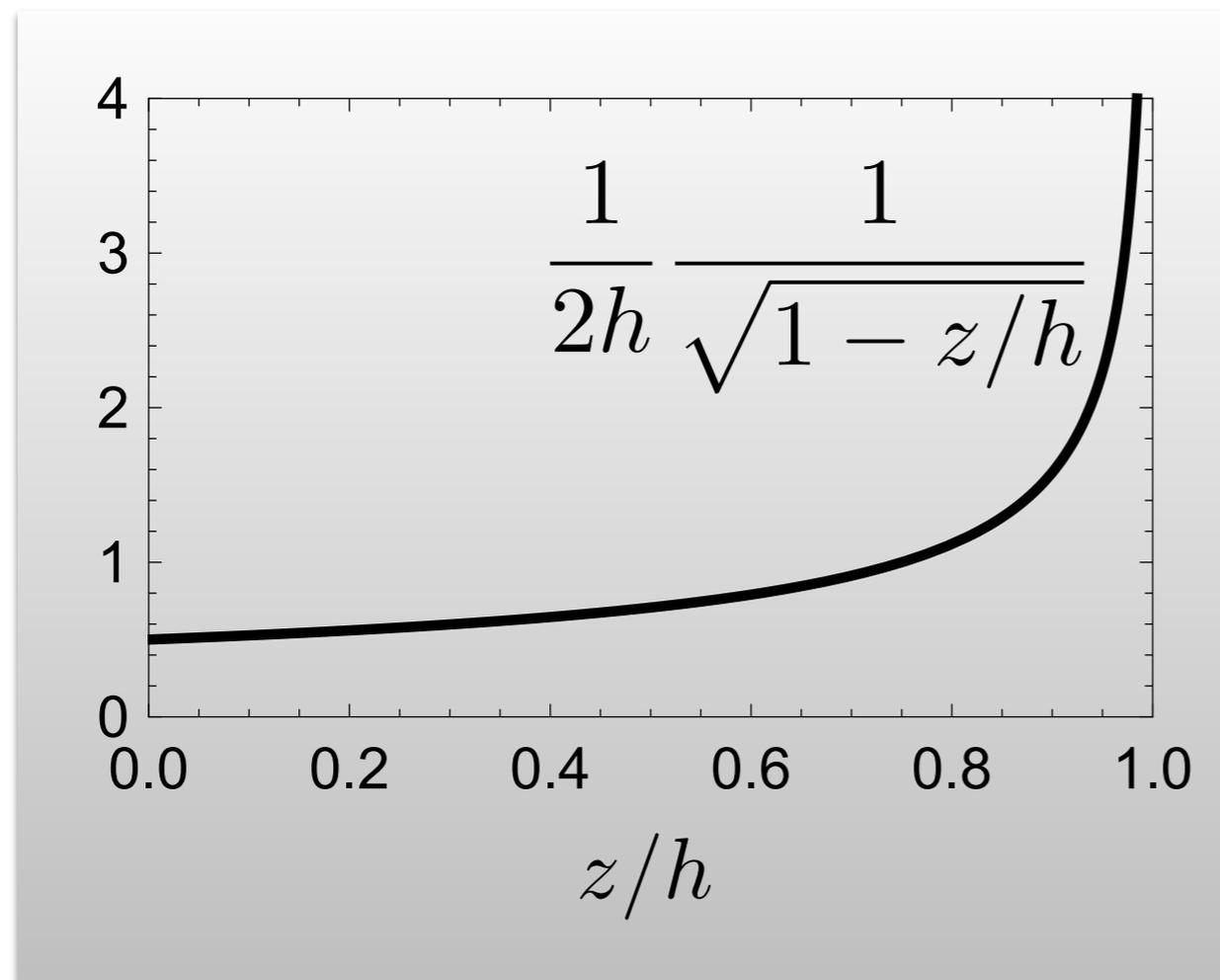
# particle distribution

$$P_{\bar{h}}(z, h) = P(z|h)P_{\bar{h}}(h)$$



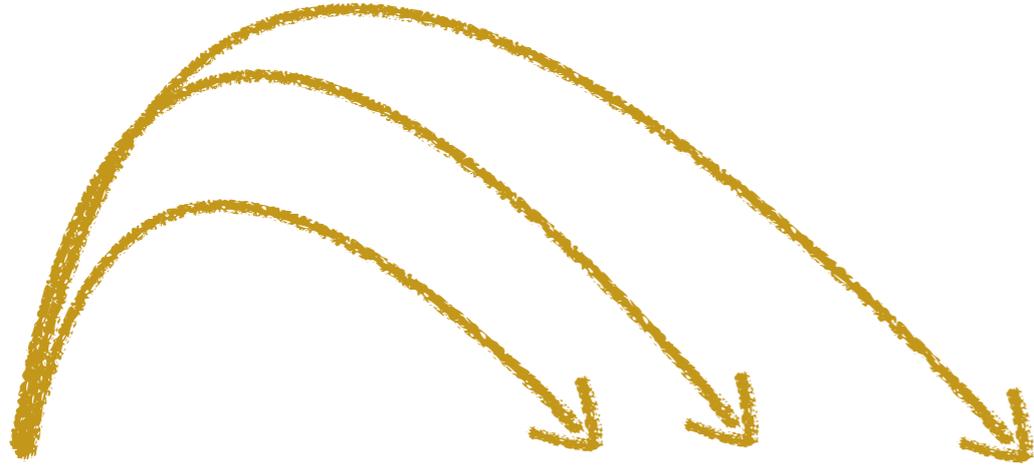
# particle distribution

$$P_{\bar{h}}(z, h) = P(z|h)P_{\bar{h}}(h)$$



$$P_{\bar{h}}(h) \propto e^{-h/\bar{h}}$$

## Reptation/Splash



*J. Fluid Mech.* (1983), vol. 130, pp. 187–202  
*Printed in Great Britain*

187

~ **barometer formula with**  
A theory for the rapid flow of identical, smooth, nearly  
elastic, spherical particles  
**granular temperature**  
By J. I. JENKINS

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York

AND S. B. SAVAGE

# granular temperature

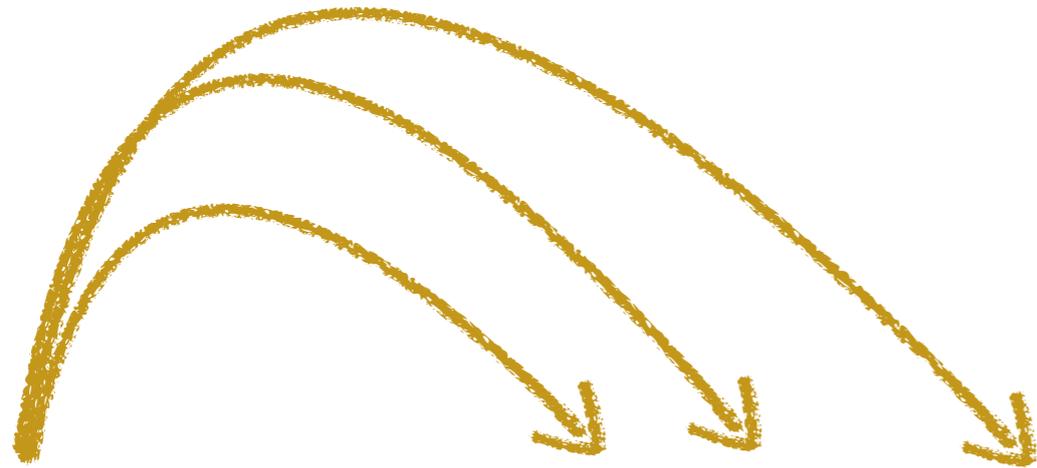
1. molecular gas with (turbulent) thermostat

$$p = nk_B T \quad -m g n(z) = \partial_z p = k_B T \partial_z n$$
$$n(z) \propto \exp(-m g z / k_B T)$$

2. splash: free granular gas released at  $z=0$

$$P(E) dE = P(v) m v dv = P(h) dh$$
$$P(h) \propto \exp(-m g h / k_B T)$$

$$P_{\bar{h}}(h) \propto e^{-h/\bar{h}} \quad \text{Reptation/Splash}$$



~ barometer formula with granular temperature

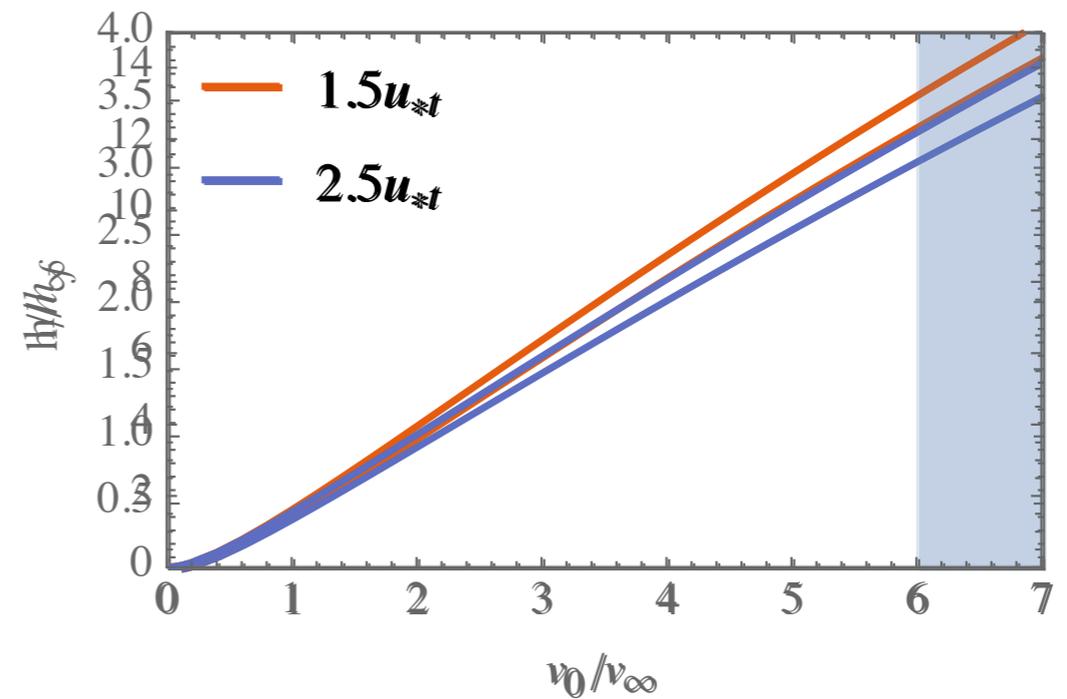
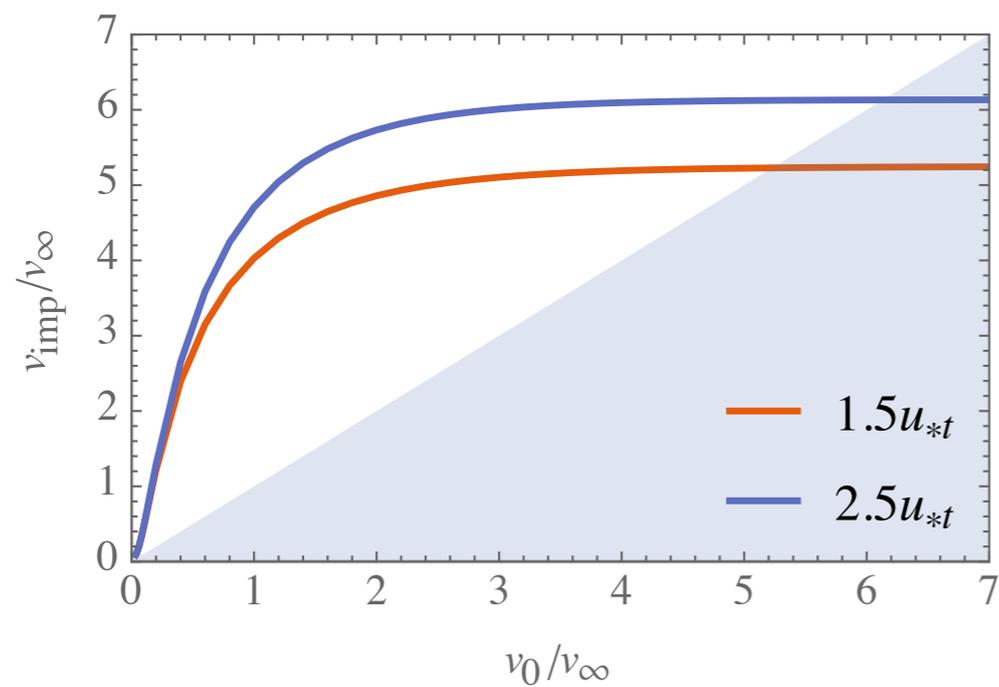
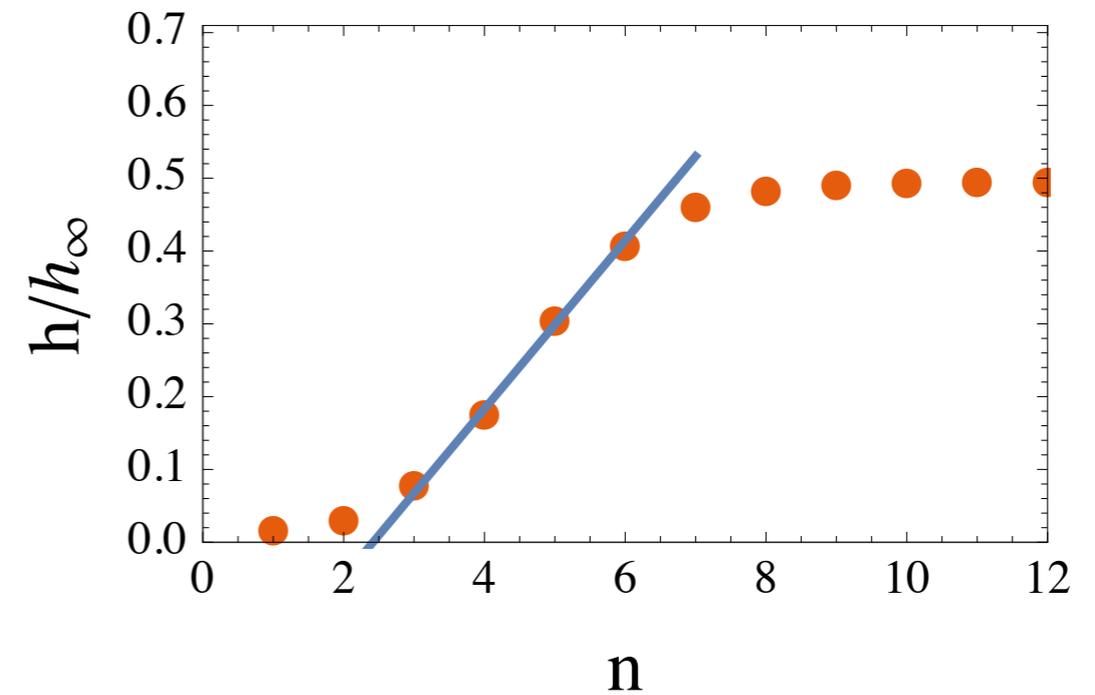
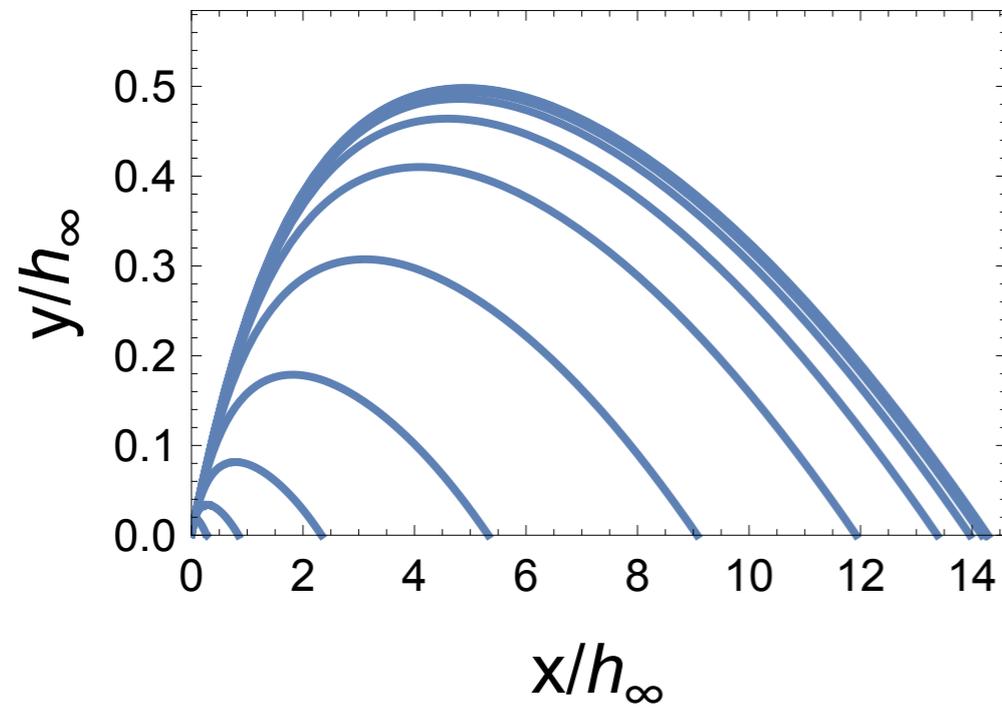
**Saltation**

$$P_{\bar{h}}(h) \propto e^{-h/\bar{h}}$$

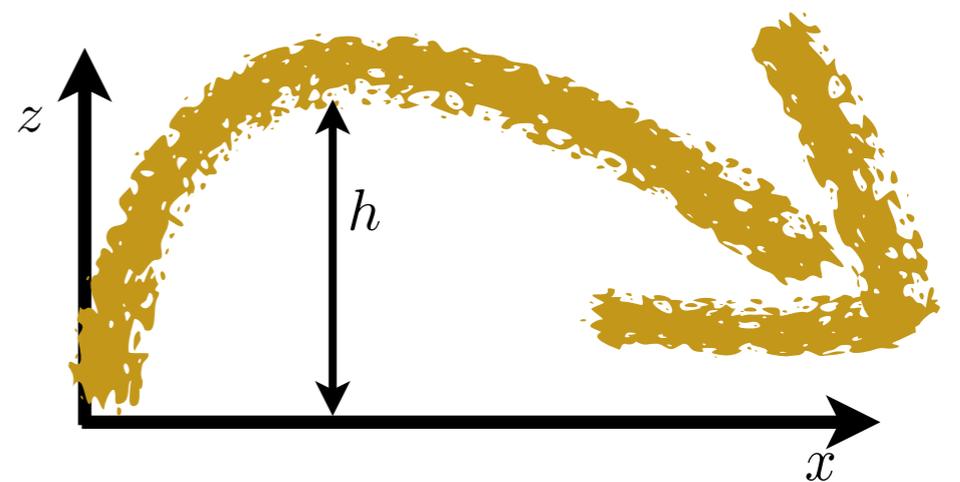
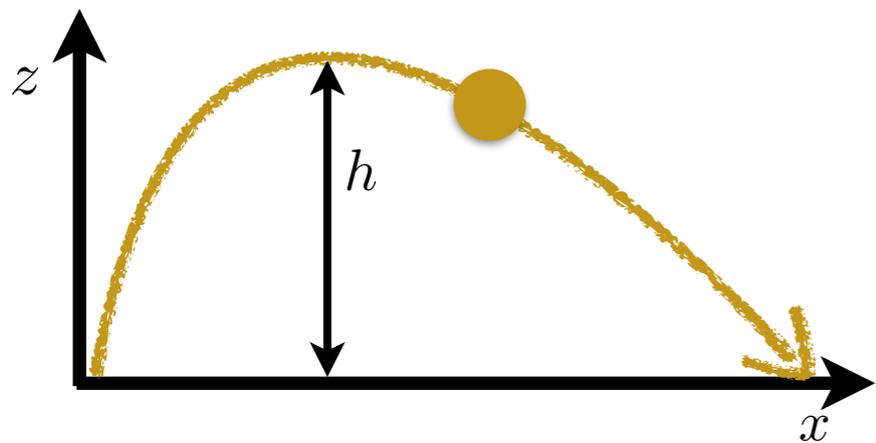


$$P(\# \text{ hops} \gtrsim N) \propto e^{-N}; \quad h \propto N?$$

# deterministic numeric solution



# turbulent diffusion — how important is it?



$$\langle \Delta z^2 \rangle \simeq Dt$$

$$D \simeq u_* h$$

diffusion coefficient

$$t_\infty \approx \sqrt{2h/g}$$

time of flight

**assume**

$$P_{\bar{h}}(h) \propto e^{-h/\bar{h}}$$

**for reptons & saltions**

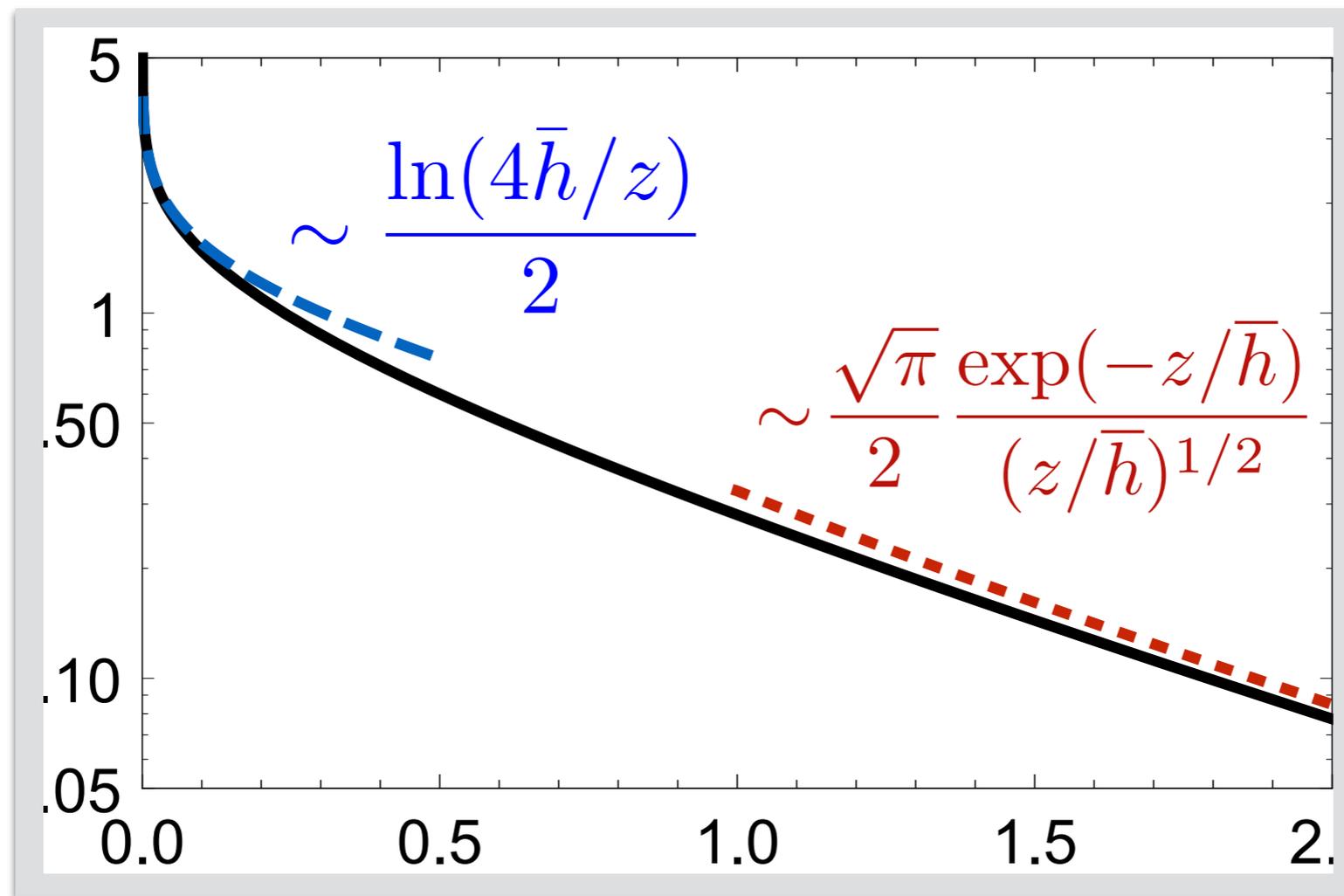
# height-resolved observables

- **grain density**  $\rho(z)$
- **horizontal flux**  $j(z)$
- **vertical flux**  $\phi(z)$
- **grain-borne stress**  $\tau_g(z)$
- **hop length distribution**  $P(z, l)$
- **hop length distribution**  $P(l) \propto -\partial_l \phi_l(0)$

# particle distribution

$$\rho_{\bar{h}}(z) = \int dh P_{\bar{h}}(z, h)$$

Prob for particle at  $z$   
for **any** trajectory

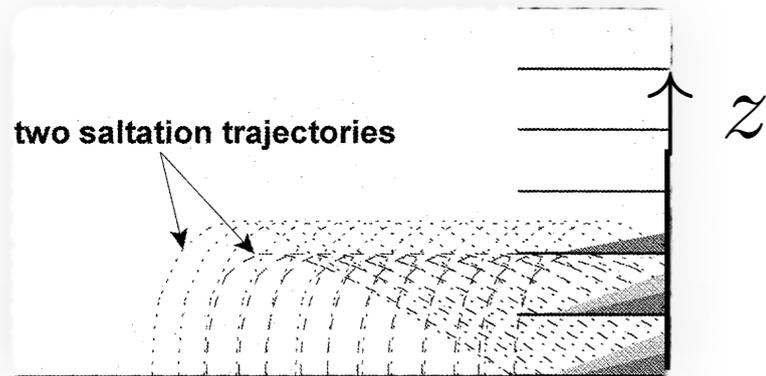


$z/\bar{h}$



wind, 2 spec

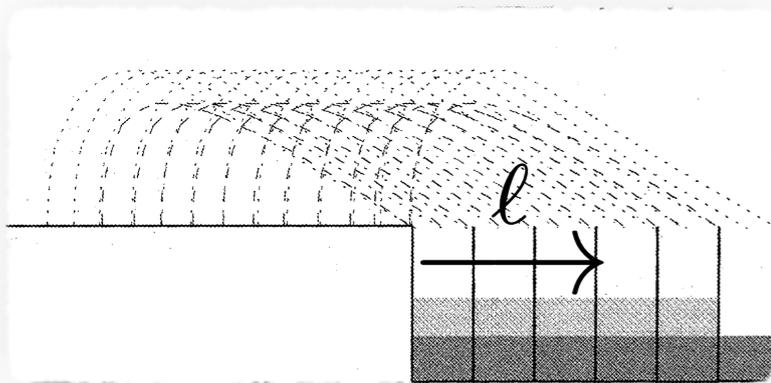
# horizontal sand flux



$$j_{\bar{h}}(z) = \int dh v_x(h) P_{\bar{h}}(z, h) = q \frac{e^{-z/\bar{h}}}{\bar{h}}$$

$$v_x(\cancel{z}, \cancel{l}, h) \approx \sqrt{2gh}/4\epsilon$$

# vertical sand flux

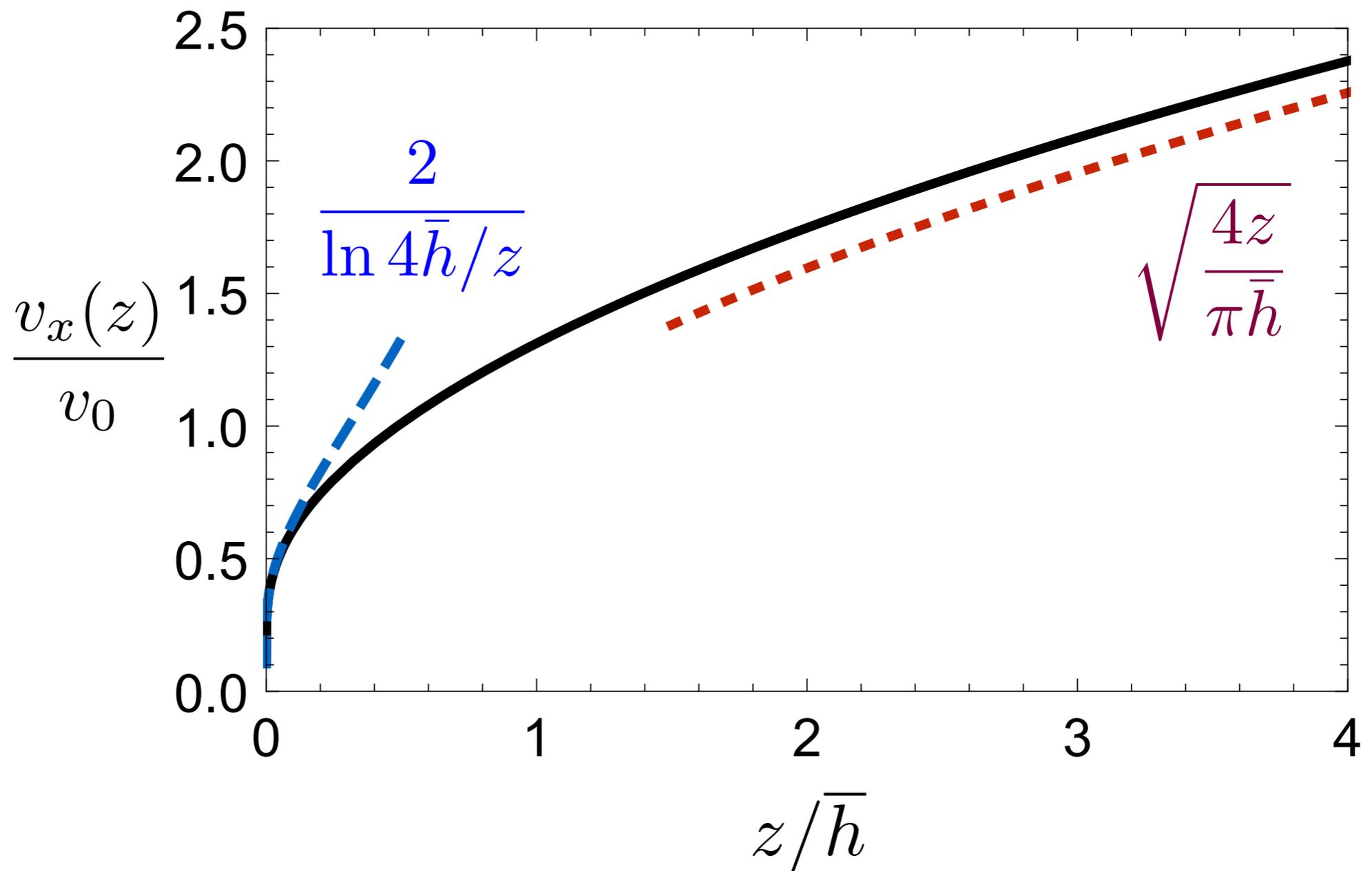


$$\phi_l(z) = \int_{l(h) > l} dh v_z(z, h) P_{\bar{h}}(z, h)$$

$$v_z(z, h) = \sqrt{2g(h - z)} \quad \phi_l(z = 0) = q \frac{\text{erfc} \sqrt{\ell \epsilon / \bar{h}}}{\bar{h} / \epsilon}$$

# particle velocity

$$v_x(z) = j(z)/\rho(z)$$



# grain-scale experiments

Rasmussen, Mikkelsen, Sedimentology (1998)

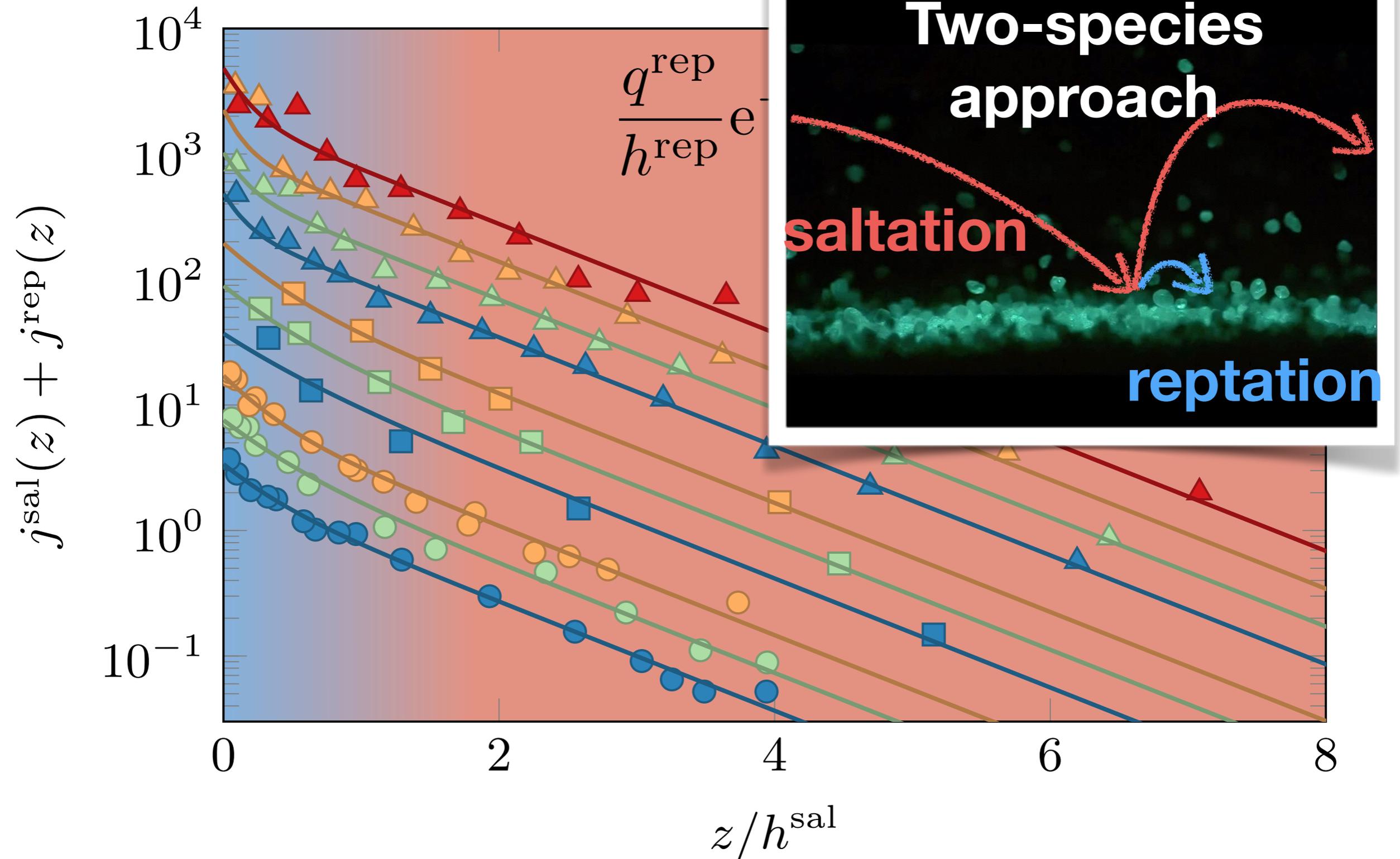
Namikas, Sedimentology (2003)

Rasmussen, Sørensen, J. Geophys. Res. (2008)

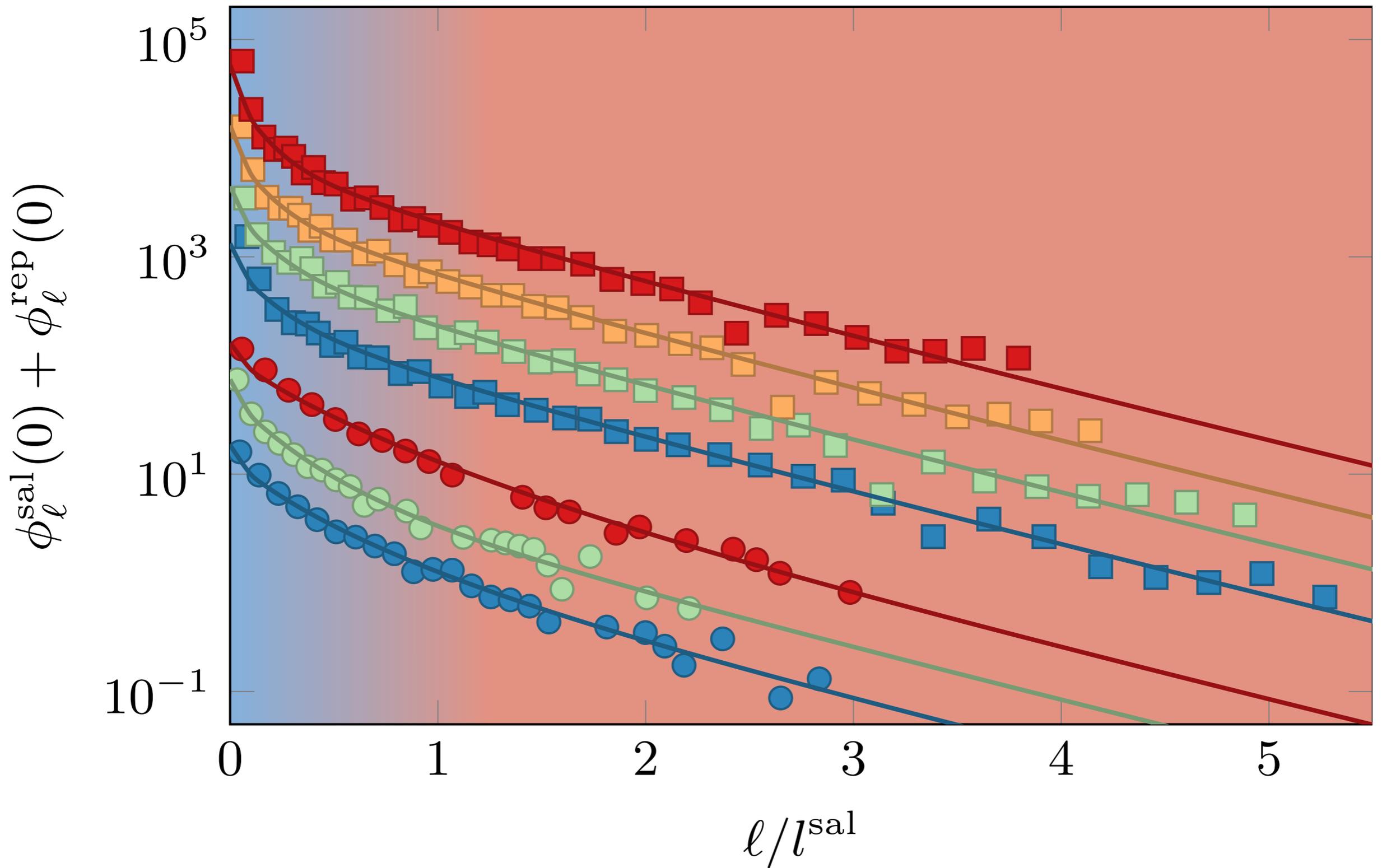
Ho, Valance, Dupont, Moctar, Aeolian Research (2014)

Durand, Claudin, Andreotti, PNAS (2014)

# horizontal sand flux

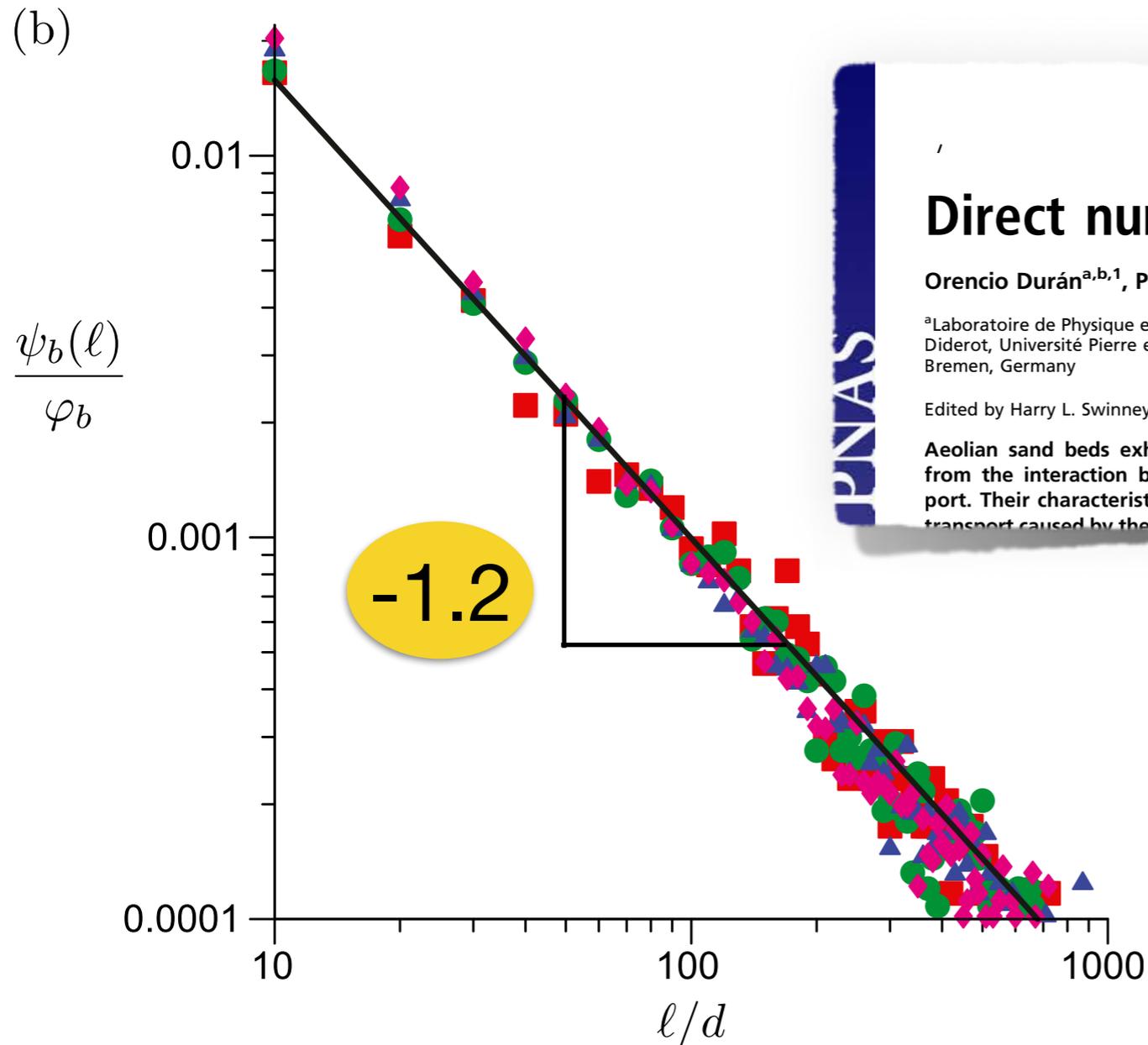


# vertical sand flux



# hop length distribution

$$P(l|z) \propto P[z, h(l)] \partial_l h(l) \propto \frac{e^{-\epsilon l / \bar{h}}}{l}$$



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← click for updates

## Direct numerical simulations of aeolian sand ripples

Orencio Durán<sup>a,b,1</sup>, Philippe Claudin<sup>a</sup>, and Bruno Andreotti<sup>a</sup>

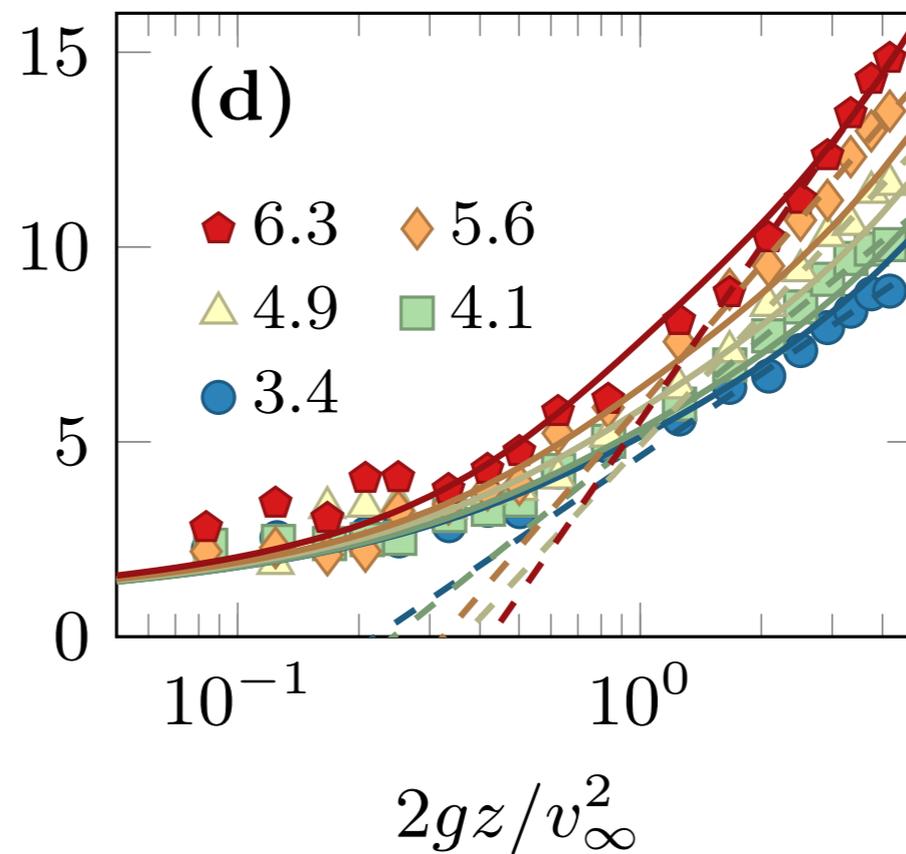
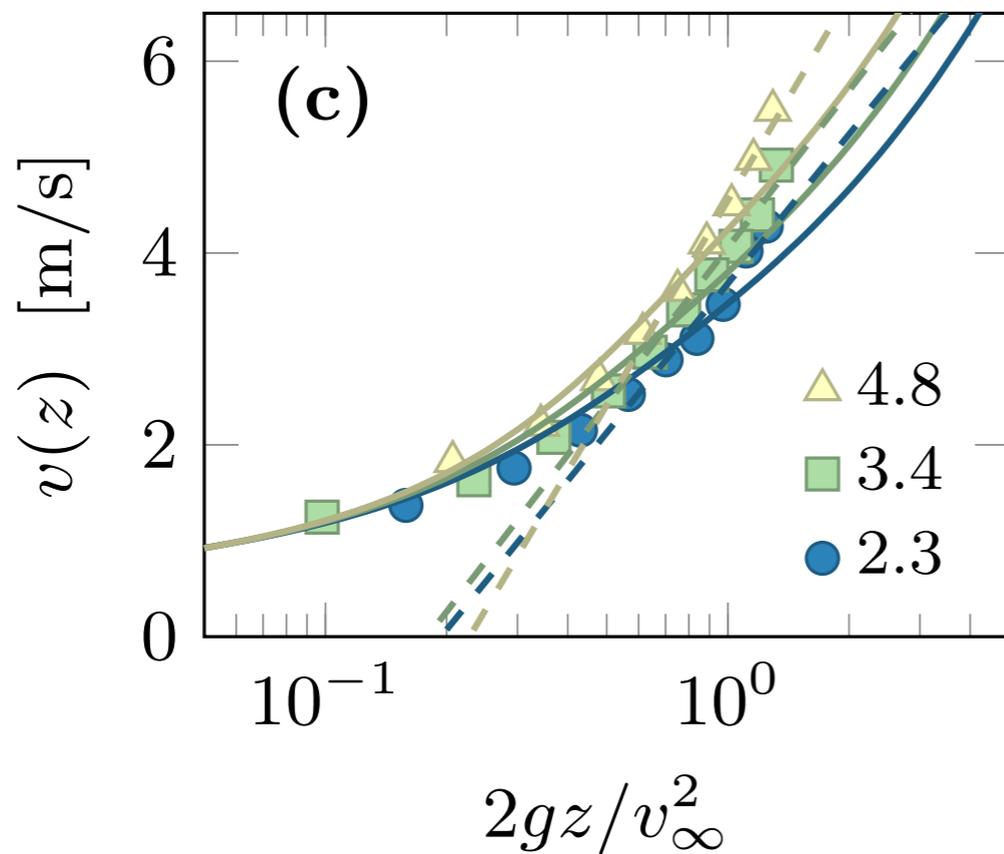
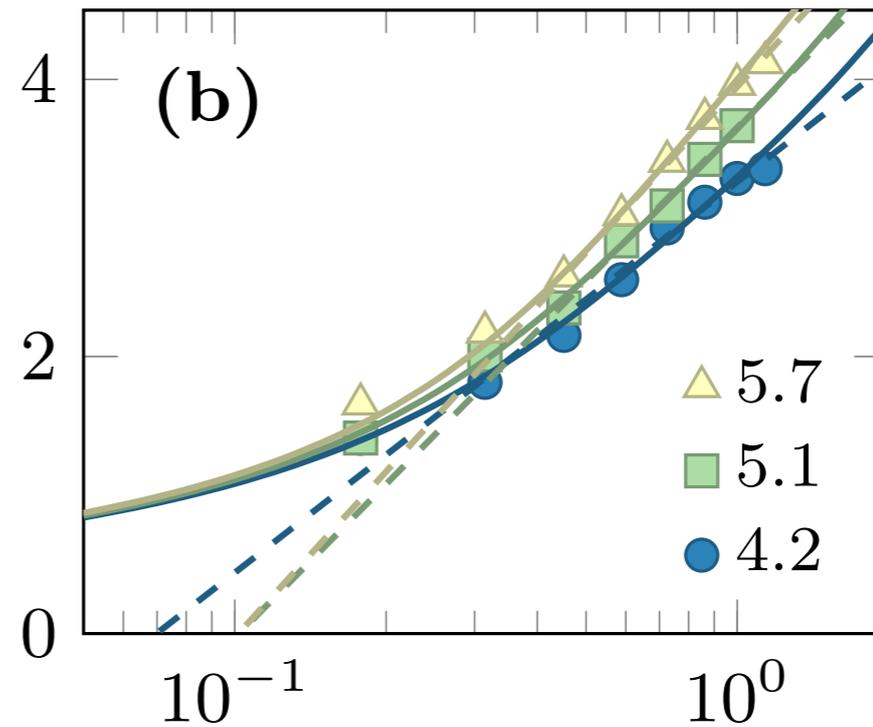
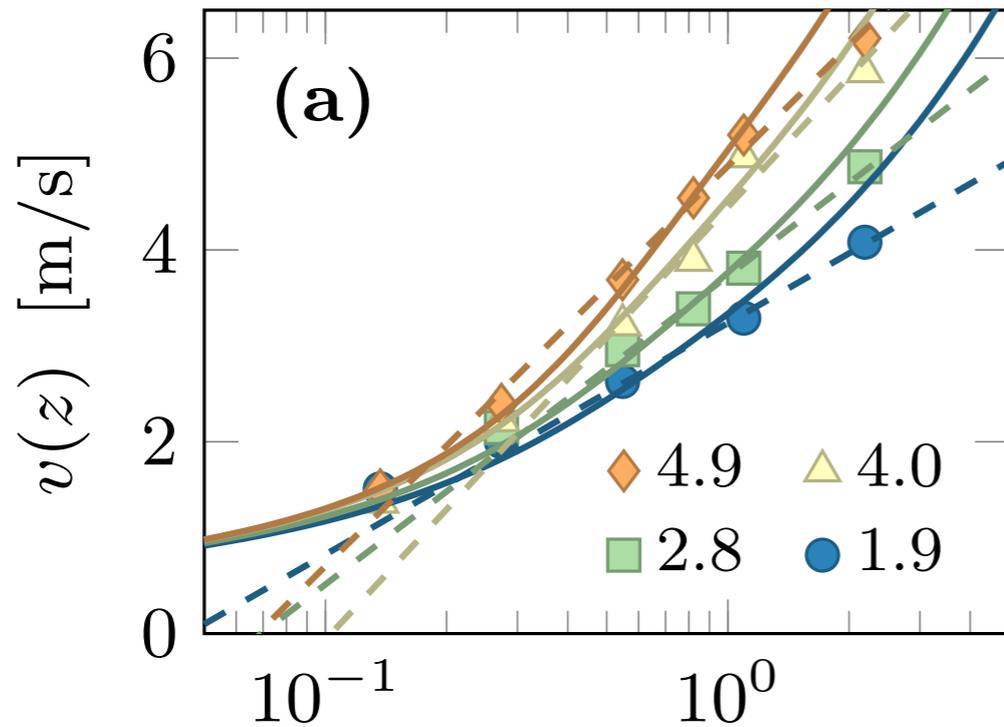
<sup>a</sup>Laboratoire de Physique et Mécanique des Milieux Hétérogènes, UMR 7636, CNRS, Ecole Supérieure de Physique et de Chimie Industrielles, Université Paris Diderot, Université Pierre et Marie Curie, 75005 Paris, France; and <sup>b</sup>MARUM—Center for Marine Environmental Sciences, University of Bremen, D-28359 Bremen, Germany

Edited by Harry L. Swinney, The University of Texas at Austin, Austin, TX, and approved September 17, 2014 (received for review July 10, 2014)

**Aeolian sand beds exhibit regular patterns of ripples resulting from the interaction between topography and sediment transport. Their characteristics have been so far related to reptation transport caused by the impacts on the ground of grains entrained**

presented in ref. 26, we explicitly implement a two-way coupling between a discrete element method for the particles and a continuum Reynolds averaged description of hydrodynamics, coarse-grained at a scale larger than the grain size. This coupling

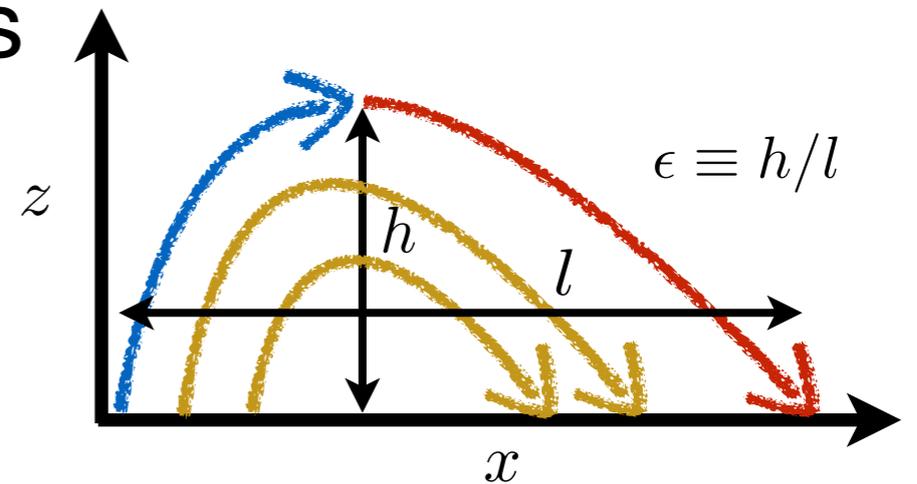
# particle velocity



# Summary

## ► analytical mesoscale model of aeolian transport

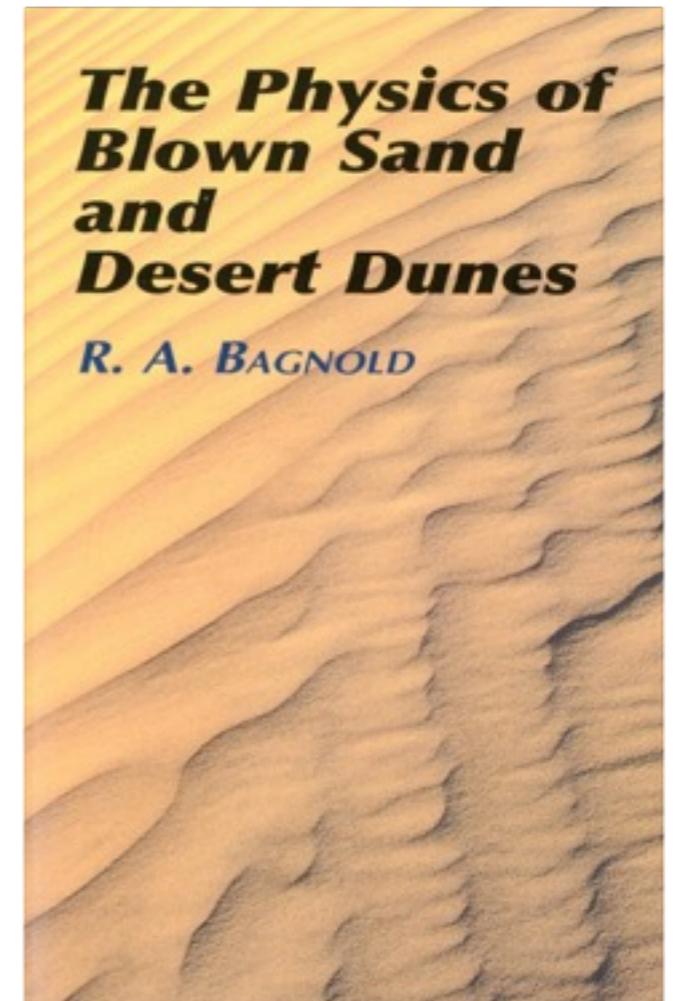
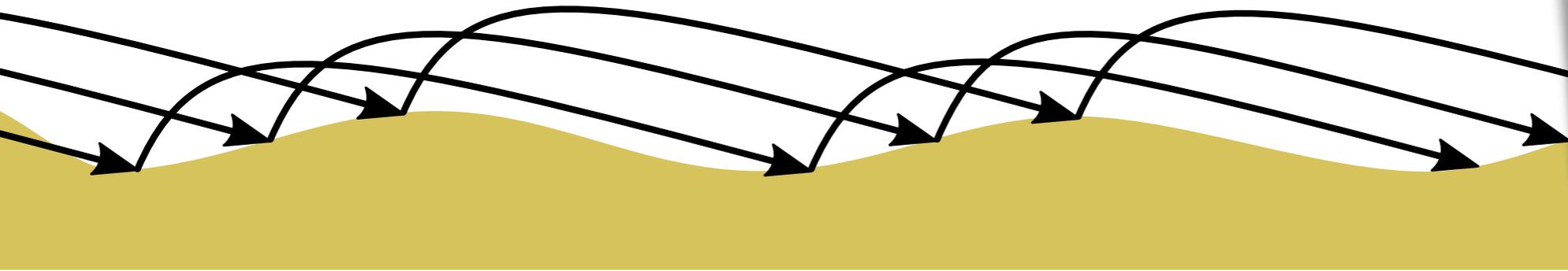
- based on grain scale physics
- ensemble of trajectories
- & two-species
- height-resolved observables
- applications to turbulent closure  
& data analysis & various mesoscale phenomena



# Analytical Mesoscale Modeling of Aeolian Megaripples

Marc Lämmel, Anne Meiwald, Klaus Kroy

# normal ripples



**PNAS**

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click for updates

## Direct numerical simulations of aeolian sand ripples

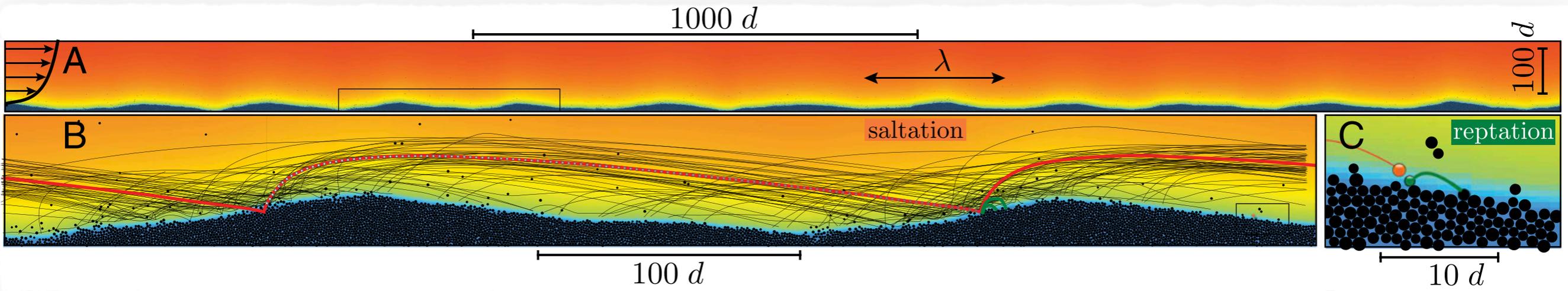
Orencio Durán<sup>a,b,1</sup>, Philippe Claudin<sup>a</sup>, and Bruno Andreotti<sup>a</sup>

<sup>a</sup>Laboratoire de Physique et Mécanique des Milieux Hétérogènes, UMR 7636, CNRS, Ecole Supérieure de Physique et de Chimie Industrielles, Université Paris Diderot, Université Pierre et Marie Curie, 75005 Paris, France; and <sup>b</sup>MARUM—Center for Marine Environmental Sciences, University of Bremen, D-28359 Bremen, Germany

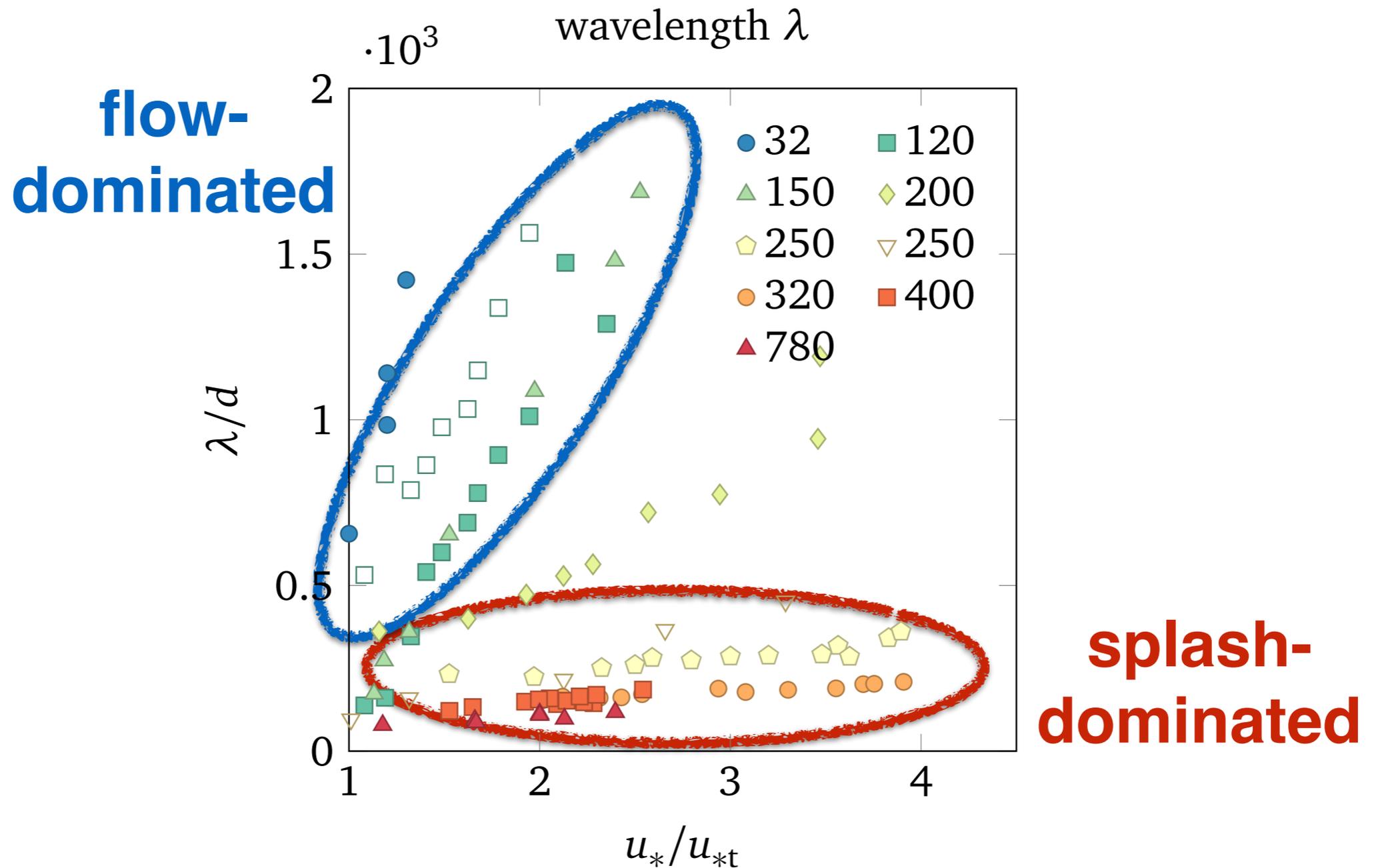
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# normal ripples



Andreotti et al., PRL 96, 028001 (2006): field  $180\mu\text{m}$ , wind tunnel  $120\mu\text{m}$

Bagnold (1941):  $250\mu\text{m}$

Walker, MA thesis (1981): 200, 250, 320, 400, 780  $\mu\text{m}$

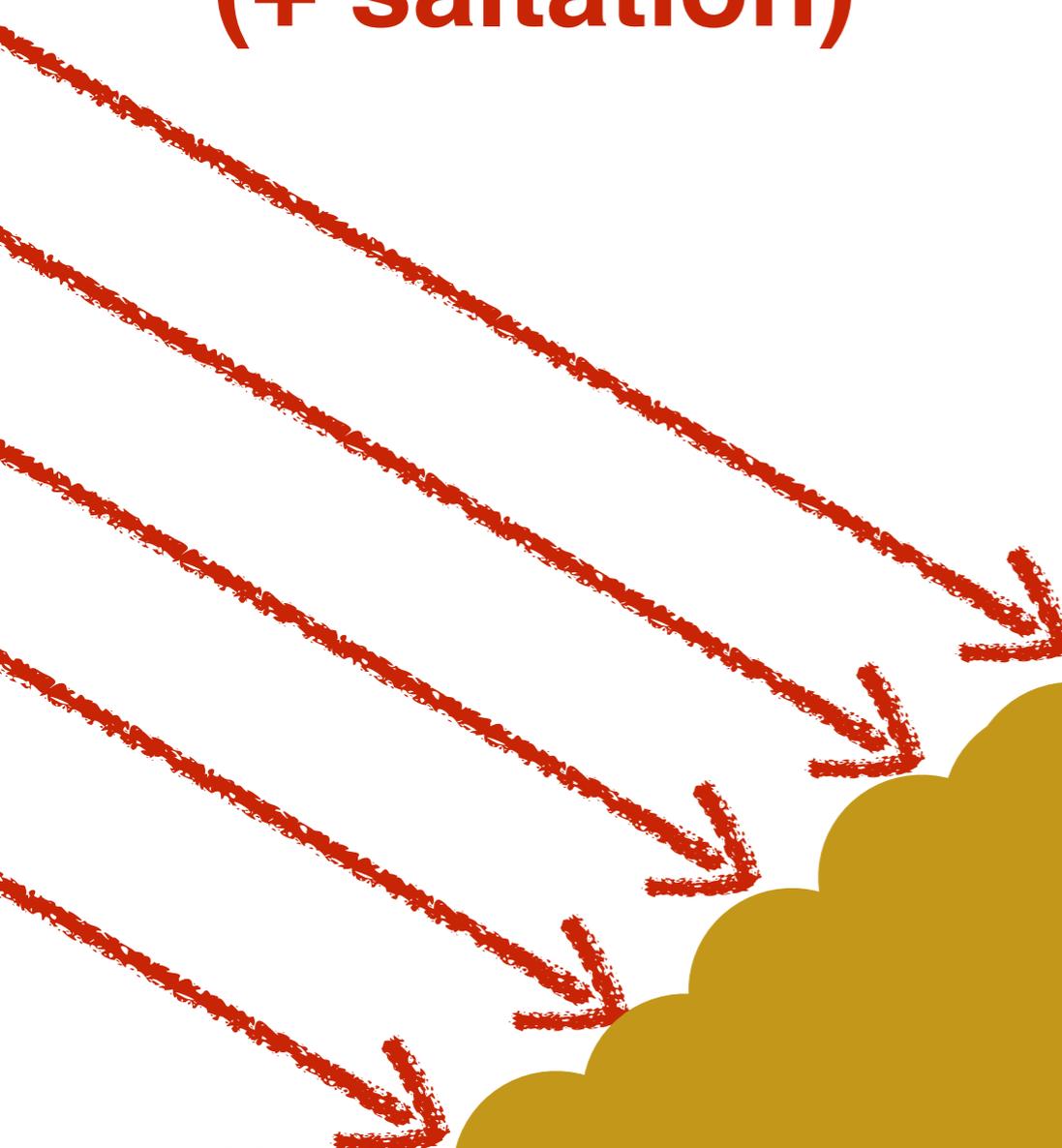
Seppälä and Lindé, Geografiska Annaler. A (1978):  $150\mu\text{m}$

# megaripple morphology

wind strength  
(+ saltation)

**megaripple**  
=  
**repton dune**

transport  
(reptation)



# megaripple morphology



**megaripple**  
**=**  
**repton dune**

## WIND RIPPLES<sup>1</sup>

ROBERT P. SHARP

California Institute of Technology<sup>2</sup>

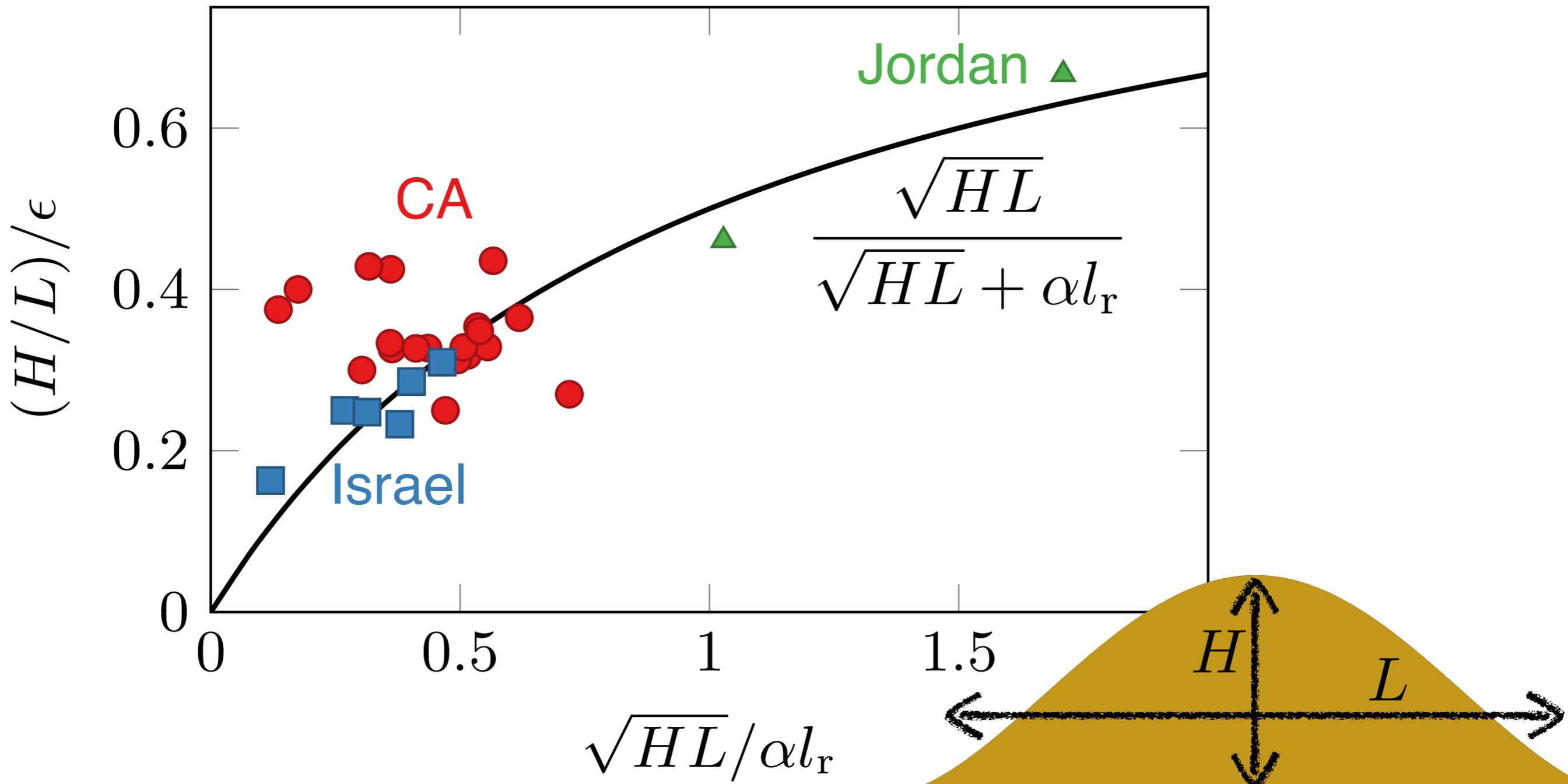
### ABSTRACT

Two types of wind ripples are distinguished; sand ripples composed of material with a median diameter roughly between 0.30 and 0.35 mm., and granule ripples composed of material approaching granule size 2–4 mm. The planimetric patterns and facing directions of the two types are compared.



# aspect ratio

reptation length:  $l_r \propto u_* \sqrt{d_2/g}$



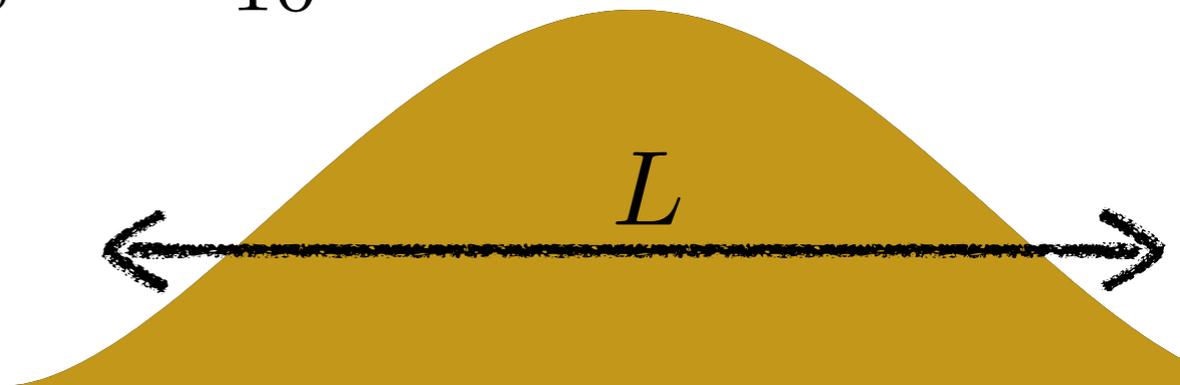
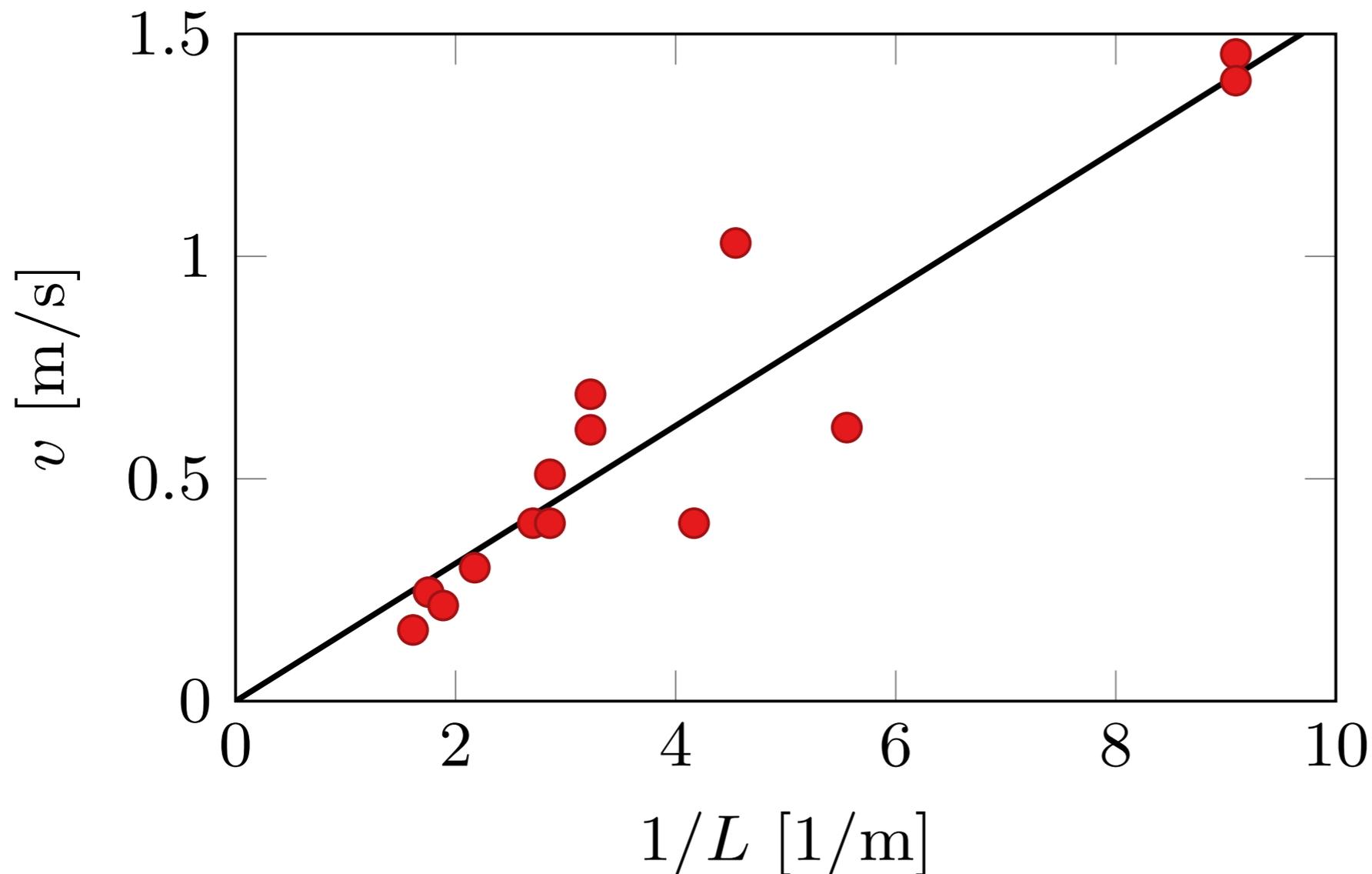
# migration speed

$$v = -\frac{\partial_t h}{\partial_x h} \propto \frac{\partial_x q}{\partial_x h} \propto 1/L$$

$$\partial_x q \sim q'(\tau) \partial_x \tau$$

$$\tau \propto H/L$$

$$\text{“}\partial_x \sim 1/L\text{”}$$



# Summary

- ▶ **analytical mesoscale model of aeolian ripples**
  - ripples vs megaripples
  - two-species ~ two particle sizes
  - megaripples = repton dunes
  - disintegration of wave structures
  - velocity scaling with length (not separation)
  - aspect ratio vs mass as predicted for dunes

**thank you!**