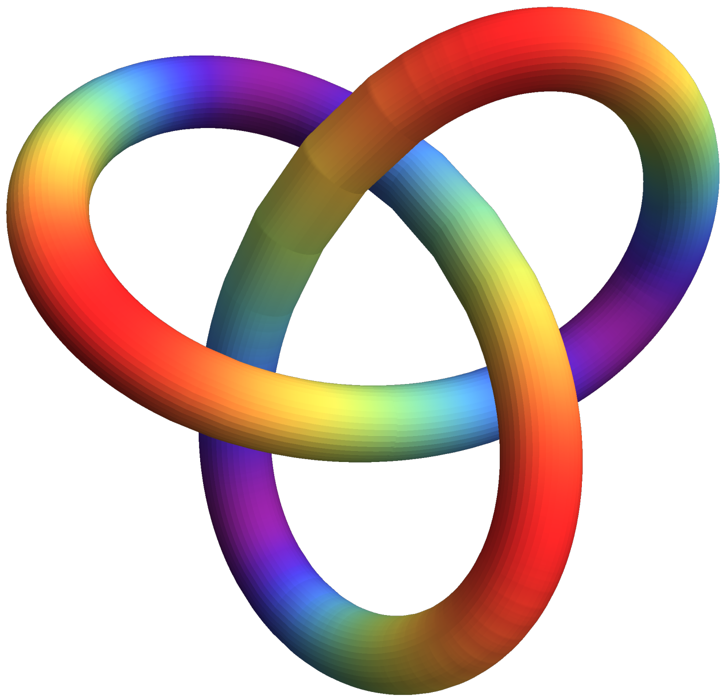
Topological rainbow chains



Germán Sierra
(IFT, Madrid)

Anyons in Quantum Many-Body Systems

MPI-PKS Dresden 21-25 January 2019

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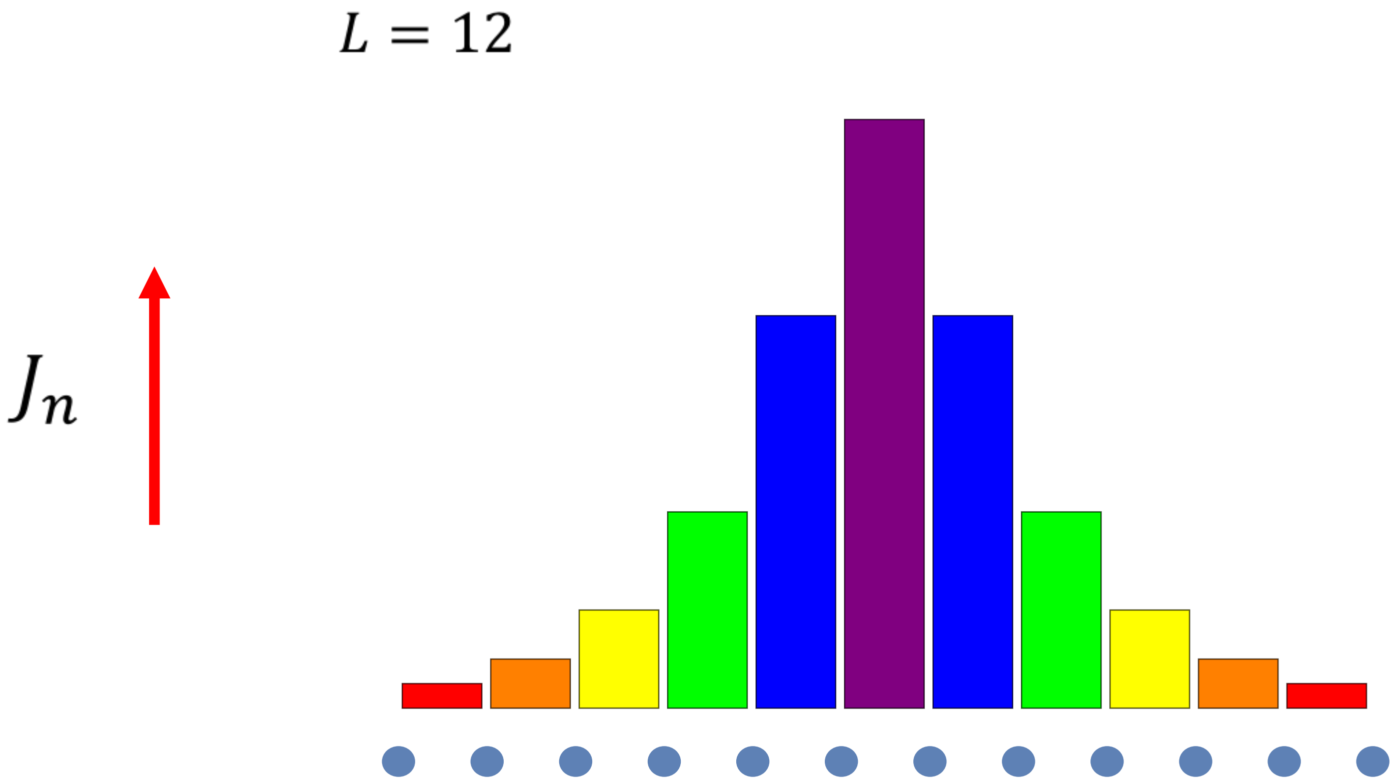
$$H = \sum_n h_{n,n+1}$$

Critical spin chain described by CFT

Entanglement entropy

$$S_A = \frac{c}{6}\ln L + \cdots$$

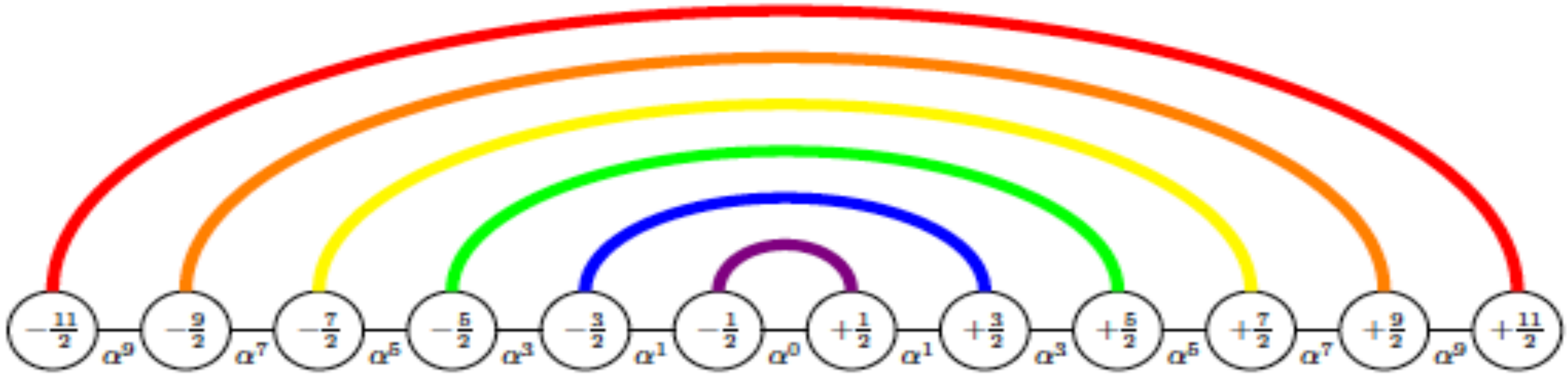
$$H = \sum_n J_n \, h_{n,n+1}$$



Critical spin chain described by CFT in curved spacetime

$$S_A \propto L$$

Ground State is a Valence Bond



- Concentric singlet phase (Vitigliano et al. 2010)
- Rainbow state (Ramirez et al. 2015)
- Matrioska state (Di Franco et al 2008)
- Nested Bell state (Alkurtass, et al 2014)

Topological properties of the rainbow model

- The parity of the number of sites the chain (even/odd)
- Symmetry of the couplings J_n (bond center/site center)

↓

Symmetry Protected Topological phases

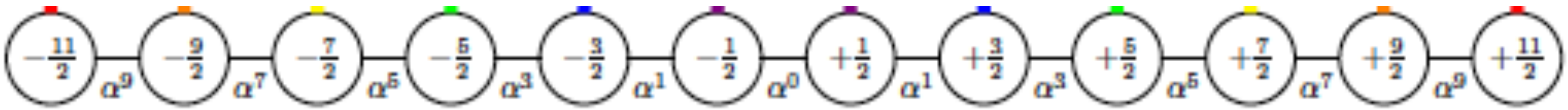
Su-Schrieffer-Heeger and Haldane phases

Outline

- Review of the rainbow XX chain
- Topological properties of XX and Heisenberg rainbows
- Relations to other works

Rainbow XX model

Vitigliano, Riera and Latorre (2010)
Ramirez, Rodriguez-Laguna, GS (2015)



$$H = \sum_n J_n \left(S_n^X S_{n+1}^X + S_n^Y S_{n+1}^Y \right)$$

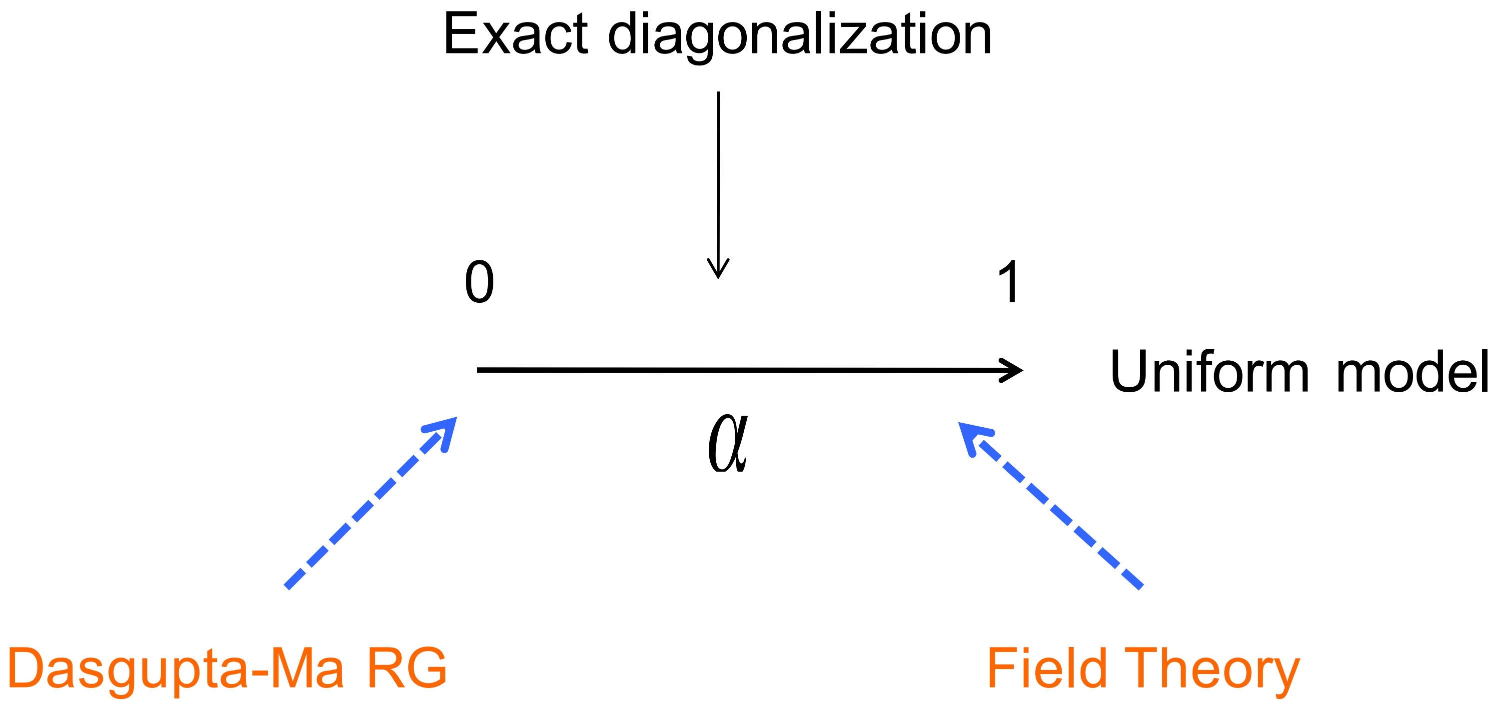
Jordan Wigner

$$H \equiv -\frac{J_0}{2} c_{\frac{1}{2}}^\dagger c_{-\frac{1}{2}} - \sum_{n=\frac{1}{2}}^{L-\frac{3}{2}} \frac{J_n}{2} \left[c_n^\dagger c_{n+1} + c_{-n}^\dagger c_{-(n+1)} \right] + \text{h.c.} \,,$$

$$\begin{cases} J_0(\alpha) & = \quad 1, \\ J_n(\alpha) & = \alpha^{2n}, \; n = \frac{1}{2}, \ldots, L - \frac{3}{2}. \end{cases}$$

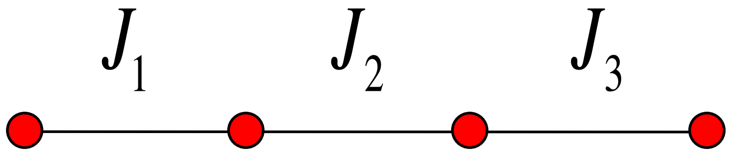
$$0 < \alpha \leq 1$$

Methods:

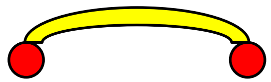

Exact diagonalization
0
1
Uniform model
α
Dasgupta-Ma RG
Field Theory

Dasgupta-Ma RG

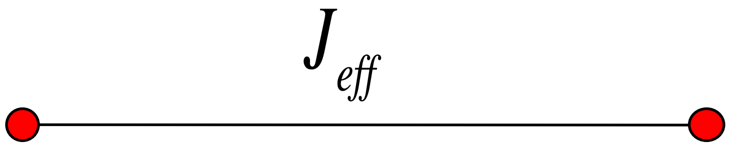
XX Hamiltonian



$$J_2 >> J_1, J_3$$



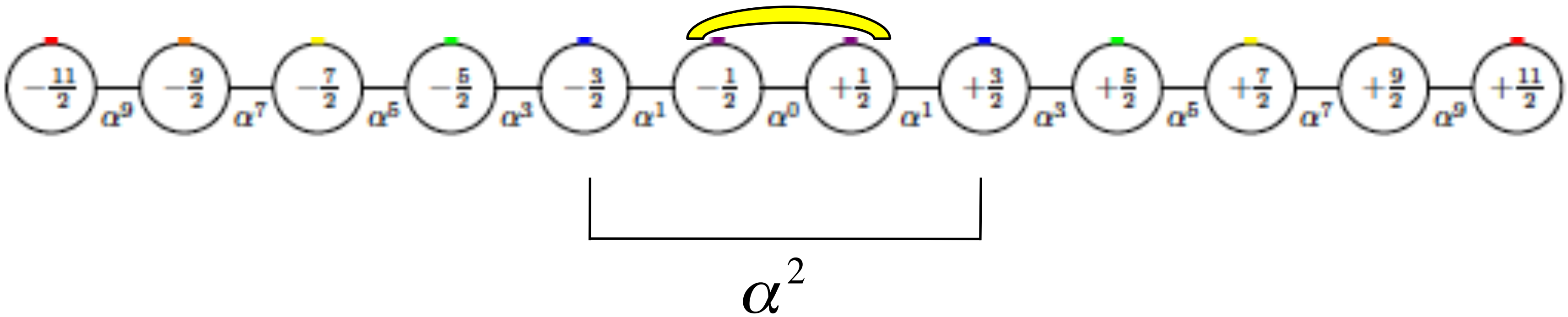
$$|bond\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



$$J_{eff} = \frac{J_1 J_3}{J_2}$$

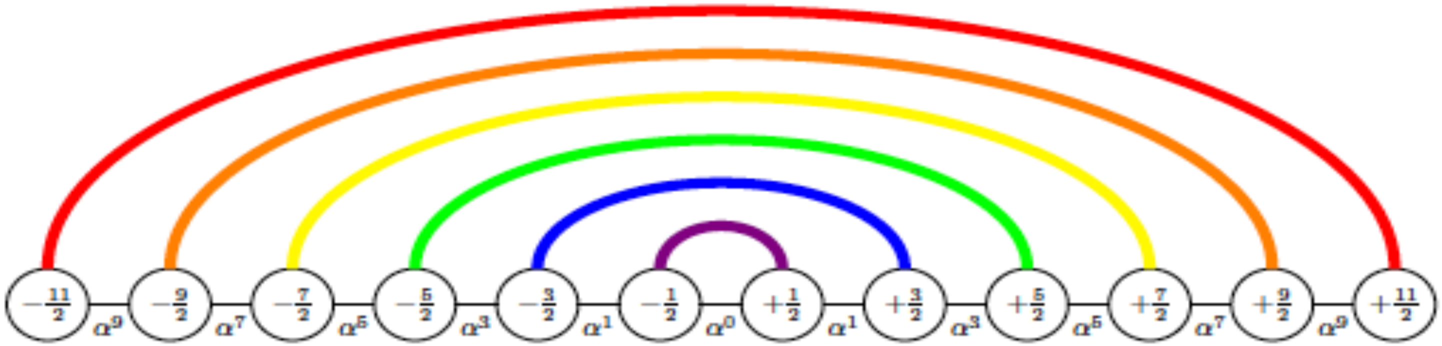
In the limit $\alpha \to 0^+$

Effective coupling: $$J_{eff} = \frac{J_{1/2} J_{1/2}}{J_0} = \alpha^2$$



This new bond is again the strongest one because $\alpha^2 > \alpha^3$

Iterate the RG ⟹ GS: valence bond state

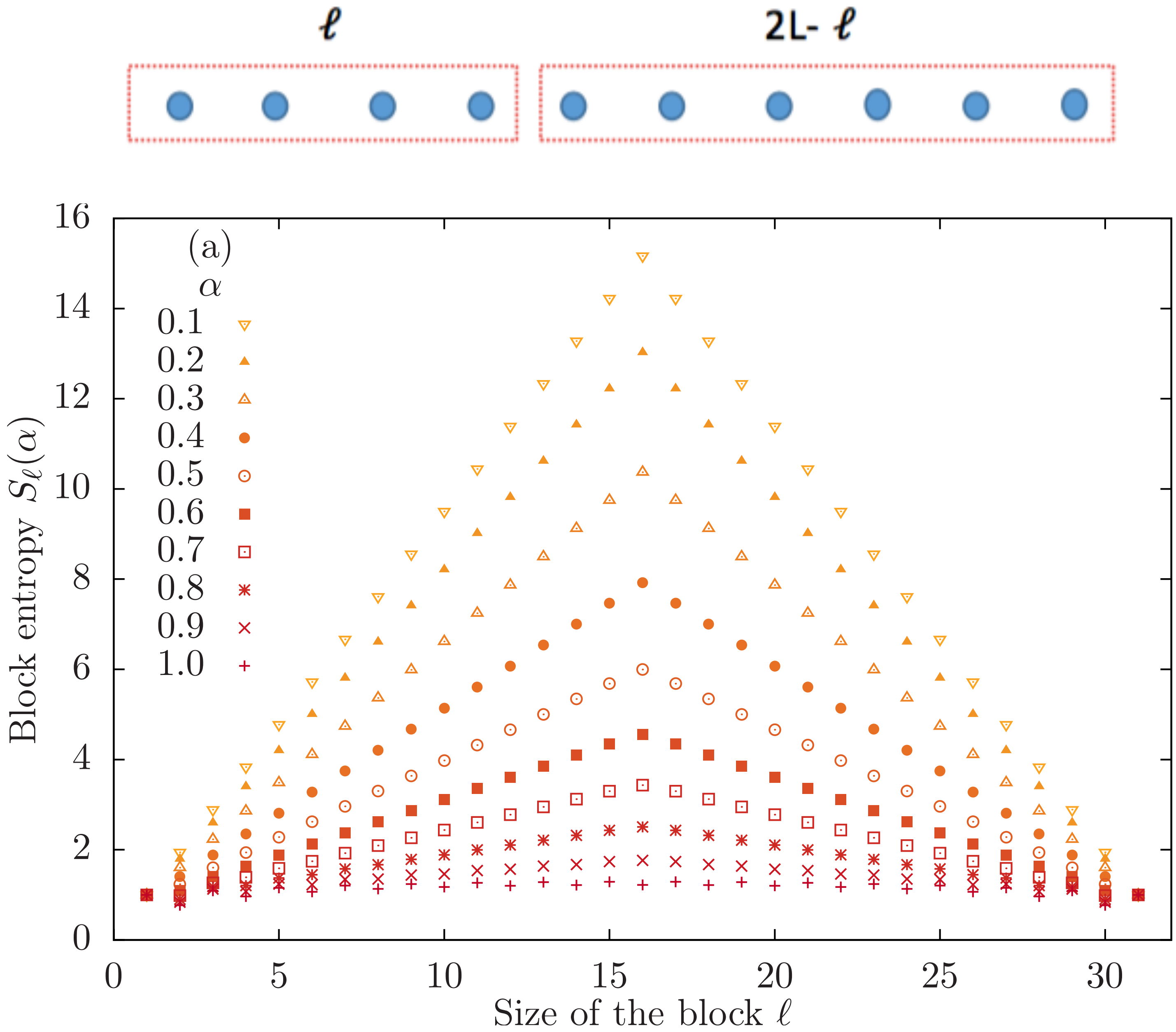


Exact in the limit $\alpha \to 0^+$

Half-block entanglement entropy

$$S_A = L \ln 2$$

All inhomogeneities : Exact Diagonalization



Weak inhomogeneity : Field theory

$$\alpha = e^{-h/2} \to 1$$

Fast-slow separation of degrees of freedom

$$\frac{c_m}{\sqrt{a}} \simeq e^{ik_F x}\psi_L(x) + e^{-ik_F x}\psi_R(x)$$

$$H \simeq iJa \int_{-aL}^{aL} dx\, e^{-\frac{h|x|}{a}} \left[\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L - \frac{h}{2a}\mathrm{sign}(x)(\psi_R^\dagger \psi_R - \psi_L^\dagger \psi_L)\right]$$

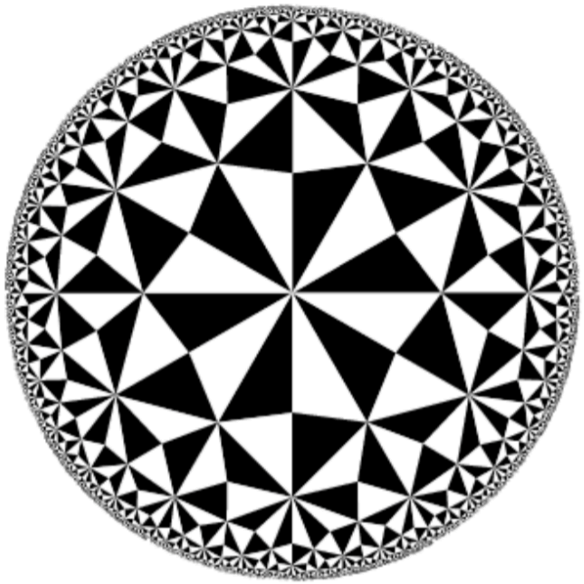
$$\psi_R(\pm L) = \mp i\, \psi_L(\pm L)$$

Massless Dirac fermion in curved spacetime

$$z = \tilde{x} + i\,t \qquad \tilde{x} = sign(x)\frac{e^{h|x|} - 1}{h}$$

$$ds^2 = e^{\sigma}\, dz\, d\bar{z}, \quad e^{\sigma} = e^{-h|x|}$$

Curvature $\quad R(x) = -\,h^2 + 4\,h\,\delta(x)$



Half-block entropy

$$S_L \approx \frac{1}{6} \ln \tilde{L} = \frac{1}{6} \ln \frac{e^{hL} - 1}{h}$$

Uniform chain $\quad S_L \approx \frac{1}{6} \ln L$

$h\, L \gg 1 \;\rightarrow\; S_L \approx \frac{h\, L}{6}$ Volumen law

More results

- General expressions of entanglement entropies
- Entanglement spectrum and Entanglement Hamiltonian
- Finite temperature
- Random + Rainbow

Thermofield double

$$h\,L \gg 1 \quad \rightarrow \quad S_L \approx \frac{h\,L}{6}$$

Entropy at finite temperature T

$$S_{\rm CFT}(L,T) \approx \frac{c}{6} \ln\left(\frac{\sinh\left(2\pi L T\right)}{\pi T}\right) \rightarrow \frac{c}{3}\pi T L$$

$$T_R = \frac{h}{2\pi}$$

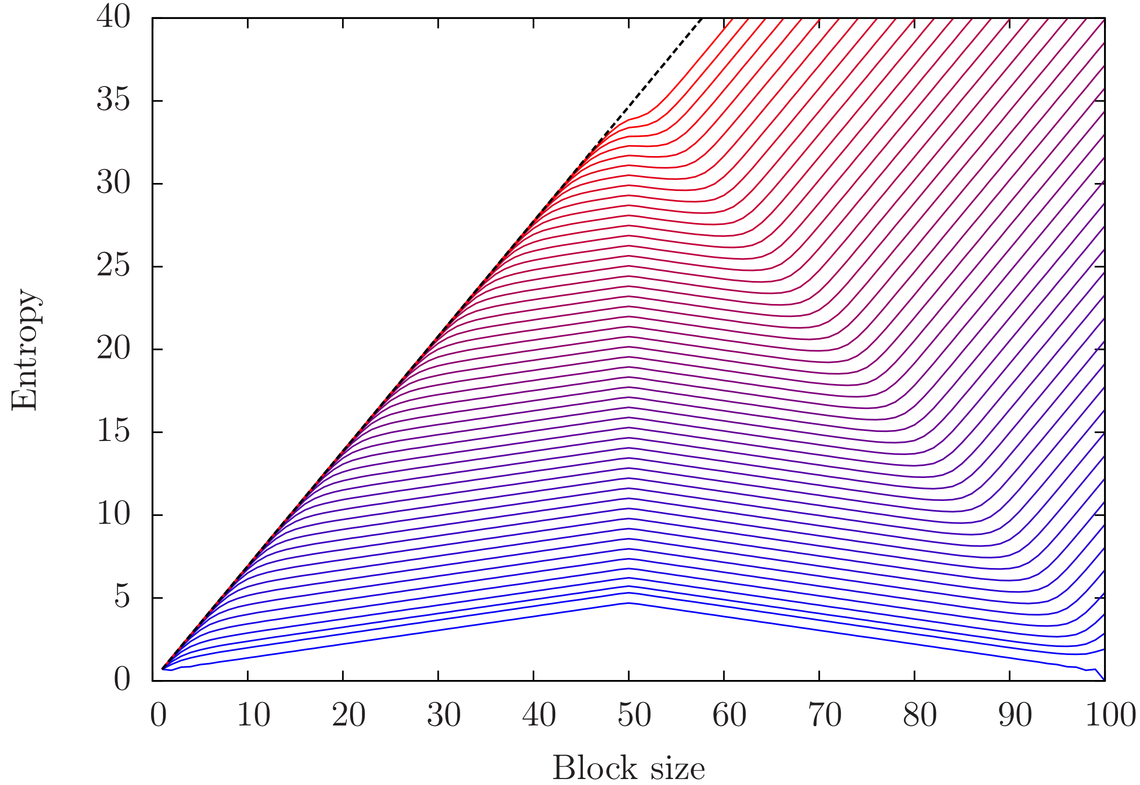
$$|\psi\rangle = \sum_n e^{-\frac{1}{2}\beta_R E_n}\, |n\rangle_l\, |n\rangle_r$$

$$H_{CFT}\,|n\rangle = E_n|n\rangle$$

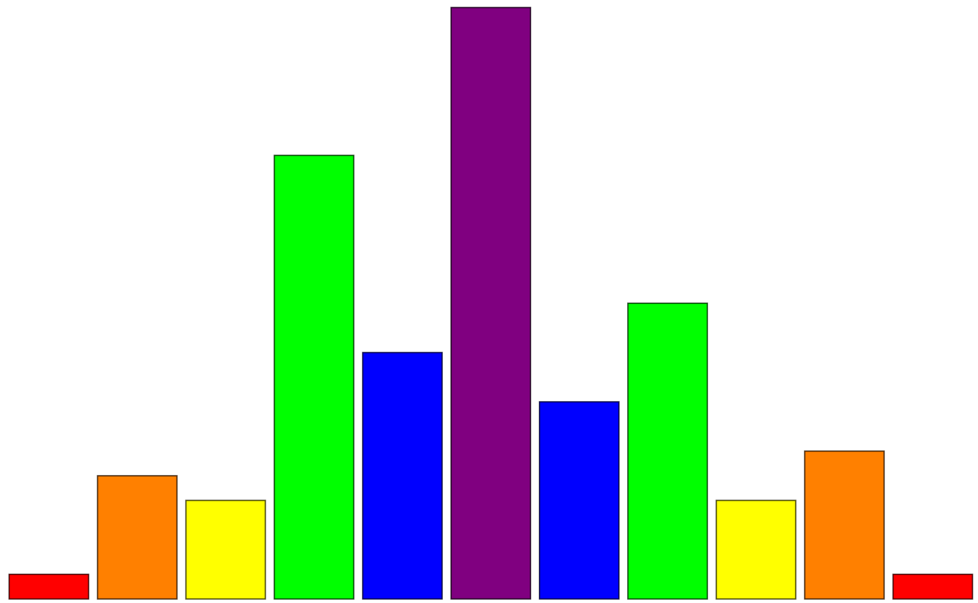
Finite temperature

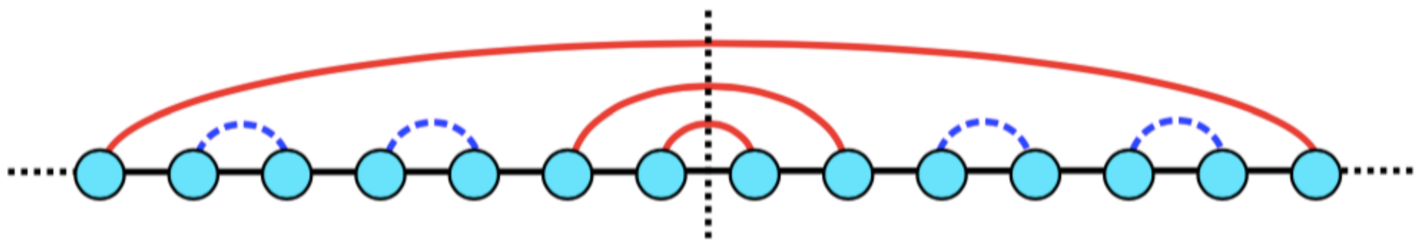
$$T = J(x_0)$$

$$S(x) \sim \begin{cases} (L-|x|)\ln 2, & x \in (-L, -x_0), \\ (L-x_0)\ln 2 + (x_0 - |x|)h/6, & x \in (-x_0, x_0), \\ (L - 2x_0 + x)\ln 2, & x \in (x_0, L). \end{cases}$$



rainbow + random = ranbow





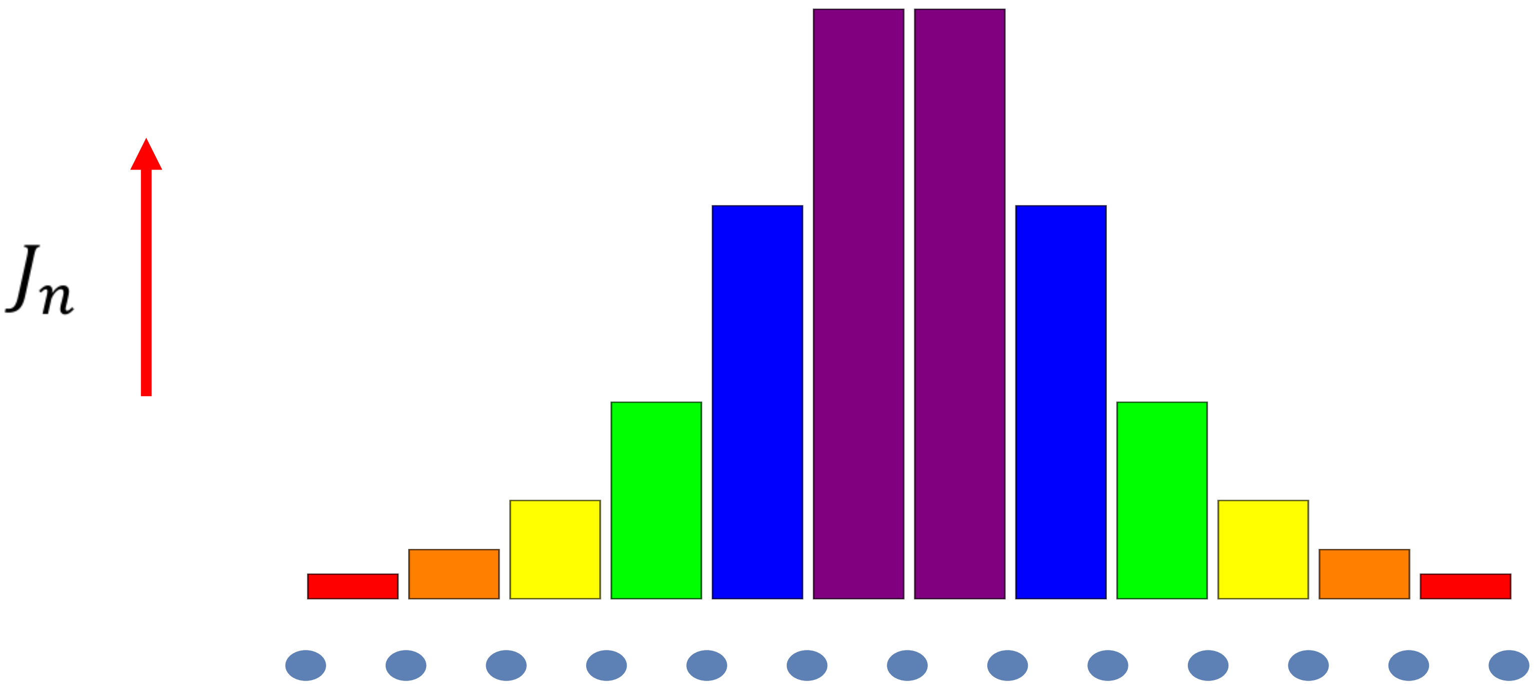
XX model: $S_A \propto \sqrt{L}$

V. Alba, S. Santalla, P. Ruggiero, J. Rodriguez-Laguna, P. Calabrese, G.S 2018

Topological rainbow

$$H = \sum_n J_n \, h_{n,n+1}$$

Open odd chain



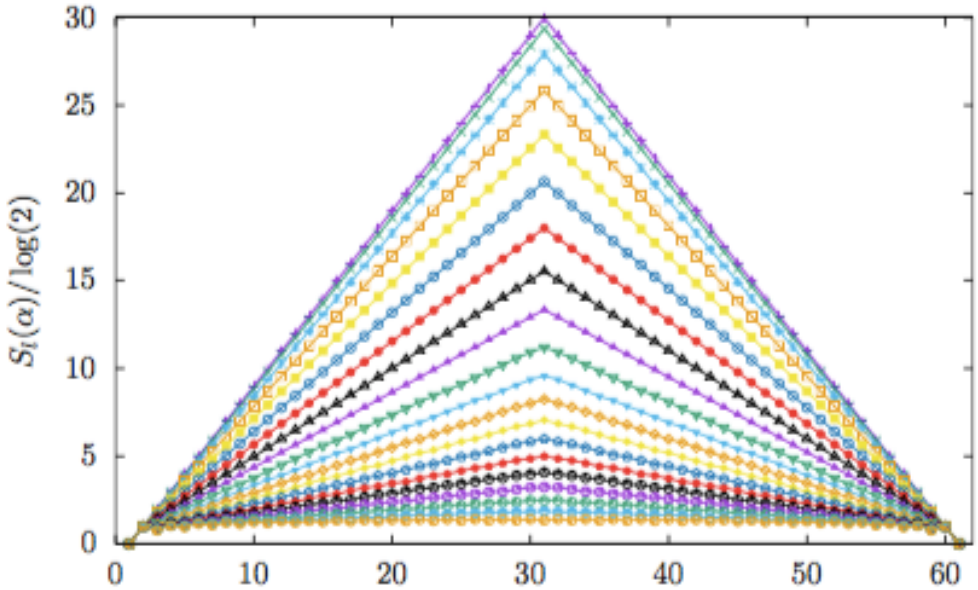
(N. Samos, S. Santallana, J.Rodriguez-Laguna, GS, 2018)

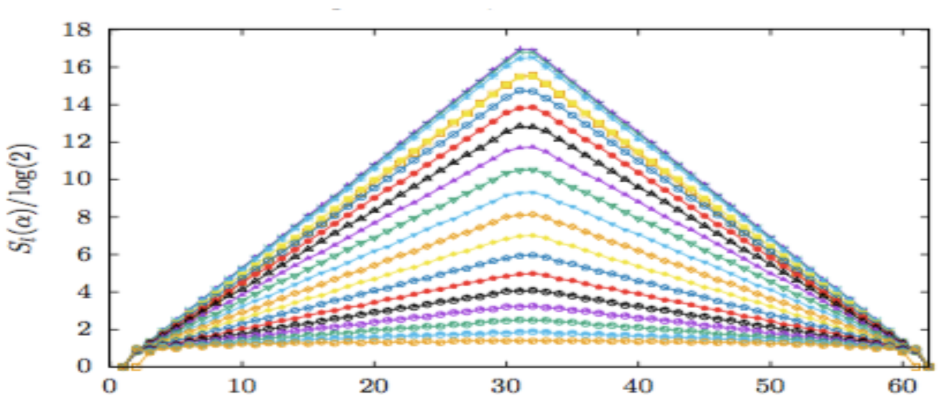
Topological rainbow

XX chain

2L sites

2L+1 sites





block entanglement entropies

$$S_A = \ell \ \ln 2$$

$$S_A = (\ell + \frac{1}{2})\,(2\ln 2 - 1)$$

Strong inhomogeneity

AF Heisenberg model

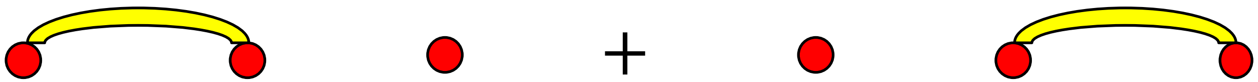
$$H_{\text{center}} = J_0 \left(\vec{\mathbf{S}}_0 \cdot \vec{\mathbf{S}}_1 + \vec{\mathbf{S}}_0 \cdot \vec{\mathbf{S}}_{-1} \right)$$

ground state
S= 1/2

$$|\psi_+\rangle = \frac{1}{\sqrt{6}} \left(|++-\rangle - 2|+-+\rangle + |-++\rangle \right)$$

$$|\psi_-\rangle = \frac{1}{\sqrt{6}} \left(-|+--\rangle + 2|-+-\rangle - |--+\rangle \right)$$

Resonating
valence bond

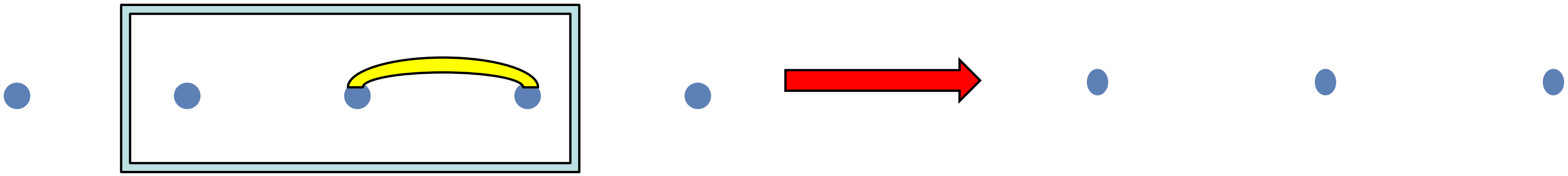


Renormalization

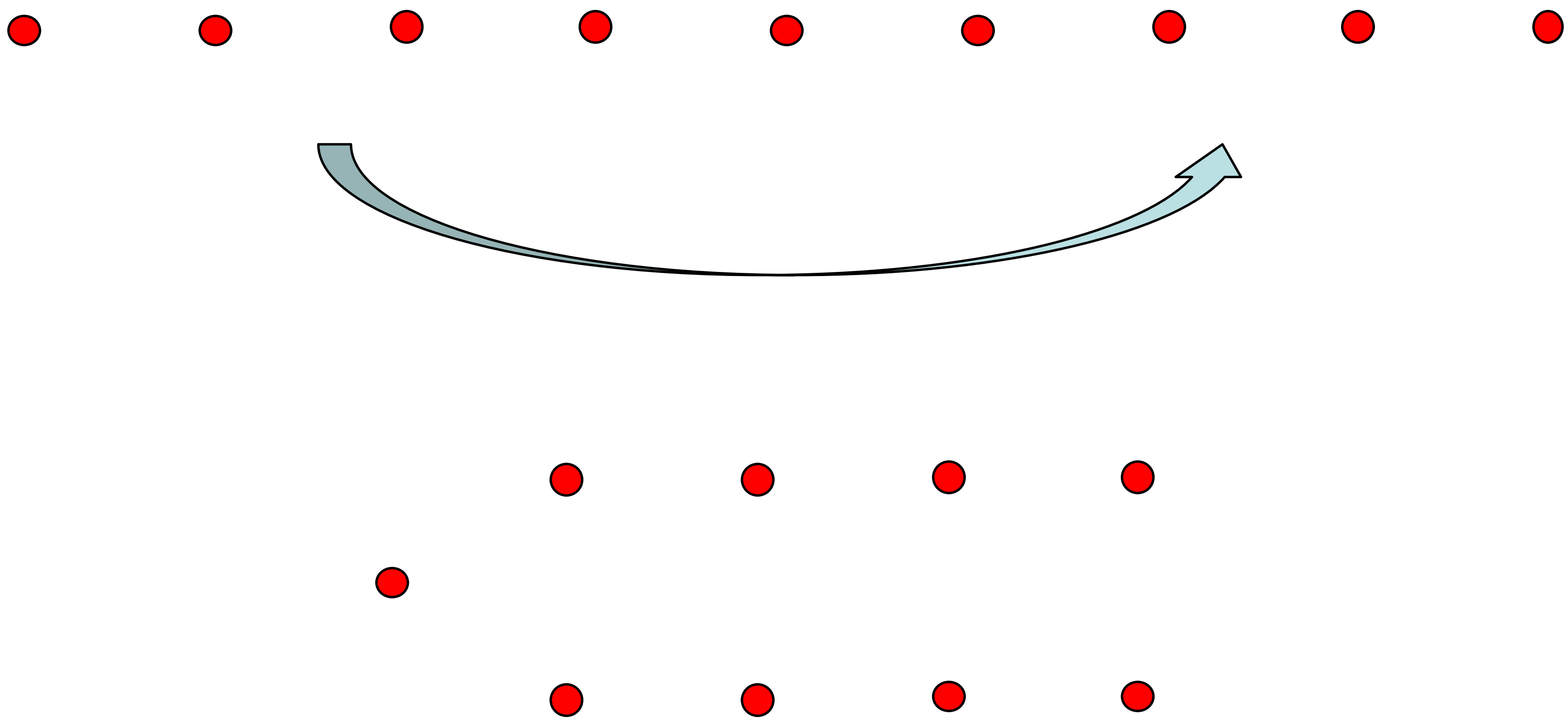
$$\vec{\mathbf{S}}_i \to \xi_i \, \vec{\mathbf{S}}_0^{(1)}, \qquad i = 0, \pm 1$$

$$\xi_1 = \xi_{-1} = \frac{2}{3}, \quad \xi_0 = -\frac{1}{3}$$

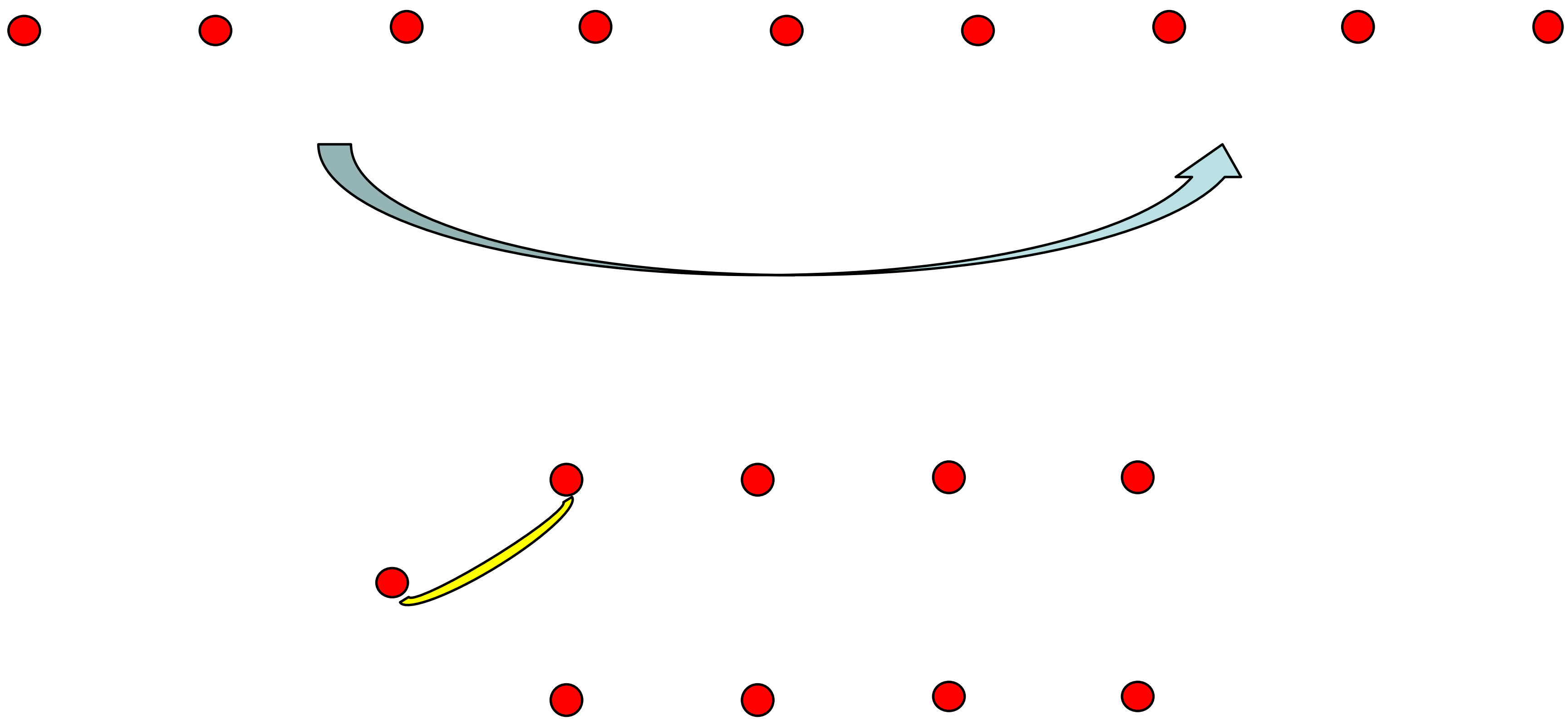
$$J_1(\vec{\mathbf{S}}_1 \cdot \vec{\mathbf{S}}_2 + \vec{\mathbf{S}}_{-1} \cdot \vec{\mathbf{S}}_{-2}) \to \frac{2J_1}{3}(\vec{\mathbf{S}}_0^{(1)} \cdot \vec{\mathbf{S}}_2 + \vec{\mathbf{S}}_0^{(1)} \cdot \vec{\mathbf{S}}_{-2})$$



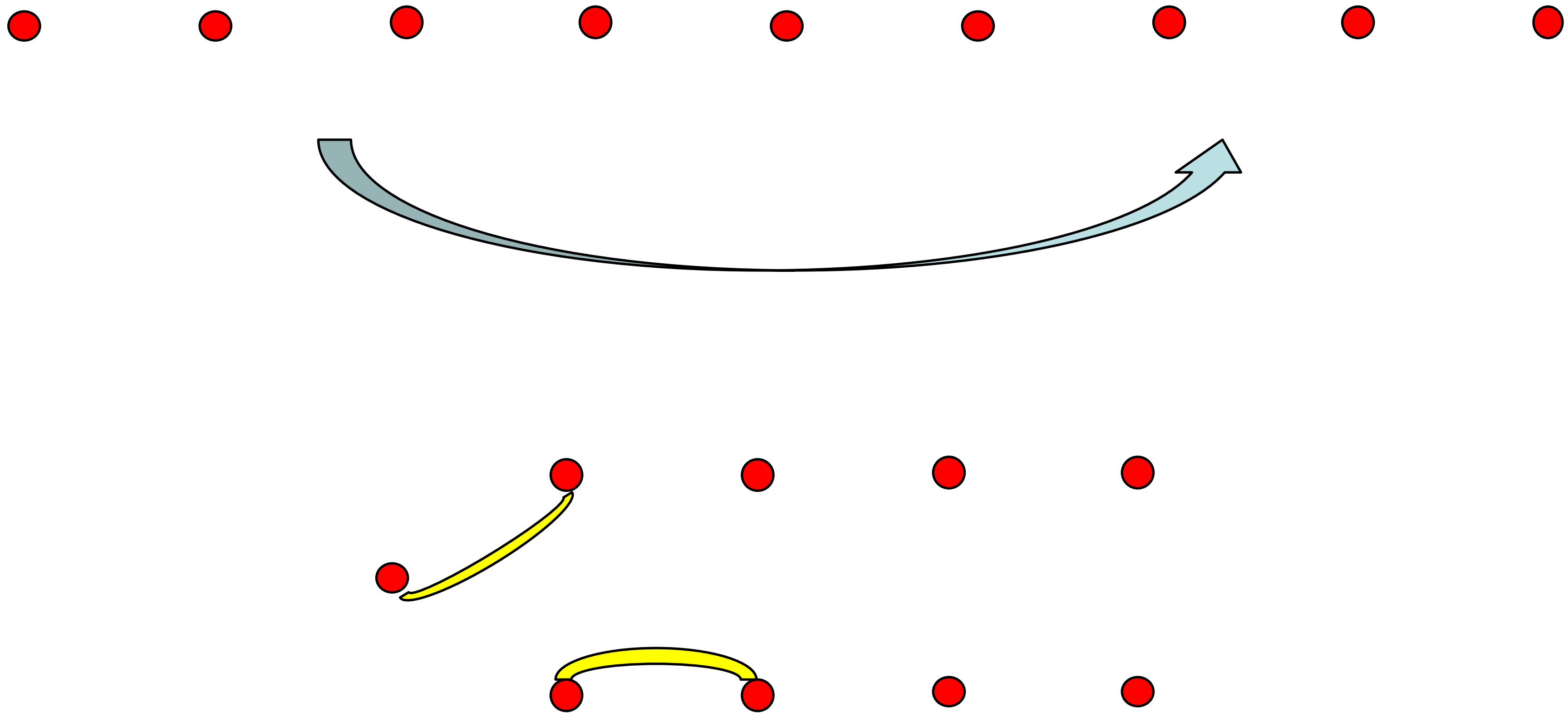
Folding the chain



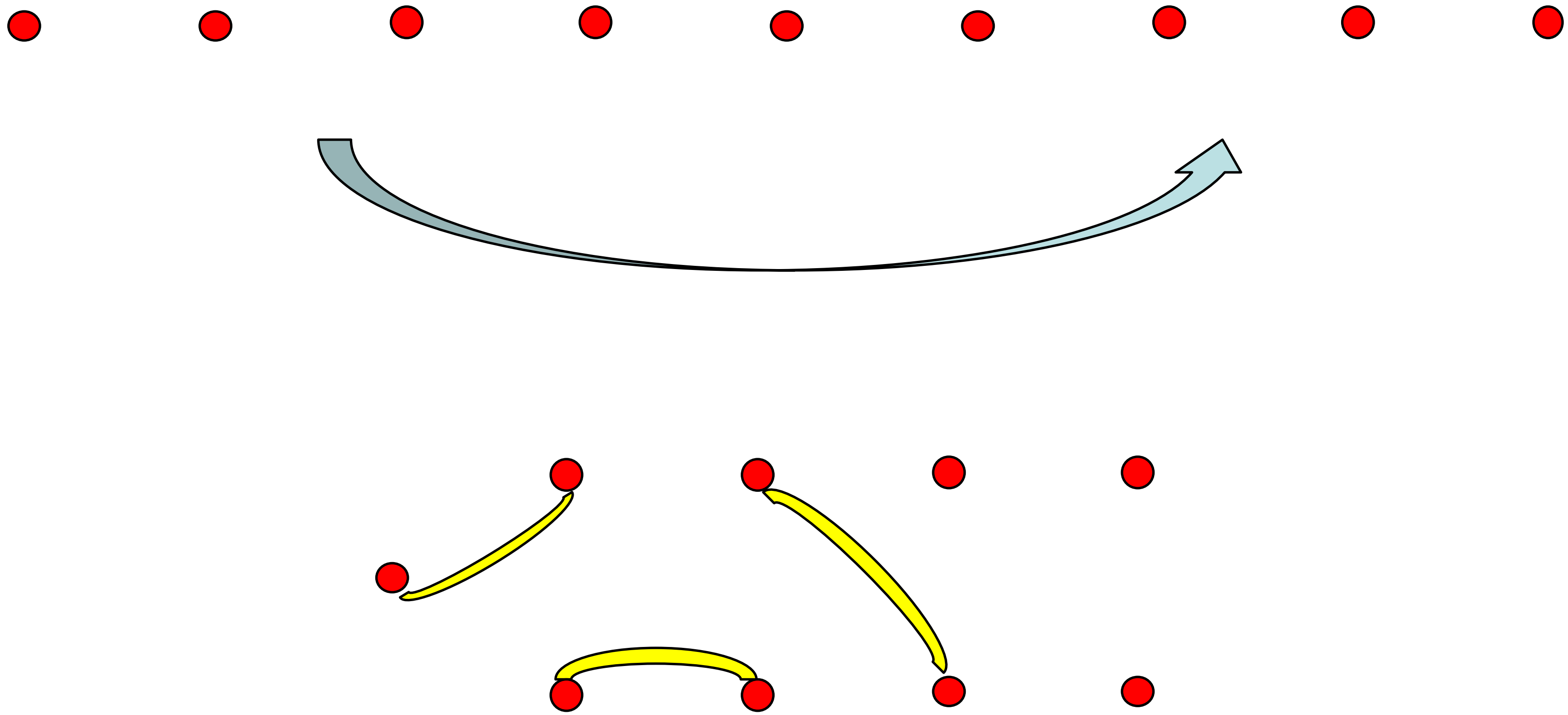
Folding the chain



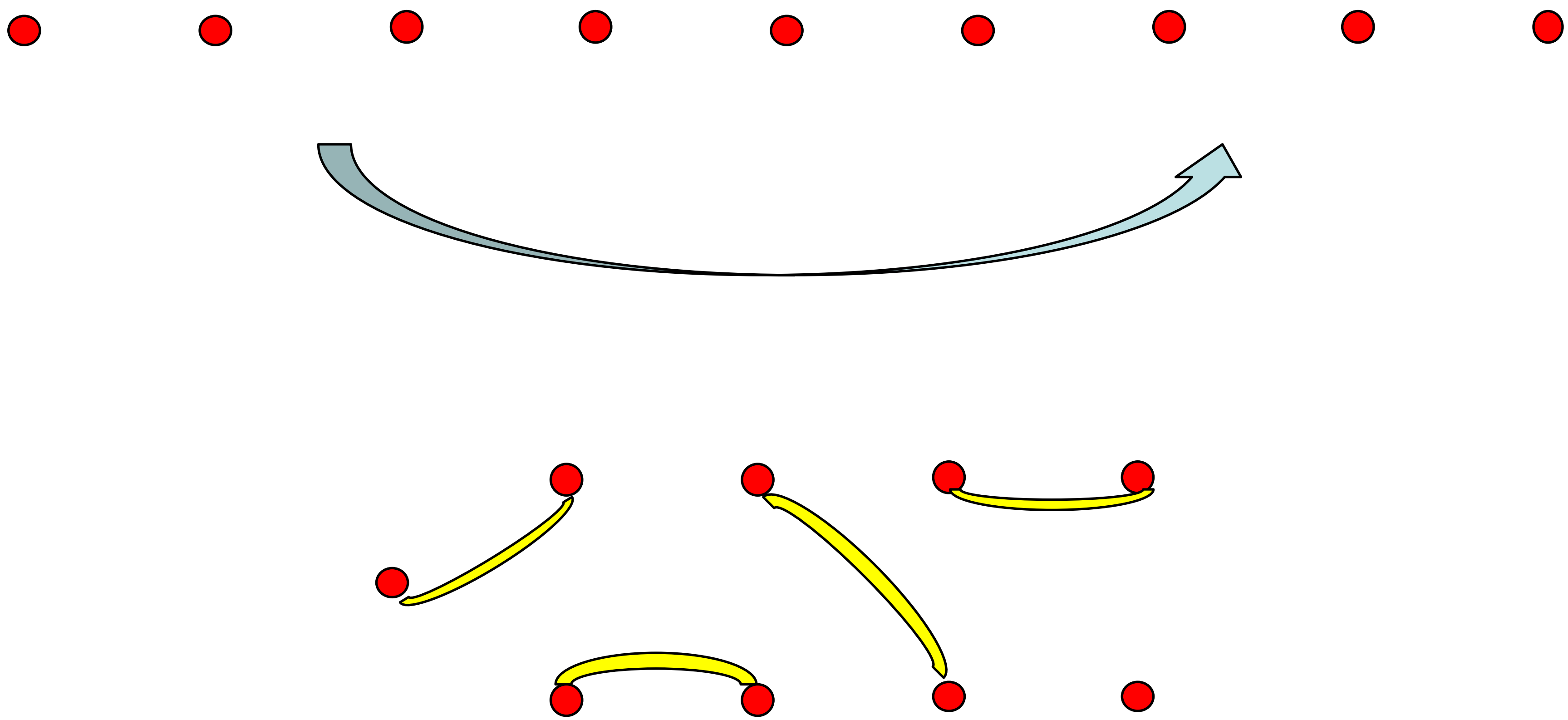
Folding the chain

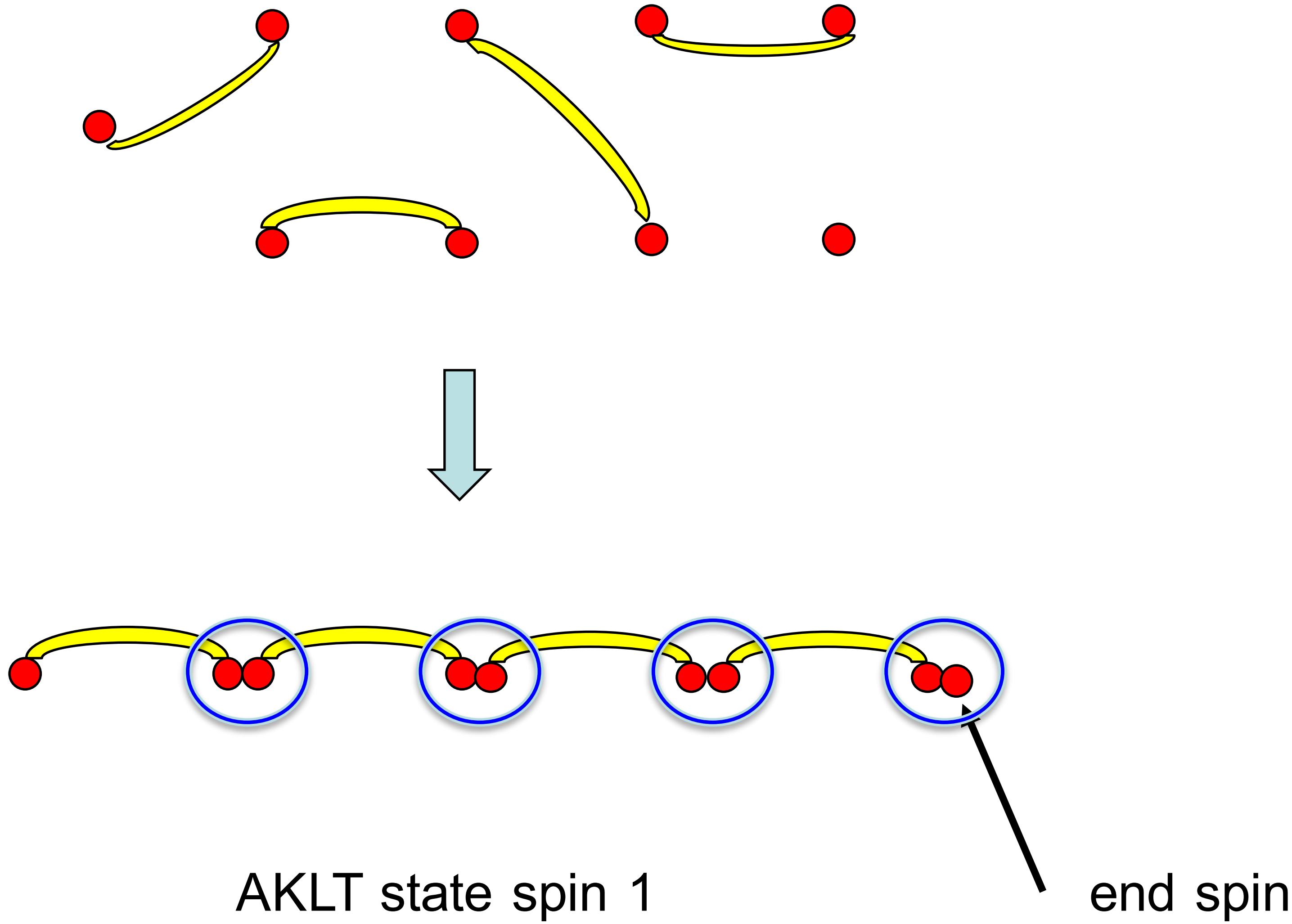


Folding the chain

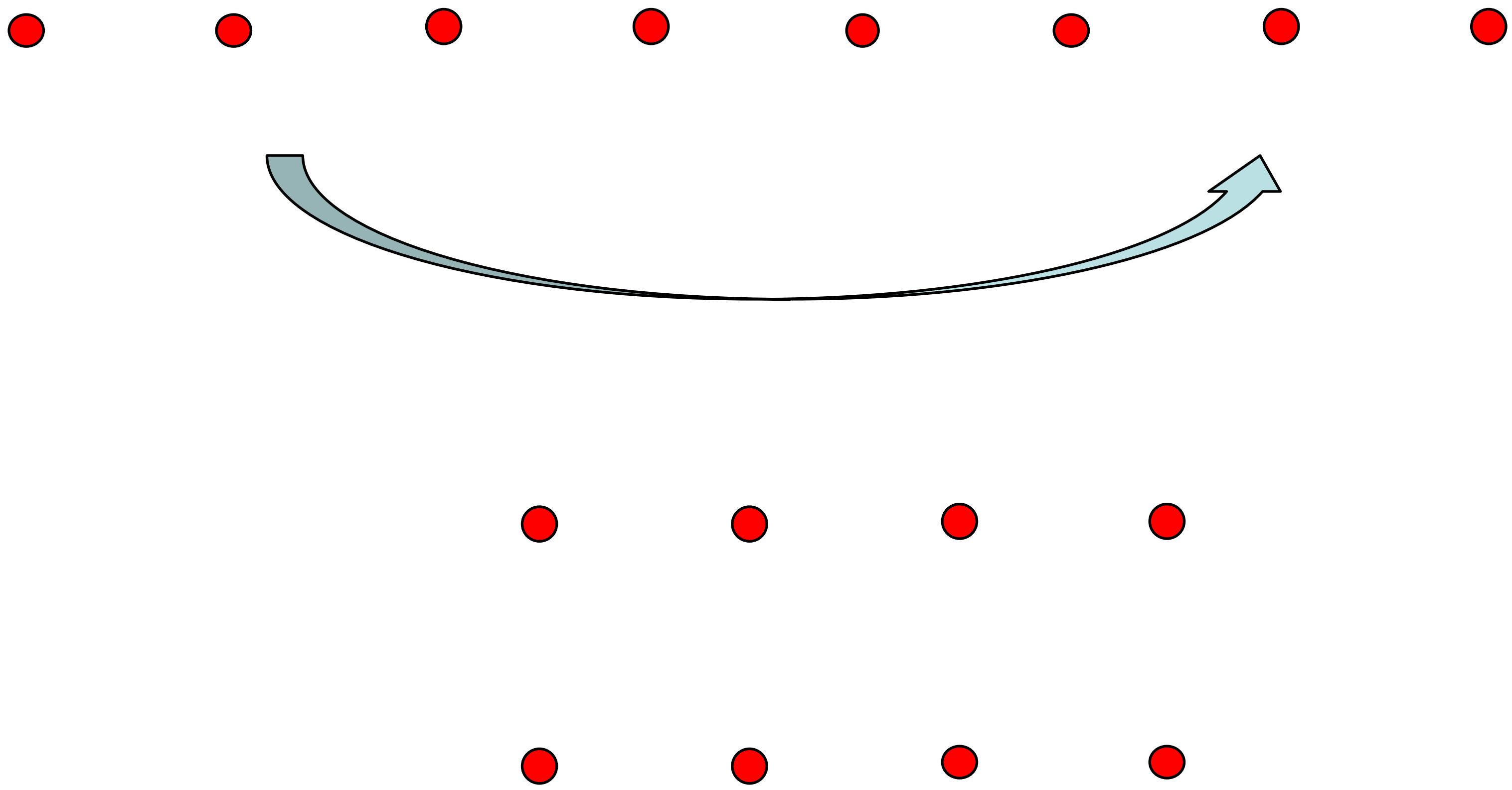


Folding the chain

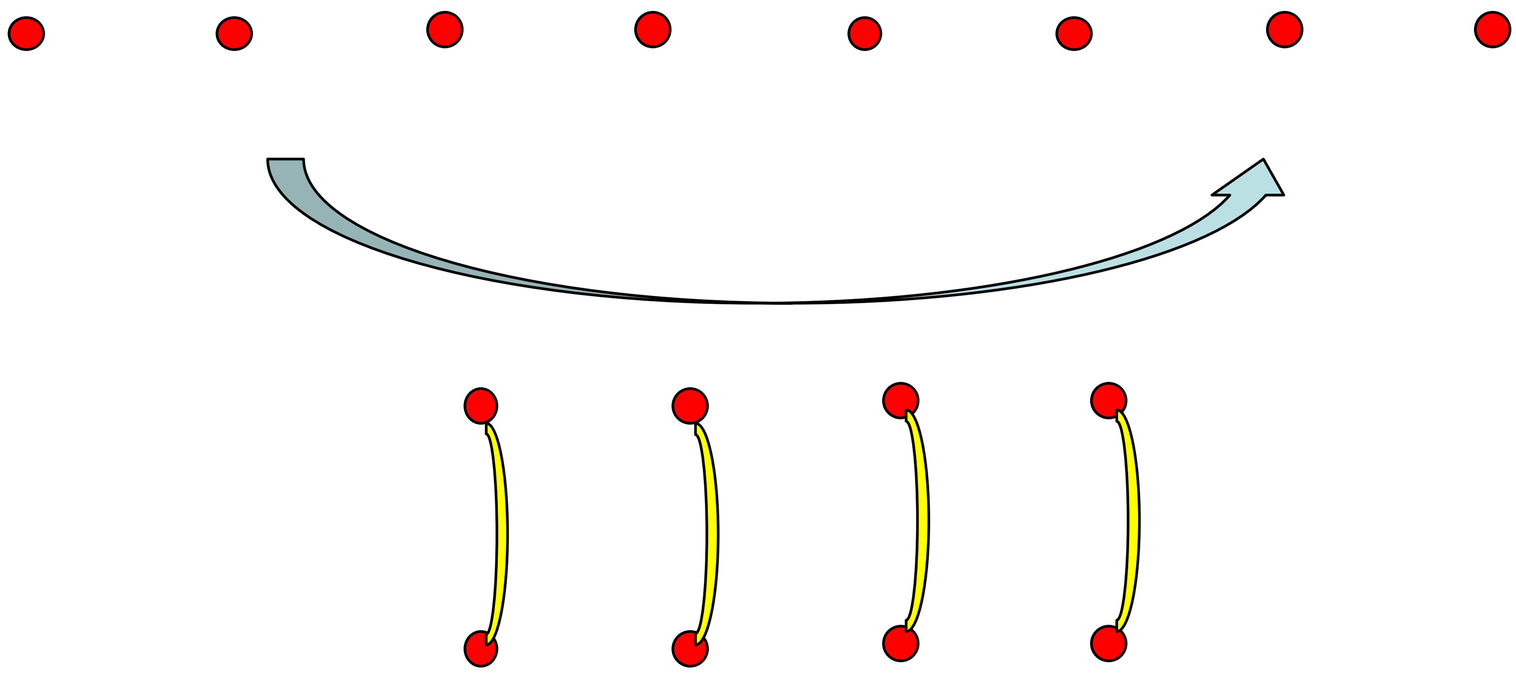



AKLT state spin 1
end spin

Folding the even chain



Folding the even chain



Even rainbow = product state in the folded basis

Even rainbow = trivial

Odd rainbow = non trivial

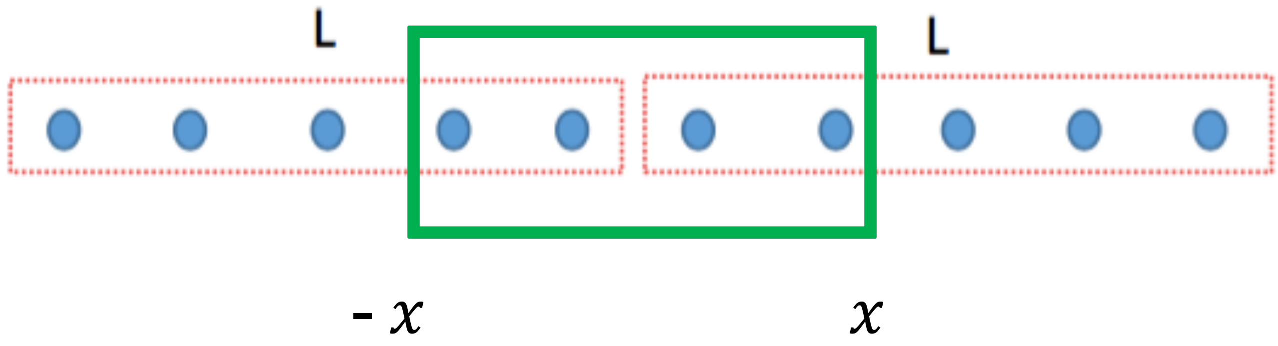
delocalized spinon ➡ localized edge spin

uniform strong inhomogeneity

Folding

long-range entanglement ➡ short-range entanglement

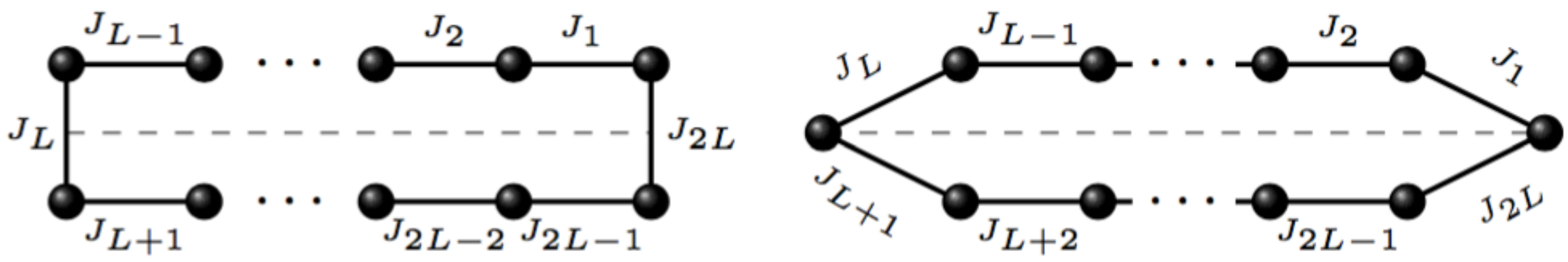
$$S_L \approx \frac{h\,L}{6}$$

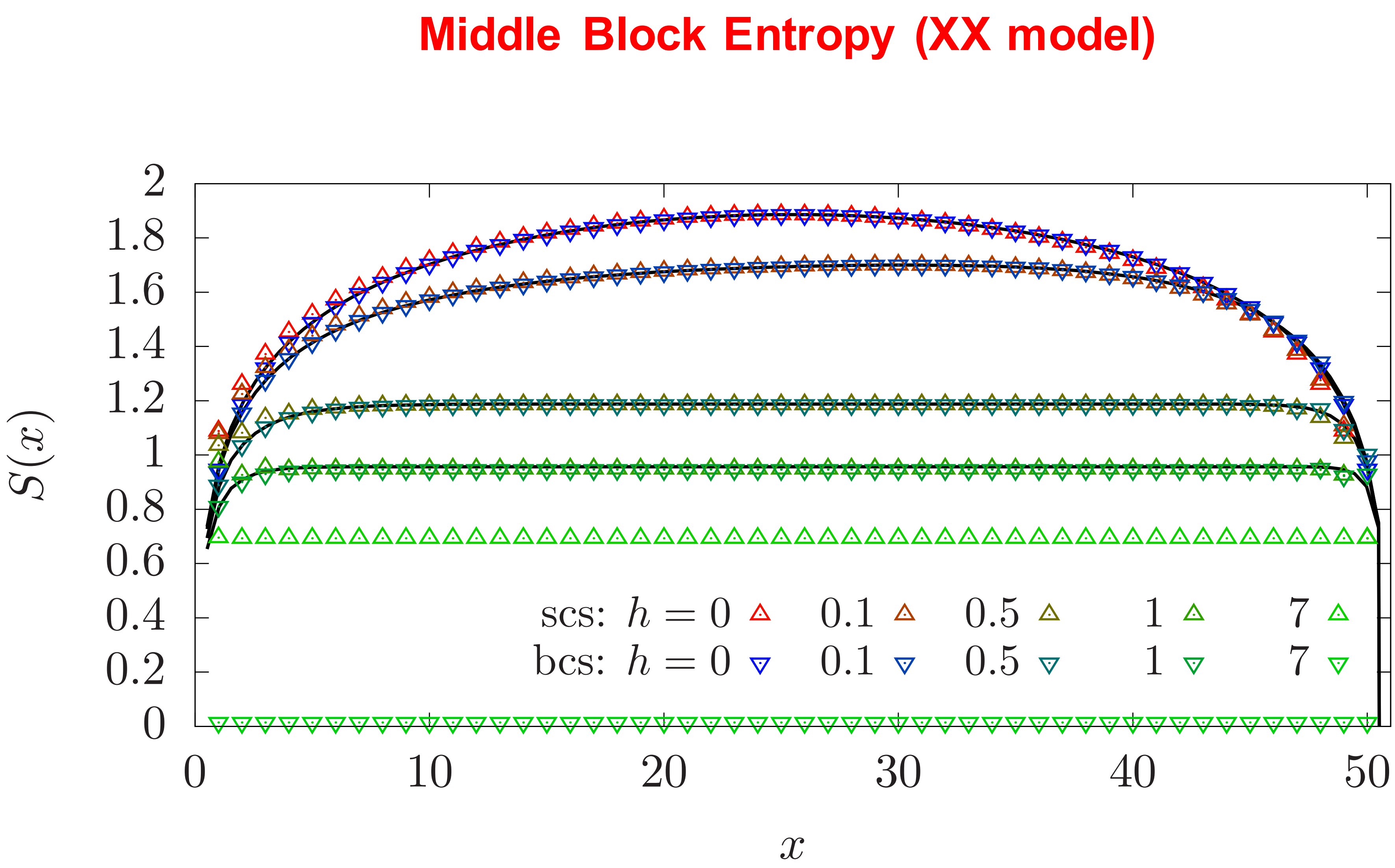


$$S(x) < \frac{1}{3} \ln \frac{4}{h}$$

even/odd chains → bond/site centered symmetry

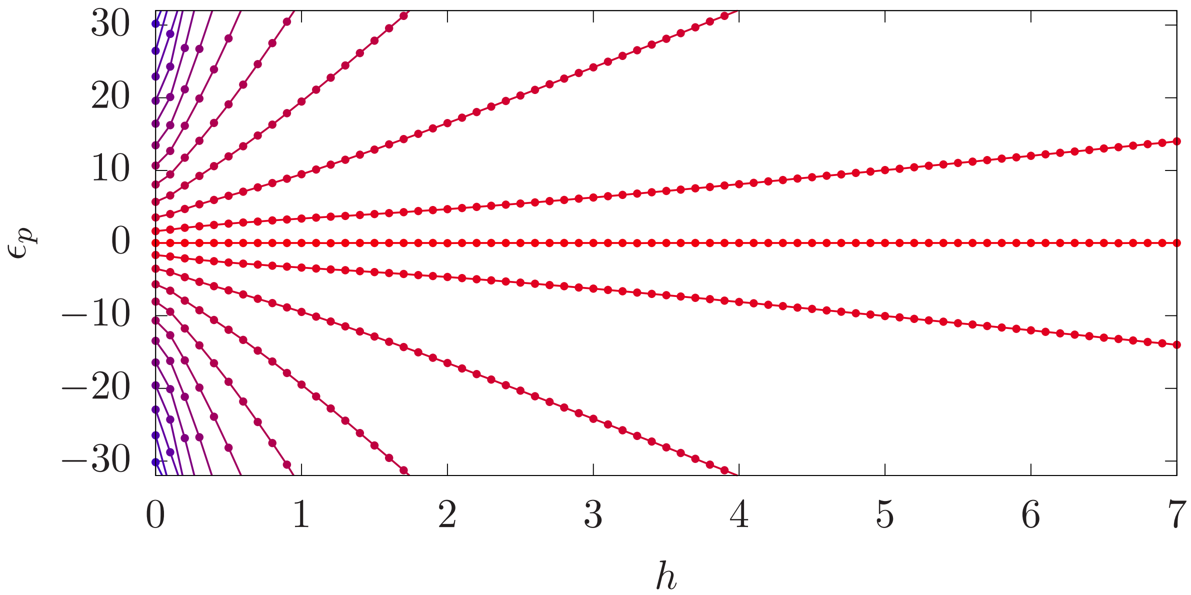
$$J_n = J_{2L-n} \; (\text{bcs}), \quad J_n = J_{2L+1-n} \; (\text{scs})$$




Middle Block Entropy (XX model)
S(x)
x
scs: h = 0
0.1
0.5
1
7
bcs: h = 0
0.1
0.5
1
7
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
10
20
30
40
50

Entanglement spectrum (XX model)

site-centered symmetry

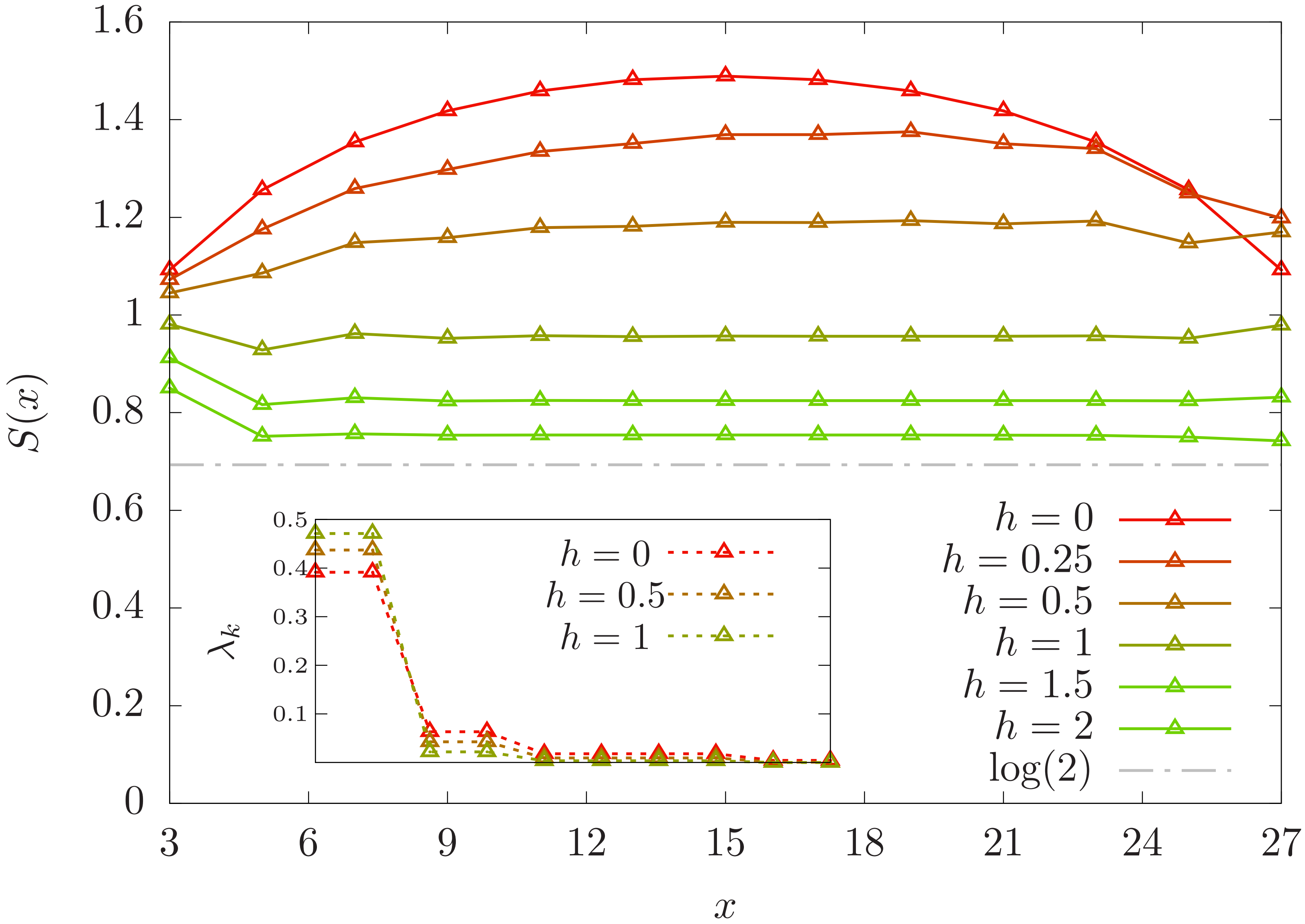


zero entanglement energy → degeneracy of EE

Time reversal + Particle hole preserve the degeneracy

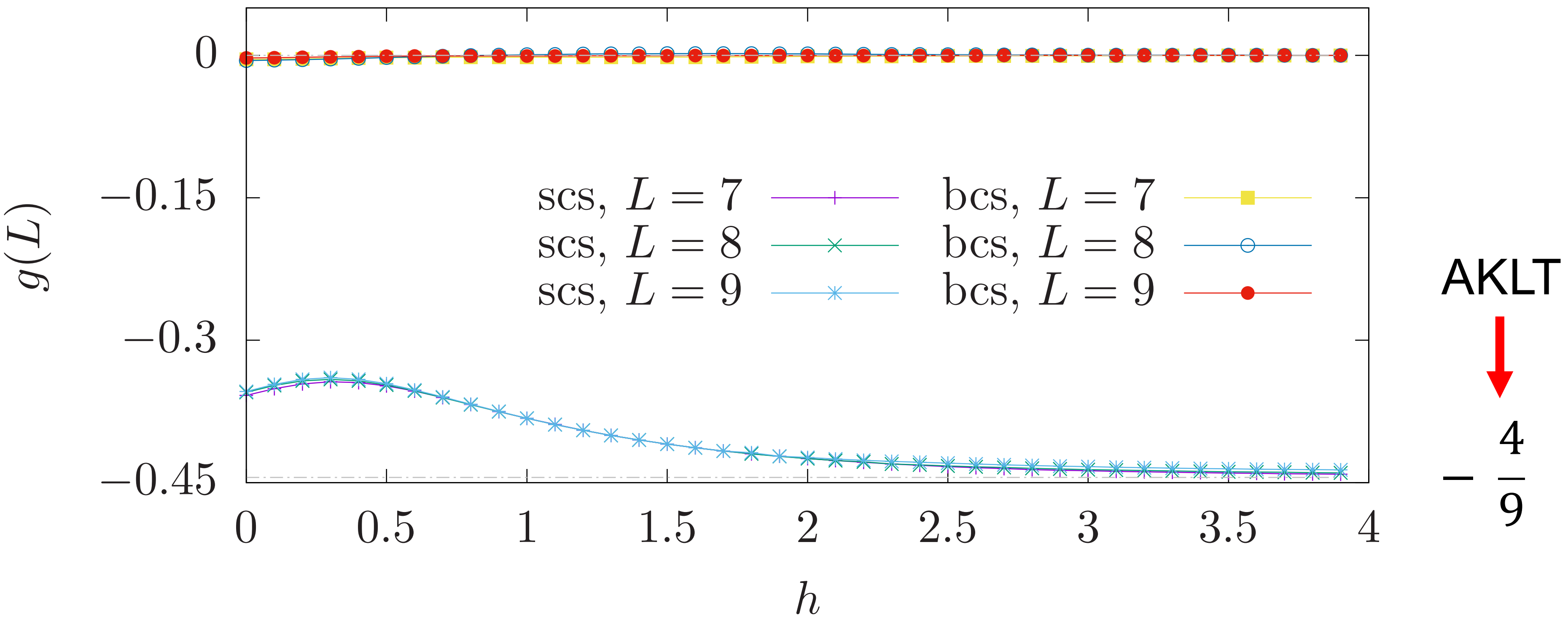
SSH phase (BDI class)

Middle Block Entropy and Entanglement Spectrum (Heisenberg)



String order parameter (Heisenberg)

$$g(L) = \langle S_1^z \, e^{i\pi \sum_{j=2}^{L-1} S_j^z} \, S_L^z \rangle$$



Higher spin chains

Heisenberg chains
S = 1 /2, 3/2, 5/2 …

WZW SU(2)@k=1

→ AKLT S =1, 3, 5,..

Non trivial SPT

Heisenberg chains
S = 1, 2, 3 …

gapped

→ AKLT S =2, 4, 6,..

Trivial SPT

Relation between gapped and “gapless” SPT phases

Global anomalies Furuya and Oshikawa (2017)

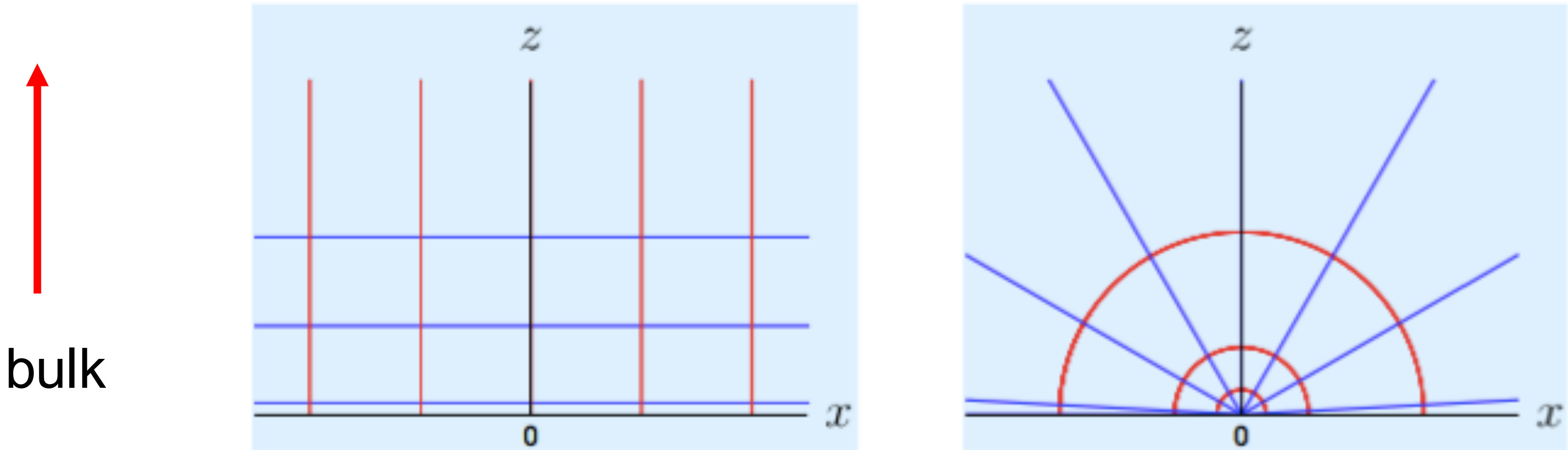
SU$(2)@k$ k: even (trivial), odd (non trivial)

Holography and the rainbow

Cormack, Liu, Nozaki, Ryu (2018)

Rainbow: CFT spacetime with curvature $R(x) = -h^2$

AdS_2 is a foliation of AdS_3



Ryu-Takayanagi

$$S_A(\ell) = \frac{c}{3}\log\left[\frac{2}{\epsilon h}\sinh\left(\frac{h\ell}{2}\right)\right]$$

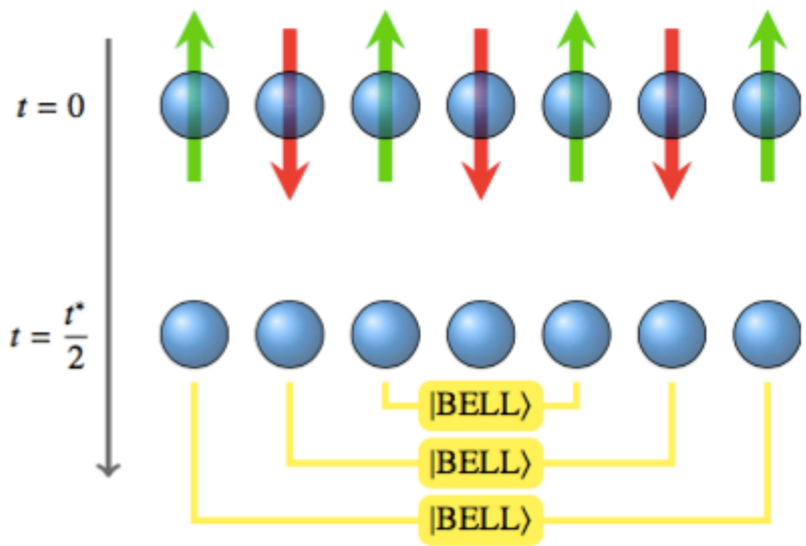
Nested entangled states for distributed quantum channels

C. Di Franco, M. Paternostro, and M. S. Kim (2004)

Optimal Quench for Distance-Independent Entanglement and Maximal Block Entropy

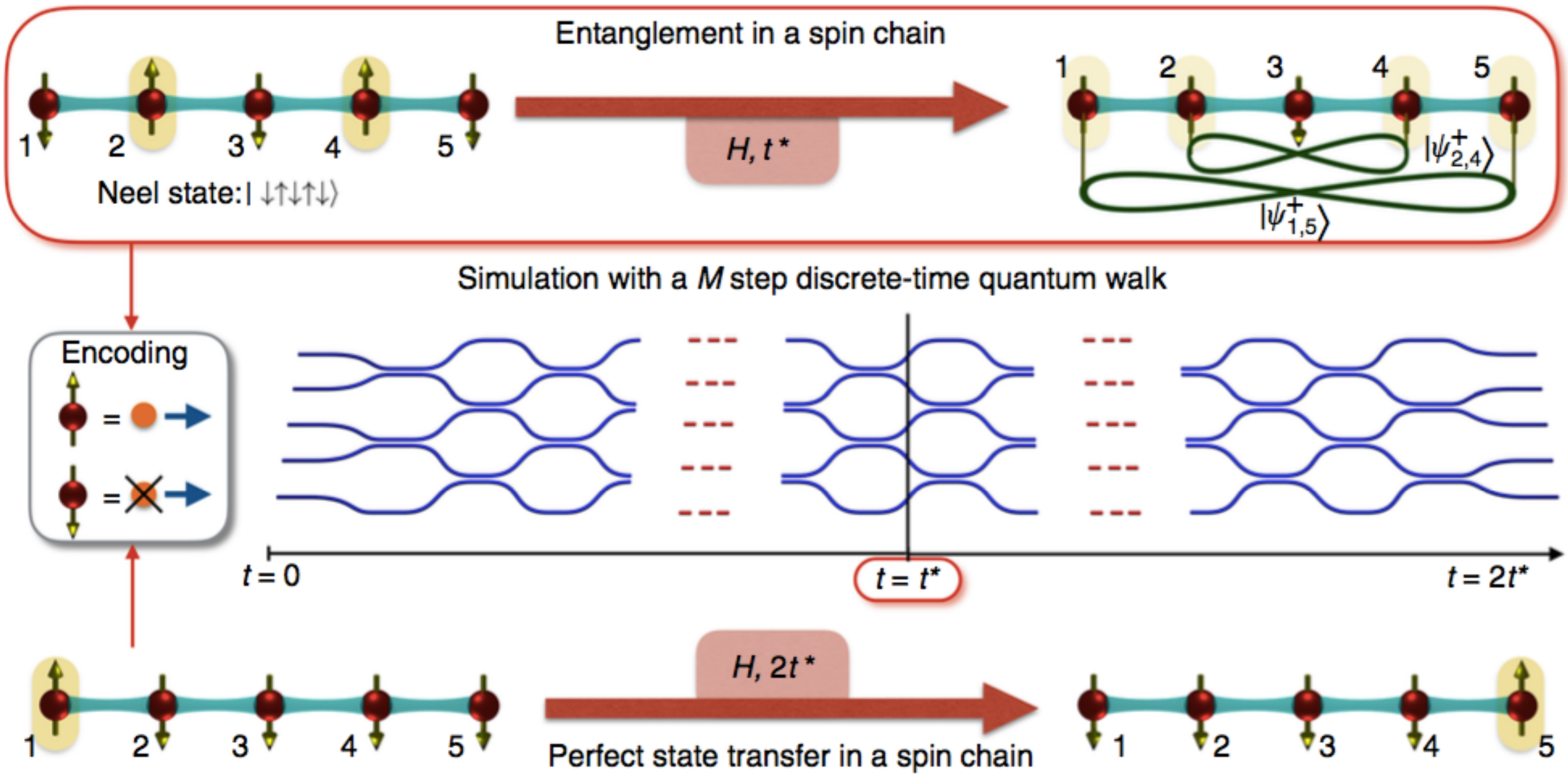
Bedoor Alkurtass,[1,2] Leonardo Banchi,[1] and Sougato Bose[1] (2018)

$$H = \sum_{i=1}^{N-1} \frac{1}{2} J_i \left(\sigma_i^X \sigma_{i+1}^X + \sigma_i^Y \sigma_{i+1}^Y \right) \qquad J_i = \sqrt{i(N-i)}$$



Photonic simulation of entanglement growth and engineering after a spin chain quench

Ioannis Pitsios[1,2], Leonardo Banchi[3], Adil S. Rab[4], Marco Bentivegna[4], Debora Caprara[4], Andrea Crespi[1,2], Nicolò Spagnolo[4], Sougato Bose[3], Paolo Mataloni[4], Roberto Osellame[1,2] & Fabio Sciarrino[4]



Works

From conformal to volume-law for the entanglement entropy in exponentially deformed critical spin 1/2 chains
G. Ramírez, J. Rodríguez-Laguna, G.S. (2015).

Entanglement over the rainbow
G. Ramírez, J. Rodríguez-Laguna, G.S. (2015).

More on the rainbow chain: entanglement, space-time geometry and thermal states
J. Rodríguez-Laguna, J. Dubail, G. Ramírez, P. Calabrese, G.S. (2016).

Entanglement hamiltonian and entanglement contour in inhomogeneous 1D critical systems
E. Tonni, J. Rodríguez-Laguna, G.S. (2018).

Unusual area-law violation in random inhomogeneous systems
V. Alba, S. Santalla, P. Ruggiero, J. Rodriguez-Laguna, P. Calabrese, G.S. (2018)

Symmetry protected phases in inhomogeneous spin chains
N. Samos Sáenz de Buruaga, S. Santalla, J. Rodriguez-Laguna, G.S. (2018)

Thanks for your attention