Time-of-flight measurements to observe anyonic statistics

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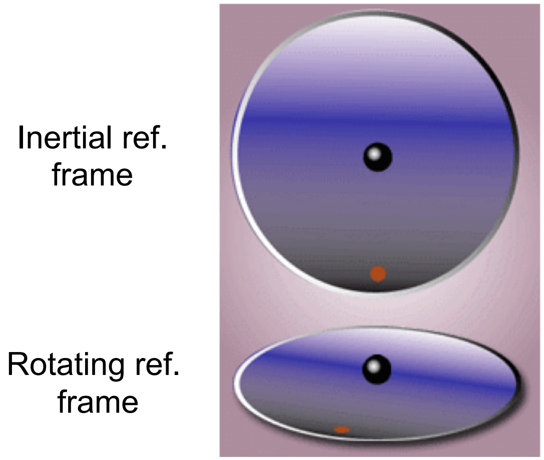
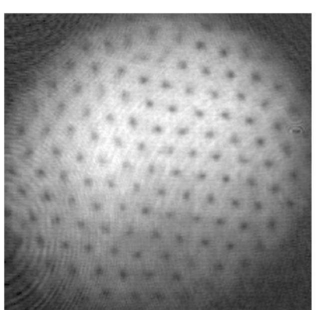
Artificial gauge fields for neutral particles (in continuum)

Coriolis effect



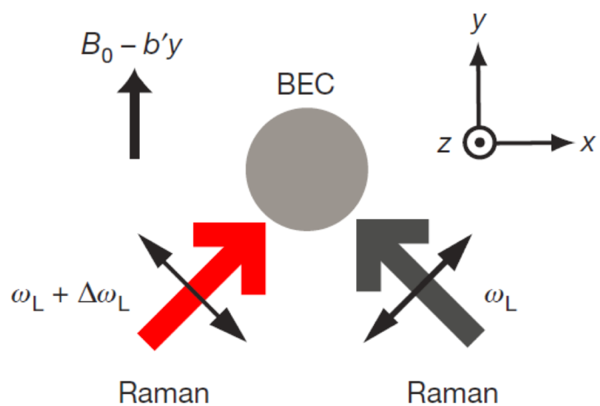
(Image from Wikipedia)

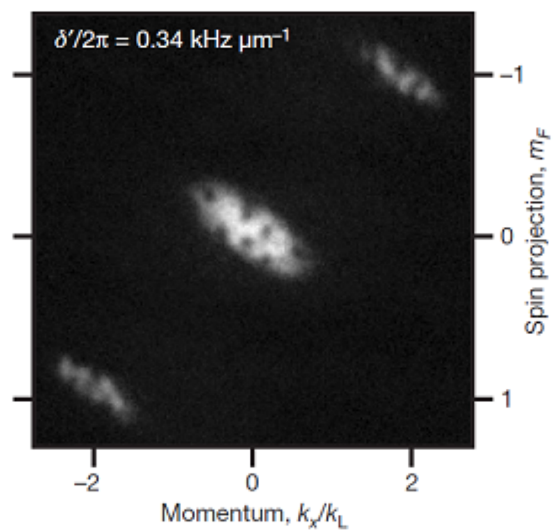
Vortex lattice in a **rotating** Na condensate



J. R. Abo-Shaeer *et al.*, **Science** 292, 476 (2001)

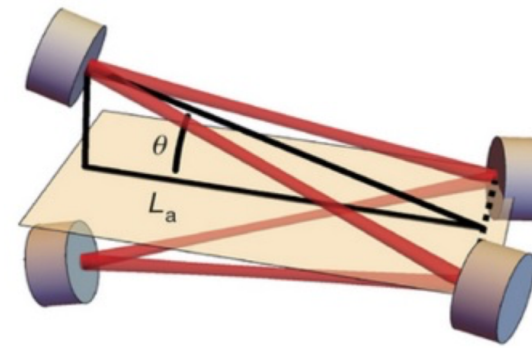
Berry phase approach

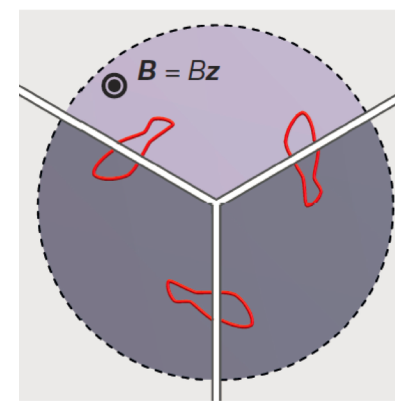


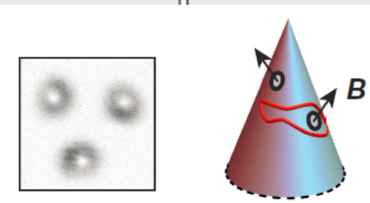


Y.-J. Lin *et al.*, **Nature** 462, 628 (2009)

Photonic Landau levels





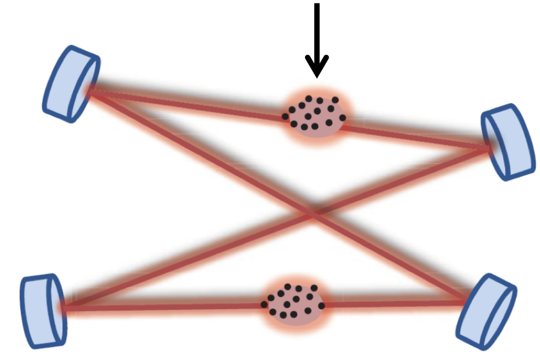


N. Schine *et al.*, **Nature** 534, 671 (2016)

Adding interactions to reach the FQH regime

$$H_{\rm FQH} = \sum_{i=1}^{N} \frac{(-i\hbar\nabla_i - \mathbf{A})^2}{2M} + g_{\rm int} \sum_{i<j} \delta^{(2)}(\mathbf{r}_i - \mathbf{r}_j)$$

See, e.g., B. Paredes *et al.*, **PRL** 87, 010402 (2001)



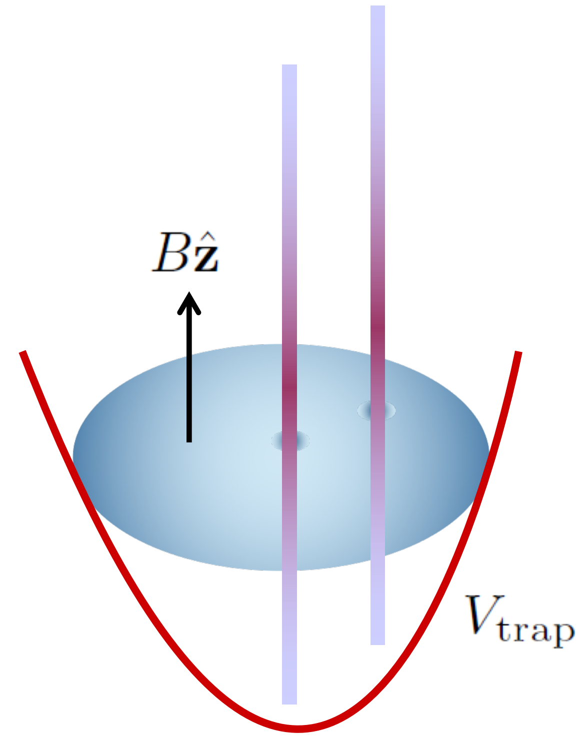
P. A. Ivanov *et al.*, **PRA** 98, 013847 (2018)

Symmetric gauge: $\mathbf{A}(\mathbf{r}) = B\hat{\mathbf{z}} \times \mathbf{r}/2$, LLL: $g_{\rm int}/\ell_{\rm B}^2 \ll \Delta E$

Exact ground state for contact interactions:

$$\Psi_{\rm FQH}(\zeta_1, \ldots, \zeta_N) \propto \prod_{j<k} (\zeta_j - \zeta_k)^m e^{-\sum_{i=1}^N |\zeta_i|^2/4}$$

R. B. Laughlin, **PRL** 50, 1395 (1983)

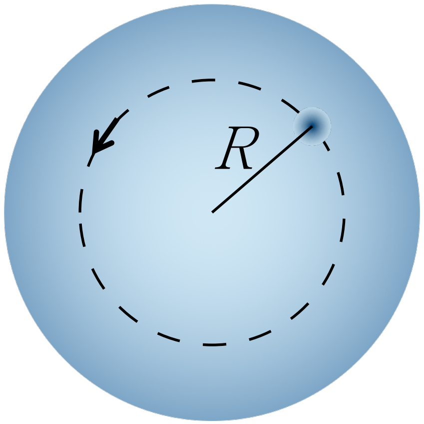


Pinning potential: $V_{\rm qh} = V_0 \sum_{i=1}^{N_{\rm qh}} \sum_{j=1}^{N} \delta^{(2)}(\mathbf{r}_j - \mathbf{R}_i)$

$$H_{\rm qh} = H_{\rm FQH} + V_{\rm qh} + V_{\rm trap}$$

$$\Psi_{\rm 1qh} \propto \prod_{i=1}^{N} (\zeta_i - \mathcal{R}_1) \Psi_{\rm FQH} \qquad \Psi_{\rm 2qh} \propto \prod_{i=1}^{N} (\zeta_i - \mathcal{R}_1)(\zeta_i - \mathcal{R}_2) \Psi_{\rm FQH}$$

Quasihole braiding and total angular momentum



Berry phase: $\varphi_{\mathrm{B}}(R) = i \oint_R \langle \Psi(\theta)|\partial_\theta|\Psi(\theta)\rangle d\theta$

M. V. Berry, **Proc. R. Soc. Lond. A** 392, 45 (1984)

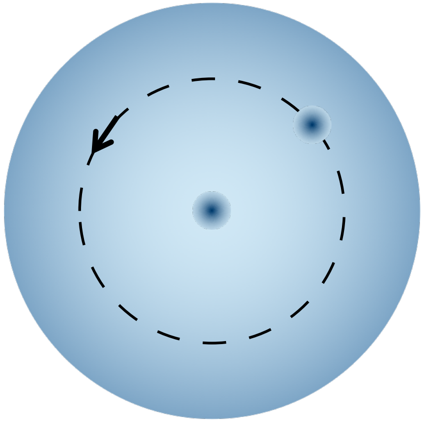
A different look at Berry phase:

$$\partial_\theta|\Psi(\theta)\rangle = \lim_{\delta\theta\to 0}\{[|\Psi(\theta+\delta\theta)\rangle - |\Psi(\theta)\rangle]/\delta\theta\}$$

$$|\Psi(\theta+\delta\theta)\rangle = \exp(-iL_z\delta\theta/\hbar)|\Psi(\theta)\rangle \qquad \exp(-iL_z\delta\theta/\hbar) \simeq 1 - iL_z\delta\theta/\hbar$$

$$\partial_\theta|\Psi(\theta)\rangle = -(iL_z/\hbar)|\Psi(\theta)\rangle \qquad \varphi_{\mathrm{B}}(R) = \frac{1}{\hbar}\oint_R \langle\Psi(\theta)|L_z|\Psi(\theta)\rangle d\theta = \frac{2\pi}{\hbar}\langle L_z\rangle$$

ROU, I. Carusotto, **Phys. Lett. A** 377, 2074 (2013) **(LLL case)**
ROU, E. Macaluso, T. Comparin, I. Carusotto, **PRL** 120, 230403 (2018)



Braiding phase: $\phi_{\mathrm{br}}(R) = \varphi_{\mathrm{B}}^{2\mathrm{qh}}(R) - \varphi_{\mathrm{B}}^{1\mathrm{qh}}(R)$

Exchange phase: $\phi_{\mathrm{st}}(R) = \phi_{\mathrm{br}}(R)/2 \ \ (= \nu\pi)$

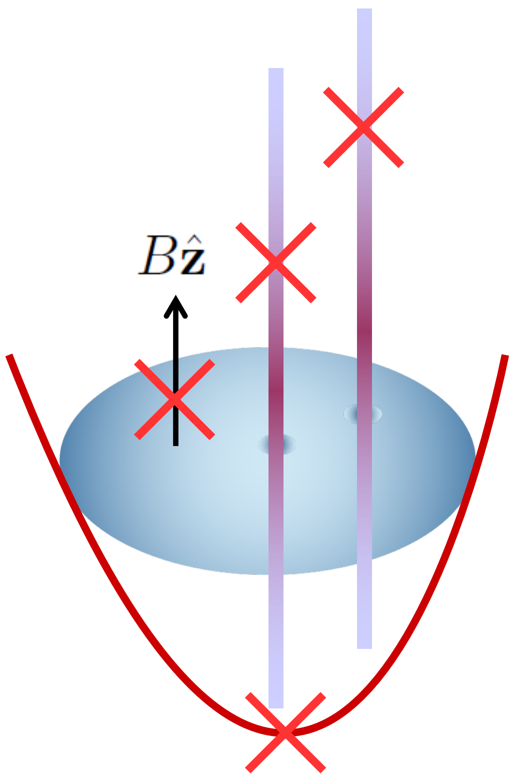
D. Arovas *et al.*, **PRL** 53, 722 (1984)

$$\phi_{\mathrm{br}}(R) = \frac{2\pi}{\hbar}(\langle L_z\rangle^{2\mathrm{qh}} - \langle L_z\rangle^{1\mathrm{qh}})$$

Measuring angular momentum indirectly

$$\langle r^2 \rangle_{\text{trap}} = \frac{2l_B^2}{N}\left(\frac{\langle L_z \rangle_{\text{trap}}}{\hbar} + N\right)$$ (exact in the LLL manifold)

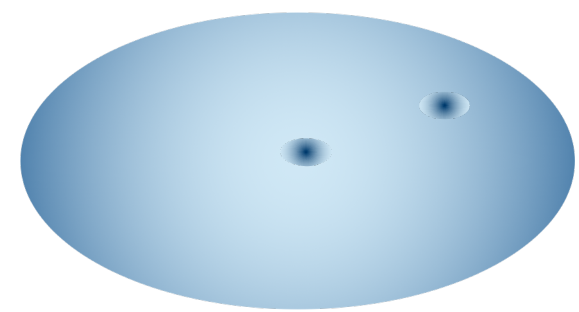
Time-of-flight measurement:



$$\hbar\mathbf{k} \simeq M\mathbf{r}/t$$

ballistic expansion

$$\langle n(\mathbf{r}) \rangle_{\text{TOF}} \simeq (M/\hbar t)^2 \langle \tilde{n}(\mathbf{k}) \rangle_{\text{trap}}$$



self-similar expansion: $\langle r^2 \rangle_{\text{TOF}} \propto t^2 \langle r^2 \rangle_{\text{trap}}$

T.-L. Ho, E. J. Mueller, **PRL** 89, 050401 (2002)
N. Read, N. R. Cooper, **PRA** 68, 035601 (2003)

Experimental observable: $\phi_{\text{br}}(R) \simeq 2\pi N \left(\frac{\sqrt{2}Ml_{\text{B}}}{\hbar t}\right)^2 (\langle r^2 \rangle_{\text{TOF}}^{2\text{qh}} - \langle r^2 \rangle_{\text{TOF}}^{1\text{qh}})$

Monte Carlo approach to calculate $\langle r^2 \rangle$

Laughlin's plasma analogy:

$$e^{-\beta U} = |\Psi|^2, \ \beta = 2\nu$$

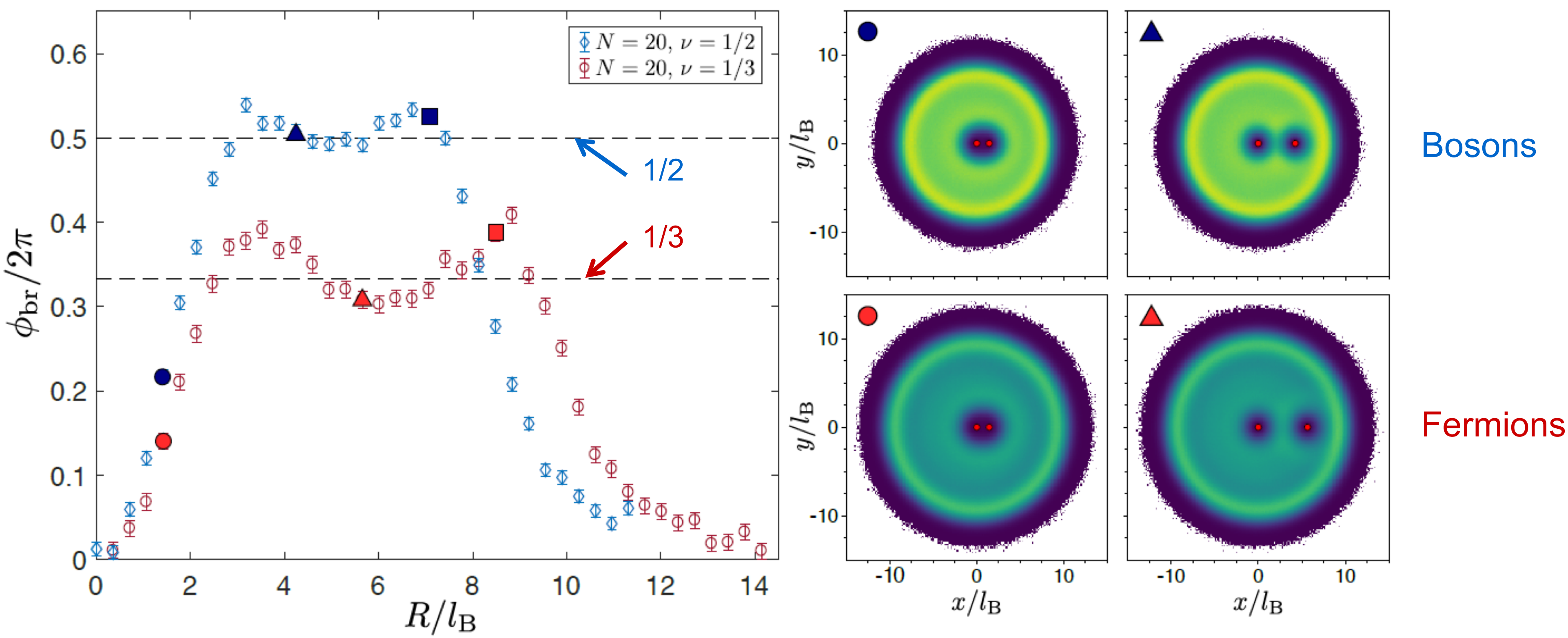
U is the energy of a 2D plasma with extra repulsive charges accounting for qhs

R. Morf, B. I. Halperin, **PRB** 33, 2221 (1986)

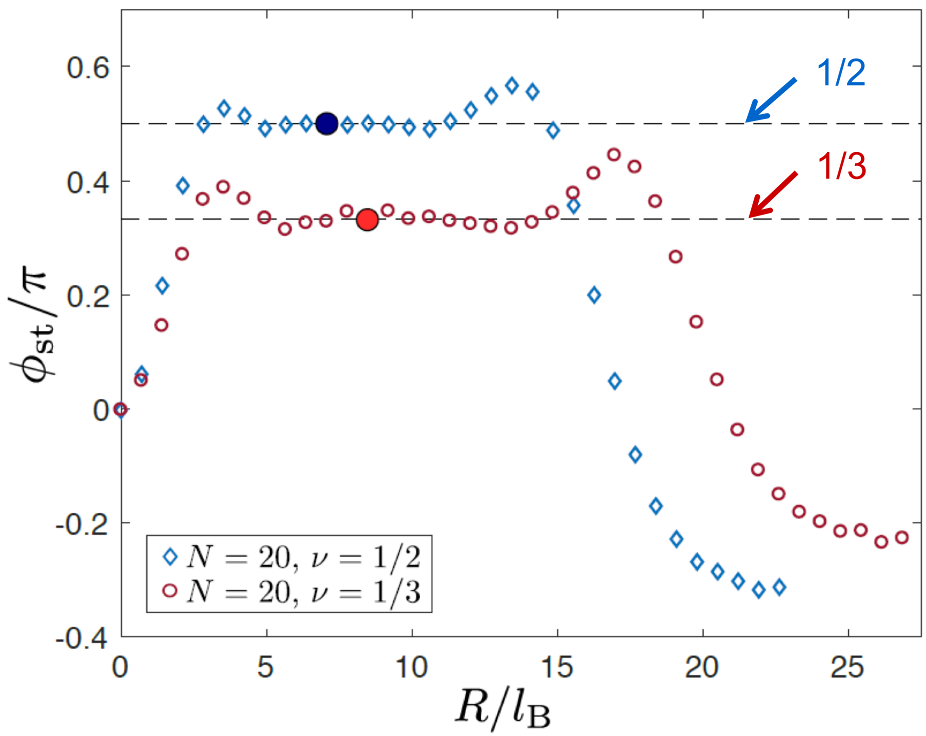
$$\langle \Psi_{\rm qh} | \frac{1}{N} \sum_{i=1}^{N} r_i^2 | \Psi_{\rm qh} \rangle$$

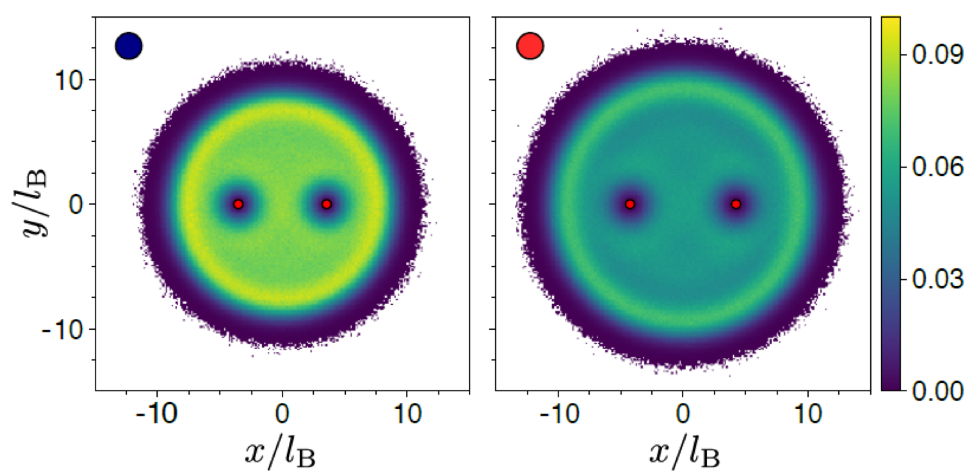
$$= \frac{\int d\zeta_1 d\zeta_1^* \ldots d\zeta_N d\zeta_N^* |\zeta_1|^2 e^{-\beta U}}{\int d\zeta_1 d\zeta_1^* \ldots d\zeta_N d\zeta_N^* e^{-\beta U}}$$

$$\phi_{\rm st}(R) = \phi_{\rm br}(R)/2 \qquad \phi_{\rm st} = \nu\pi$$



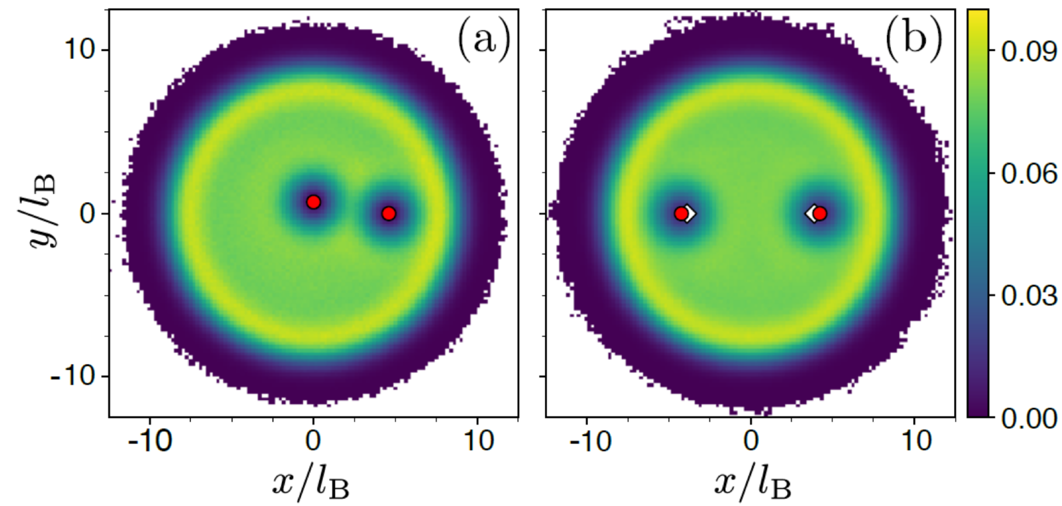
Symmetrically-positioned qhs for a larger plateau:





Possible imperfections:

Uncertainty in the qh positions



(a) One qh not exactly at the center. Arbitrary configurations require three average values:

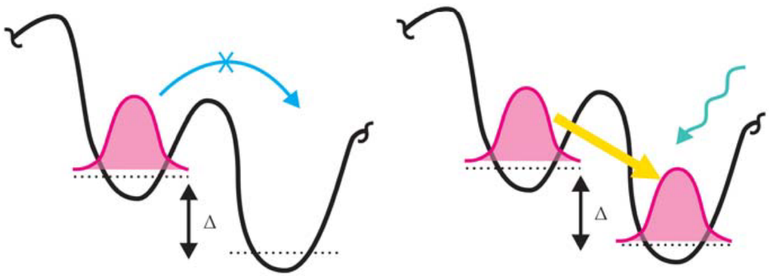
$$\langle L_z\rangle^{2\rm qh},\ \langle L_z\rangle^{1\rm qh}_{(1)},\ \langle L_z\rangle^{1\rm qh}_{(2)}$$

(b) Deviations in qh positions due to thermal fluctuations: include qh positions as additional coordinates in the MC algorithm

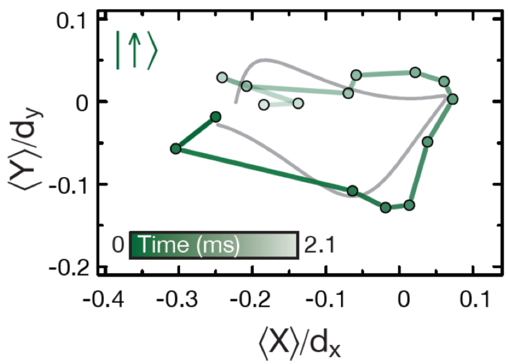
Exchange phase is robust against small deviations $(\sim l_{\rm B})$

Hofstadter-Hubbard Model

$$H_0 = -t \sum_{\langle ij \rangle} \left(e^{i2\pi\phi_{ij}} c_i^\dagger c_j + \text{h.c.} \right)$$

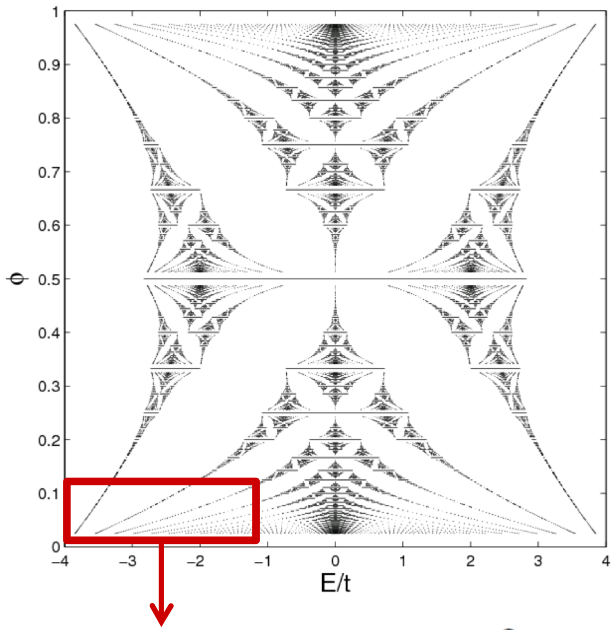


V. Galitski *et al.*, **Physics Today** 72, 39 (2019)

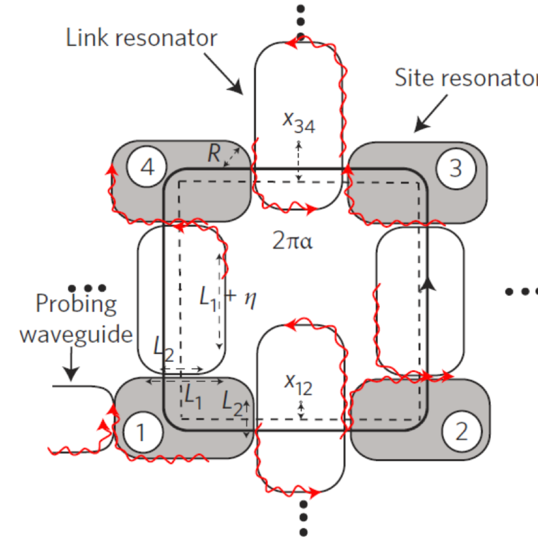


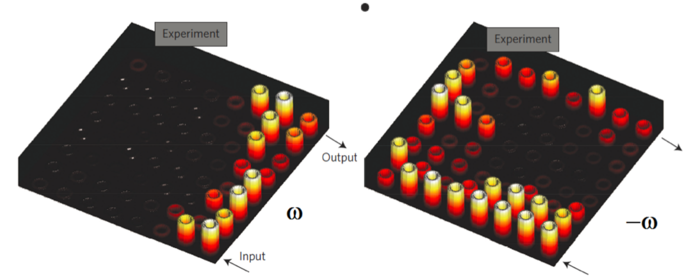
M. Aidelsburger *et al.*, **PRL** 111, 185301 (2013)
H. Miyake *et al.*, **PRL** 111, 185302 (2013)

D. R. Hofstadter, **PRB** 14, 2239 (1976)



Landau levels $\phi = Ba^2/\phi_0$





Theo: M. Hafezi *et al.*, **Nat. Phys.** 7, 907 (2011)
Exp: M. Hafezi *et al.*, **Nat. Phot.** 7, 1001 (2013)

With on-site repulsive interactions:

$$H_{\text{FQH}} = H_0 + (U/2) \sum_i n_i(n_i - 1)$$

Supports bosonic Laughlin states for low flux

A. S. Sørensen *et al.*, **PRL** 94, 086803 (2005)

Quasihole Hamiltonian

$$H_{1\text{qh}} = H_{\text{FQH}} + \boxed{V n_{i_0}}$$

qh pinning potential

Z. Liu *et al.*, **PRB** 91, 045126 (2015)

Real-space probe for lattice quasiholes

ROU, **PRA** 98, 063629 (2018)

$$\langle r^2\rangle_{\rm L}^{N,\phi} = 2N\ell_B^2 = \frac{N}{\pi\phi}a^2 \qquad \langle r^2\rangle_{\rm QH}^{N,\phi} = 2(N+1)\ell_B^2 = \frac{N+1}{\pi\phi}a^2$$

Continuum expectations

$$R_{\rm L/QH} \equiv \langle r^2\rangle_{\rm L}^{N,\phi}/\langle r^2\rangle_{\rm QH}^{N,\phi} = N/(N+1)$$

$$R_{\rm QH/QH} \equiv \langle r^2\rangle_{\rm QH}^{N,\phi}/\langle r^2\rangle_{\rm QH}^{N+1,\phi'} = \frac{(N+1)\phi'}{(N+2)\phi}$$

Our choice:

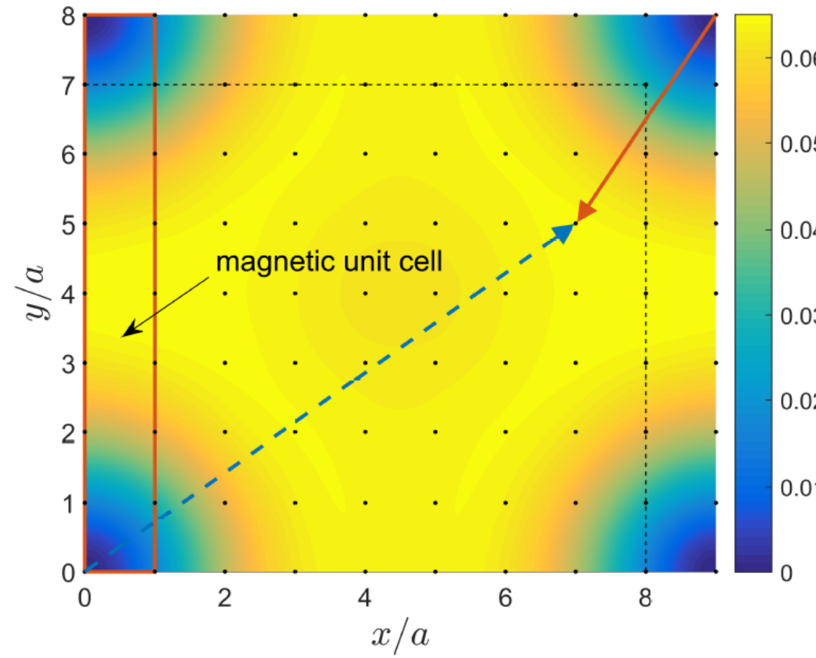
$$\phi = 1/2N$$

$$\phi' = 1/2(N+1)$$

(Periodic boundary conditions assumed)

	$\langle r^2\rangle_{\rm L}^{N,\phi}/a^2$		$\langle r^2\rangle_{\rm QH}^{N,\phi}/a^2$		$R_{\rm L/QH}$		$R_{\rm QH/QH}$	
	cont.	lat.	cont.	lat.	cont.	lat.	cont.	lat.
N=2	2.547	3.000	3.820	3.571 (4.050)	0.667	0.840 (0.741)	0.500	0.437 (0.493)
N=3	5.730	6.333	7.639	8.171 (8.210)	0.750	0.775 (0.771)	0.600	0.603 (0.606)
N=4	10.19	11.00	12.73	13.54	0.800	0.812	0.667	0.670
N=5	15.92	17.00	19.10	20.21	0.833	0.841	0.714	0.717
N=6	22.92	24.33	26.74	28.20	0.857	0.863	0.750	0.752
N=7	31.19	33.00	35.65	37.52	0.875	0.879	-	-

Continuum limit: $\phi \ll 1$



$N = 4,\ L_x = 9,\ L_y = 8,\ \phi = 1/8$

For a non-Abelian generalization
see **poster # 10** by Elia Macaluso