The flow of fluidised particles

Andr gg

Centr ronment cal Flows

Unive ristol

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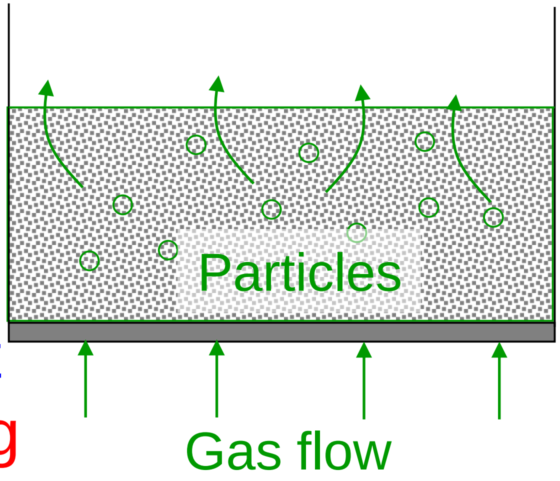
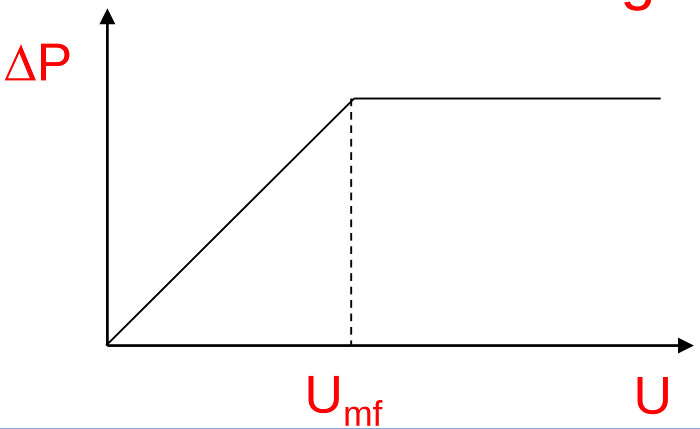
Introduction

- Granular materials are fluidised when their weight is borne by interstitial fluid.
- In a static fluidised bed
  - Gas flow is passed through a porous plate below the particles.
  - When the gas flow reaches a sufficient rate, the vertical component of the drag balances the weight of the particles





Pressure drop across the bed, ΔP, increases with gas flow until minimum velocity of fluidization, u_{mf}, and then remains constant.

Demonstration of fluidisation (i)

Royal Instituition:*Tales from the Prep Room: Making Sand Swim*

Demonstration of fluidisation (ii)

- When fluidised, the granular material can no longer support relatively dense objects

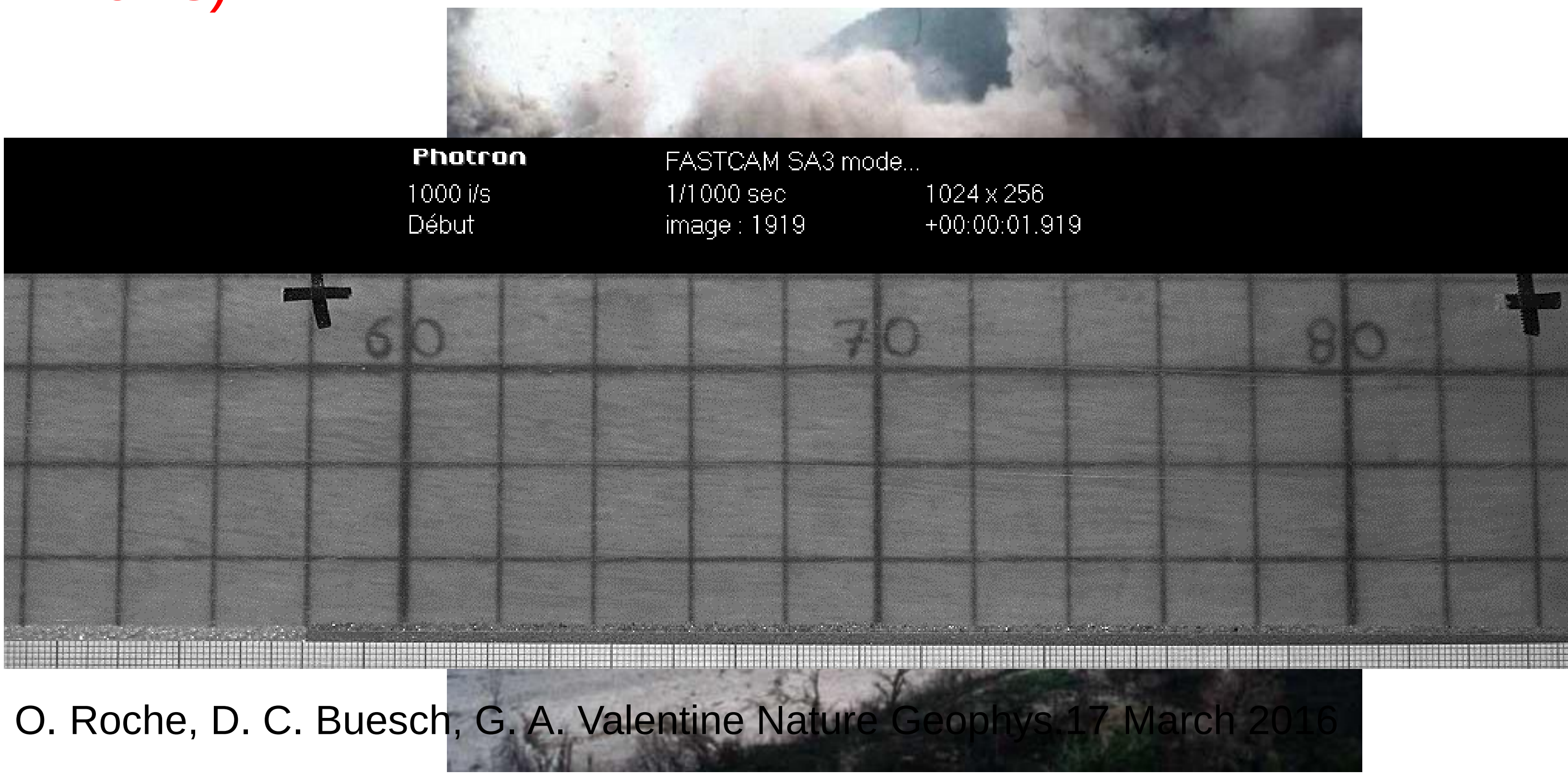
Crazy fluidisers

Flowing fluidised materials

- Gas flow can support the weight of the grains
  - The flows are highly mobile
  - What determines the resistance?
- Industrial application:
  The *dyna-slide* (air-slide) is used for conveying fine particulate along gradients less than their angle of repose

Geophysical application

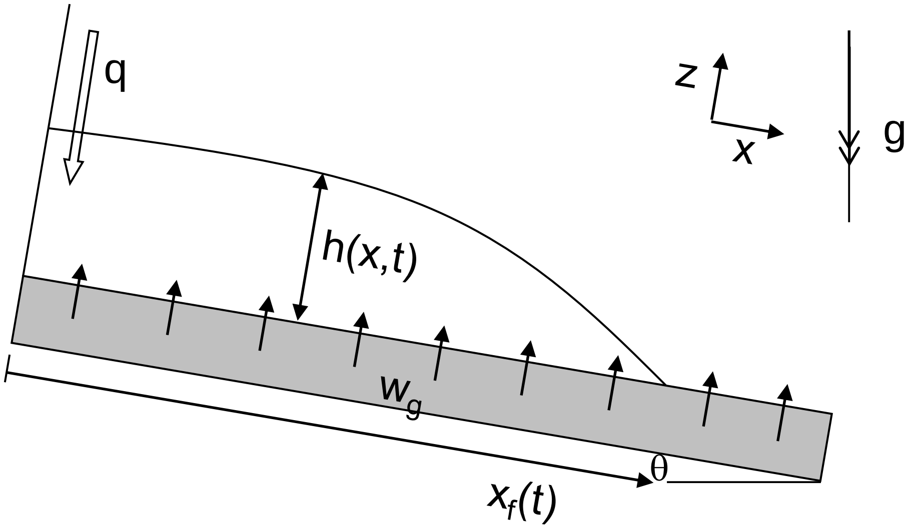
- Enhanced mobility of volcanic flows associated with gas release through the particulate material (Pyroclastic Flows)



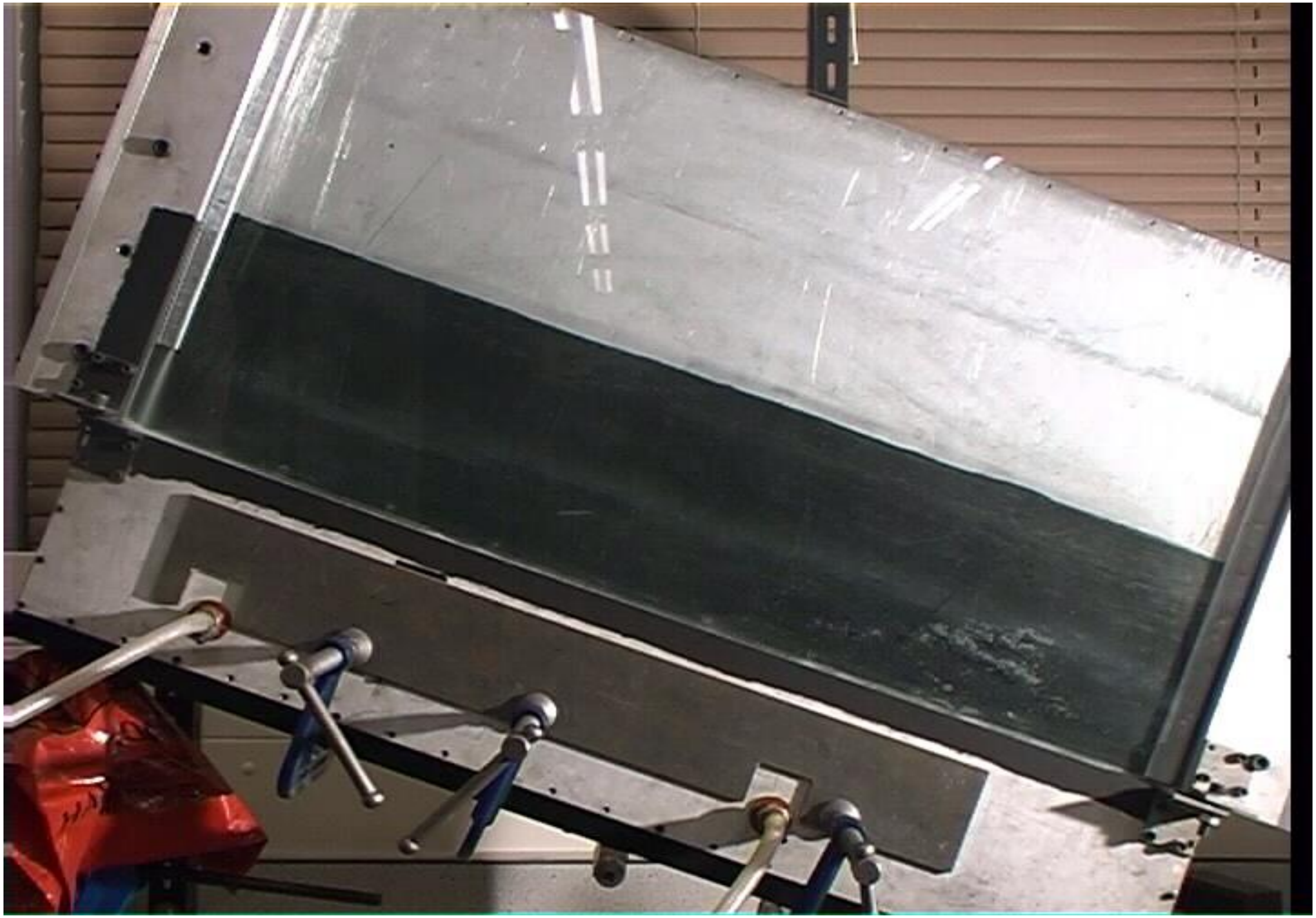
O. Roche, D. C. Buesch, G. A. Valentine Nature Geophys 17 March 2016

Experiments

- Particles were introduced at the end of a sloping, narrow channel (length 1m,width 1cm) at constant rates (q).
- The entire channel was fluidised with a flow rate, w_g, exceeding u_{mf}.
- Measurements were taken from video footage (+ PIV)



The effects of fluidisation

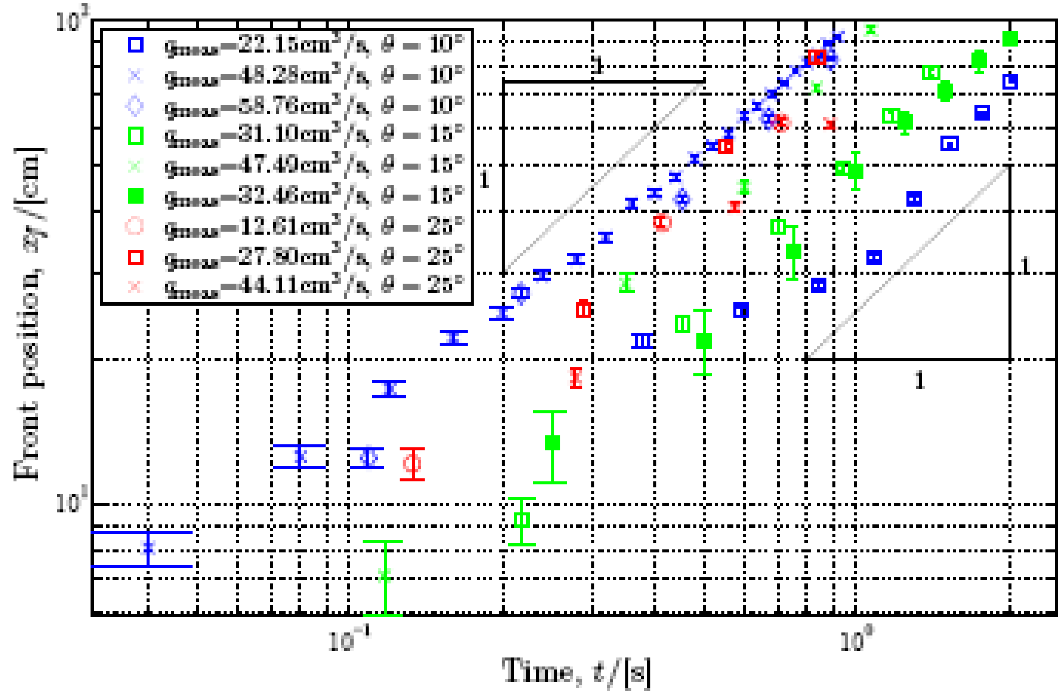


Dimensionless flow parameters

- Glass ballotini particles: Size $d=350\mu m$, Density $\rho_s = 2.5\ gcm^{-3}$, Geldart Class B
- Flows of depth h are characterised by 5 dimensionless groups
  - Slope $S=\tan\theta = 10^{-1}$
  - Density ratio $R=\rho_g/\rho_s=10^{-3}$
  - Flow thickness $\delta=d/h=10^{-2}-10^{-1}$
  - Fluidisation strength $W_g=\mu_g w_g/(\rho_s d^2 g\cos\theta)=10^{-3}$
  - Reduced Stokes number $St=\delta^2\rho_s(g\sin\theta h)^{1/2}/\mu_g=10^{-1}-10$
  - Particle Reynolds Number $Re_p=\rho_f w_g d/\mu_f \sim 1$

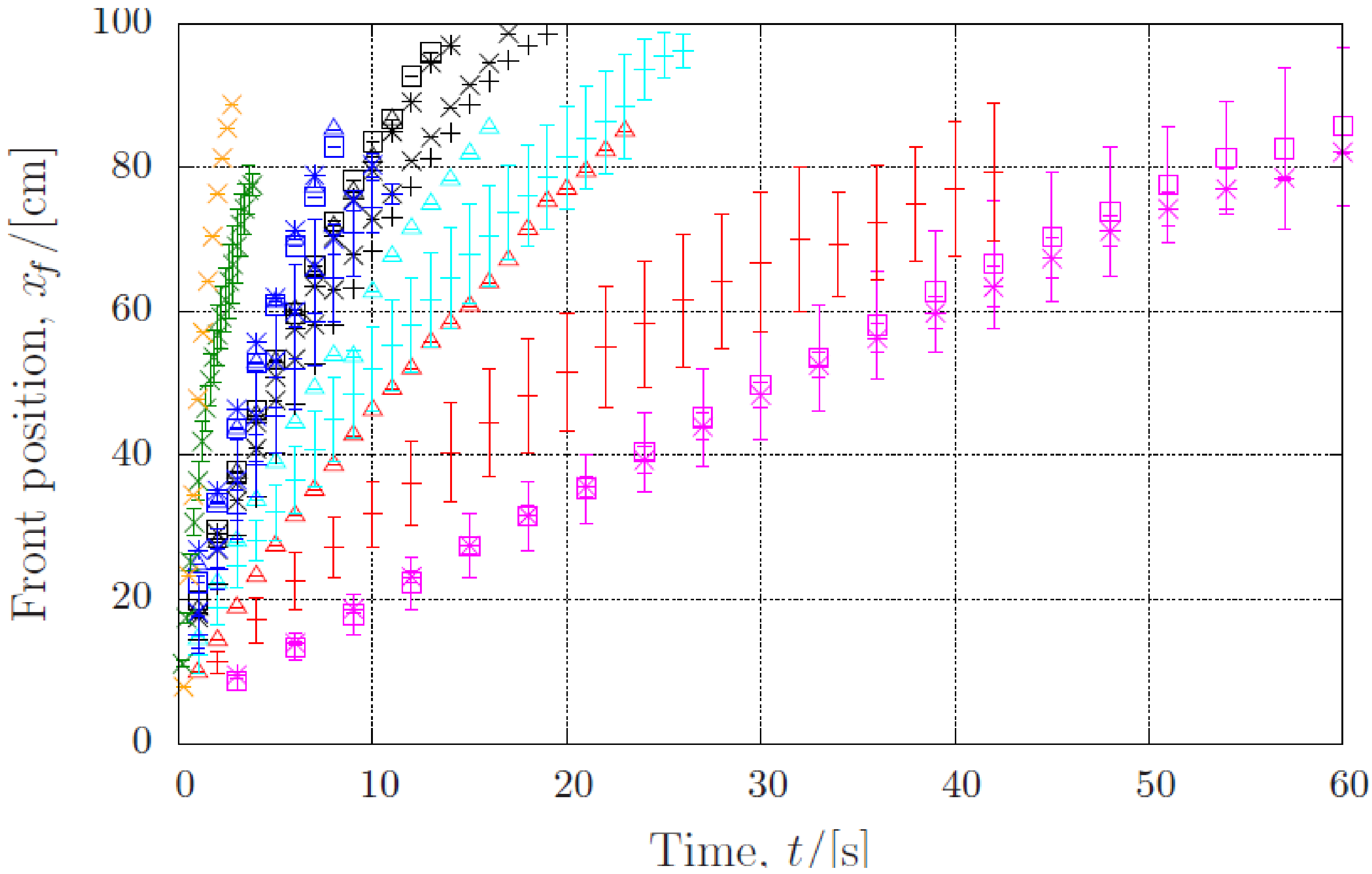
Steady fluidised current

- Flows down slopes reach a fully developed steady-state.
- By varying the inclination of the channel, we can use the experiment as a rheometer for fluidised flows.
- Measure h as function of q and θ.



Flows over horizontal surfaces

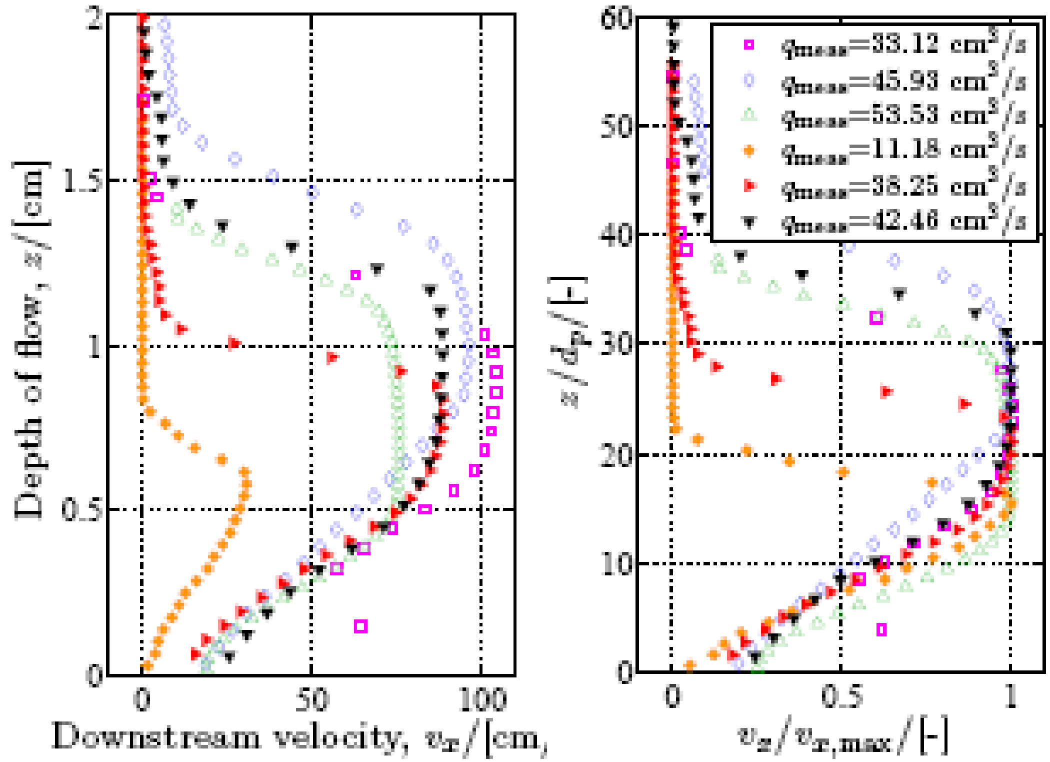
- The currents do not attain a steady state



Experiments with different volume fluxes at source

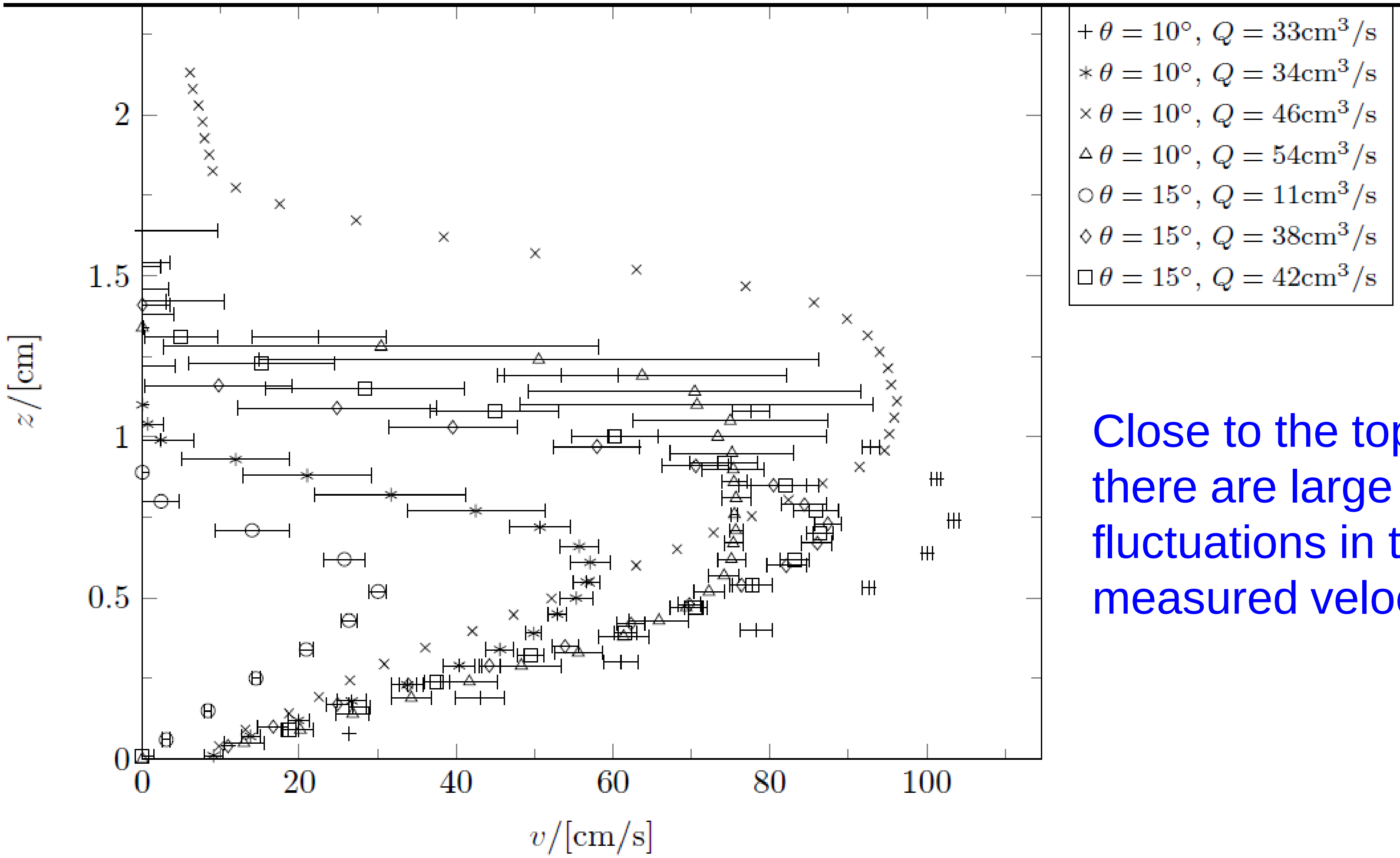
Velocity profile

- Using high-speed video footage and PIV techniques, it is possible to measure the velocity profile.
- 3 key features:
  - Slip velocity at z=0
  - Shearing zone (~15d)
  - Plug –flow region
- Horizontal gas flow speed is approximately same as speed of solids.



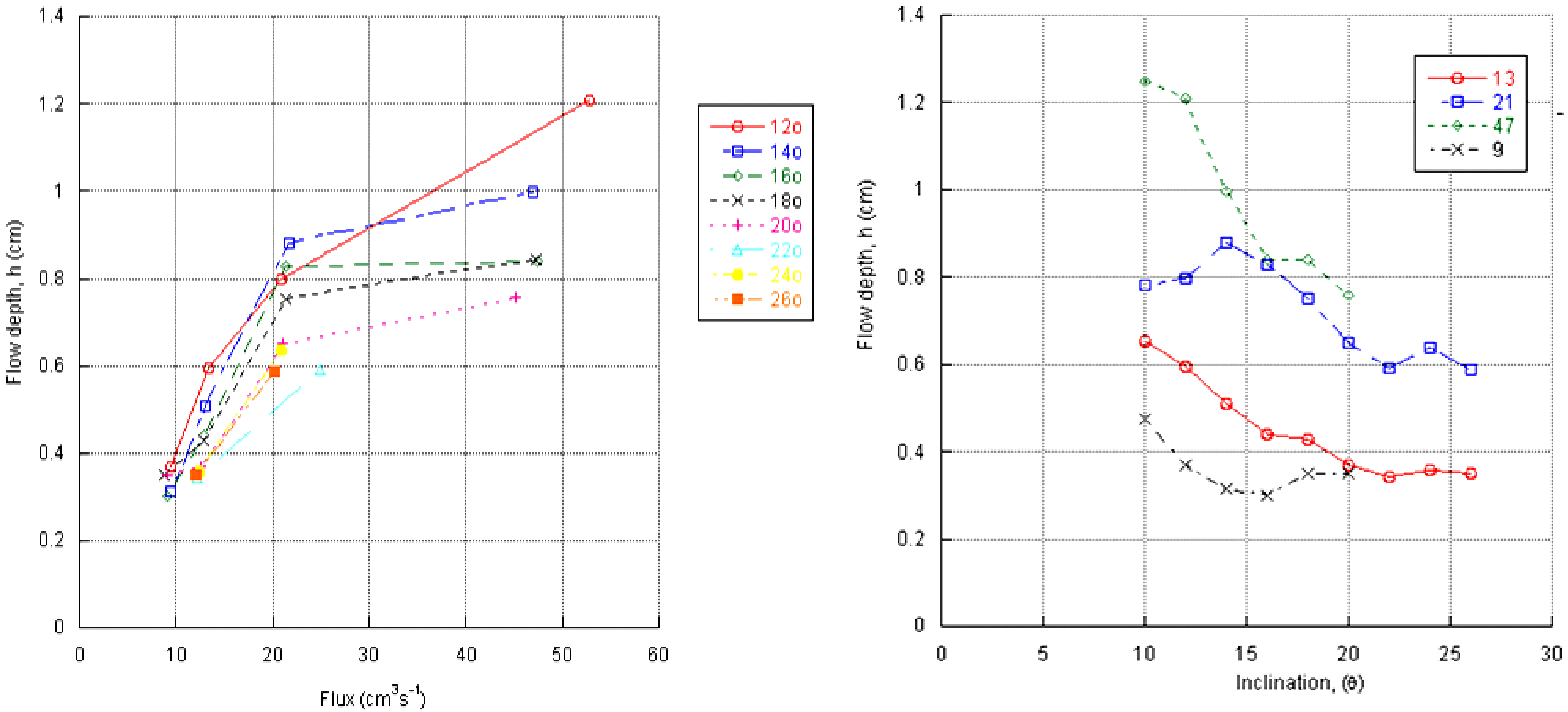
Results for steady flows down 10^o & 15^o inclines for various fluxes

Velocity profiles



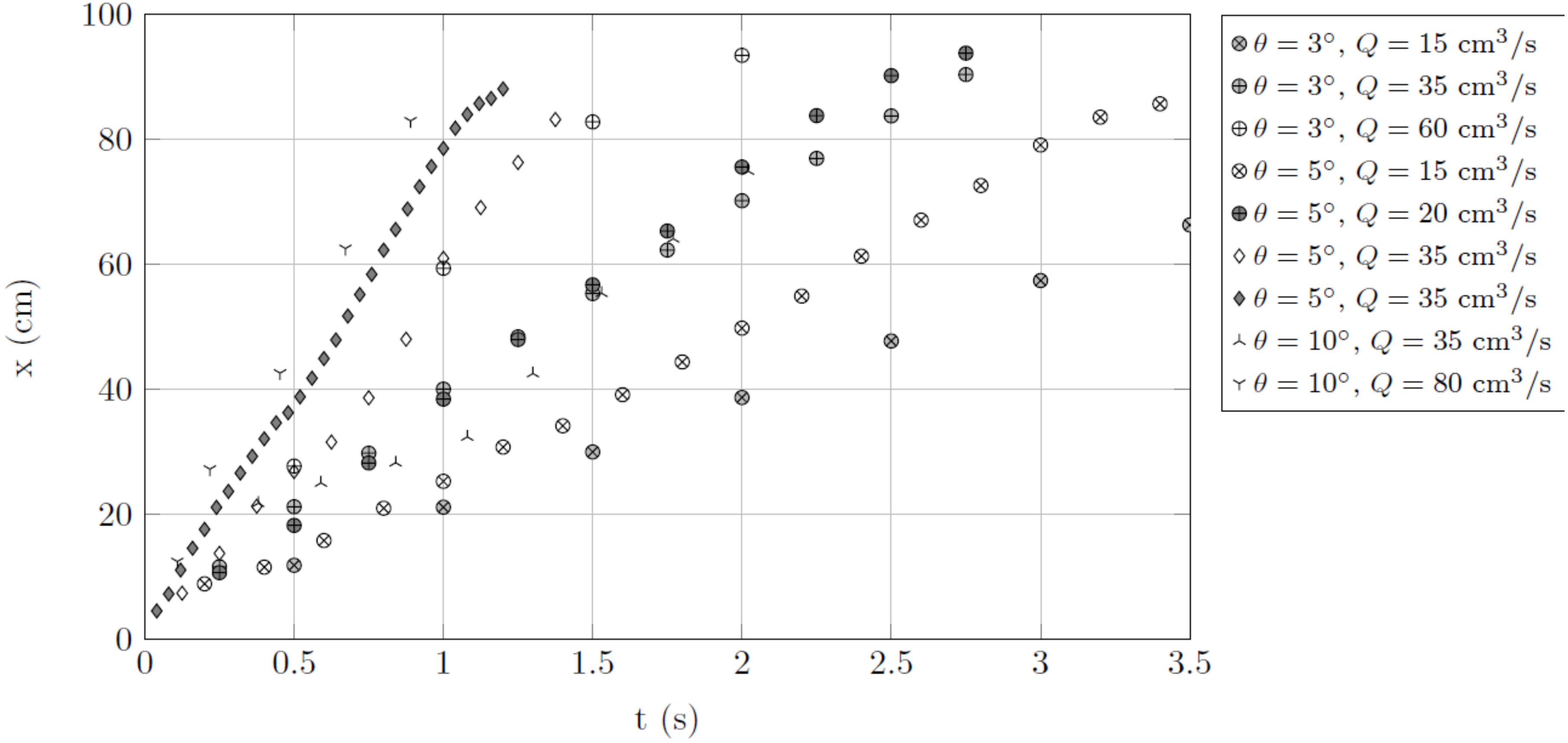
Close to the top surface there are large fluctuations in the measured velocity.

Results: *dependence of h*



Flow depth h as a function of (i) Flux q; and (ii) Inclination θ

Effects of source flux



Faster front speeds with higher fluxes

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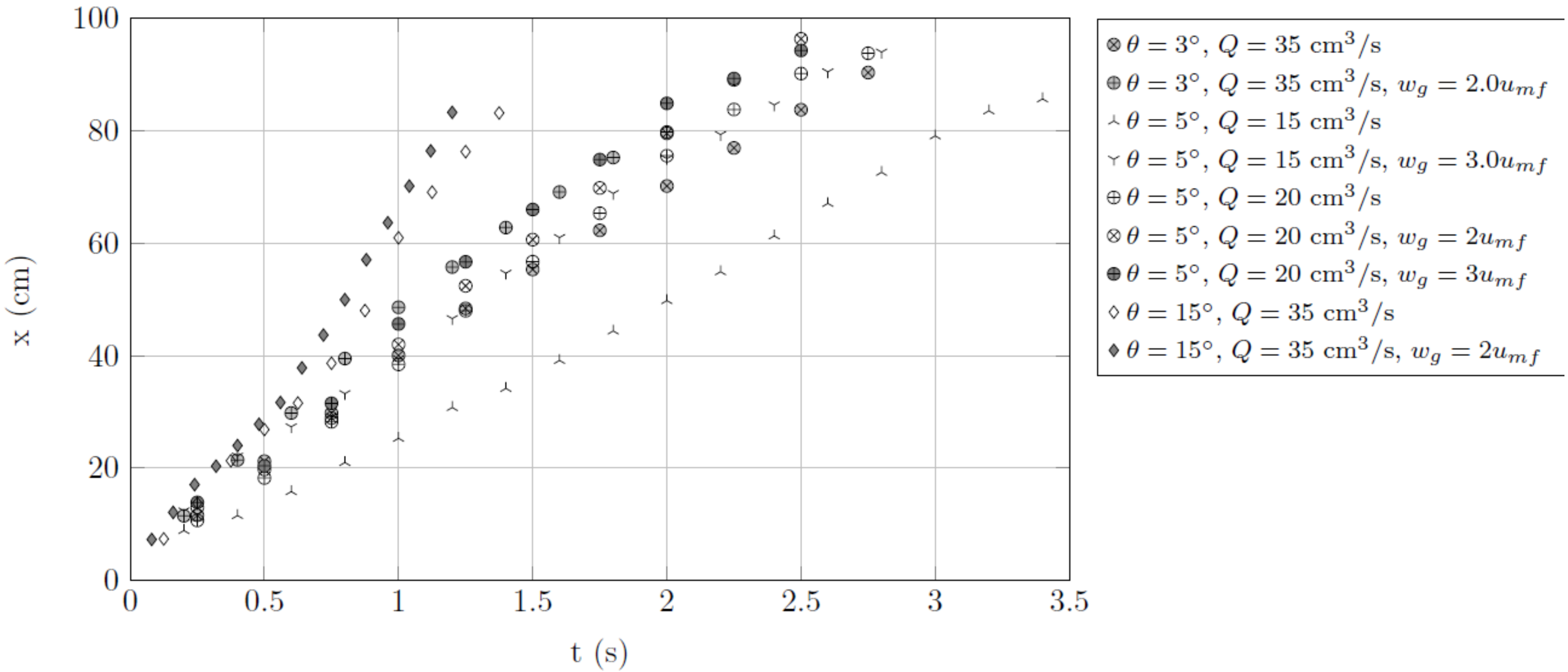
Effects of fluidisation velocity



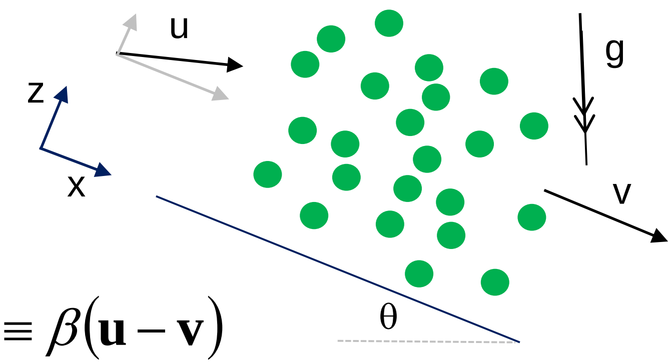
Faster front speeds with higher fluidisation velocities

Theoretical formulation: *Drag Law*

- We adopt a two-phase model and express mass & momentum (& energy) conservation for each phase.
- The key dynamical feature is the interaction between the solid and gas phases due to the imposed gas flow.
  - The vertical gas flow supports the weight of the particles.
- The Ergun equation for drag force between phases:

$$\mathbf{F}_{drag} = \left[\frac{150\mu_f \phi^2}{d^2(1-\phi)^3} + \frac{1.75\rho_f \phi|\mathbf{u}-\mathbf{v}|}{d(1-\phi)^3}\right](\mathbf{u}-\mathbf{v})$$

Since Re~1, the interaction force is dominated by the linear term



$$\mathbf{F}_{drag} \equiv \beta(\mathbf{u}-\mathbf{v})$$

Theoretical model: *Fully developed flow*

- Particle velocity **v**=(v(z),0), Gas velocity **u**=(u(z),w_g/(1-ϕ(z)))
  - Mass conservation automatically satisfied
- Normal component of momentum equation:

Fluid
$$\rho_g w_g \frac{\partial w}{\partial z} = -\rho_g(1-\phi)g\cos\theta - \frac{\partial p}{\partial z} + (1-\phi)\frac{4\mu_g}{3}\frac{\partial^2 w}{\partial z^2} - \mathbf{f}_{gs}.\hat{\mathbf{z}}$$

Solid
$$0 = \rho_s \phi g \cos\theta + \frac{\partial \sigma_{zz}}{\partial z} + \mathbf{f}_{gs}.\widehat{\mathbf{z}}$$

Interaction force between phases $\mathbf{f}_{gs} = -\phi\nabla p + \mathbf{F}_{drag}$

- Combined normal momentum balance for uniformly fluidised material (ϕ, *w* constant)

$$\frac{\partial}{\partial z}(p - \sigma_{zz}) = -(\phi\rho_s + (1-\phi)\rho_g)g\cos\theta$$

- Normal stress in solid fraction

$$\frac{\partial \sigma_{zz}}{\partial z} = \phi(\rho_s - \rho_f)g\cos\theta - \frac{\beta w_g}{(1-\phi)^2}$$

Theoretical model: *Fully developed flow*

- Fluid phase horizontal momentum:

$$\rho_g w_g \frac{\partial u}{\partial z} = \rho_g (1-\phi) g \sin\theta - \beta(u-v) + \mu_g (1-\phi) \frac{\partial^2 u}{\partial z^2}$$

- Solid phase horizontal momentum

$$0 = \rho_s \phi \sin\theta + \beta(u-v) + \frac{\partial \sigma_{xz}}{\partial z}$$

- Combined momentum equation reveals that solids' shear stress (σ_{xz}) must be non-negligible to achieve balance.

$$\rho_f w_g \frac{\partial u}{\partial z} = \left(\rho_f (1-\phi) + \rho_s \phi\right) g \sin\theta + \mu_f (1-\phi) \frac{\partial^2 u}{\partial z^2} + \frac{\partial \sigma_{xz}}{\partial z}$$

  - Size of shear stress: $\sigma_{xz} \sim \rho_s gh$, but can not be Coulomb-like as there is no normal stress in solid phase.

- Need a rheology for fluidised material!

Kinetic theory: *Granular temperature*

- The Stokes number of the particle motion is large
  - Particles interact directly with each other through collisions
- Granular temperature (T) measures the average fluctuations of the particle velocity.
- Granular temperature is generated by shear in the velocity field and dissipated through inelastic collisions.
- Kinetic theory provides closures for the particle stresses
- $\sigma_{xz} = \rho_s d T^{1/2} f_1(\phi, e) \frac{\partial v}{\partial z}$ $\qquad \sigma_{zz} = \rho_s d \mathrm{T} f_2(\phi, e)$

Granular temperature

- Conservation of granular temperature expressed by

$$0 = \frac{\partial q_T}{\partial z} + \sigma_{xz}\frac{\partial v}{\partial z} - f_3(\phi, e)\frac{T^{3/2}}{d}$$

Conduction Generation Dissipation

- Boundary conditions:
  - At base

$$u = 0 \qquad v = f_5 d\frac{\partial v}{\partial z} \qquad f_4 d\frac{\partial T}{\partial z} = f_6 v^2 - f_7 T$$

No slip (fluid) Slip (particles) Energy production/dissipation

  - At top surface

$$\frac{\partial u}{\partial z} = 0 \qquad p - \frac{4\mu}{3}\frac{\partial w}{\partial z} = 0 \qquad (\sigma_{xz}, \sigma_{zz}) = \frac{\pi}{6}\left(\frac{\phi}{\phi_m}\right)^{2/3}\rho_s d\boldsymbol{g} \qquad \frac{\partial T}{\partial z} = 0$$

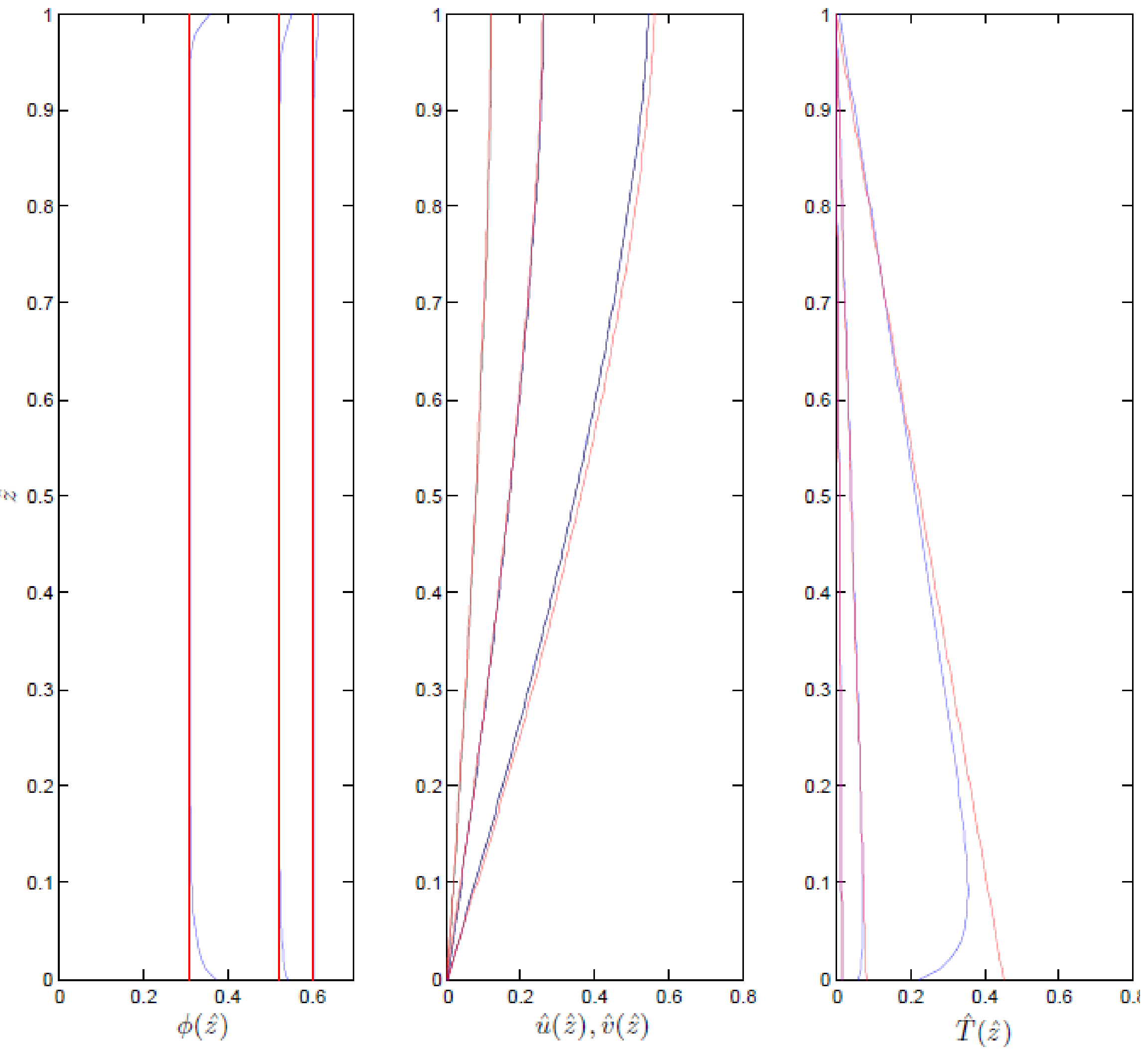
No fluid stresses Small particle stresses No energy flux

Dynamical regime

- Balance between downslope acceleration and particle shear stress
$$\rho_f w_g \frac{\partial u}{\partial z} = \left(\rho_f(1-\phi) + \rho_s \phi\right) g \sin\theta + \mu_f \frac{\partial^2 u}{\partial z^2} + \frac{\partial \sigma_{xz}}{\partial z}$$
- Balance between production and dissipation of granular temperature
$$0 = \frac{\partial q_T}{\partial z} + \sigma_{xz}\frac{\partial v}{\partial z} - f_3(\phi, e)\frac{T^{3/2}}{d}$$
- Velocity scale $(g\sin\theta \; h^3/d^2)^{1/2}$,Temperature scale $g\sin\theta\, h$
- Differs from non-fluidised flows: these require the solids normal stress to balance the weight ($T \sim g\cos\theta\, h$).
  - For fluidised flows the temperature is lower
  - The effective viscosity is lower
  - The flows are faster more mobile.

Numerical solution

- Boundary value problem (8th order) with 5 dimensionless parameters
- Typical solutions for different values of W_g
- Uniform volume fraction
- Negligible velocity difference
- Linear decrease of T



Approximate solution (i)

- Away from boundaries

$$u = v \qquad\qquad f_1\left(\frac{\partial v}{\partial z}\right)^2 = f_3 T$$

No slip between phases Balance between production and dissipation

- Momentum balances normal and parallel to slope

$$-S\frac{\partial}{\partial z}(f_2 T) = \phi - W_g\frac{f_0}{(1-\phi)^2} \qquad\qquad \frac{\partial}{\partial z}\left((f_1 f_3)^{1/2} T\right) = -\phi$$

Normal stresses Shear stresses

- These admit a solution with uniform volume fraction $\phi(z) = \bar{\phi}$

$$\frac{S\bar{\phi} f_2}{(f_1 f_3)^{1/2}} = \bar{\phi} - W_g\frac{f_0}{(1-\bar{\phi})^2} \qquad\qquad \frac{\partial T}{\partial z} = -\frac{\bar{\phi}}{(f_1 f_3)^{1/2}}$$

Approximate solution (ii): *v & T*

- Temperature and Velocity

$$T = -\frac{\bar{\phi}}{(f_1 f_3)^{1/2}}(1-z) \qquad v = \left(\frac{\bar{\phi}^2 f_3}{f_1^3}\right)^{1/4} \frac{2}{3}\left(1-(1-z)^{3/2}\right)$$

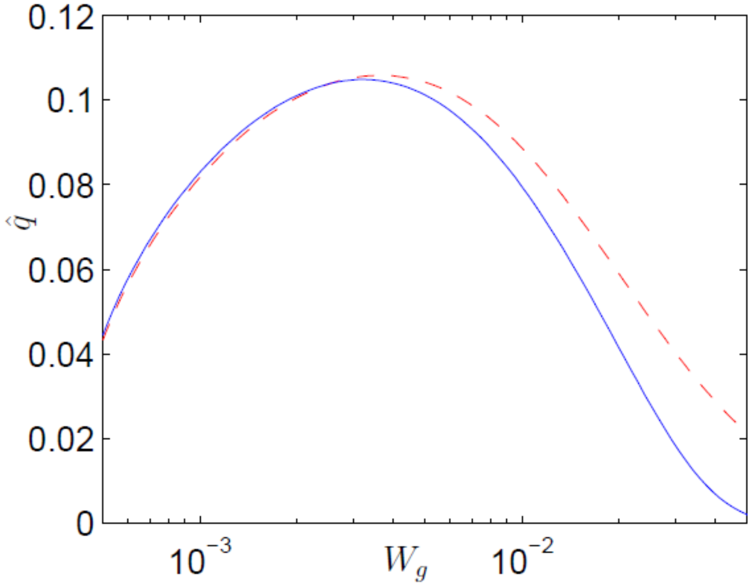
- Volume flux of particles

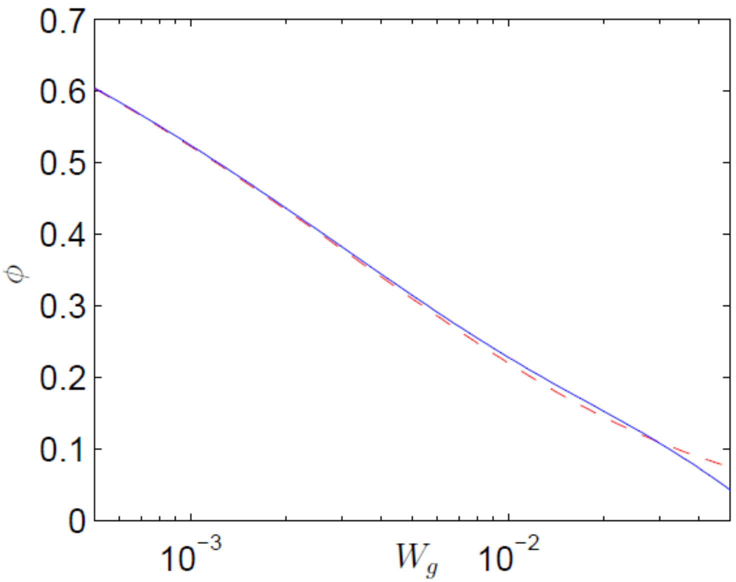
$$q = \int_0^h \bar{\phi} v dz = \frac{2}{5}\left(\frac{\bar{\phi}^6 f_3}{f_1^3}\right)^{1/4}$$

Dimensional form

$$q = \frac{2}{5}\left(\frac{\bar{\phi}^6 f_3}{f_1^3}\right)^{1/4} \left(\frac{h^5 g \sin\theta}{d^2}\right)^{1/2}$$

Numerical and asymptotic solutions





Approximate solution

- Volume flux (per unit width) carried layer

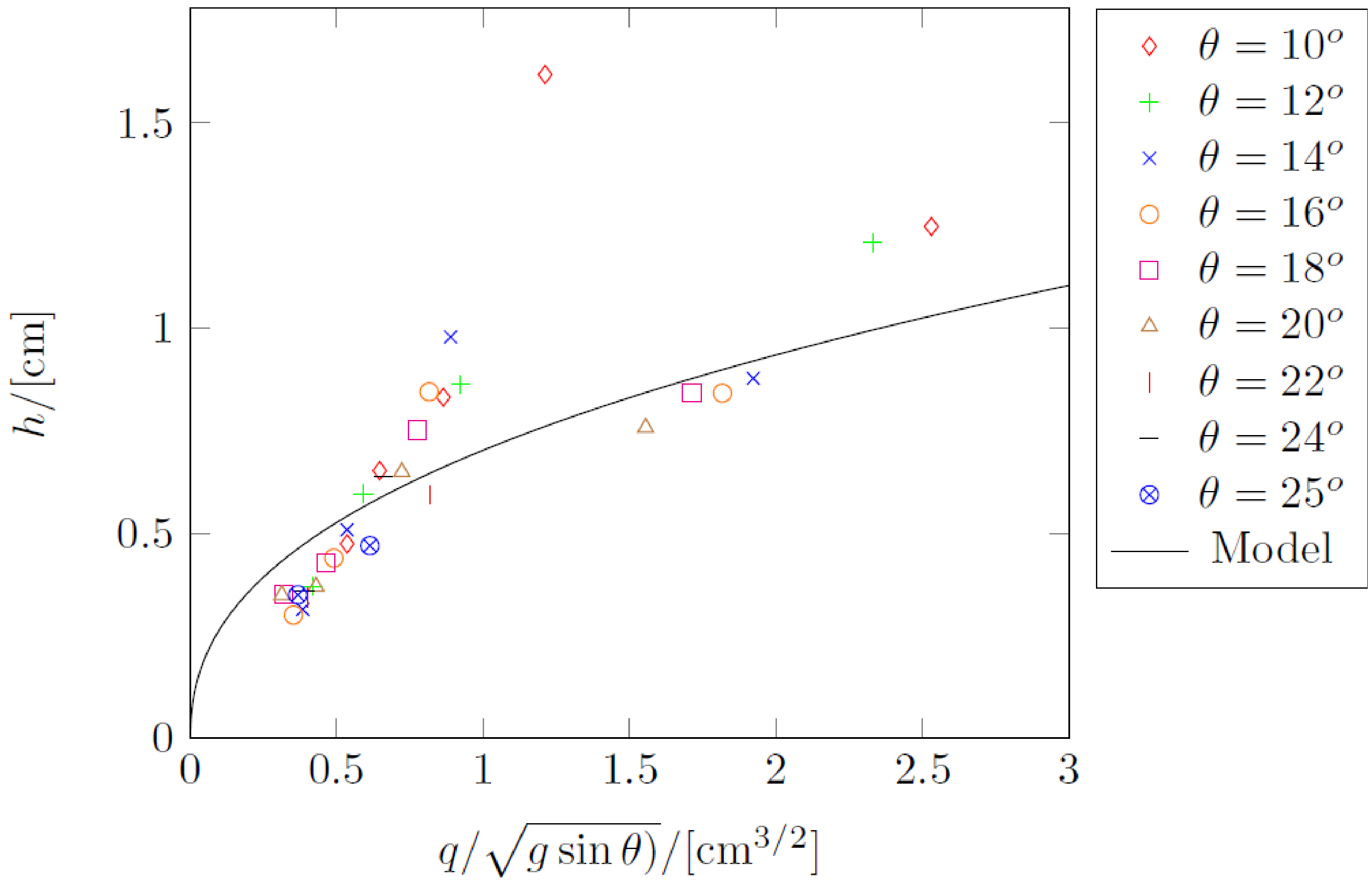
$$q = \frac{2}{5}\left(\frac{\bar{\phi}^6 f_3}{f_1^3}\right)^{1/4}\left(\frac{h^5 g \sin\theta}{d^2}\right)^{1/2}$$

Mobility F(ϕ)

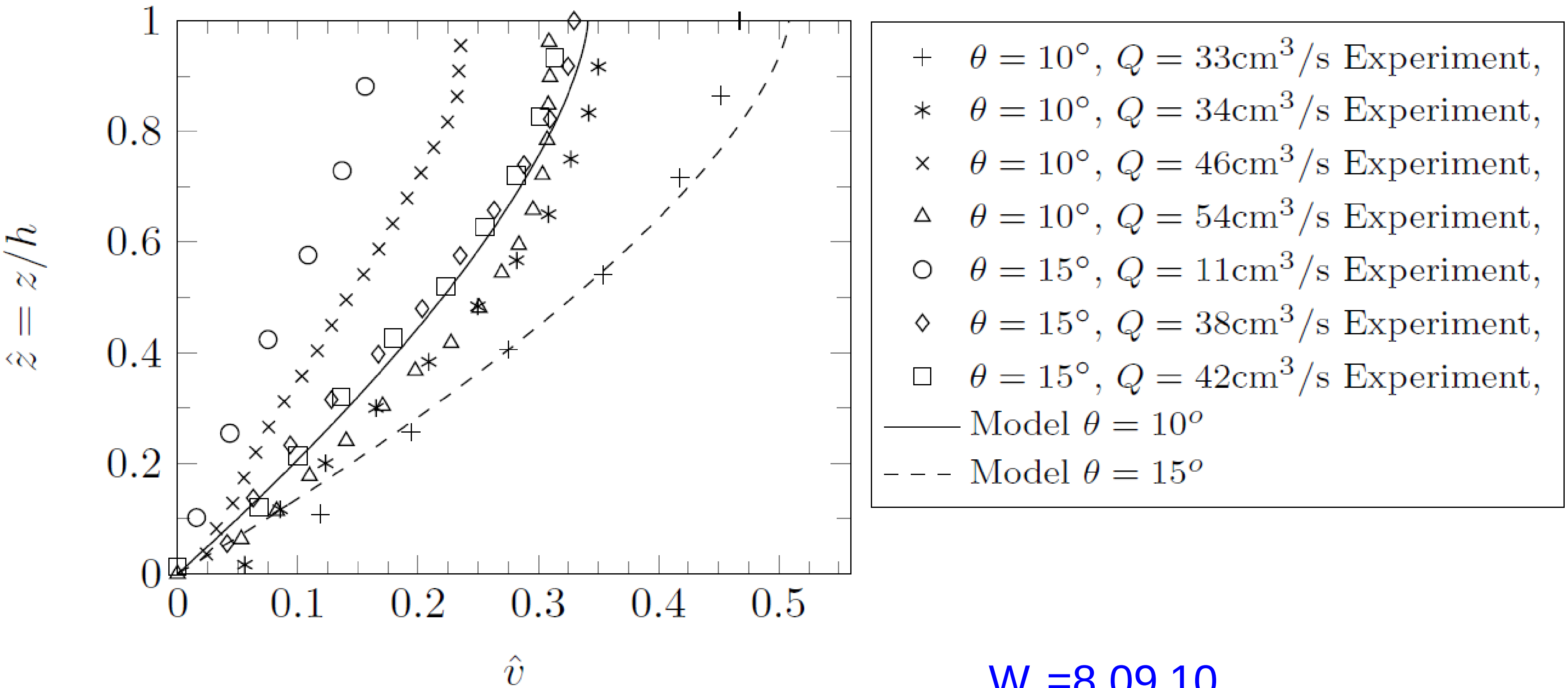
Bagnold scaling

- Mobility F(ϕ) determined by fluidising gas flow

Comparison with data



Velocity profiles



$W_g = 8.09\ 10$
$e = 0.75^{-4}$

Unsteady, developing flows *(dimensional variables)*

- The flows are shallow; vertical accelerations are negligible

$$\frac{\partial}{\partial z}(p-\sigma_{zz}) = \rho_s g\ \cos\theta\,\phi$$

- Dominant terms in downslope momentum balance

$$\rho_s\,\phi\frac{Dv}{Dt} = \rho_s\phi\,\mathrm{g}\sin\theta - \frac{\partial p}{\partial x} + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xz}}{\partial z}$$

- In depth-integrated form ($\sigma_{xx}=\sigma_{zz}$)

$$\rho_s\frac{\partial}{\partial t}\int_0^h \phi v\,dz + \rho_s\frac{\partial}{\partial x}\int_0^h \phi v^2\,dz = \rho_s \mathrm{g}\sin\theta\int_0^h \phi\,dz + \int_0^h -\frac{\partial p}{\partial x} + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xz}}{\partial z}\,dz$$

- Magnitude of inertial to 'viscous' terms $\frac{h\rho_s v^2}{L\sigma_{xz}} \sim \frac{h^3}{f(\phi)Ld^2}$

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Depth-averaged model

- Volume fraction, ϕ, determined by W_g
- Expressions for height of flow, *h(x,t),* and depth-averaged velocity, *v(x,t)*

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hv) = 0$$

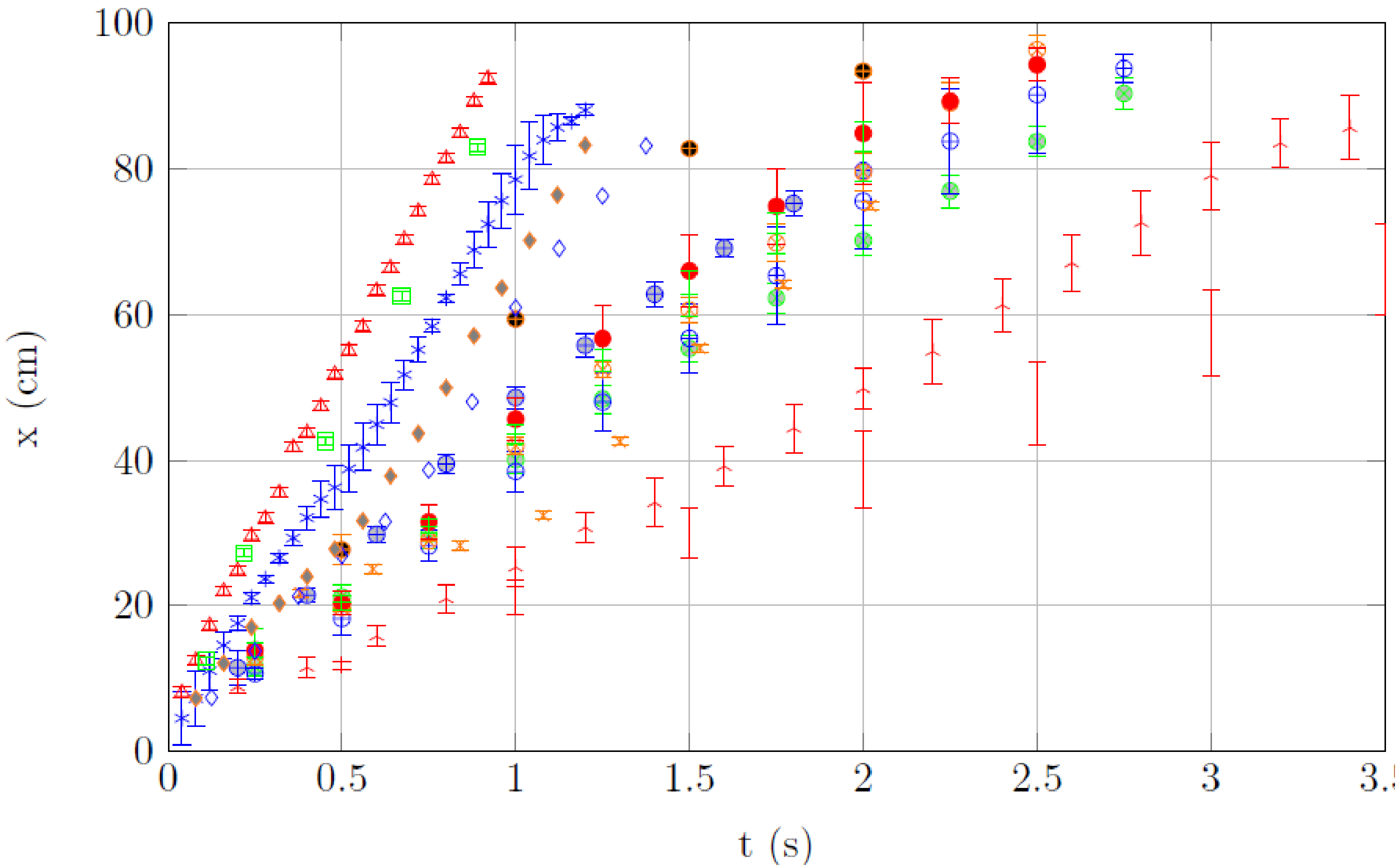
$$\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(hv^2) + \frac{g\cos\theta}{2}\frac{\partial}{\partial x}(h^2) = \mathrm{g}\sin\theta h - F(\phi)d^2\left(\frac{2v}{5h}\right)^2$$

$$\phi hv = q \ at \ x = 0$$

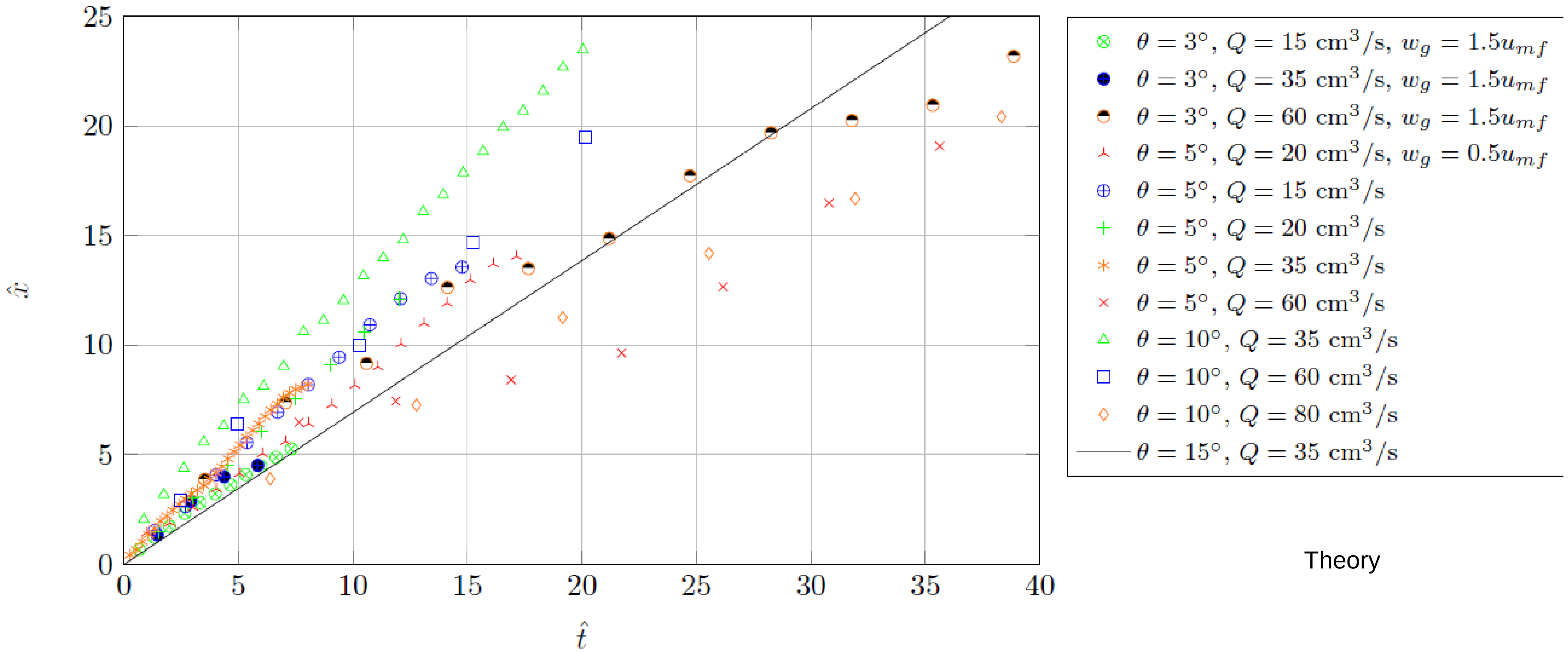
- Seek travelling wave solution, *h(x,t)=H(x- ct), v(x,t)=V(x- ct)*
- Find that *h(x,t)* and *v(x,t)* uniform throughout most of the domain and the front speed given by $\mathrm{g}\sin\theta h = F(\phi)d^2\frac{4v^2}{25h^2}$

Unsteady, developing flows: *down slopes*

- After initial transients, flow attains steady balance between
  downslope acceleration = basal drag



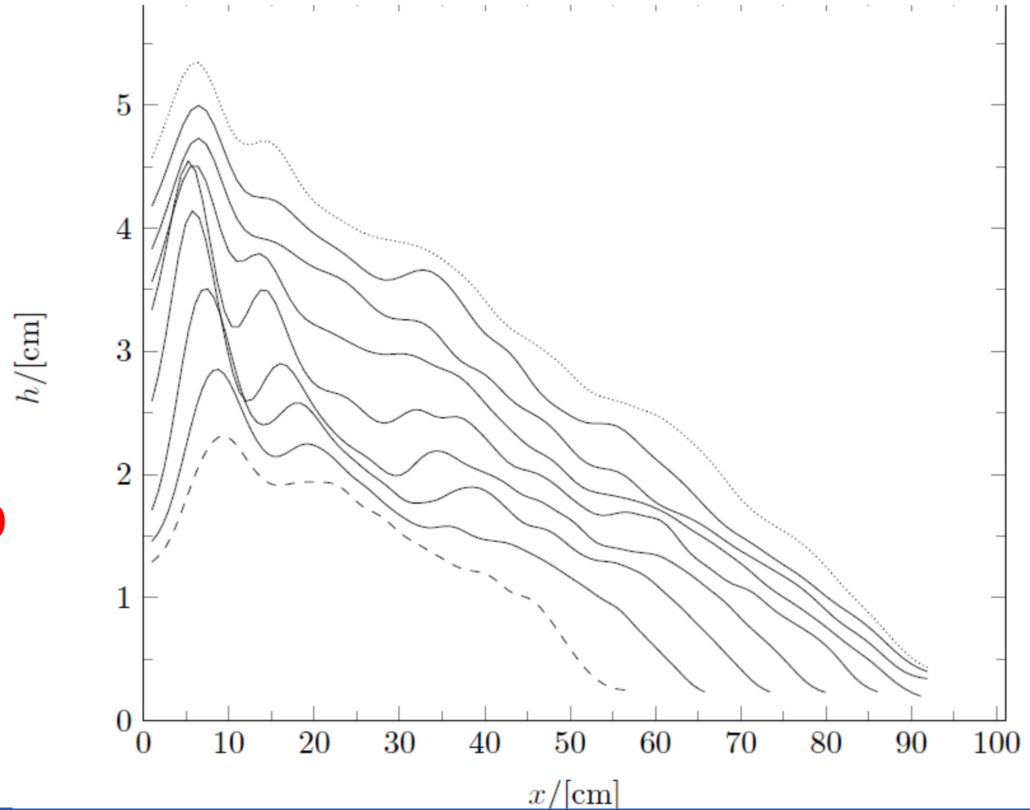
Downslope flow: *theory vs data*



- Dimensional $x_f = \left(\frac{4}{25}\frac{q^3 g \sin\theta F^2}{d^2\phi^3}\right)^{1/5} t$

Unsteady flows on horizontal surfaces

- Flows are slower and decelerate along channel
- Shear is localised to small basal region, with much larger plug-flow region.
- Height profiles at successive times



- Resistive stress due to side walls not base

Horizontal flows with side-wall drag

- Flows length (L) >> Flow depth (h) >>Flow width (B)
- Grains are supported by fluid drag
- Fluid pressure is hydrostatic
- Horizontal pressure gradients drive the motion and are resisted by cross-stream stresses.

$$\rho_s \, \phi \frac{Dv}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}$$

- When inertia negligible and granular temperature determined by local balance of production and dissipation

$$Q = \int \phi \, \rho_s v \, dzdy = \frac{2h}{5}\left(\frac{\phi^6 f_1^3}{f_3}\right)^{1/2}\left(\frac{B^5 g \sin\theta}{8d^2}\right)^{1/2}\left(-\frac{\partial h}{\partial x}\right)^{1/2}$$

Horizontal motion: *similarity solution*

- Mass conservation $$B\phi\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=0$$ subject to $Q(0,t)=Q_0$

- Dimensionless variables:
  scale h & x by H and t by $\phi H^2B/Q_0$ $$H=\frac{Q_0 d}{(gB^5)^{1/2}F\bar{\phi}}$$

- Governing equation
  $$\frac{\partial h}{\partial t}=\frac{1}{5\sqrt{2}}\frac{\partial}{\partial x}\left(h\left(-\frac{\partial h}{\partial x}\right)^{1/2}\right),$$ subject to $h\left(-\frac{\partial h}{\partial x}\right)^{1/2}=5\sqrt{2}$ at $x=0$

- Similarity solution:
  gearing between x & t: $h/t \sim (h/x)^{3/2}$ and $h^{3/2} \sim x^{1/2}$

- $x \sim t^{3/4}$ and $h \sim t^{1/4}$

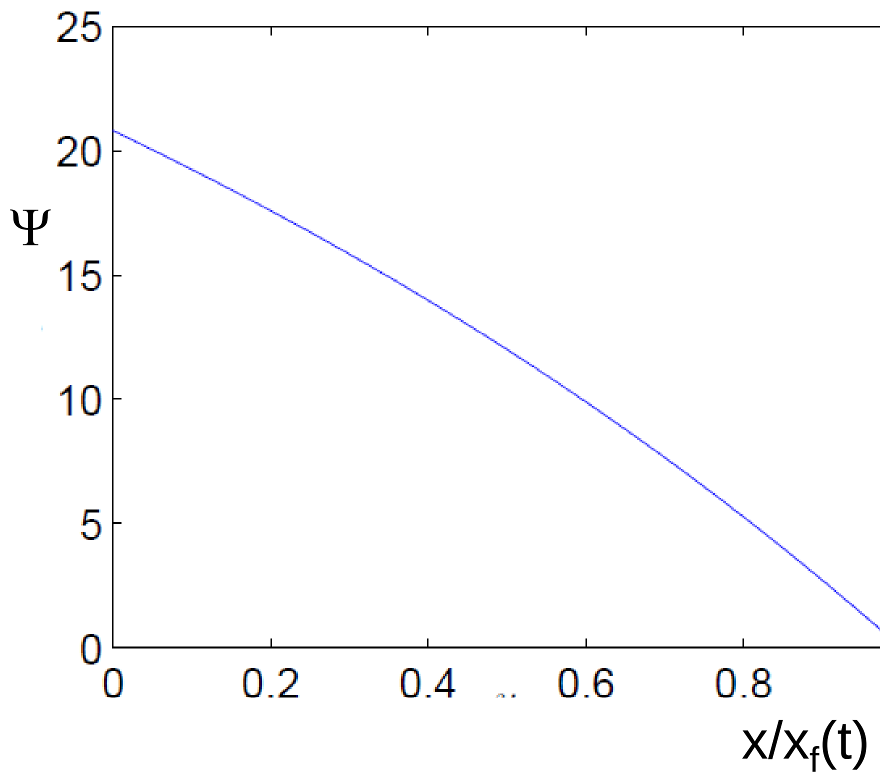
Similarity solution

- Dimensionless Solution: (Numerical solution of ODE with boundary conditions)

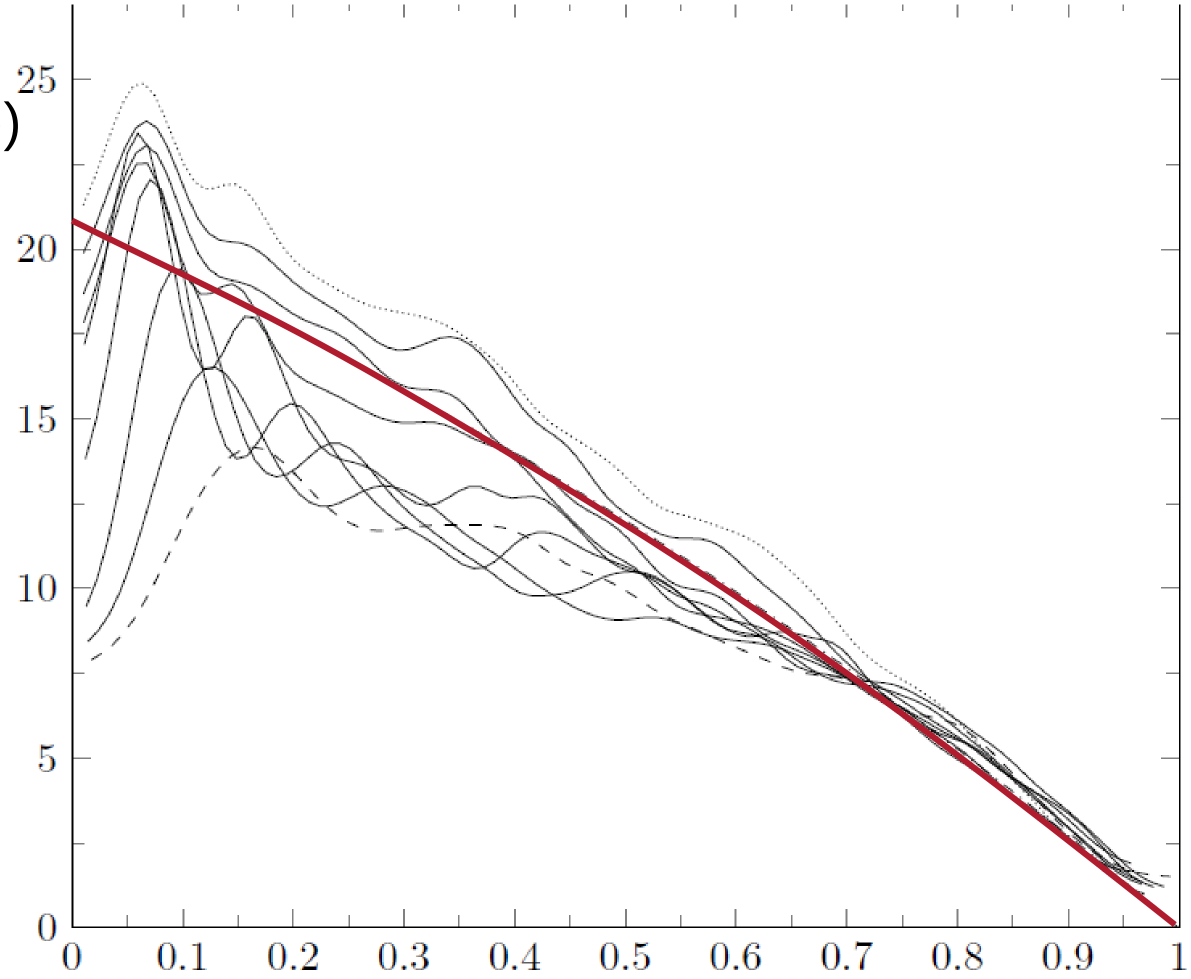
$$x_f(t) = Kt^{3/4}$$

$$h(x,t) = t^{1/4}\Psi\left(\frac{x}{x_f}\right)$$

- K=0.5434



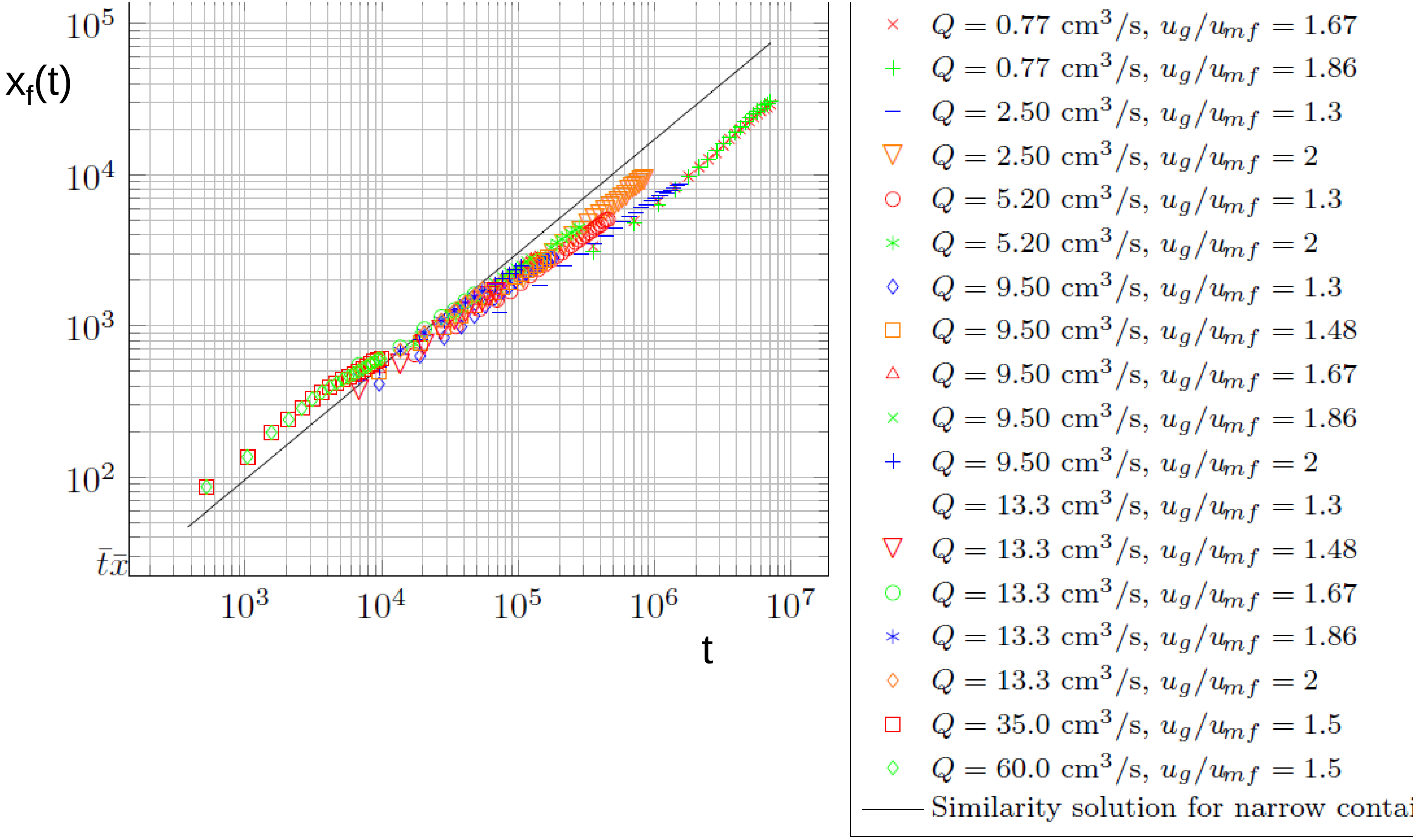
Horizontal motion: *theory and experiment*



$\Psi(x/x_f(t))$

$x/x_f(t)$

Horizontal motion: *theory and experiment*



Conclusions

- We have experimentally investigated gravitationally-driven fluidised flow down slopes and over horizontal surfaces
  - Fully developed steady state
  - Velocity profile
- A two-phase model of the motion, featuring the drag between the phases, can not be balanced unless the shear stresses from the solid-phase are included.
- In this experimental regime, the particles flow at high Stokes number and so collisions dominate.
- Predictive model of unsteady motion without any fitted parameters

Mixtures of materials