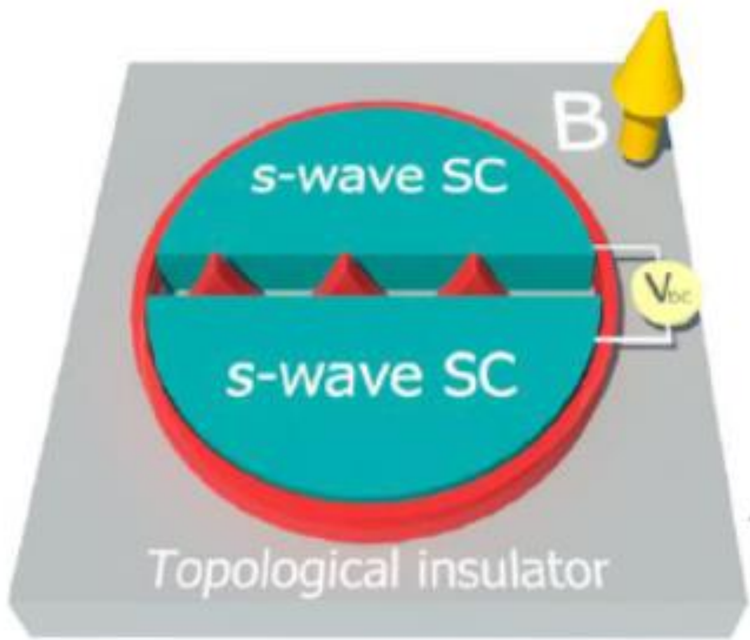
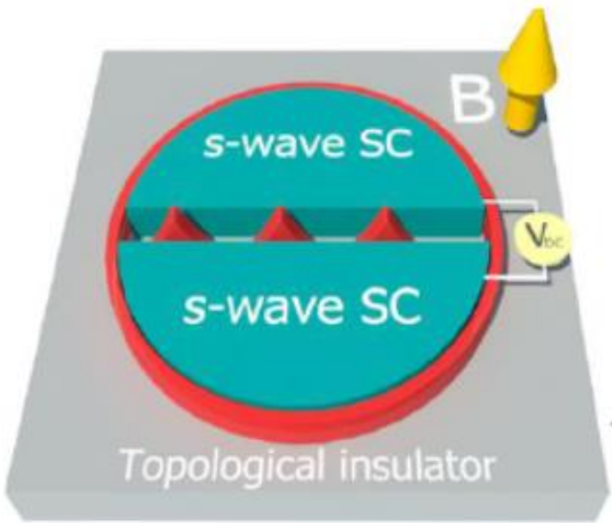
Anyons in Quantum Many-Body Systems
(Anyon19) Jan 25 2019

Non-Abelian Evolution of a Majorana Train in a Single Josephson Junction: $2n\pi$ fractional AC Josephson effect



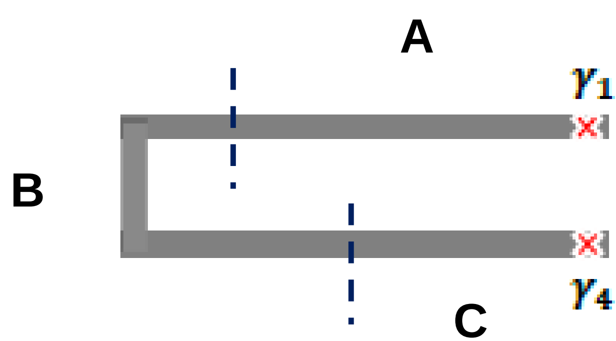
Heung-Sun Sim
Physics, KAIST

Part I: **Non-Abelian evolution of a Majorana train**



Choi, Sim, submitted (2018)

Part II: **Nonlocal entanglement (length-independent, topological) in the bulk of 1D fermions**



Park, Shim, Lee, Sim, PRL (2017)

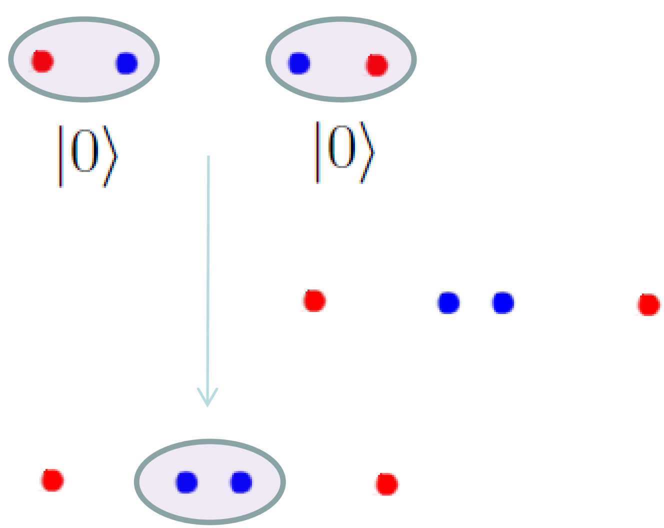
Non-Abelian Fusion of Majoranas

Majorana fermions

- Real fermions (particle = anti-particle) $\gamma_i^\dagger = \gamma_i$
- Two Majoranas (fusion) = a complex fermion $\psi = \gamma_1 + i\gamma_2 \quad \psi^\dagger = \gamma_1 - i\gamma_2$

$$\psi^\dagger |0\rangle = |1\rangle$$

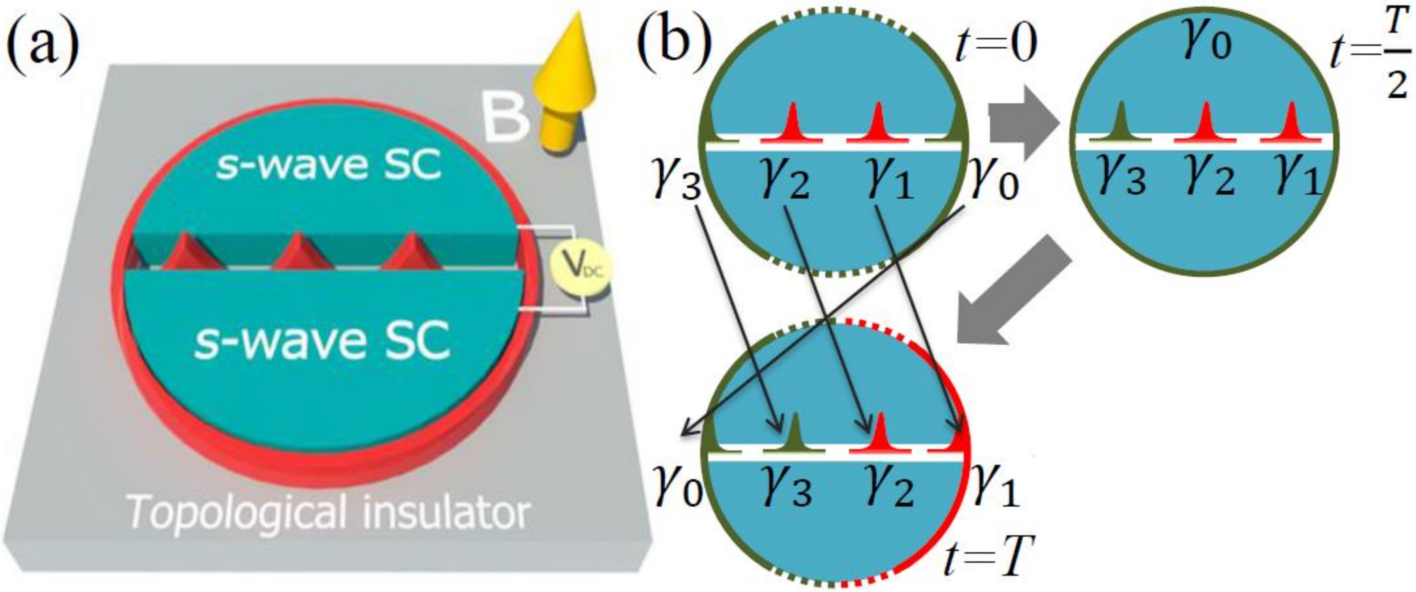
- Interchanging fusion partners: four Majoranas



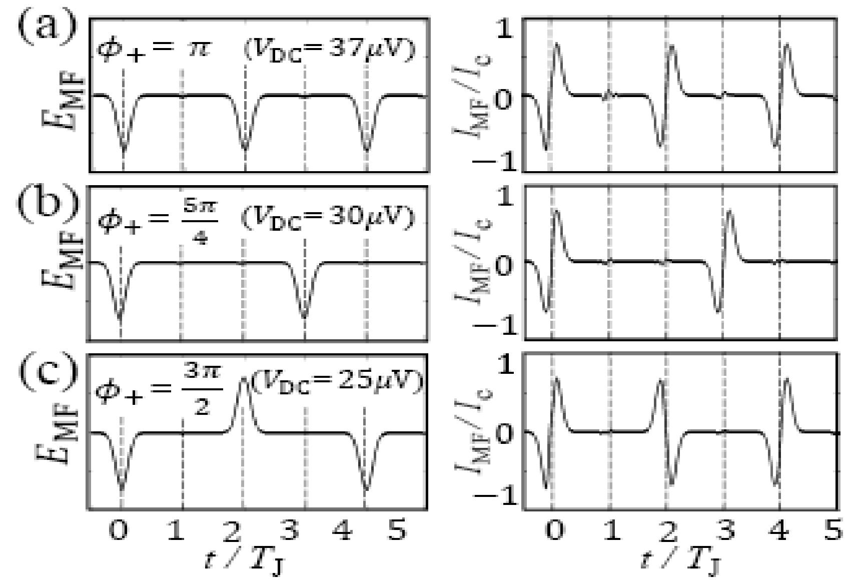
$$|00\rangle \rightarrow |00\rangle + |11\rangle$$

- Generation of a form of entanglement!

Part I: **Non-Abelian evolution of a Majorana train:**
$2n\pi$ fractional AC Josephson effects



$$T_J = h/(2eV_{DC})$$

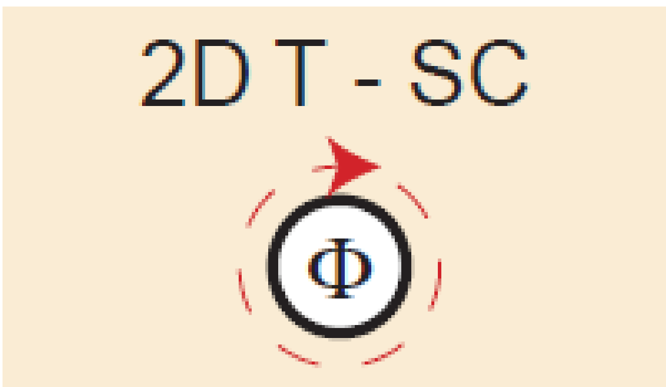


$2n\pi$ fractional AC Josephson effects

$$n \geq 2$$

Majorana fermions in topological SC

$$\gamma_i^\dagger = \gamma_i$$

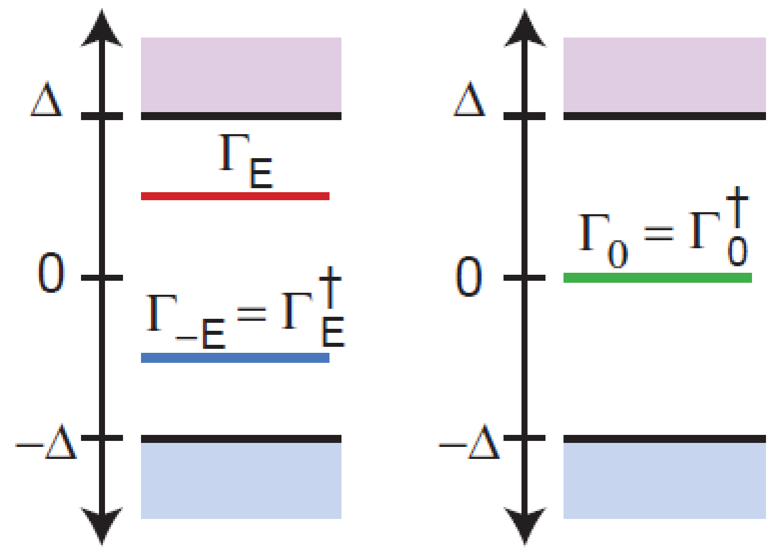


$$kL + \pi + \pi = 2n\pi$$

Magnetic flux

Berry phase

Particle-hole symmetry in SC

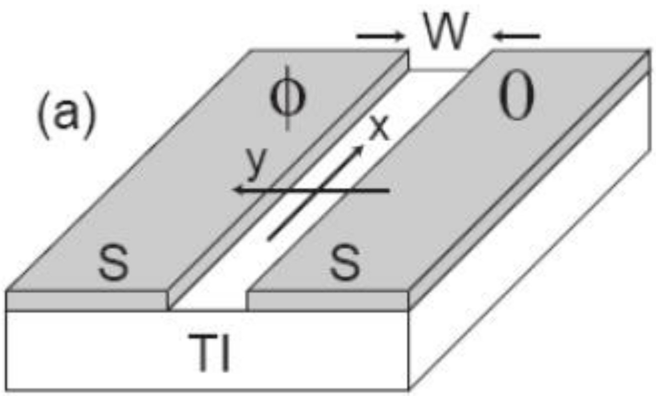


M. Z. Hasan and C. L. Kane, RMP (2010).

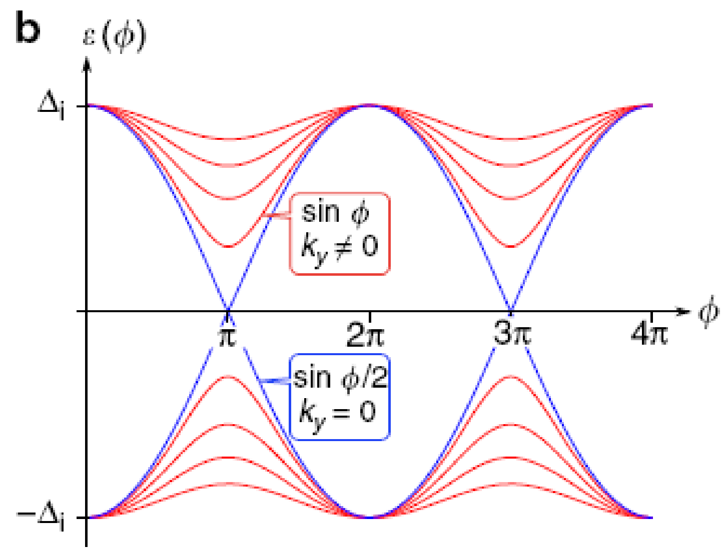
Topological Josephson junction

Fu and Kane, PRL 2008

Topological insulator



$$\phi = \pi$$



4π fractional Josephson effect

$$I_{4\pi} \sin \phi/2$$

Experiments: doubled Shapiro step

LP Rokhinson et al. Nat Phys 2012

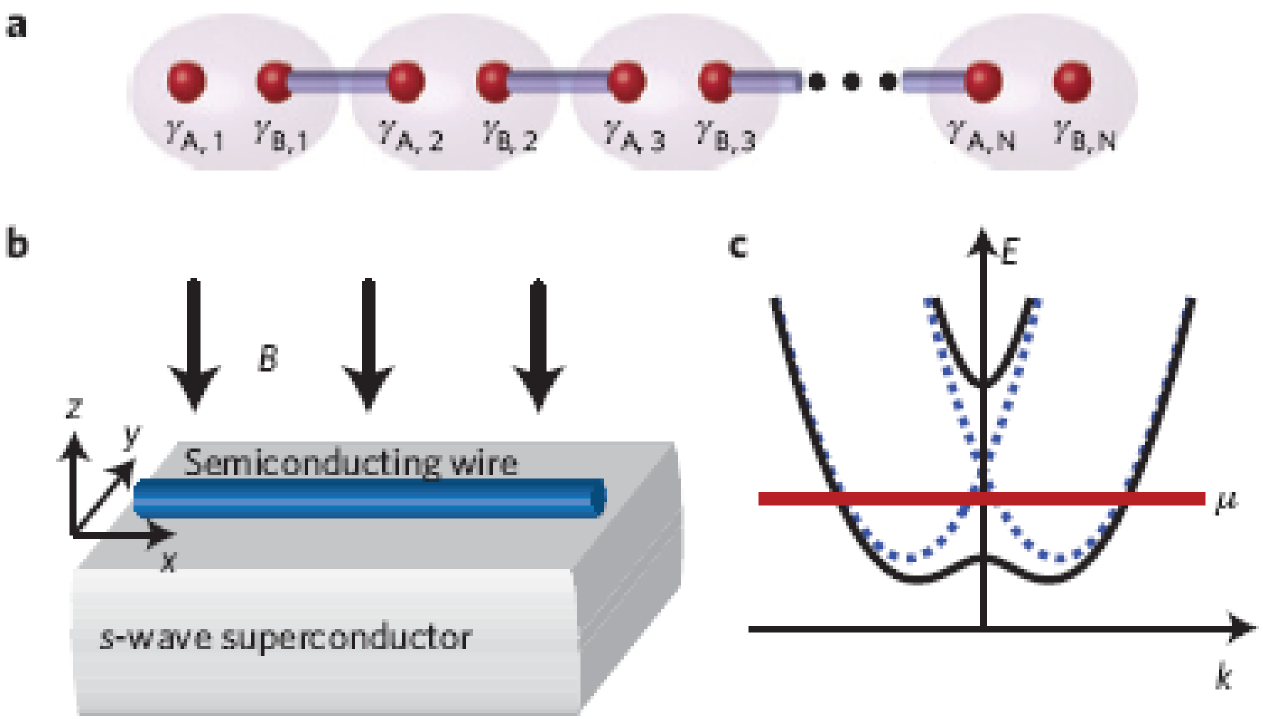
Wiedenmann et al. Nat Comm 2016

Topological Josephson junction

Nanowire + spin-orbit coupling

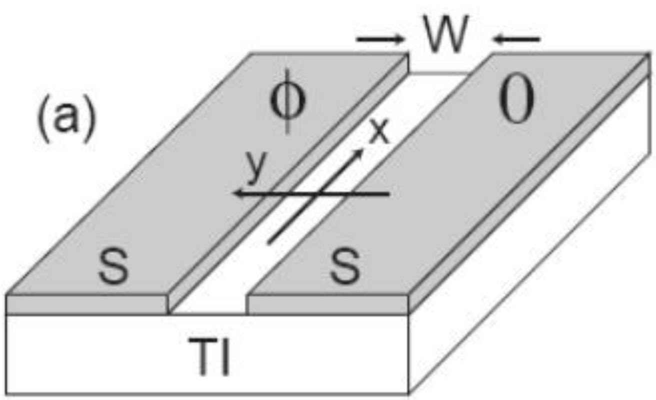
Lutchyn, Sau, Das Sarma, PRL 2010

Oreg, Refael, von Oppen, PRL 2010

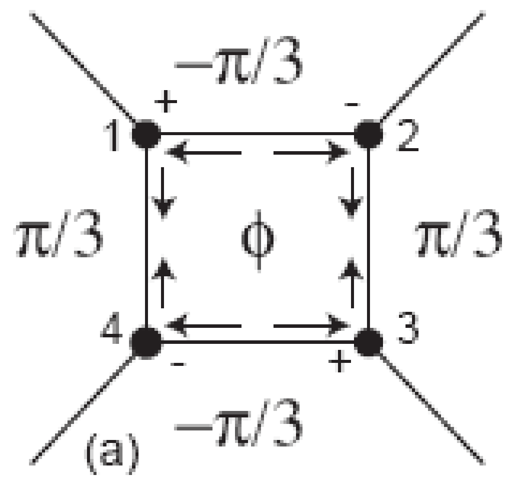


Non-abelian braiding with multiple Josephson junctions and dynamical control of system parameters

Fu and Kane, PRL 2008



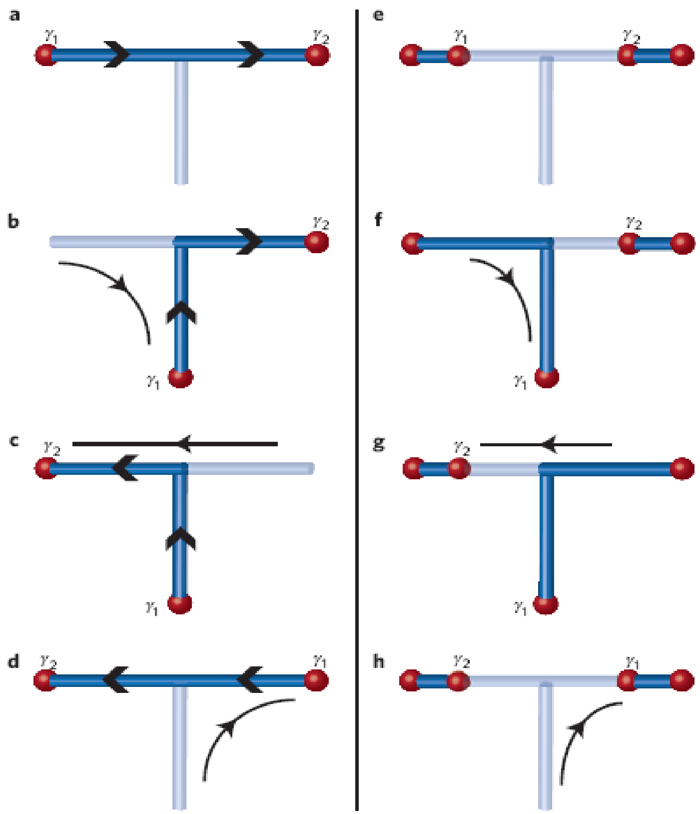
$$\phi = \pi$$

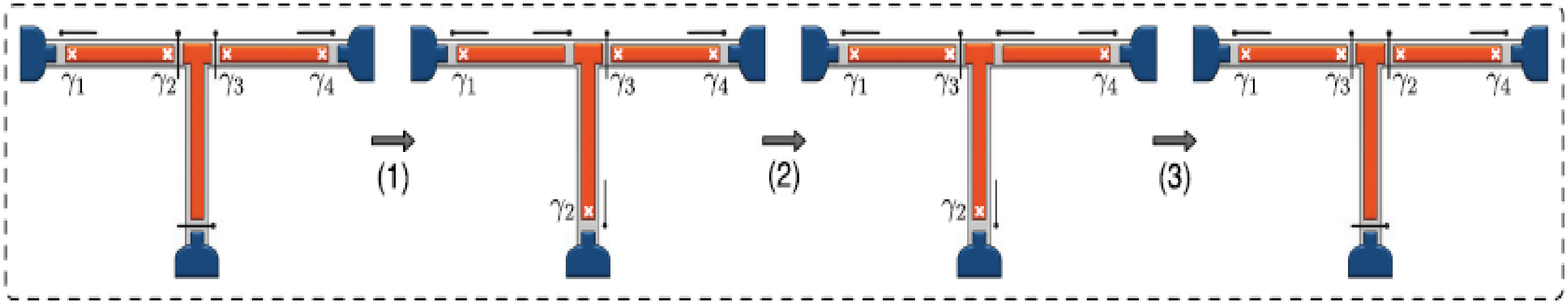


$$|0_{12}0_{34}\rangle \to (|0_{14}0_{32}\rangle + |1_{14}1_{32}\rangle)/\sqrt{2}.$$

Non-abelian braiding with multiple Josephson junctions and dynamical control of system parameters

Alicea, Oreg, Refael, von Oppen, Fisher, Nat. Phys. 2011

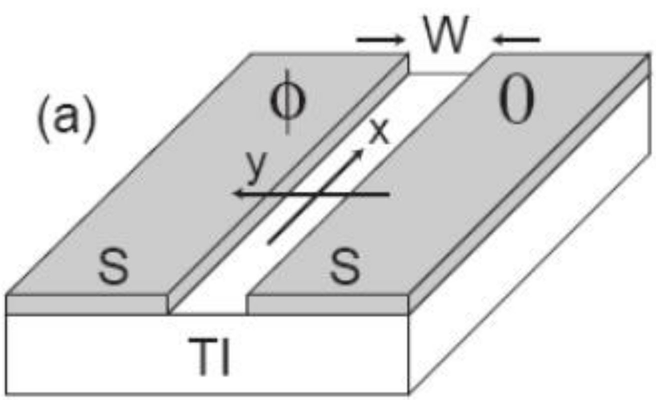




Non-abelian braiding with a single Josephson junction?

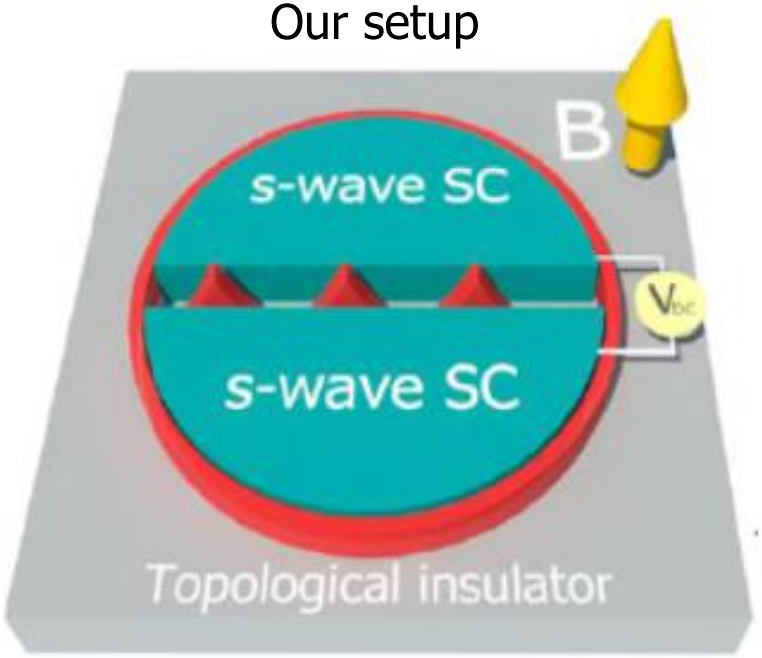
YES! (under magnetic fields)

and without dynamically tuning the system parameters



4π fractional Josephson

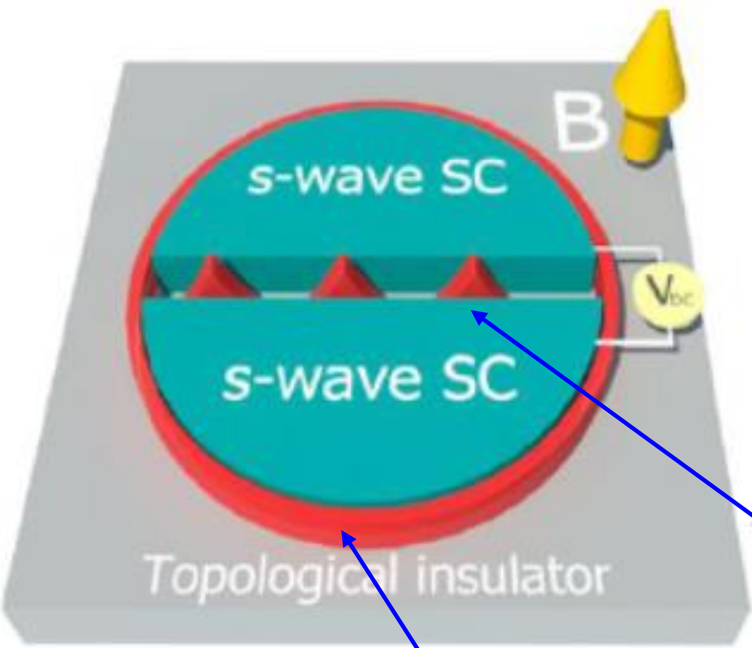
Our setup



$2n\pi$ fractional Josephson

$n \geq 2$

Conditions for Majorana zero modes



$$N = BLW/\Phi_0 = 3$$

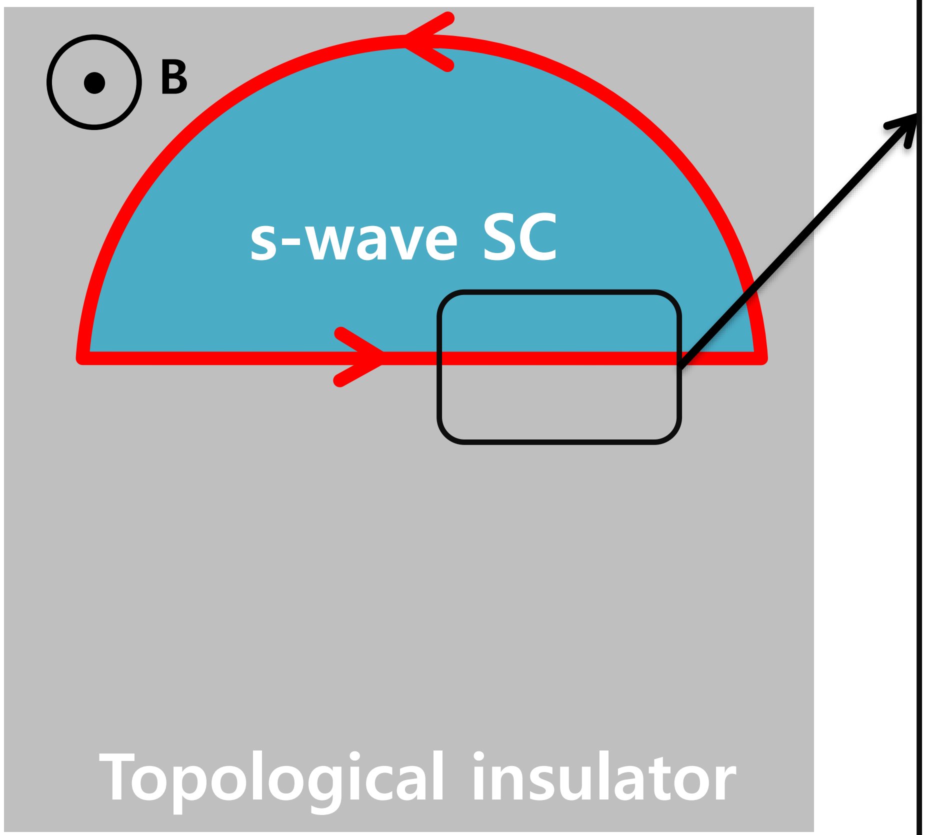
$$\Phi_0 = h/(2e)$$

Mobile localized MZMs (Majorana zero modes)

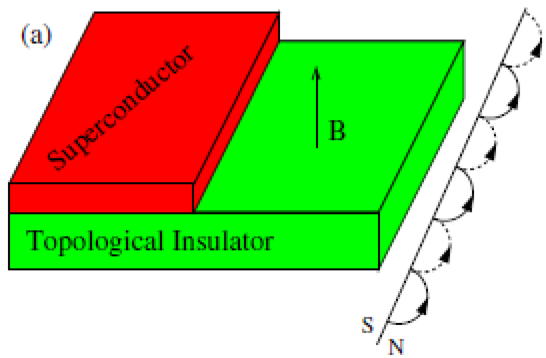
$$x_{k=1,2,\cdots}(t) = \frac{W}{N}\left(k - 1 + \frac{t}{T_{\mathrm{J}}}\right)$$

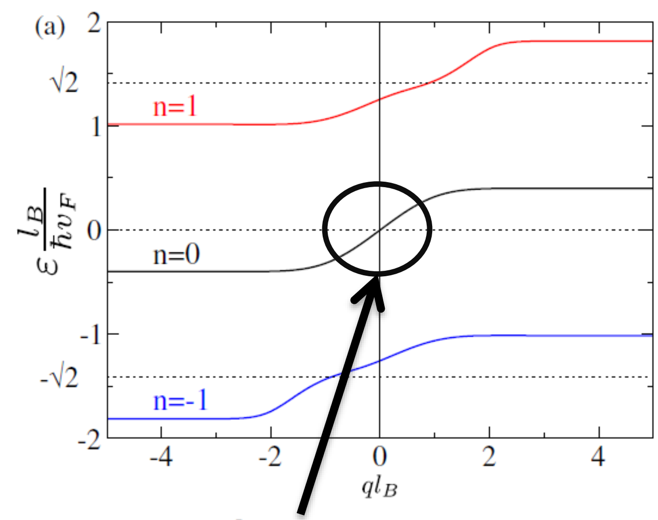
An extended MZM

$$2lE/v_{\mathrm{arc}} + \pi + \pi(M_{\mathrm{u}} + M_{\mathrm{l}}) + \arg(r_0 r_W) = 0, \pm 2\pi, \cdots$$



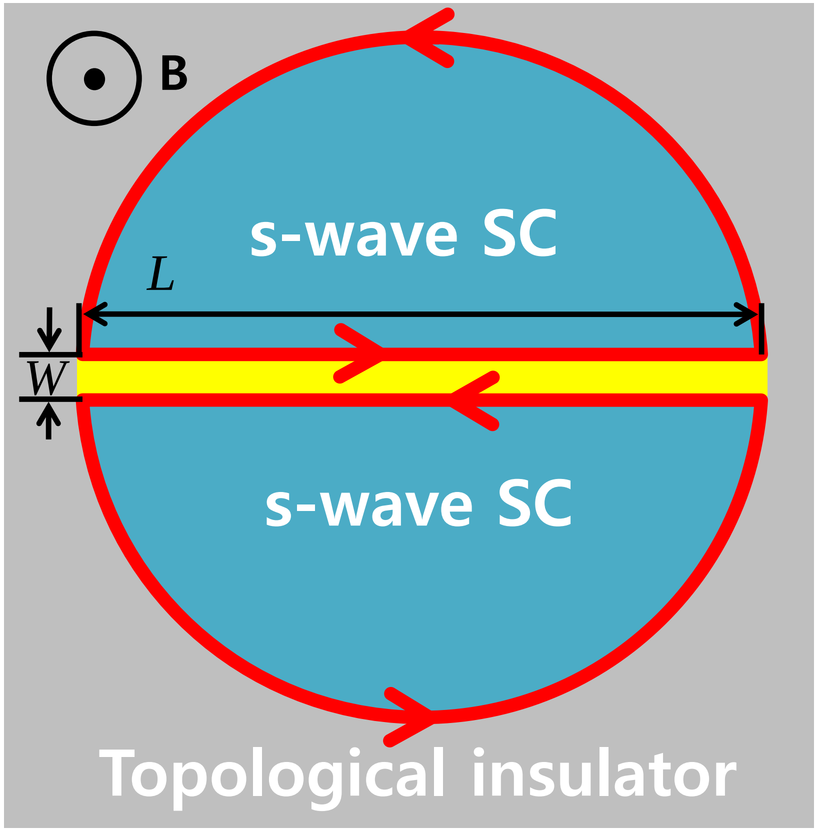
chiral Majorana modes





$$\gamma_q^\dagger = \gamma_{-q}$$

Tiwari, Zuelicke, and Bruder,
PRL **110**, 186805 (2013)

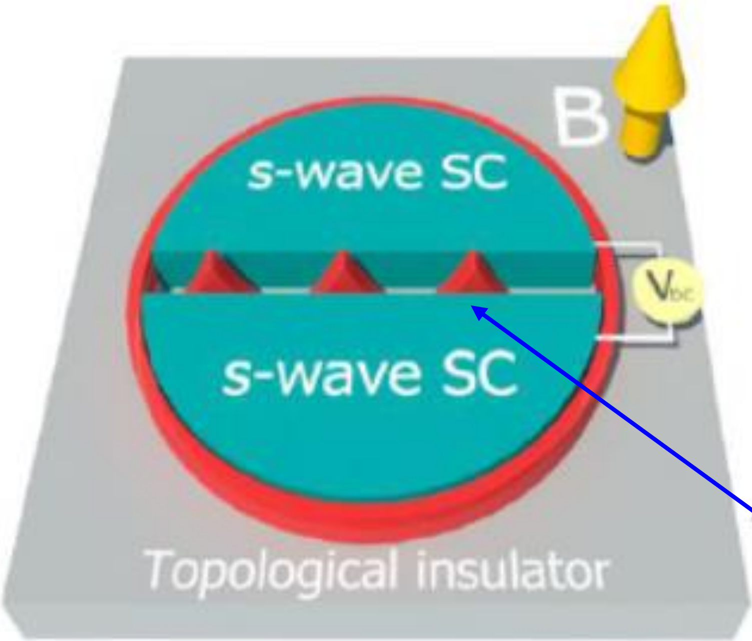


$$H(t) = \int_{-l}^{W} dx\,\Gamma(x)^{\top}\left(-iv(x)\sigma_z\partial_x + m(x,t)\sigma_y\right)\Gamma(x)$$

$$m(x,t) = \Delta_0 \sin\left(\frac{N\pi x}{W} - \frac{eV_{\mathrm{DC}}t}{\hbar}\right)$$

$$i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = H(t)\left|\psi(t)\right\rangle \qquad E_{tot}(t) = \left\langle\psi(t)\right|H(t)\left|\psi(t)\right\rangle \qquad I_J = \frac{1}{V_{DC}}\frac{\partial E_{tot}}{\partial t}$$

Conditions for Majorana zero modes



$$N = BLW/\Phi_0 = 3$$

Mobile localized MZMs (Majorana zero modes)

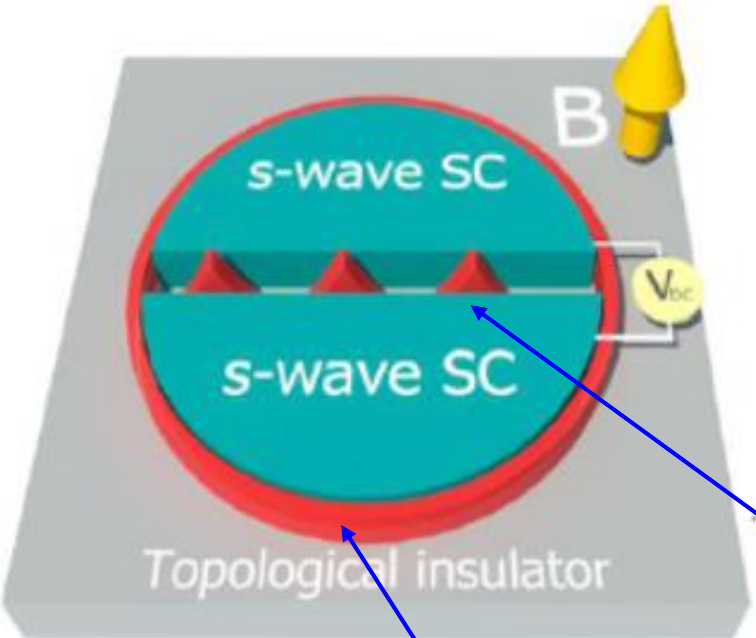
$$x_{k=1,2,\cdots}(t) = \frac{W}{N}\left(k - 1 + \frac{t}{T_{\rm J}}\right)$$

$$\lambda \equiv \sqrt{\frac{\hbar v W}{\pi N \Delta}} \ll W/N$$

$$T_{\rm J} = h/(2eV_{\rm DC})$$

The Kitaev chains and trivial chains alternately appear in the junction.

Conditions for Majorana zero modes



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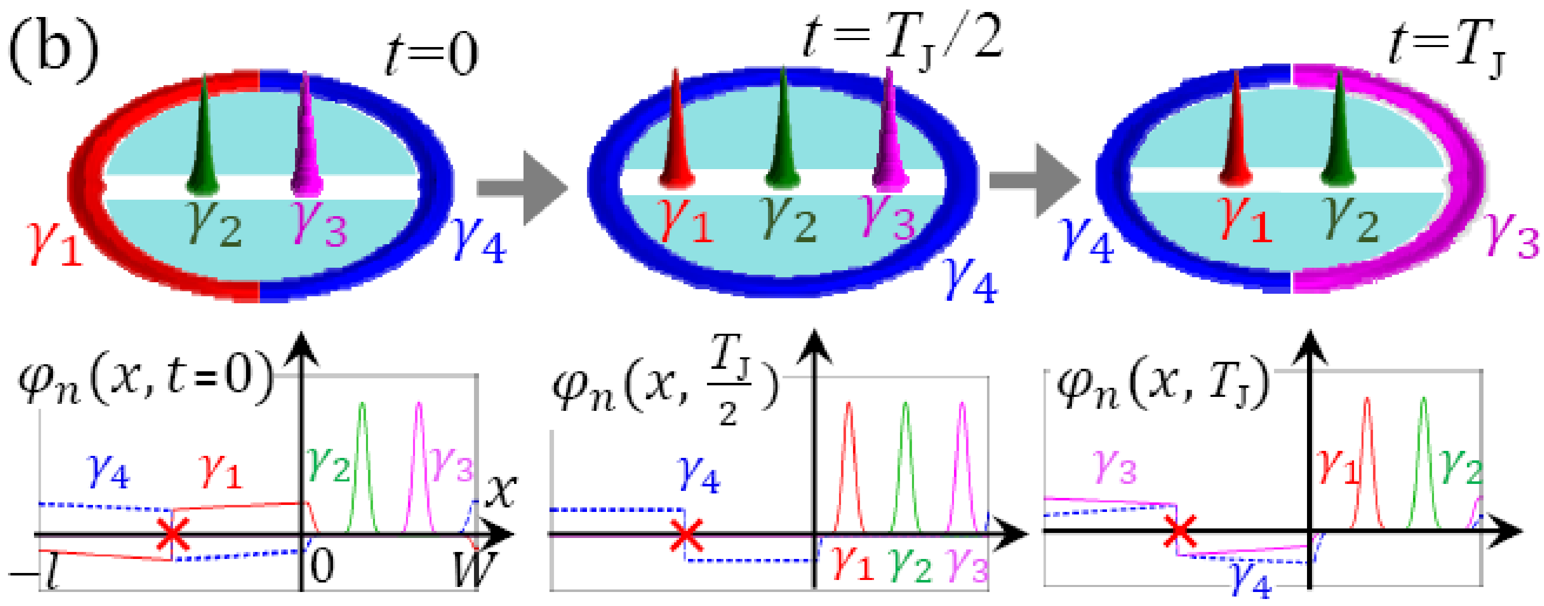
$$x_{k=1,2,\cdots}(t) = \frac{W}{N}\left(k-1+\frac{t}{T_{\mathrm{J}}}\right)$$

$$2lE/v_{\mathrm{arc}} + \pi + \pi(M_{\mathrm{u}} + M_{\mathrm{l}}) + \arg(r_0 r_W) = 0, \pm 2\pi, \cdots$$

An extended MZM appears along the arcs
when there are an odd number of MZMs inside (e.g., along the junction)

Mobile Majorana train & Fusion



After one Hamiltonian period $T_{\rm J} = h/(2eV_{\rm DC})$:

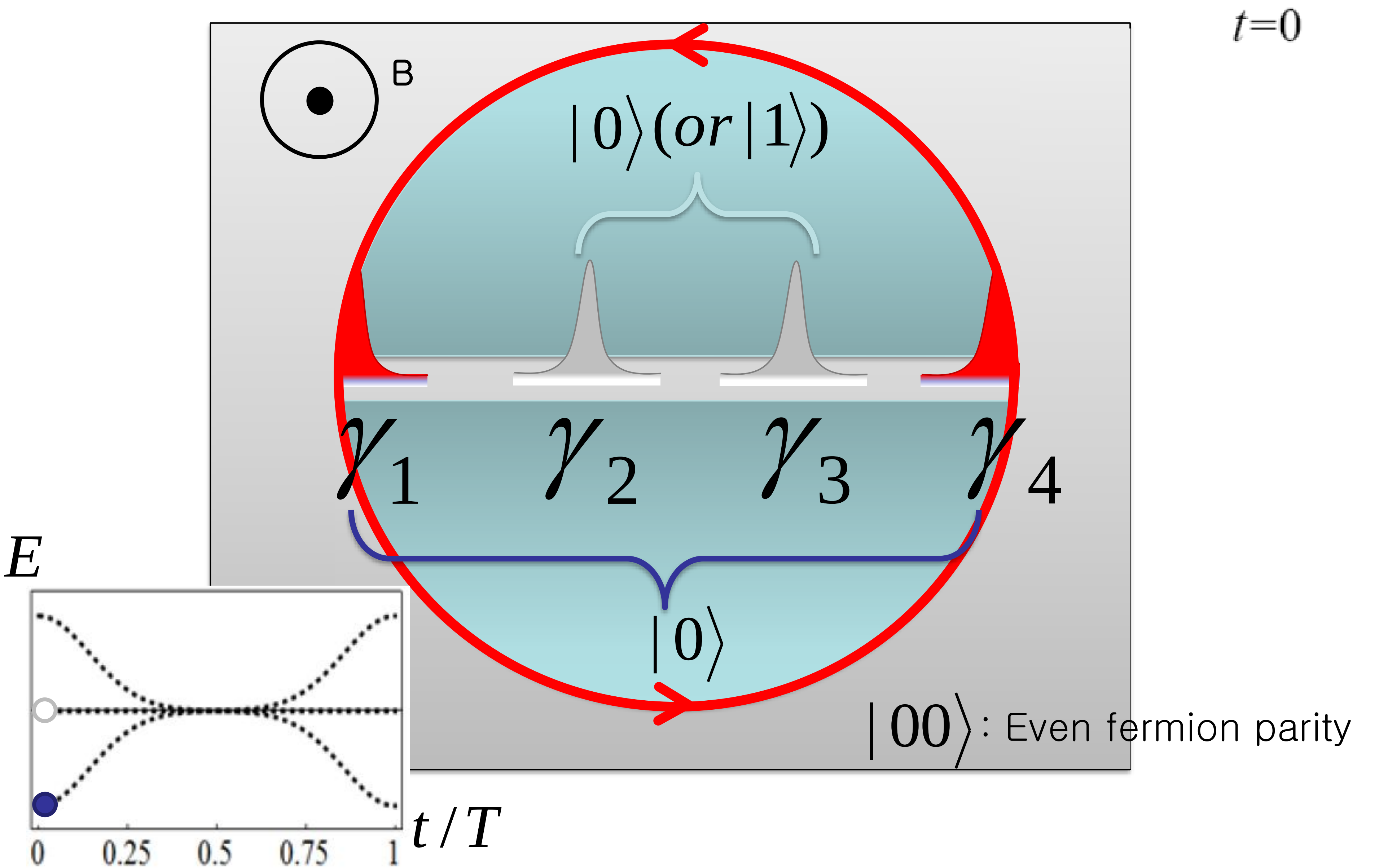
$$\gamma_1(T_{\rm J}) = \gamma_2(0)$$

$$\gamma_2(T_{\rm J}) = \gamma_3(0)$$

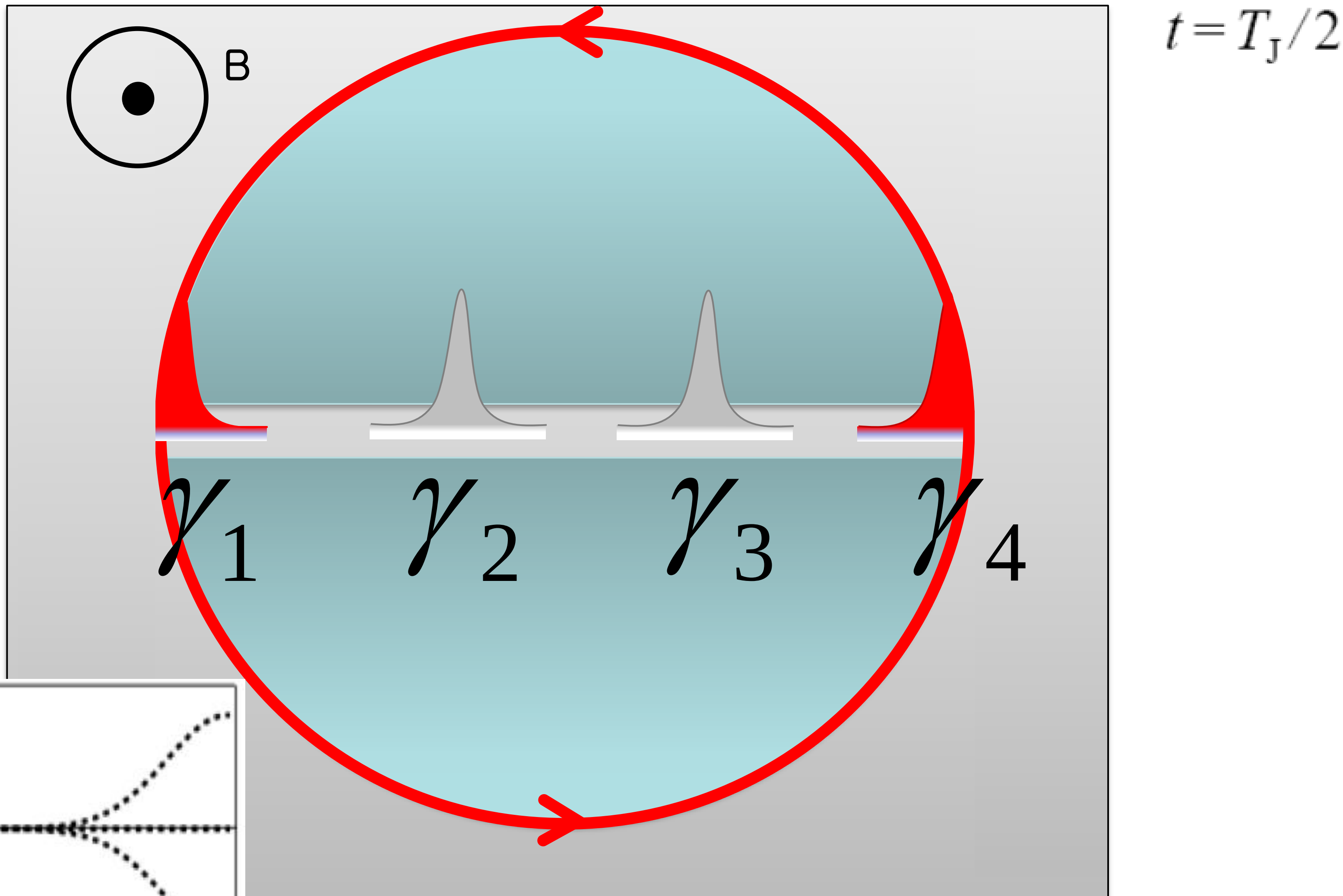
$$\gamma_3(T_{\rm J}) = \gamma_4(0)$$

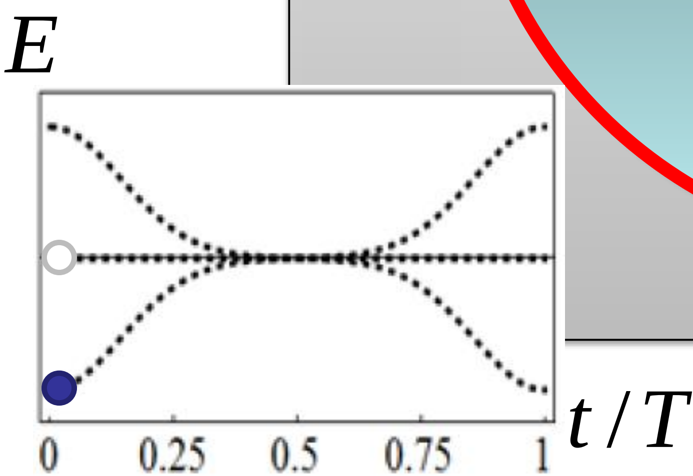
$$\gamma_4(T_{\rm J}) = -\gamma_1(0)$$

Non-Abelian braiding of MZMs

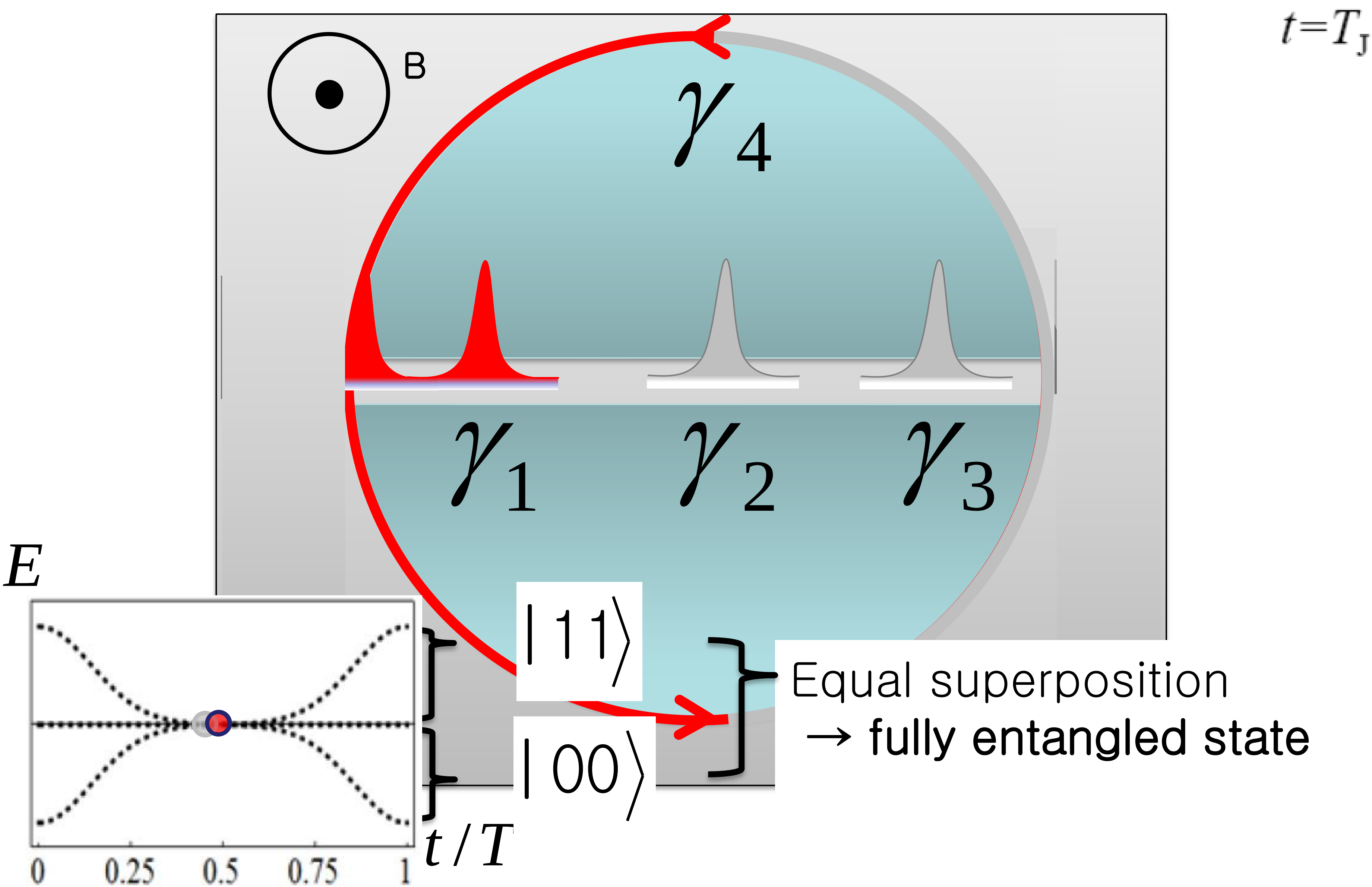


Non-Abelian braiding of MZMs

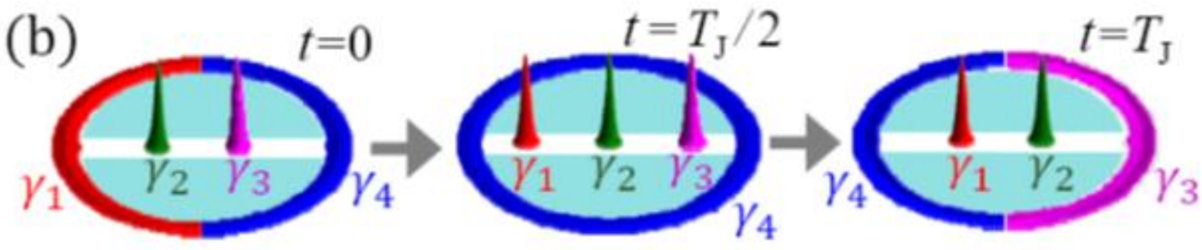




Non-Abelian braiding of MZMs

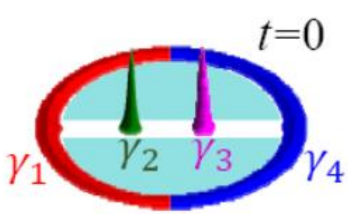


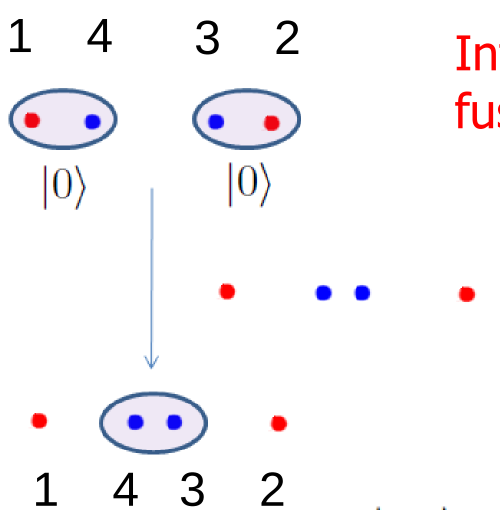
Mobile Majorana train & Fusion



$t{=}0$ $\qquad\qquad$ $t{=}T_J$

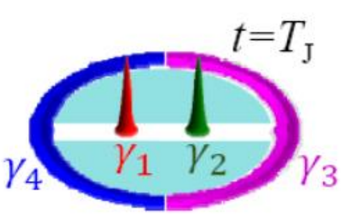
$$|0_{41}0_{32}\rangle_0 \mapsto \frac{e^{i\phi}}{\sqrt{2}}\left(e^{i\phi'}|0_{41}0_{32}\rangle_0 - ie^{-i\phi'}|1_{41}1_{32}\rangle_0\right)$$





Interchanging fusion partners

$$\gamma_1(T_J) = \gamma_2(0)$$
$$\gamma_2(T_J) = \gamma_3(0)$$
$$\gamma_3(T_J) = \gamma_4(0)$$
$$\gamma_4(T_J) = -\gamma_1(0)$$

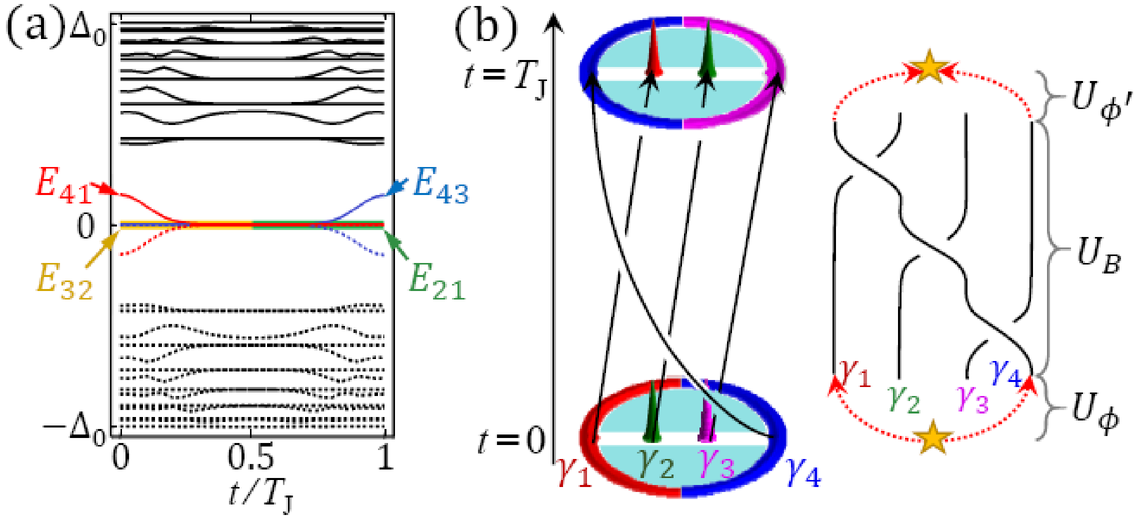


$$|00\rangle \to |00\rangle + |11\rangle$$

Mobile Majorana train & Fusion

$$|0_{41}0_{32}\rangle_0 \mapsto \frac{e^{i\phi}}{\sqrt{2}}(e^{i\phi'}|0_{41}0_{32}\rangle_0 - ie^{-i\phi'}|1_{41}1_{32}\rangle_0)$$

$$U = U_{\phi'}U_{\mathrm{B}}U_{\phi}$$

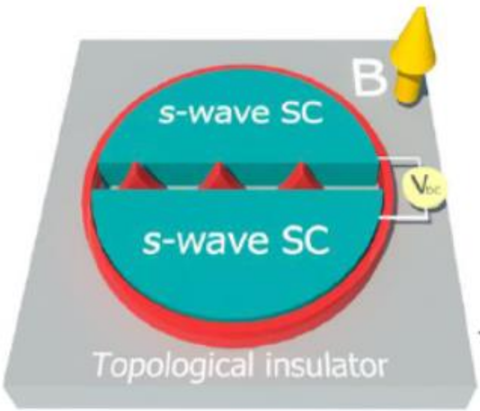


$$U_\phi = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & e^{i\phi} & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$

$$U_{\mathrm{B}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i & 0 & 0 \\ -i & -1 & 0 & 0 \\ 0 & 0 & -i & 1 \\ 0 & 0 & 1 & -i \end{pmatrix}$$

$$\{|0_{41}0_{32}\rangle_0, |1_{41}1_{32}\rangle_0, |0_{41}1_{32}\rangle_0, |1_{41}0_{32}\rangle_0\}$$

$2n\pi$ Fractional Josephson effects



$2n\pi$ periodic in time, $\quad n \geq 2$

Signature of the non-Abelian braiding statistics:
n is determined by the bias voltage

$$T_{\mathrm{J}} = h/(2eV_{\mathrm{DC}})$$

n = 2

$$|\psi(0)\rangle = |0_{41}0_{32}\rangle_0 \overset{U}{\mapsto} -|0_{41}0_{32}\rangle_0 - ie^{i\phi_-}|1_{41}1_{32}\rangle_0 \qquad t = T_J$$

$$\overset{U}{\mapsto} |0_{41}0_{32}\rangle_0 \qquad t = 2T_J$$

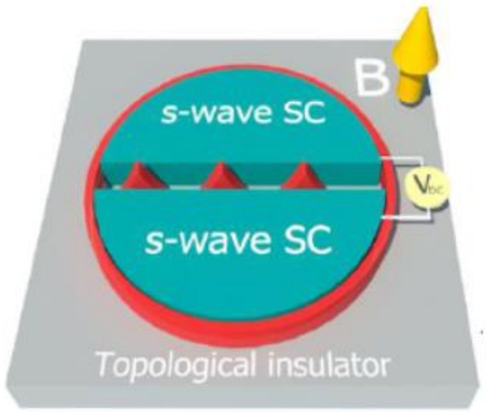
n = 3

$$|\psi(0)\rangle = |0_{41}0_{32}\rangle_0 \overset{U}{\mapsto} \frac{e^{i\pi/4}}{\sqrt{2}}|0_{41}0_{32}\rangle_0 + \frac{ie^{i\phi_-}}{\sqrt{2}}|1_{41}1_{32}\rangle_0 \qquad t = T_J$$

$$\overset{U}{\mapsto} \frac{e^{i\pi/4}}{\sqrt{2}}|0_{41}0_{32}\rangle_0 - \frac{e^{i\phi_-}}{\sqrt{2}}|1_{41}1_{32}\rangle_0 \qquad t = 2T_J$$

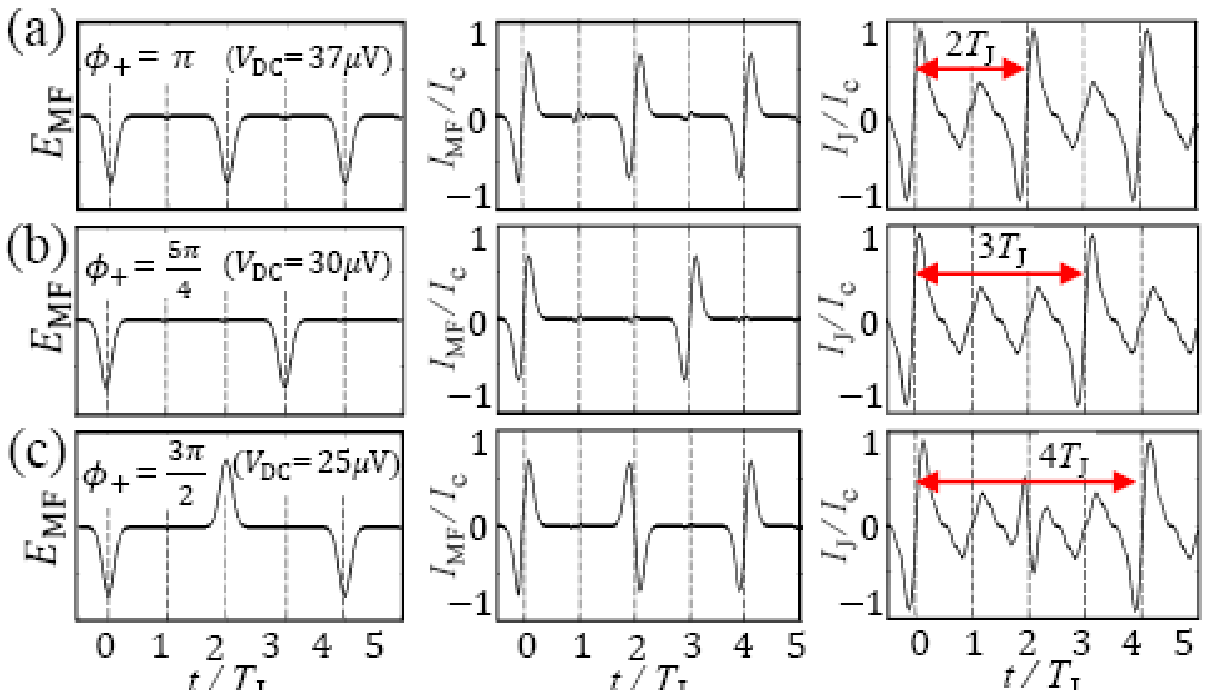
$$\overset{U}{\mapsto} |0_{41}0_{32}\rangle_0 \qquad t = 3T_J$$

$2n\pi$ Fractional Josephson effects



$2n\pi$ periodic in time, $\quad n \geq 2$

Signature of the non-Abelian braiding statistics:
n is determined by the bias voltage



4π fractional AC Josephson

6π fractional AC Josephson

8π fractional AC Josephson

$$T_J = h/(2eV_{DC})$$

Summary of Part I

Non-Abelian evolution of a Majorana train in a single Josephson junction

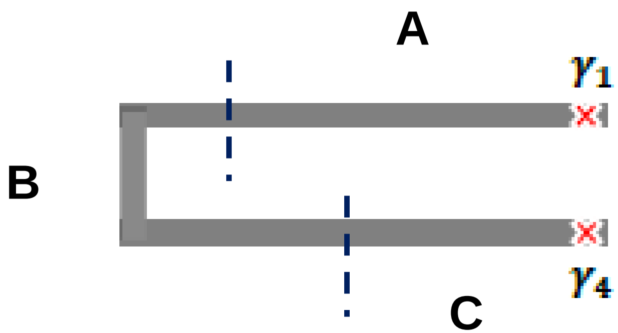
- Generation and Braiding of mobile Majorana fermions
- Signature of Non-Abelian effects: $2n\pi$ fractional AC Josephson effect

Nonlocal entanglement in 1D bulk at finite temperature

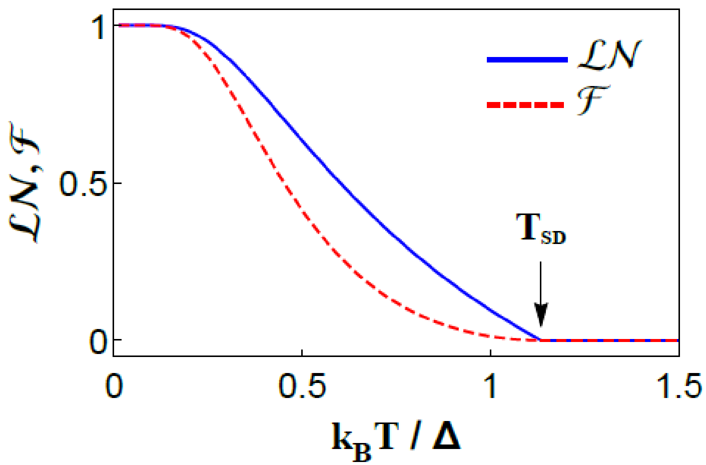
- Entanglement by non-Abelian fusion
- Dependent on topological classes (the number of the end Majorana fermions)
- Sudden death and birth of nonlocal entanglement

Nonlocal entanglement in 1D	Park, Shim, Lee, Sim, PRL (2017)	
Non-Abelian evolution of a Majorana train	Choi, Sim	submitted (2018)

Part II: - **Nonlocal entanglement** **(topological) in bulk of 1D fermions**



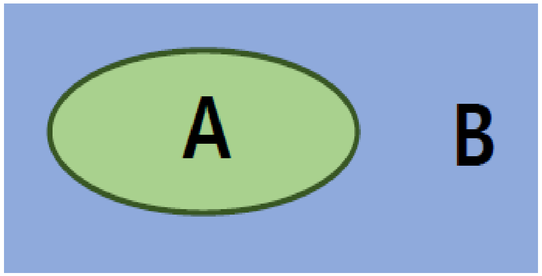
Entanglement B|AC



- Non-Abelian anyonic statistics + Fermi statistics
- Topological-class dependent entanglement

Park, Shim, Lee, Sim, PRL (2017)

Topological order and entanglement



Entanglement entropy

$$\mathcal{E}_{\mathrm{E}}(|\psi\rangle) = -\mathrm{Tr}(\rho_{\mathrm{A}} \log_2 \rho_{\mathrm{A}})$$

$$\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{B}} |\psi\rangle\langle\psi|$$

Quantum correlation between A and B generates entropy even at zero temperature

Topological entanglement entropy

$$\mathcal{E}_E = \alpha L - \gamma + \cdots$$

Global constant

$$\gamma = \log D$$

(D: quantum dimension; topological ground-state degeneracy)

Identifying anyonic topological order

non-topological order: $\gamma = 0$

fractional quantum Hall with filling 1/q (Abelian anyon) :

$$\gamma = 0.5 \log q$$

Kitaev & Preskill,
Levin & Wen

Our motivation: 1. topological entanglement at finite temperature ?
2. 1D version

Our target systems: Kitaev chain and its variants

$$\hat{H}_{\mathrm{I}} = -\sum_{j=1}^{N-1}\left[\frac{t}{2}(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \frac{\Delta}{2}(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger)\right] + \sum_{j=1}^{N} \mu c_j^\dagger c_j$$

$$t = \Delta$$
$$\mu = 0$$

- One Majorana fermion at each end
- Two degenerate ground states

$$|0\rangle_{\mathrm{I}} \qquad |1\rangle_{\mathrm{I}} = f_{14}^\dagger |0\rangle_{\mathrm{I}}$$

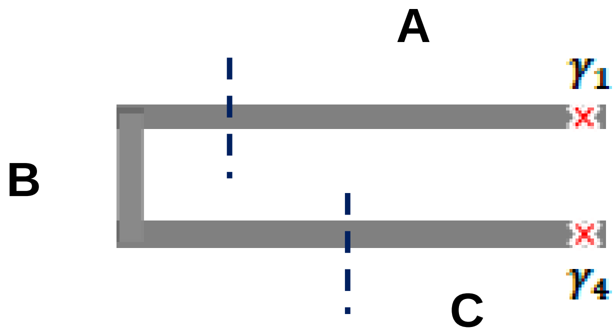
$$f_{ab} \equiv (\gamma_a + i\gamma_b)/\sqrt{2}$$

$$\hat{H}_{\mathrm{II}} = -\frac{\Delta}{2}\sum_{j=1}^{N-2}(c_j + c_j^\dagger)(c_{j+2} - c_{j+2}^\dagger)$$

Two Majoranas at each end

Fidkowski and Kitaev PRB 2011
Turner, Pollmann, and Berg, PRB 2011

What we compute: Entanglement between B and AC



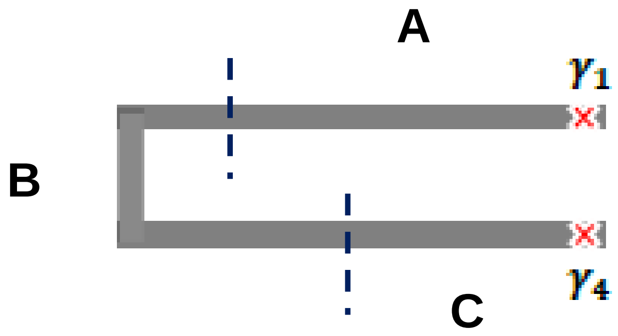
$$\rho = e^{-H/(k_B T)} / \mathrm{Tr} e^{-H/(k_B T)}$$

Thermal mixture of different-parity states

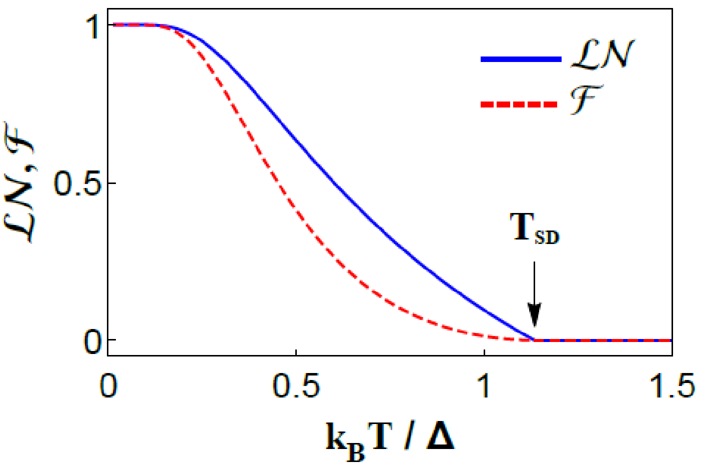
Entanglement B|AC

Mixed-state entanglement measure
- Entanglement of formation: generalization of entanglement entropy
- Logarithmic entanglement negativity: computable measure for mixed states

Entanglement between B and AC: Results

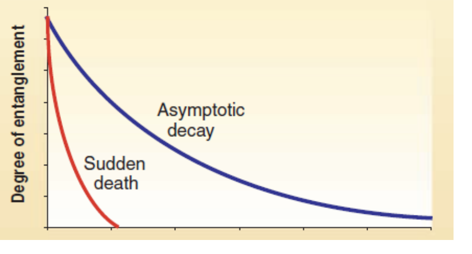


Entanglement B|AC



Entanglement of formation
Logarithmic negativity

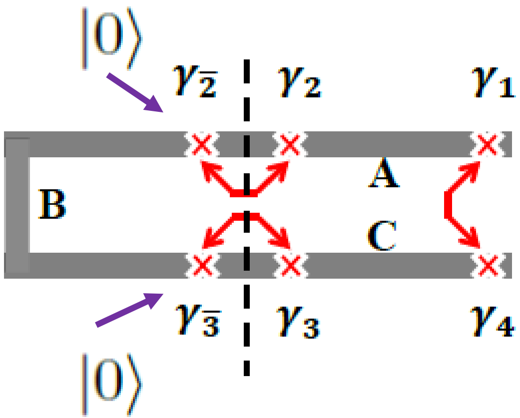
- T → 0: Entanglement = 1
- Nonlocal, independent of length (≫ correlation length)
- Sudden death of the entanglement at certain T



Corresponding spin systems (Wigner Jordan)
- No entanglement in the thermal states

T Yu & JH Eberly, Science 323, 598 (2009)

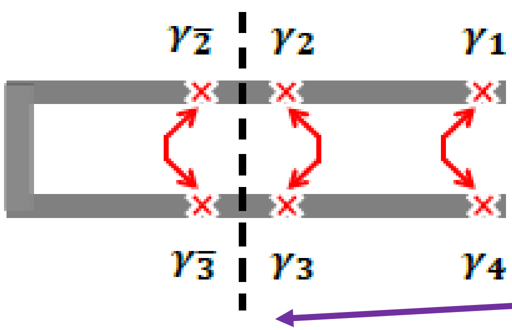
Entanglement between B and AC: Bell entanglements



Ground states $|0\rangle_{\mathrm{I}}$ $\quad |1\rangle_{\mathrm{I}} = f_{14}^{\dagger}|0\rangle_{\mathrm{I}}$

$$f_{ab} \equiv (\gamma_a + i\gamma_b)/\sqrt{2}$$

Mapping fermion occupation states to qubits



$$|0\rangle_{\mathrm{I}} \mapsto |\mathrm{Bell}\rangle^q |0_{14}\rangle^q$$

$$|1\rangle_{\mathrm{I}} \mapsto |\mathrm{Bell}\rangle^q |1_{14}\rangle^q$$

$$|\mathrm{Bell}\rangle^q = \frac{1}{\sqrt{2}}(|0_{\bar{2}\bar{3}}\rangle^q |0_{23}\rangle^q + i|1_{\bar{2}\bar{3}}\rangle^q |1_{23}\rangle^q)$$

Nonlocal entanglement by interchanging Majorana fusion partners

The zero-temperature thermal state has the Bell entanglement!

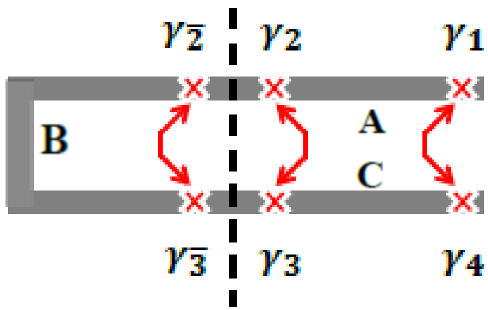
$$\rho_{\mathrm{I}}(T=0) = (|0\rangle\langle 0|_{\mathrm{I}} + |1\rangle\langle 1|_{\mathrm{I}})/2$$

A nontrivial step…

To define entanglement in fermion occupation states,
we need to map the fermion states into qubit states.

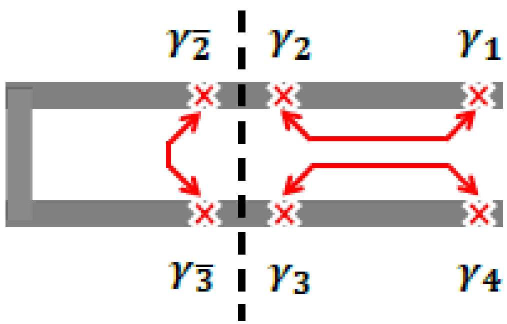
One must treat the fermion-exchange sign (-1) properly.

Role of fermion statistics



$$|0\rangle_{\rm I} \mapsto |\text{Bell}\rangle^q |0_{14}\rangle^q \qquad |1\rangle_{\rm I} \mapsto |\text{Bell}\rangle^q |1_{14}\rangle^q$$

$$|\text{Bell}\rangle^q = \frac{1}{\sqrt{2}}\left(|0_{\bar{2}\bar{3}}\rangle^q |0_{23}\rangle^q + i|1_{\bar{2}\bar{3}}\rangle^q |1_{23}\rangle^q\right)$$

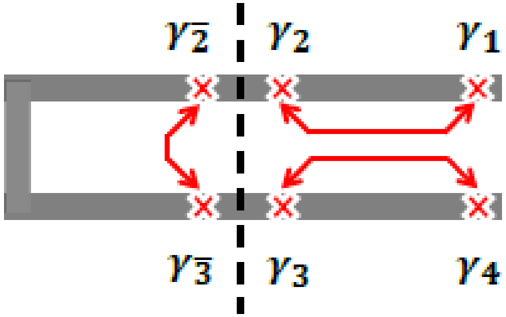


$$|0\rangle_{\rm I} = \frac{1}{2}\left(1 + f_{12}^\dagger f_{34}^\dagger + f_{12}^\dagger f_{\bar{2}\bar{3}}^\dagger + f_{\bar{2}\bar{3}}^\dagger f_{34}^\dagger\right)|0_{12}0_{\bar{2}\bar{3}}0_{34}\cdots\rangle,$$

$$|1\rangle_{\rm I} = \frac{1}{2}\left(f_{12}^\dagger + f_{34}^\dagger + f_{\bar{2}\bar{3}}^\dagger + f_{12}^\dagger f_{\bar{2}\bar{3}}^\dagger f_{34}^\dagger\right)|0_{12}0_{\bar{2}\bar{3}}0_{34}\cdots\rangle.$$

$f_{ab} \equiv (\gamma_a + i\gamma_b)/\sqrt{2}$

Role of fermion statistics



$$|0\rangle_{\rm I} = \frac{1}{2}(1 + f^\dagger_{12} f^\dagger_{34} + f^\dagger_{12} f^\dagger_{\bar{2}\bar{3}} + f^\dagger_{\bar{2}\bar{3}} f^\dagger_{34})|0_{12} 0_{\bar{2}\bar{3}} 0_{34} \cdots\rangle,$$

$$|1\rangle_{\rm I} = \frac{1}{2}(f^\dagger_{12} + f^\dagger_{34} + f^\dagger_{\bar{2}\bar{3}} + f^\dagger_{12} f^\dagger_{\bar{2}\bar{3}} f^\dagger_{34})|0_{12} 0_{\bar{2}\bar{3}} 0_{34} \cdots\rangle.$$

We must collect operators belonging to B and those to AC,
before mapping fermion occupation states to qubit tensor products

$$|0\rangle_{\rm I} = \tfrac{1}{2}(1 + f^\dagger_{12} f^\dagger_{34} - f^\dagger_{\bar{2}\bar{3}} f^\dagger_{12} + f^\dagger_{\bar{2}\bar{3}} f^\dagger_{34})|0_{\bar{2}\bar{3}} 0_{12} 0_{34} \ldots\rangle,$$

$$|1\rangle_{\rm I} = \tfrac{1}{2}(f^\dagger_{12} + f^\dagger_{34} + f^\dagger_{\bar{2}\bar{3}} - f^\dagger_{\bar{2}\bar{3}} f^\dagger_{12} f^\dagger_{34})|0_{\bar{2}\bar{3}} 0_{12} 0_{34} \ldots\rangle.$$

$$\begin{aligned}|0\rangle_{\rm I} \mapsto |0\rangle^q_{\rm I} =& \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q + |1_{12}\rangle^q|1_{34}\rangle^q) \\ & - |1_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q - |0_{12}\rangle^q|1_{34}\rangle^q)),\end{aligned}$$

$$\begin{aligned}|1\rangle_{\rm I} \mapsto |1\rangle^q_{\rm I} =& \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q + |0_{12}\rangle^q|1_{34}\rangle^q) \\ & + |1_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q - |1_{12}\rangle^q|1_{34}\rangle^q)).\end{aligned}$$

Role of fermion statistics

Fermion case:

$$|0\rangle_{\mathrm{I}} \mapsto |0\rangle^q_{\mathrm{I}} = \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q + |1_{12}\rangle^q|1_{34}\rangle^q) - |1_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q - |0_{12}\rangle^q|1_{34}\rangle^q)),$$

$$|0\psi\rangle + |1\chi\rangle$$

$$|1\rangle_{\mathrm{I}} \mapsto |1\rangle^q_{\mathrm{I}} = \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q + |0_{12}\rangle^q|1_{34}\rangle^q) + |1_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q - |1_{12}\rangle^q|1_{34}\rangle^q)).$$

$$|0\phi\rangle + |1\omega\rangle$$

$\rho_{\mathrm{I}}(T=0) = (|0\rangle\langle 0|_{\mathrm{I}} + |1\rangle\langle 1|_{\mathrm{I}})/2$, the zero-temperature thermal state has the maximum entanglement!

Spin case:

$$|0\rangle^s_I = \frac{1}{2}[|0_{\bar{2}\bar{3}}\rangle^s(|0_{12}\rangle^s|0_{34}\rangle^s + |1_{12}\rangle^s|1_{34}\rangle^s) + |1_{\bar{2}\bar{3}}\rangle^s(|1_{12}\rangle^s|0_{34}\rangle^s + |0_{12}\rangle^s|1_{34}\rangle^s)],$$

$$|0\psi\rangle + |1\phi\rangle$$

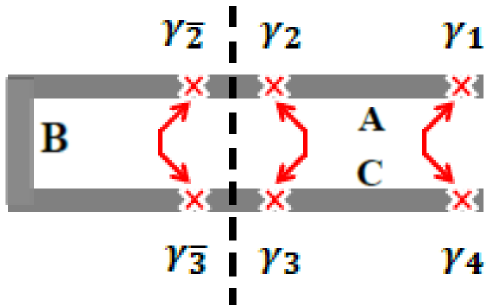
$$|1\rangle^s_I = \frac{1}{2}[|0_{\bar{2}\bar{3}}\rangle^s(|1_{12}\rangle^s|0_{34}\rangle^s + |0_{12}\rangle^s|1_{34}\rangle^s) + |1_{\bar{2}\bar{3}}\rangle^s(|0_{12}\rangle^s|0_{34}\rangle^s + |1_{12}\rangle^s|1_{34}\rangle^s)],$$

$$|0\phi\rangle + |1\psi\rangle$$

$\rho^s_I(T=0) = (|0\rangle\langle 0|^s + |1\rangle\langle 1|^s)/2$ has no entanglement!

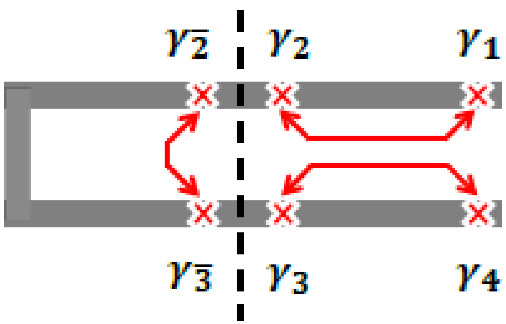
Zero temperature: Summary

Fermion case:



$$|0\rangle_{\mathrm{I}} \mapsto |\mathrm{Bell}\rangle^q |0_{14}\rangle^q \qquad |1\rangle_{\mathrm{I}} \mapsto |\mathrm{Bell}\rangle^q |1_{14}\rangle^q$$

$$|\mathrm{Bell}\rangle^q = \frac{1}{\sqrt{2}}(|0_{\bar{2}\bar{3}}\rangle^q |0_{23}\rangle^q + i|1_{\bar{2}\bar{3}}\rangle^q |1_{23}\rangle^q)$$



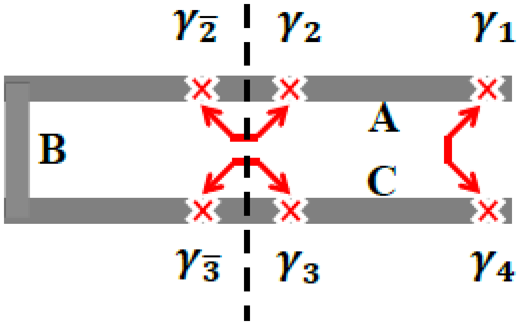
$$\begin{aligned}|0\rangle_{\mathrm{I}} \mapsto |0\rangle_{\mathrm{I}}^q =& \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q + |1_{12}\rangle^q|1_{34}\rangle^q) \\ & - |1_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q - |0_{12}\rangle^q|1_{34}\rangle^q)), \\ |1\rangle_{\mathrm{I}} \mapsto |1\rangle_{\mathrm{I}}^q =& \frac{1}{2}(|0_{\bar{2}\bar{3}}\rangle^q(|1_{12}\rangle^q|0_{34}\rangle^q + |0_{12}\rangle^q|1_{34}\rangle^q) \\ & + |1_{\bar{2}\bar{3}}\rangle^q(|0_{12}\rangle^q|0_{34}\rangle^q - |1_{12}\rangle^q|1_{34}\rangle^q)).\end{aligned}$$

1. The zero-temperature thermal state has the Bell entanglement (in any basis)!
   $\rho_{\mathrm{I}}(T=0) = (|0\rangle\langle 0|_{\mathrm{I}} + |1\rangle\langle 1|_{\mathrm{I}})/2$

2. The entanglement results from Majorana non-Abelian statistics **and** Fermi-Dirac statistics

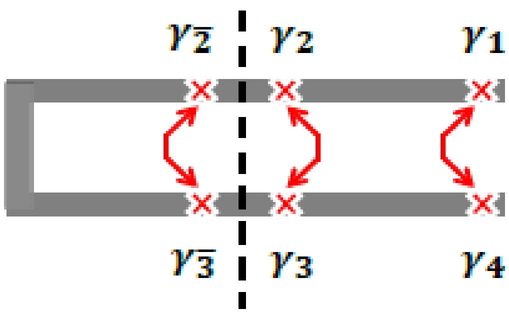
Spin case: no entanglement

Finite temperature



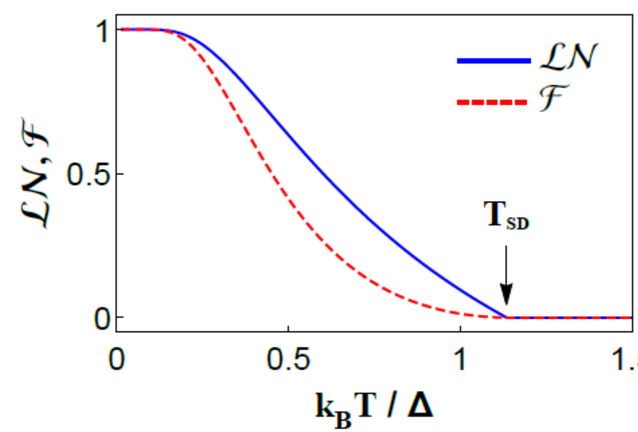
Thermal states $\quad \rho = e^{-H/(k_B T)} / \mathrm{Tr} e^{-H/(k_B T)}$

Thermal mixing of Bell entanglements



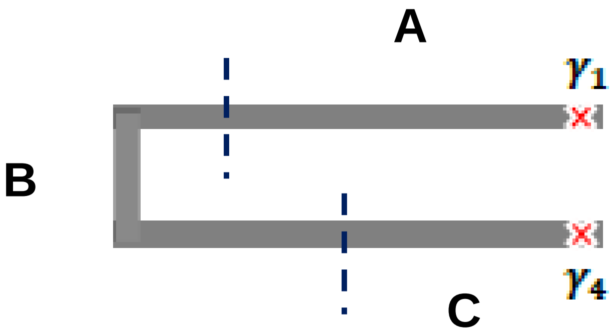
$\lvert 0\rangle_{\mathrm{I}}$, $\lvert 1\rangle_{\mathrm{I}}$	$\lvert \mathrm{Bell}\rangle^q = \frac{1}{\sqrt{2}}(\lvert 0_{\bar{2}\bar{3}}\rangle^q \lvert 0_{23}\rangle^q + i\lvert 1_{\bar{2}\bar{3}}\rangle^q \lvert 1_{23}\rangle^q)$
$f^\dagger_{2\bar{2}}$	$\lvert 0_{\bar{2}\bar{3}}\rangle^q \lvert 1_{23}\rangle^q - i\lvert 1_{\bar{2}\bar{3}}\rangle^q \lvert 0_{23}\rangle^q$
$f^\dagger_{\bar{3}3}$	$\lvert 0_{\bar{2}\bar{3}}\rangle^q \lvert 1_{23}\rangle^q + i\lvert 1_{\bar{2}\bar{3}}\rangle^q \lvert 0_{23}\rangle^q$
$f^\dagger_{2\bar{2}} f^\dagger_{\bar{3}3}$	$\lvert 0_{\bar{2}\bar{3}}\rangle^q \lvert 0_{23}\rangle^q - i\lvert 1_{\bar{2}\bar{3}}\rangle^q \lvert 1_{23}\rangle^q$

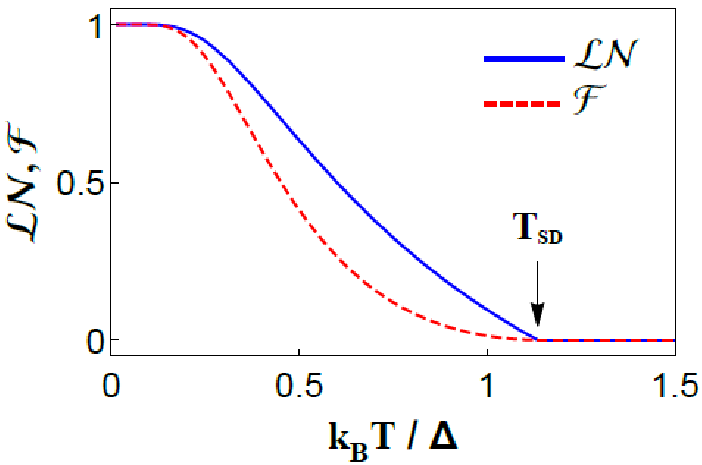
Entanglement sudden death



Comparison with the corresponding 1D spin

Fermion case:





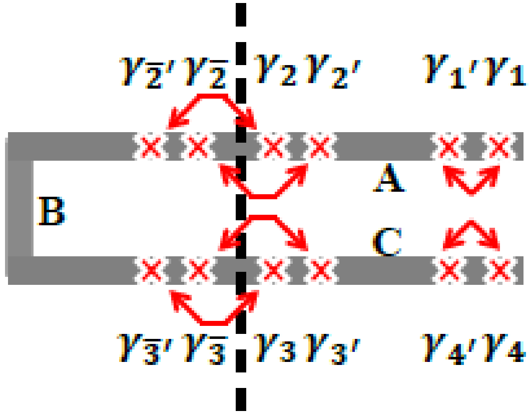
Spin case (Wigner Jordan):

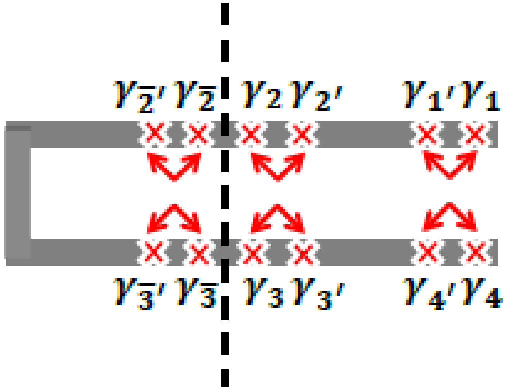
No Entanglement B|AC at any temperature

Our target systems: a different class from the Kitaev chain

$$\hat{H}_{\mathrm{II}} = -\frac{\Delta}{2}\sum_{j=1}^{N-2}(c_j + c_j^\dagger)(c_{j+2} - c_{j+2}^\dagger)$$

Two Majoranas at each end





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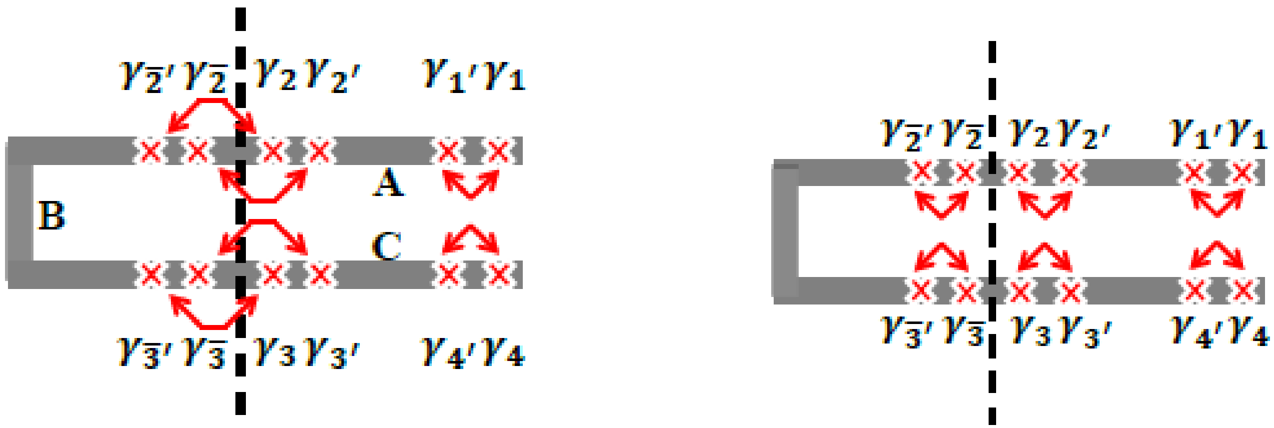


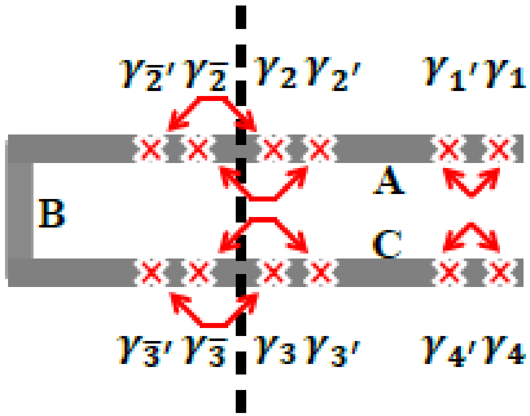
Changing Majorana fusion pairs ➔ 4-qubit cluster-state entanglement (nonlocal)

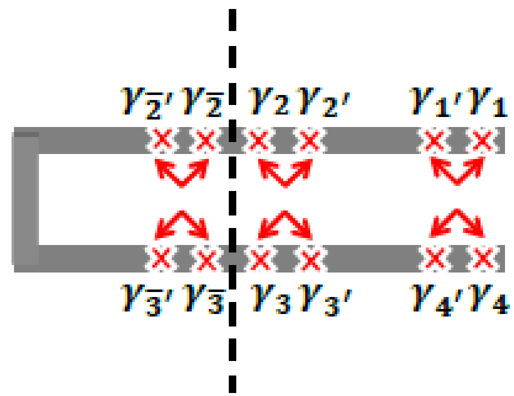
$$|n\rangle_{\mathrm{II}} \to |n\rangle_{\mathrm{II}}^q = |\mathrm{CL}\rangle^q |n_{11'}\rangle^q |n_{44'}\rangle^q,$$

$$|\mathrm{CL}\rangle^q = \frac{1}{2}[|0_{\bar{2}\bar{2}'}\rangle^q |0_{22'}\rangle^q (|0_{\bar{3}\bar{3}'}\rangle^q |0_{33'}\rangle^q + |1_{\bar{3}\bar{3}'}\rangle^q |1_{33'}\rangle^q)$$
$$- |1_{\bar{2}\bar{2}'}\rangle^q |1_{22'}\rangle^q (|0_{\bar{3}\bar{3}'}\rangle^q |0_{33'}\rangle^q - |1_{\bar{3}\bar{3}'}\rangle^q |1_{33'}\rangle^q)].$$

Our target systems: a different class from the Kitaev chain

$$\hat{H}_{\rm II} \;=\; -\frac{\Delta}{2}\sum_{j=1}^{N-2}(c_j + c_j^\dagger)(c_{j+2} - c_{j+2}^\dagger)$$





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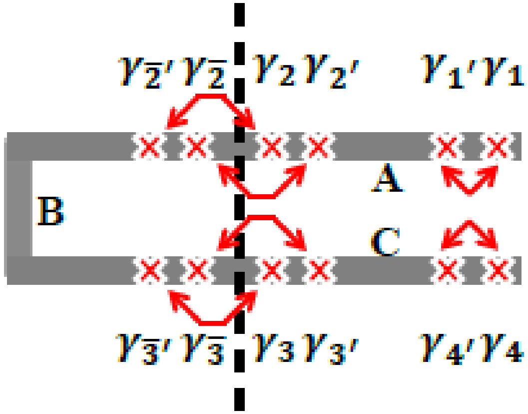
$$|n\rangle_{\rm II} \to |n\rangle_{\rm II}^q = |{\rm CL}\rangle^q |n_{11'}\rangle^q |n_{44'}\rangle^q,$$

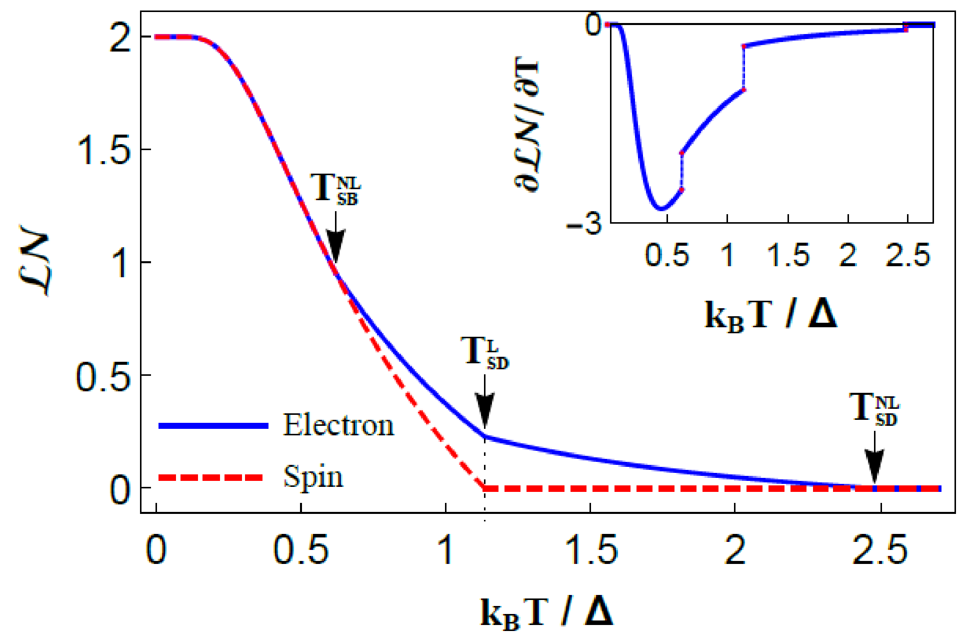
$$|{\rm CL}\rangle^q = \frac{1}{2}[|0_{\bar{2}2'}\rangle^q|0_{22'}\rangle^q(|0_{\bar{3}\bar{3}'}\rangle^q|0_{33'}\rangle^q + |1_{\bar{3}\bar{3}'}\rangle^q|1_{33'}\rangle^q)$$
$$- |1_{\bar{2}\bar{2}'}\rangle^q|1_{22'}\rangle^q(|0_{\bar{3}\bar{3}'}\rangle^q|0_{33'}\rangle^q - |1_{\bar{3}\bar{3}'}\rangle^q|1_{33'}\rangle^q)].$$

Spin case (Winger-Jordan):

Bell*Bell local entanglement in $\rho_{\rm II}^s(T=0)$ $\quad |{\rm Bell}_2\rangle^s \otimes |{\rm Bell}_3\rangle^s$

Entanglement between B and AC: finite temperature





$$\mathcal{LN}(\rho_{\mathrm{II}}(T)) - \mathcal{LN}(\rho^{s}_{\mathrm{II}}(T))$$

A marker for nonlocal entanglement originating from fermion statistics and Majorana fermions

Sudden birth and death of nonlocal entanglement

Summary

Non-Abelian evolution of a Majorana train in a single Josephson junction

- Generation and Braiding of mobile Majorana fermions
- Signature of Non-Abelian effects: $2n\pi$ fractional AC Josephson effect

Nonlocal entanglement in 1D bulk at finite temperature

- Entanglement by non-Abelian fusion and fermionic exchange statistics
- Dependent on topological classes (the number of the end Majorana fermions)
- Sudden death and birth of nonlocal entanglement

Nonlocal entanglement in 1D	Park, Shim, Lee, Sim, PRL (2017)
Non-Abelian evolution of a Majorana train	Choi, Sim submitted (2018)

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Seung-Sub B. Lee (Munchen, the group of Jan von Delft)

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