Non-Abelian topological Berry phases
Theory & Experiment

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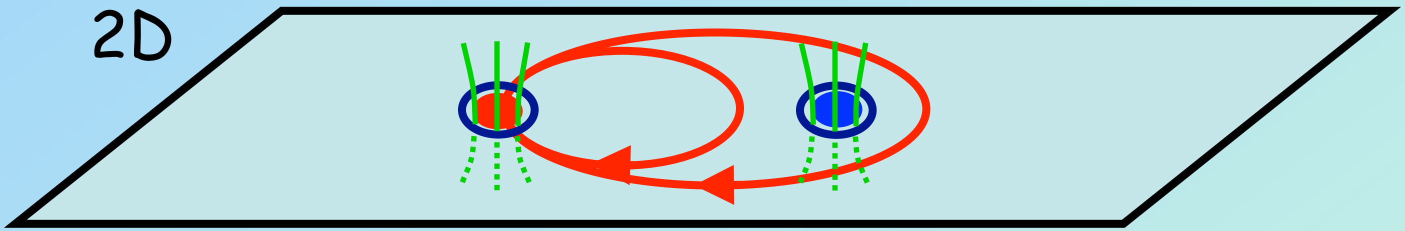
Statistics

Statistics as quantum evolution



$$|\Psi\rangle \to e^{i2\varphi}|\Psi\rangle$$

$$|\Psi\rangle \to U|\Psi\rangle$$

Anyons

Anyons: vortices with **flux** & **charge (fractional)**.

Aharonov-Bohm effect ⇔ **Geometric Phase**.

Overview

Superconducting Hamiltonians:

- Topological phase of matter

But SC are **hard to simulate** in the laboratory:

- Non-conservation of particles
- Zero energy, localisation at boundary
- **Braiding** not possible (yet)

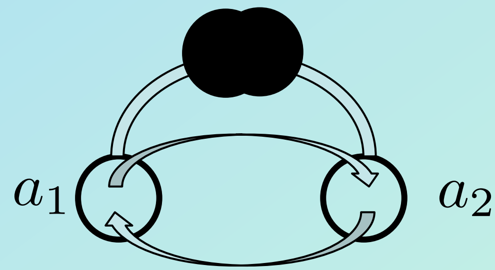
Spin-1/2 states are easy to simulate in the laboratory (photons, atoms, ions, Josephson junctions, NMR,…)

We find **spin analogs of SC** and **simulate braiding**.

Superconducting fermion chain

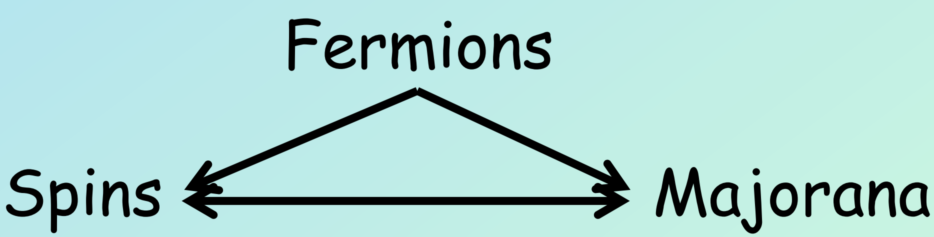
Consider two sites with **tunnelling** and **pairing** interactions

$$H_{\rm SC} = -\left(\boxed{a_1^\dagger a_2 + a_2^\dagger a_1} + \boxed{a_1^\dagger a_2^\dagger + a_2 a_1}\right)$$

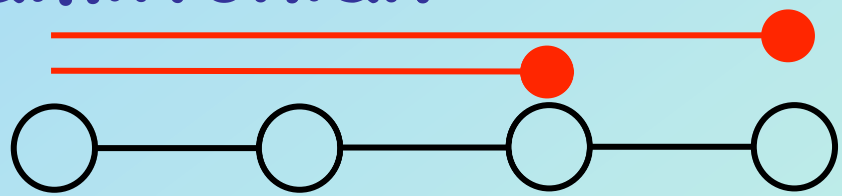


Number of fermions is not conserved
due to pairing term.
Parity of fermions is conserved.

We will treat this Hamiltonian in two ways:



The Ising Hamiltonian



For N sites we have

$$H_{\mathrm{SC}} = -\sum_{i=1}^{N-1}\left(a_i^\dagger a_{i+1} + a_i^\dagger a_{i+1}^\dagger + \mathrm{h.c.}\right) = -\sum_{i=1}^{N-1}\sigma_i^x \sigma_{i+1}^x$$

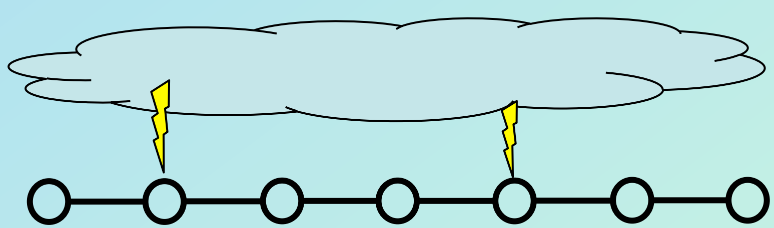
Local to local Hamiltonian by non-local JW transformation.

The ground states is still doubly degenerate

$$|++\ldots+\rangle,\ |--\ldots-\rangle$$

It takes N flips to change from one state to the other.

$$|\psi\rangle = \alpha|0\rangle_L + \beta|1\rangle_L = \alpha|++\ldots+\rangle + \beta|--\ldots-\rangle$$

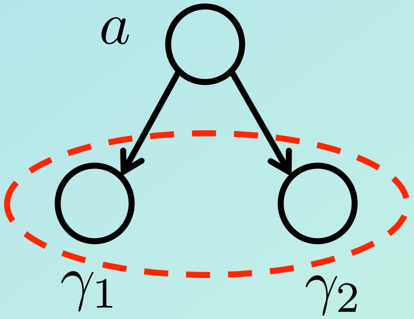


$$H_{\mathrm{error}} = B\sigma_i^x$$

will cause dephasing of the qubit state.

Majoranas from fermions

"Real" and "imaginary" decomposition gives Majoranas:



$$\gamma_1 = \frac{a + a^\dagger}{2}, \ \gamma_2 = \frac{a - a^\dagger}{2i}$$

They are fermions that **are their own anti-particles:**

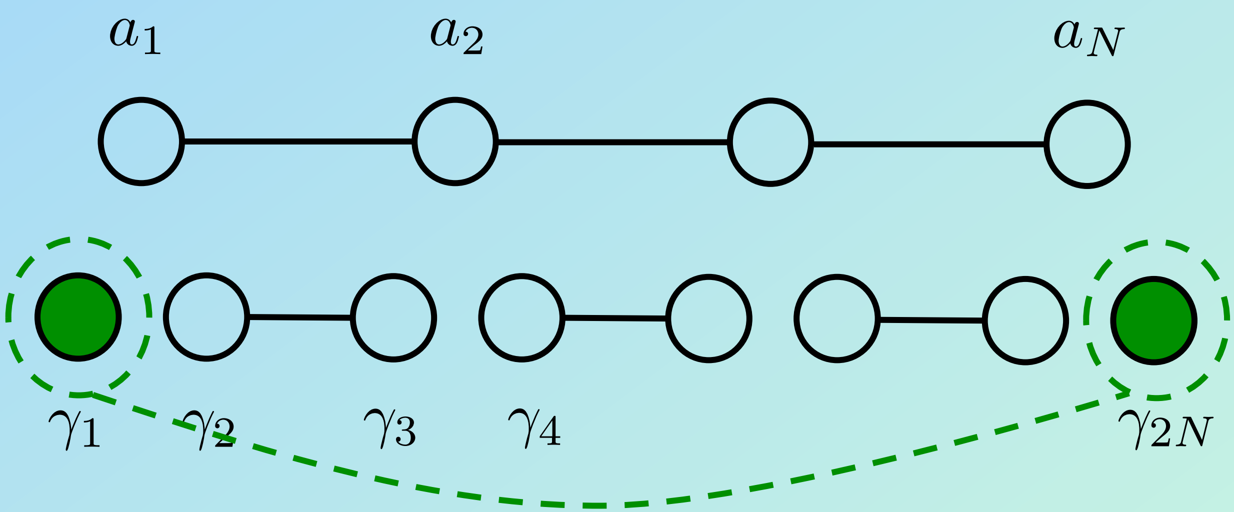
$$\gamma_j^\dagger = \gamma_j$$

Up to now **Majoranas** are just a **mathematical construction.**

The Kitaev Hamiltonian

Consider the superconducting Hamiltonian:

$$H_{\mathrm{SC}} = -\sum_{i=1}^{N-1}\left(a_i^\dagger a_{i+1} + a_i^\dagger a_{i+1}^\dagger + \mathrm{h.c.}\right) = -4i\sum_{i=1}^{N-1}\gamma_{2i}\gamma_{2i+1}$$

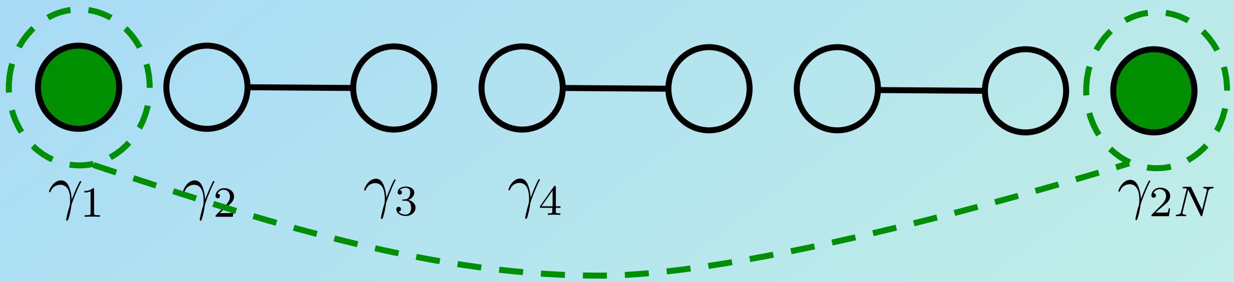


$$b = \gamma_1 + i\gamma_{2N}$$

$b^\dagger b = 0$ or 1 Degenerate states. Leave at the end-points.

The Kitaev Hamiltonian

The degenerate eigenstates is a stable qubit:

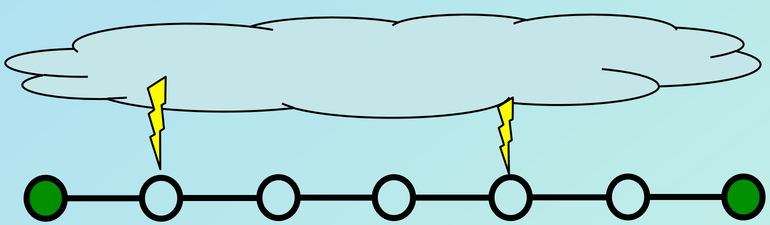


$$b = \gamma_1 + i\gamma_{2N}$$

$$b^\dagger b = 0 \text{ or } 1$$

$$|\psi\rangle = \alpha|0\rangle_L + \beta|1\rangle_L = \alpha|b^\dagger b = 0\rangle + \beta|b^\dagger b = 1\rangle$$

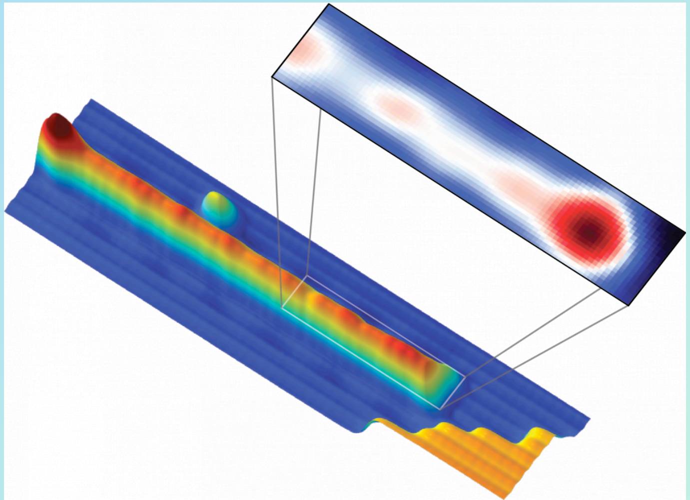
$$H_{\text{error}} = \mu a_i^\dagger a_i$$



In the presence of chemical potentials the edge modes are **exponentially** localised at the end.

QC: Manage expectations

- Tiny energy gap:
  - Temperature
- Finite extend:
  - Perturbations
  - Position inaccuracy
- Adiabatic transport
- State manipulations:
  - Preparation
  - Measurement



What are Majoranas?

Kitaev vs Ising

The JW trans between spins and Majoranas is non-local.

$$a_i = \Big(\prod_{j<i} \sigma_j^z\Big)\sigma_i^+, \quad a_i^\dagger = \Big(\prod_{j<i} \sigma_j^z\Big)\sigma_i^-,$$

Both Hamiltonians are local.

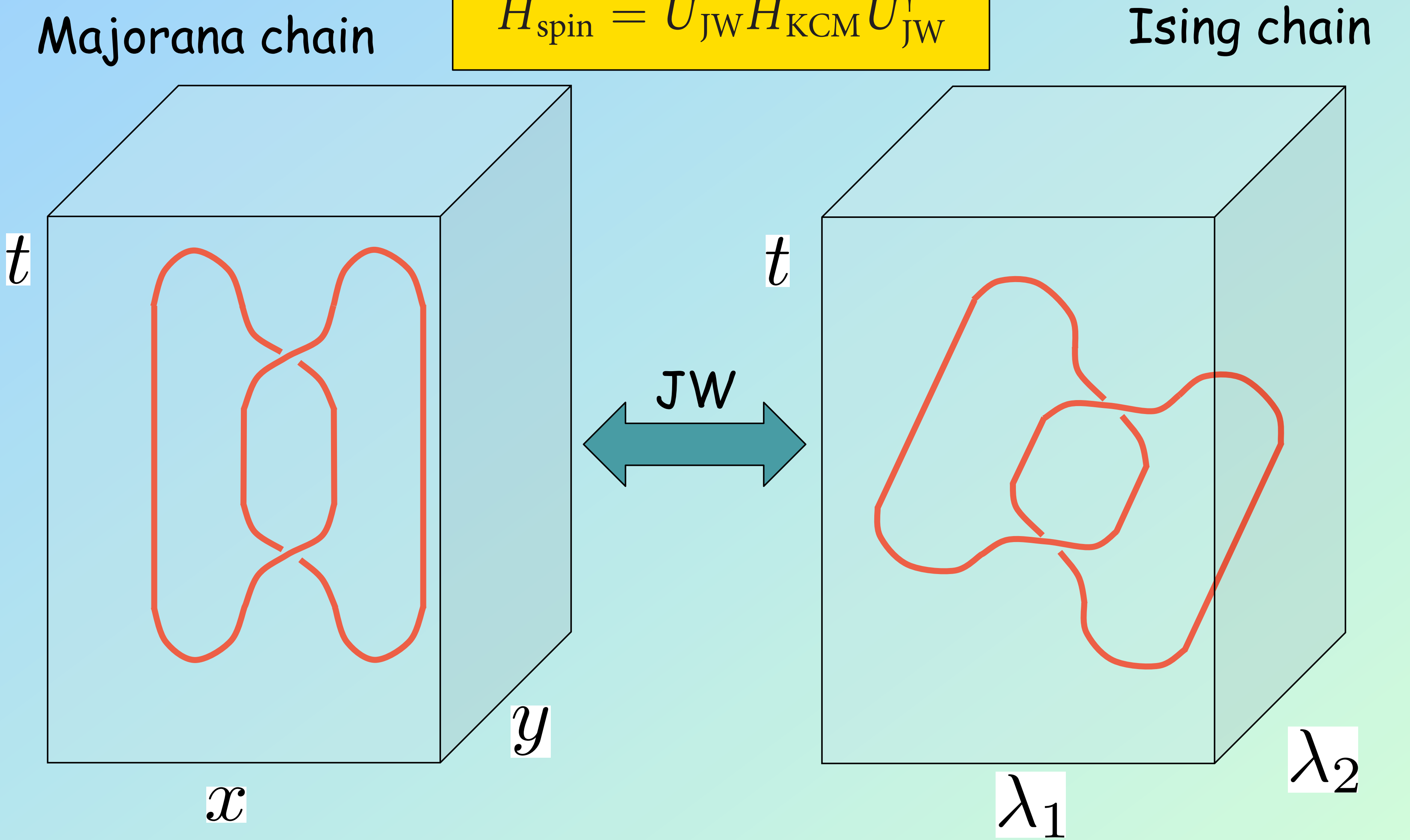
Spectrum is the same: unitary time evolution operators are the same.

Eigenstates can have different properties:

Local Majorana quasiparticles that do not overlap **map** to completely **dispersed states** of spin with complete overlap.

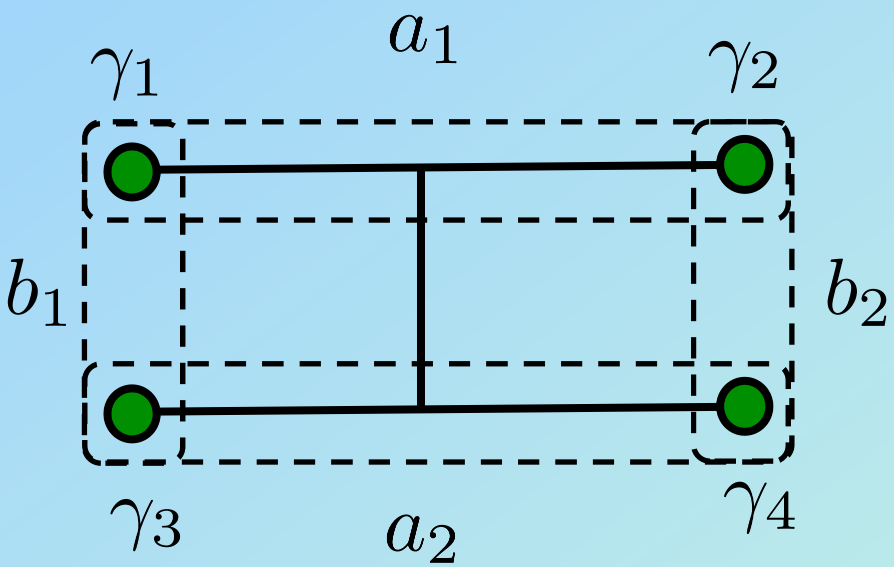
Unitary mapping

$$H_{\rm spin} = U_{\rm JW} H_{\rm KCM} U_{\rm JW}^{\dagger}$$



Majoranas as anyons

Fusion and braiding of Majorana fermions



Fusion

$$\{a_1, b_1\} = \frac{1}{2}$$

One can show:

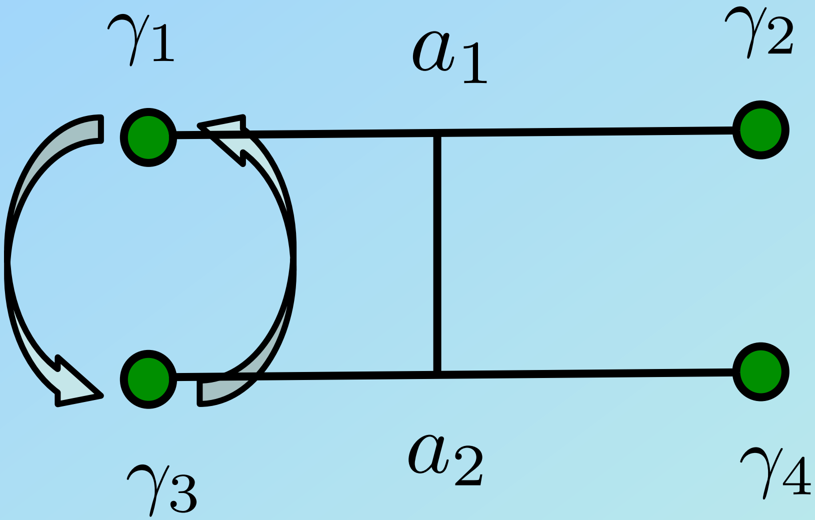
$$\Rightarrow |11\rangle_b = \sqrt{2} b_1^\dagger b_1 |00\rangle_a = \frac{1}{\sqrt{2}}(|00\rangle_a + |11\rangle_a)$$

Similarly:
$$|00\rangle_b = -\frac{1}{\sqrt{2}}(|00\rangle_a - |11\rangle_a)$$

Bases related by
$$F = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

Majoranas as anyons

Adiabatic braiding of Majorana fermions



Braiding

$$\mathcal{U} = a\mathbf{1} + b\gamma_1 + c\gamma_3 + d\gamma_1\gamma_3$$

$$\mathcal{U}^\dagger\mathcal{U} = 1,\ \ \mathcal{U}\gamma_1\mathcal{U}^\dagger \propto \gamma_3,\ \ \mathcal{U}\gamma_3\mathcal{U}^\dagger \propto \gamma_1$$

This gives two possible solutions

$\mathcal{U}^2 = e^{i\pi}$ ⟵ *Nature Commun.*

$\mathcal{U}^2 = e^{i\pi/4}(a_1a_2 + a_1a_2^\dagger + a_1^\dagger a_2 + a_1^\dagger a_2^\dagger)$ ⟵ *Science Advances*

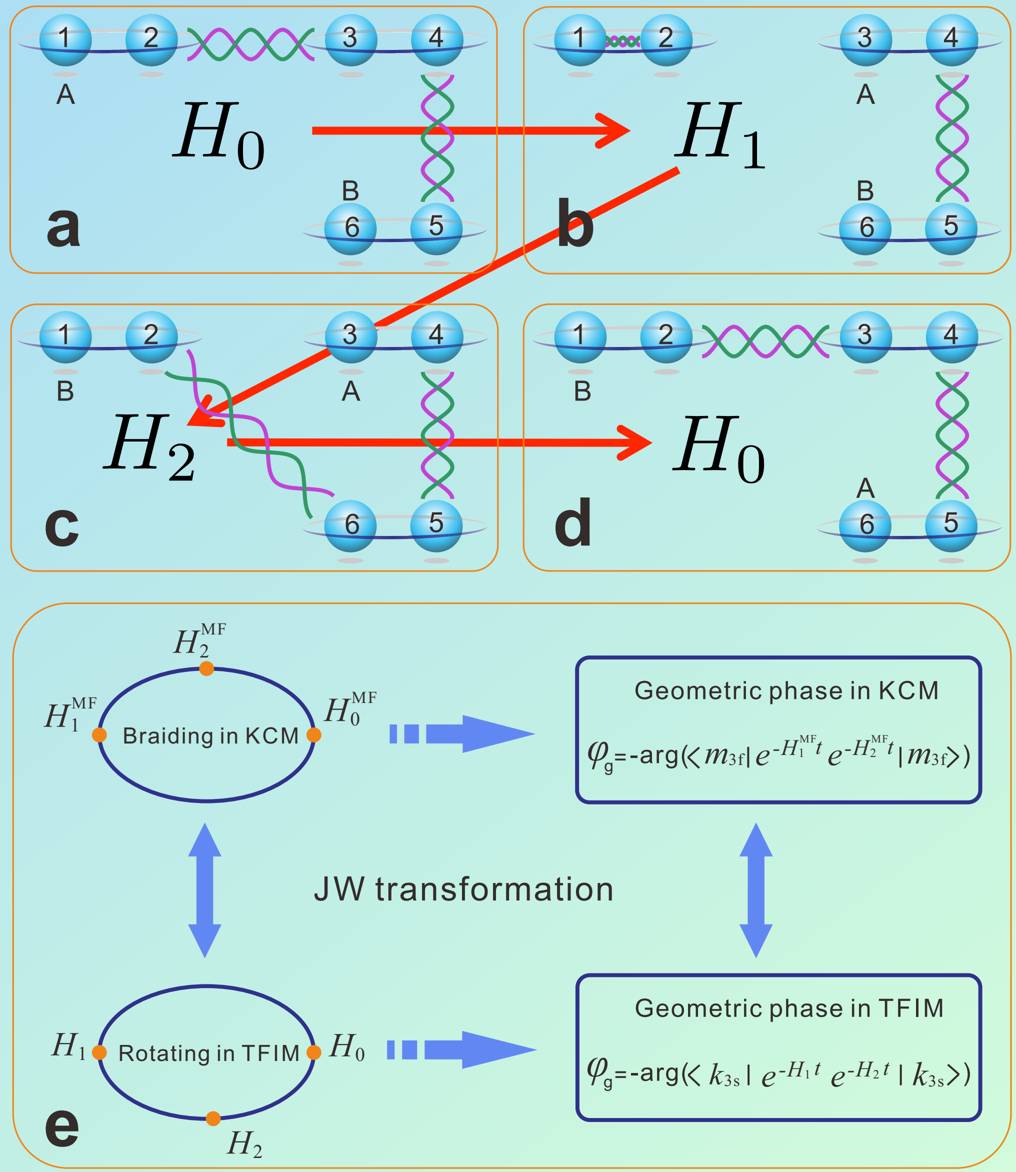
Photonic quantum simulator

Produce geometric phases:

Adiabatically change Hamiltonians -> Majoranas A and B are **exchanged**.

Translate Majoranas to spins: JW transf.

Do spin adiabatic evolution.



Photonic quantum simulator

Adiabatic dissipative evolution:

$$\varphi_{\mathrm{g}} = -\arg(\langle m_{Lf}|P_1 P_2 \cdots P_n|m_{Lf}\rangle)$$

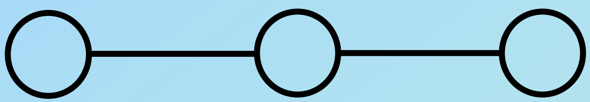
P_j project the state to the eigenstate of H_j

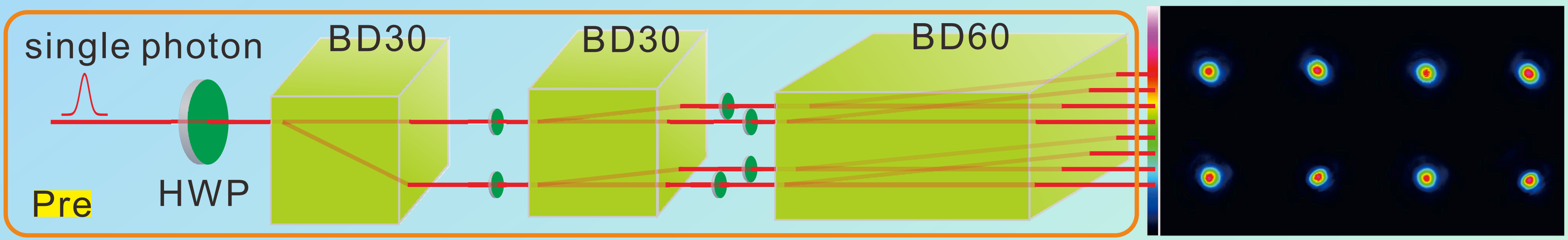
Can take $P_j \approx e^{-H_j t}$ for large t.

"Imaginary-time evolution"

Photonic quantum simulator

Three spins: $2^3 = 8$ states: $|\Psi\rangle = \sum_{j=1}^{8} c_j |j\rangle$





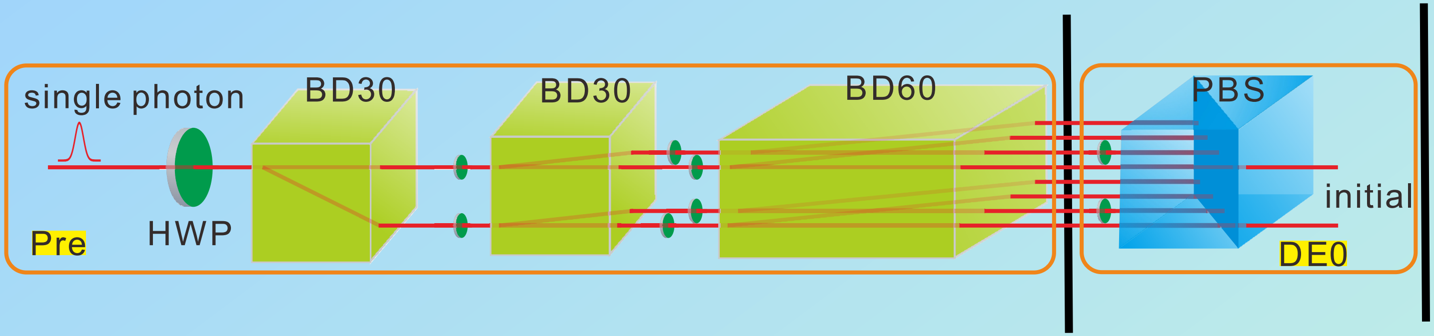
Pre: State preparation
HWP: Half Wave Plate
BD: Beam Displacer 30 or 60 mm

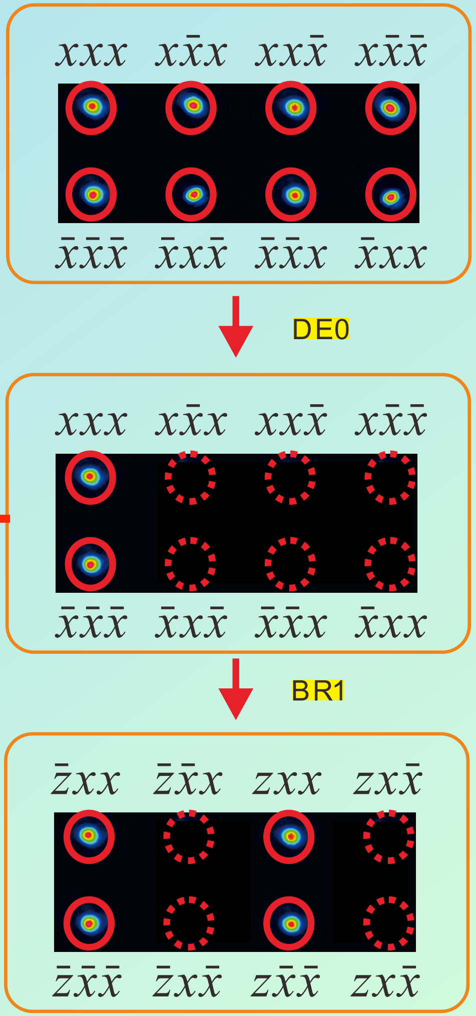
Use photonic mode for spin state
Use polarisation to couple to the environment

Photonic quantum simulator

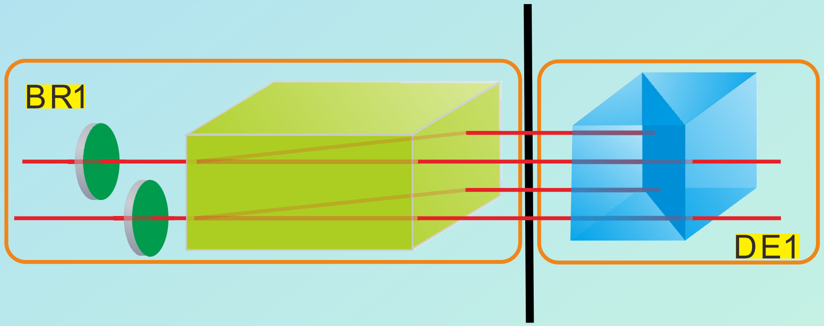
$$|\Psi\rangle = \sum_{j=1}^{8} c_j |j\rangle$$

Produce geometric phases:



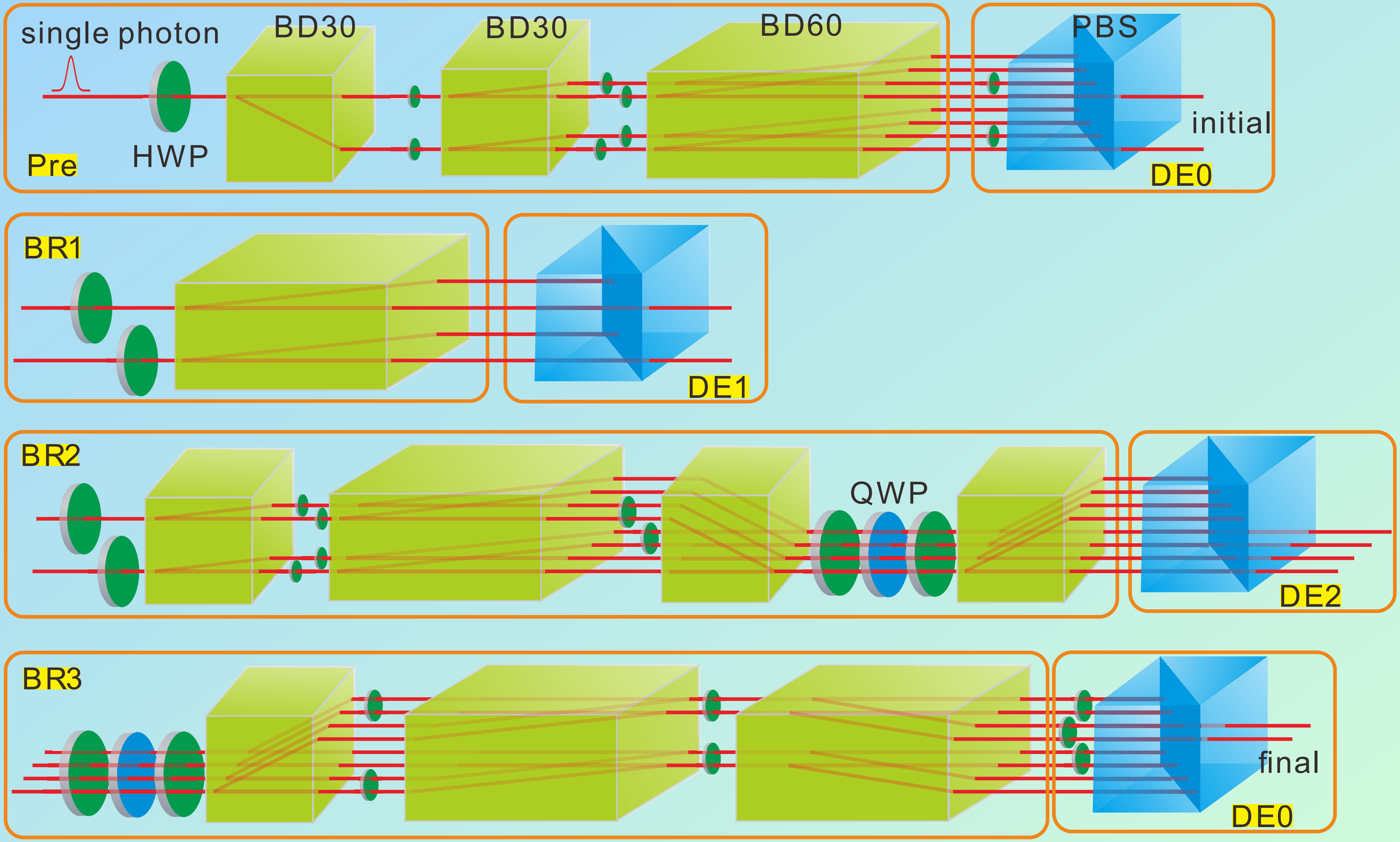


$$|\Psi\rangle = \alpha|xxx\rangle + \beta|\bar{x}\bar{x}\bar{x}\rangle$$



Photonic quantum simulator

Produce geometric phases:



Abelian Statistics

Experimentally produced geometric phases:

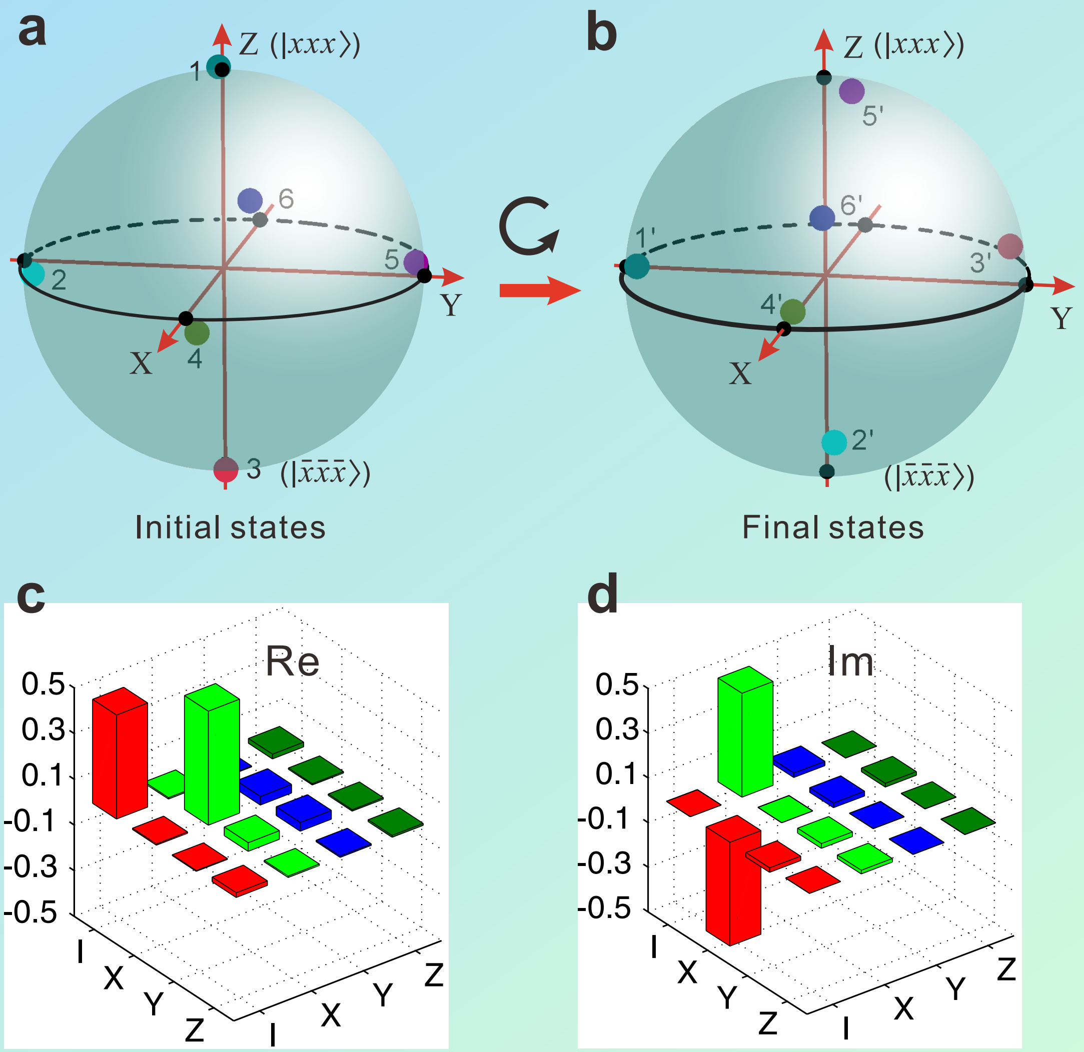
$$\varphi_g \approx \pi/2$$

Fidelity:

$$94.13 \pm 0.04\%$$

(errors deduced from Poissonian photon counting noise)

$$|\Psi\rangle = \alpha|xxx\rangle + \beta|\bar{x}\bar{x}\bar{x}\rangle$$

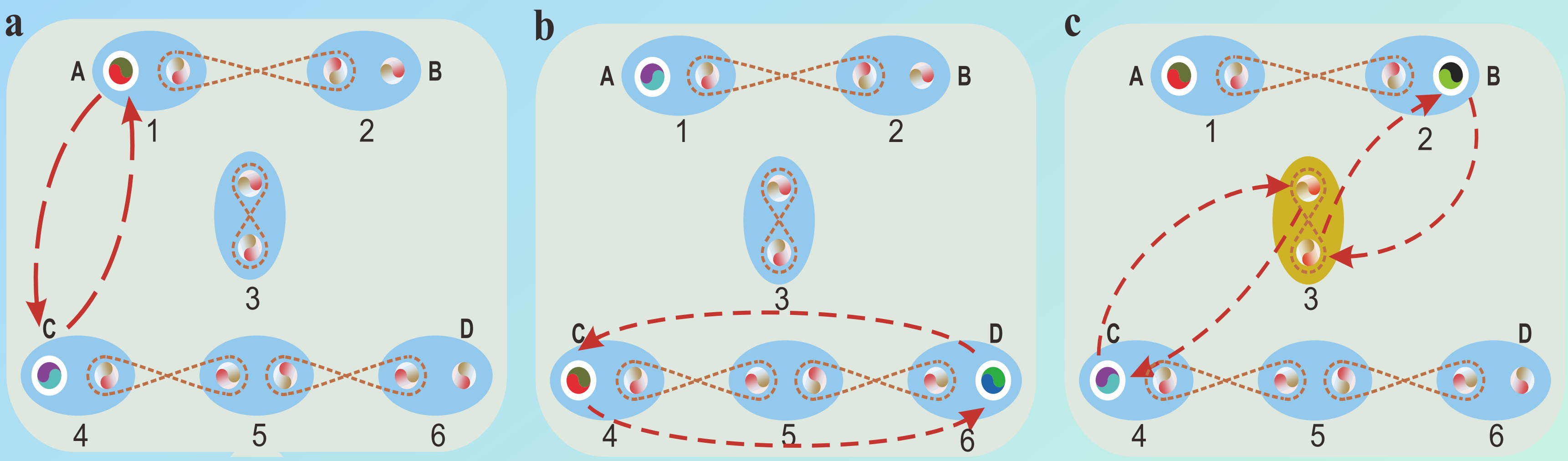


Tomography

[*Nature Commun.* **7**, 13194 (2016)]

Non-Abelian Statistics

Exchange A and C Majorana fermions

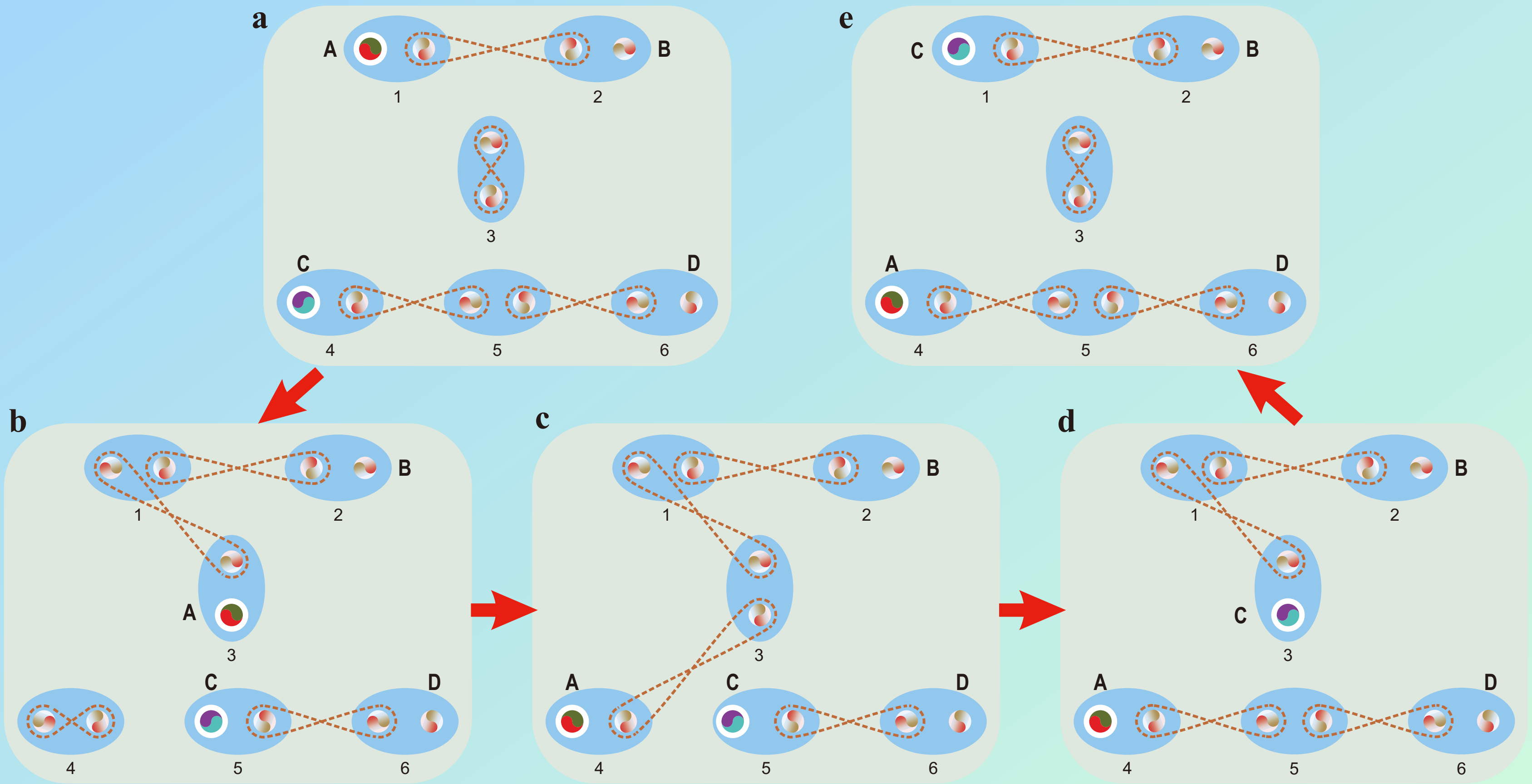


$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad R = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Non-Abelian statistics emerges as $HR \neq RH$

Non-Abelian Statistics

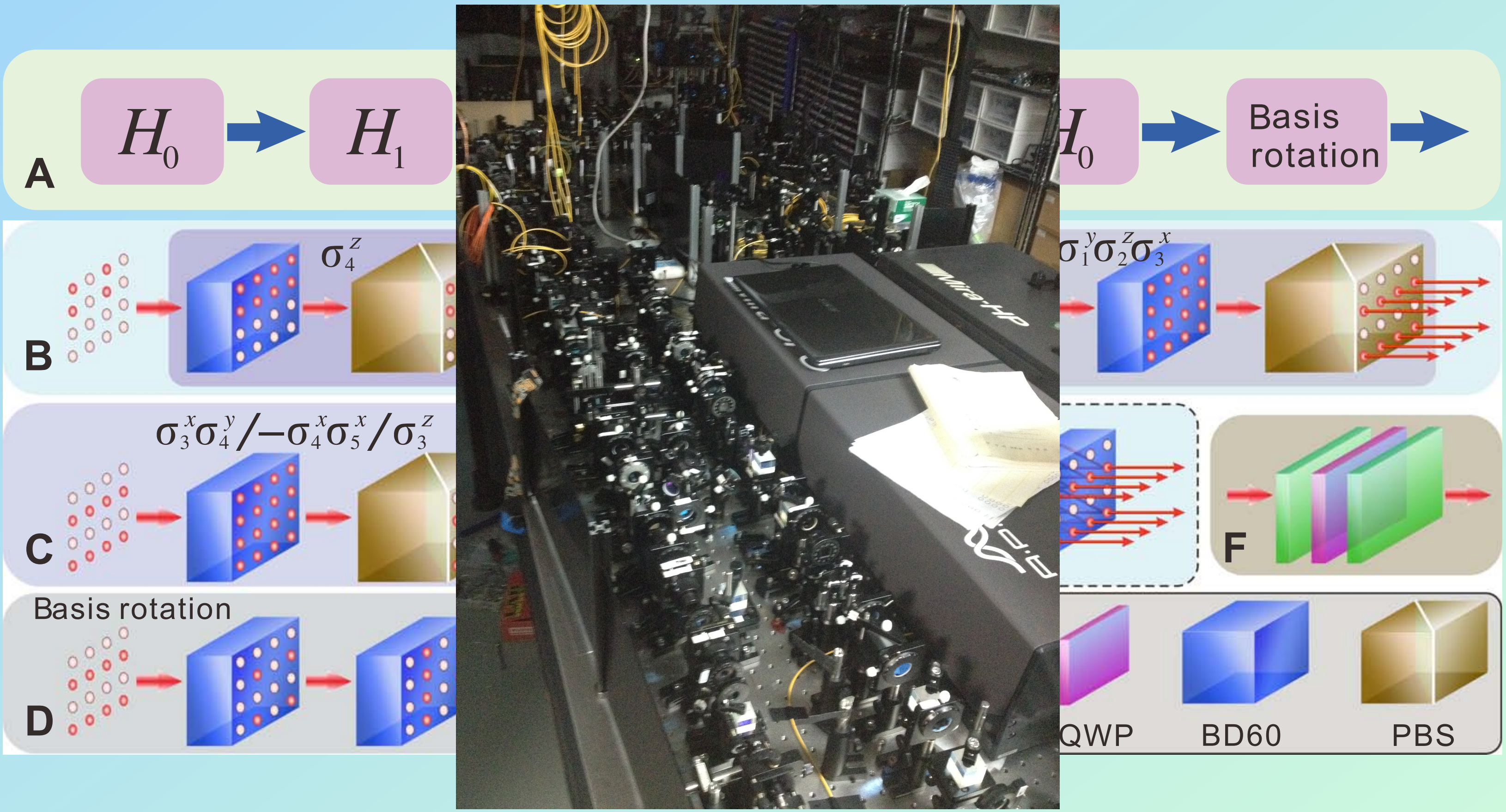
Exchange A and C Majorana fermions



Non-Abelian statistics emerges. $2^6 = 64$ states!

Non-Abelian Statistics

To implement it we use the following processes:



Non-Abelian Statistics

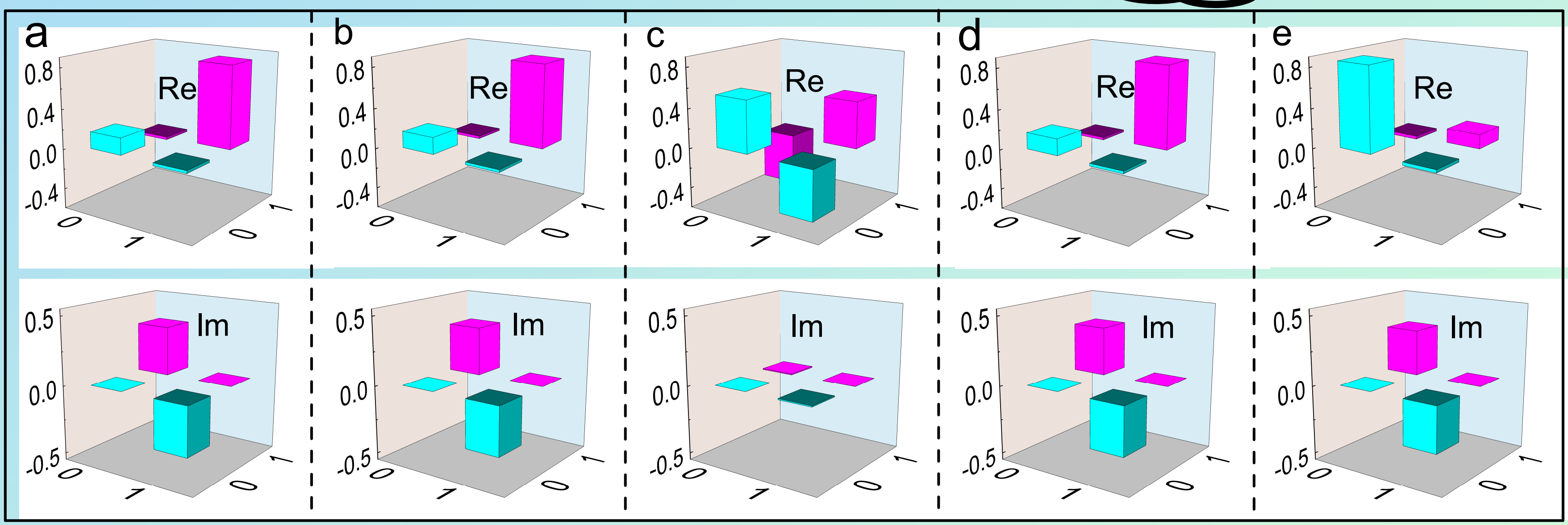
Fidelities:

Most gates F>97%

Total Fidelity >91%

Errors:

No errors Errors on 4 Errors on 3 4&5 E &4 Errors



Summary

- **Spins** are favourable for **quantum simulations** with photons, atoms, ions, Josephson junctions, NMR,…

- **Topological phases** such as SC fermionic systems exciting:
  - encoding protected quantum information
  - demonstrating new physics (anyons)

- Here we simulated their **braiding properties, construct one-qubit gates and demonstrate fault-tolerance.**

- Outlook: Quantum algorithms are similar to evaluating **Jones polynomials** ->

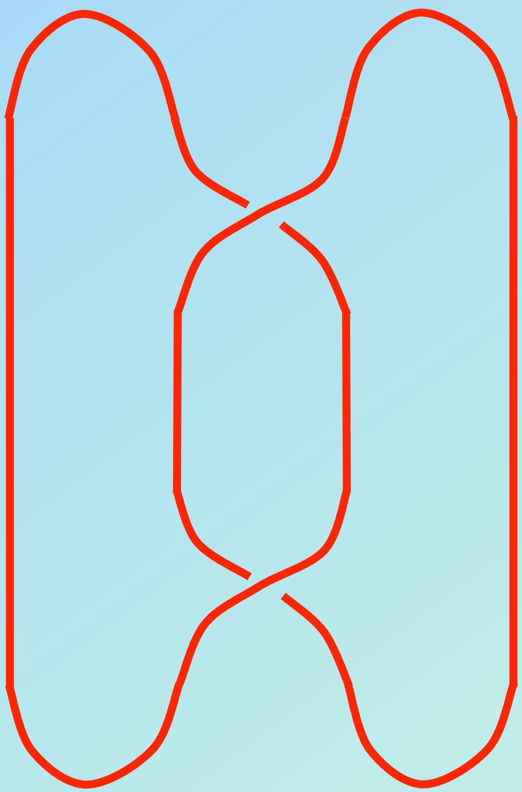
  Quantum Machine Learning…

Outlook

Deutsch-Jozsa Algorithm:

$$|0\rangle = H R^2 H |0\rangle$$

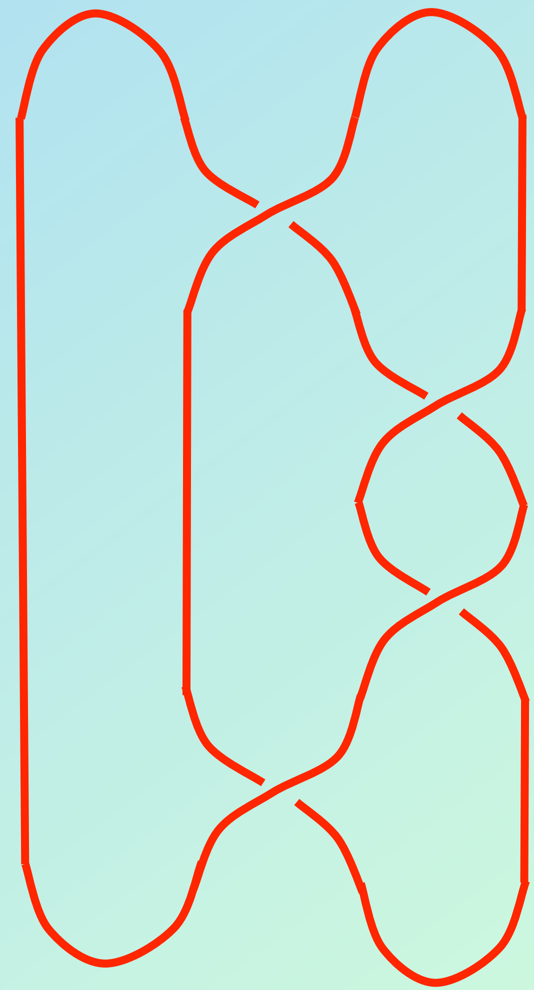
$$|1\rangle = H \mathbb{1} H |0\rangle$$



H

$\mathbb{1}$

H



H

R

R

H