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Phase diagram of the Ruby model with infinite-PEPS

Román Orús

Donostia International Physics Center (DIPC)

January 24th 2019

S. S. Jahromi, RO, M. Kargarian, A. Langari, PRB 97, 115161 (2018)

S. S Jahromi

DFG

Motivation: TO with 2-body interactions

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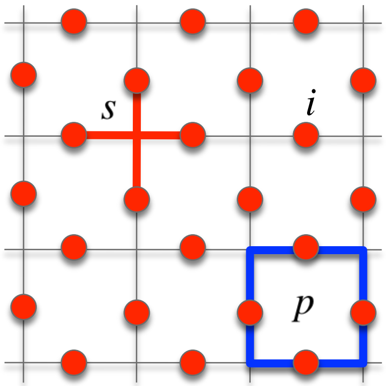
Toric Code

$$H = -J\sum_s A_s - J\sum_p B_p$$

Z_2



$$A_s = \prod_{i\in s}\sigma_i^x$$

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A. Kitaev, Annals of Physics 303, 2-30 (2003)

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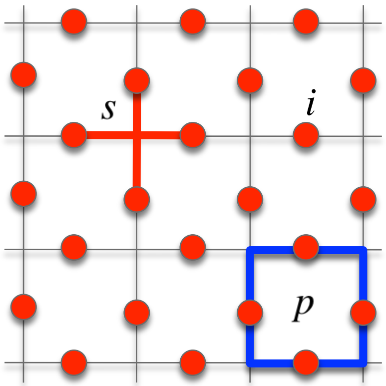

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KH model

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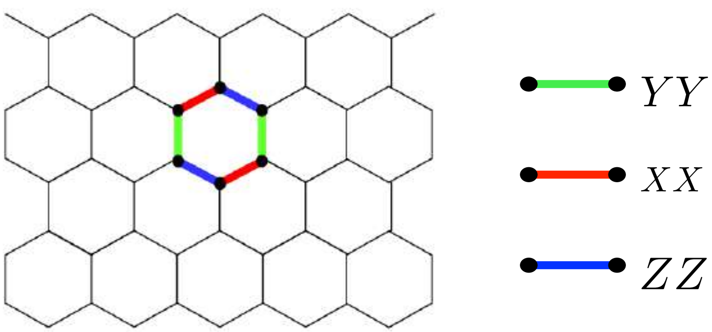

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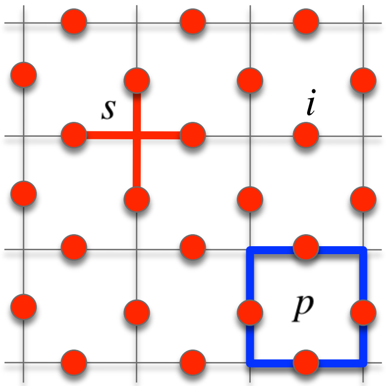

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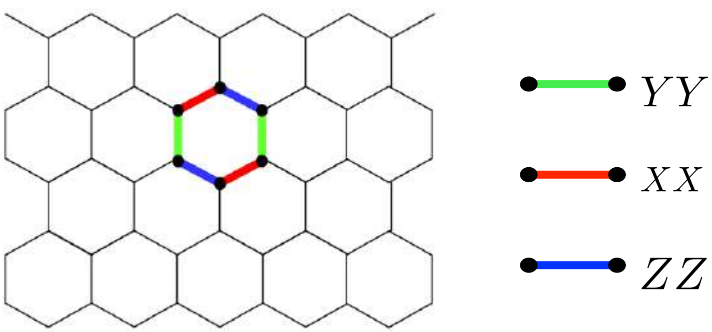
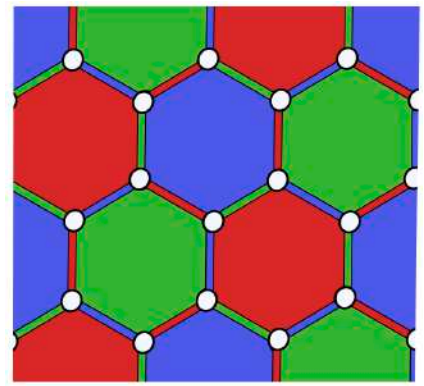


A. Kitaev, Annals of Physics 321, 2-111 (2006)

Color Code

$$H = -J\sum_p (X_p + Z_p)$$

Z_2xZ_2



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H. Bombin, M. A. Martin-Delgado, PRL 97 180501 (2006)

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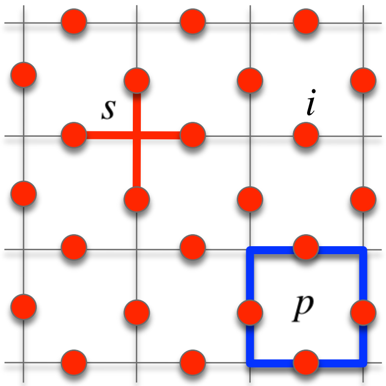

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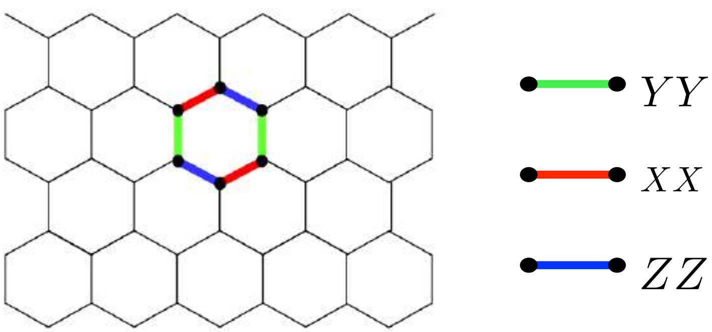

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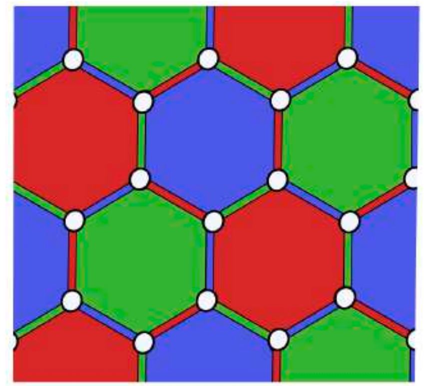
$Z_2 x Z_2$



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H. Bombin, M. A. Martin-Delgado, PRL 97 180501 (2006)

?

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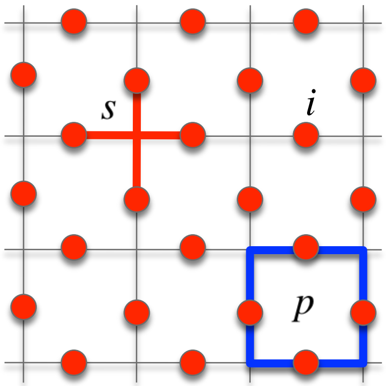

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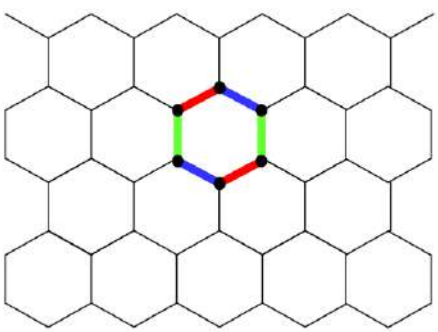

YY

XX

ZZ

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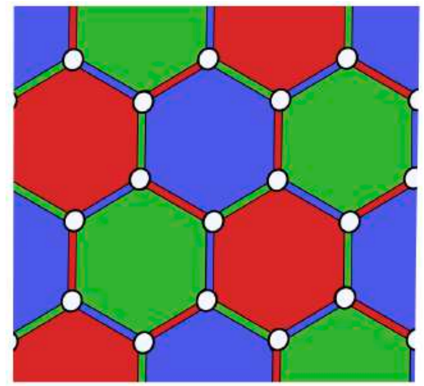
Z_2xZ_2



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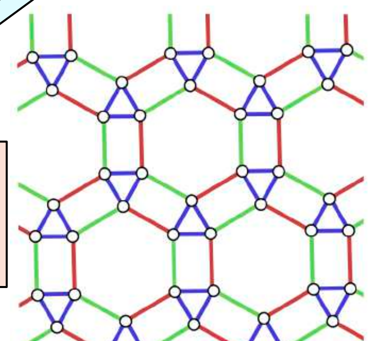
H. Bombin, M. A. Martin-Delgado, PRL 97 180501 (2006)

↑ $J_z \gg J_x, J_y$

Ruby model

$$H = -\sum_{z} J_\alpha \sum_{\alpha-\text{links}} \sigma_i^\alpha\sigma_j^\alpha$$

Not exactly solvable!



YY

XX

ZZ

S. S. Jahromi et al, PRB 94, 125145 (2016)

Goal

- Study the phase diagram of the ruby model with infinite-PEPS (tensor networks)

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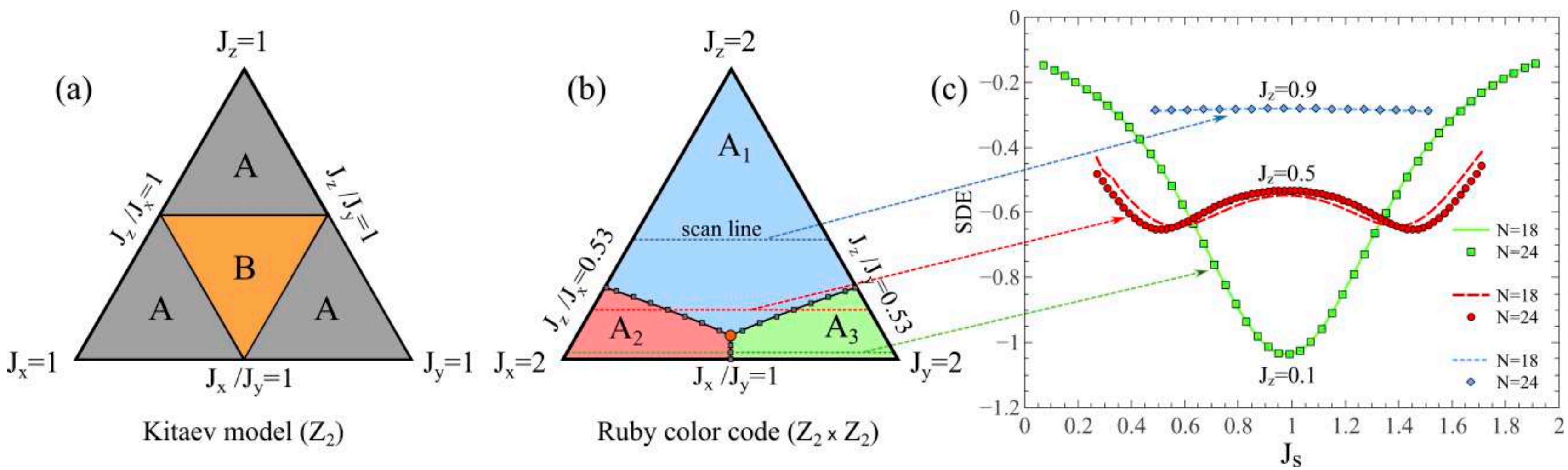
- Previous studies by exact diagonalization (18 and 24 sites):

  S. S. Jahromi et al, PRB 94, 125145 (2016)



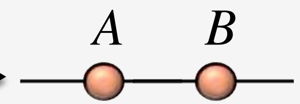
- A_1 phase: gapped, Z_2 x Z_2 TO.

- A_2 & A_3 phases: gapless, gapped by pert., conjectured Ising anyons.

- Nature of phase transitions? Accurate position?

Tensor Networks

e.g. RO, Annals of Physics 349 (2014) 117–158

$A \cdot B \rightarrow$ 

$$|\Psi\rangle = \sum_{i's} \Psi_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

p-level systems

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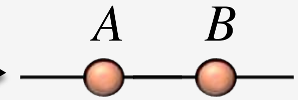
$A \cdot B \rightarrow$ 

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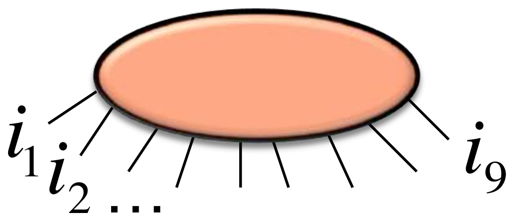
p-level systems

$$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$$



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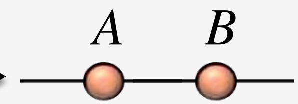

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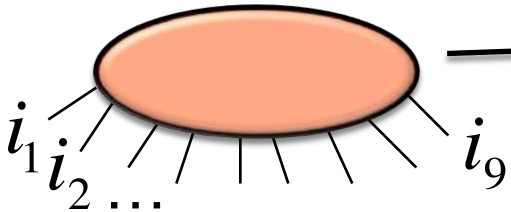
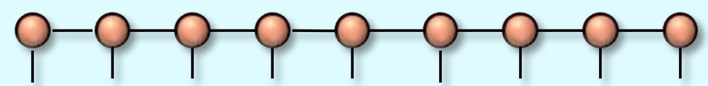


1d

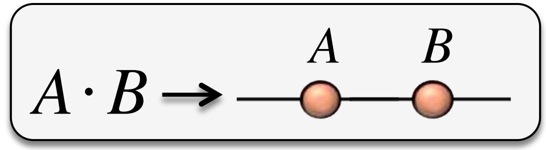
Matrix Product States (MPS)



DMRG, PWFRG, TEBD…

Tensor Networks

e.g. RO, Annals of Physics 349 (2014) 117–158



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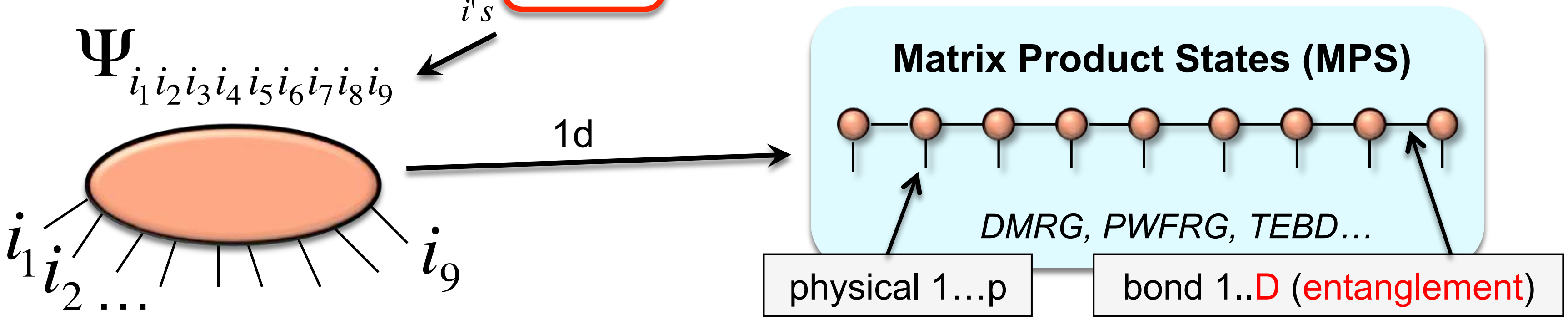

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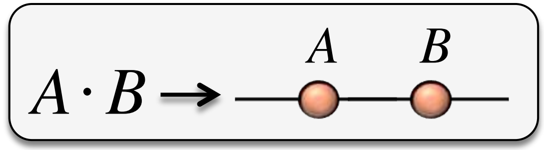

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p-level systems

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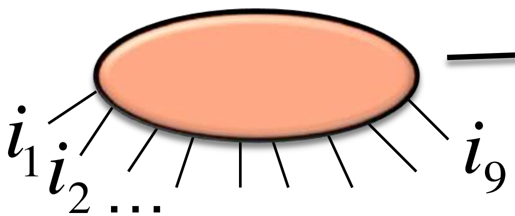

1d

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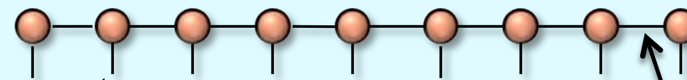


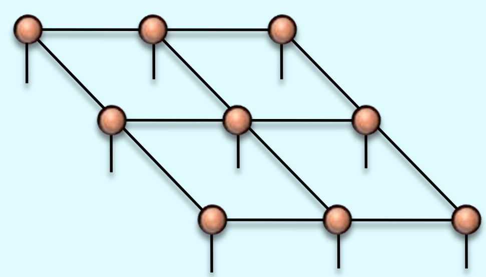
DMRG, PWFRG, TEBD…

physical 1…p

bond 1..D (entanglement)

2d, 3d...

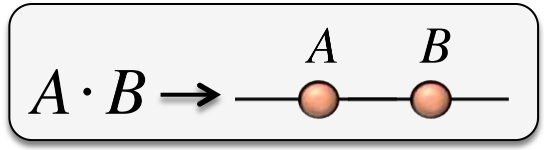
Projected Entangled Pair States (PEPS), Tensor Product States (TPS)



Tensor Product Variational Approach, PEPS & iPEPS algorithms, Tensor-Entanglement Renormalization, TRG/SRG/HOTRG/HOSRG…

Tensor Networks

e.g. RO, *Annals of Physics 349 (2014) 117–158*



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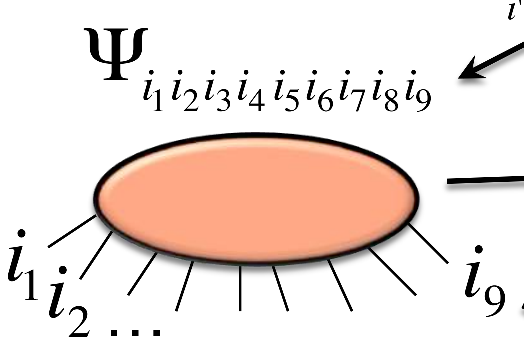
p-level systems

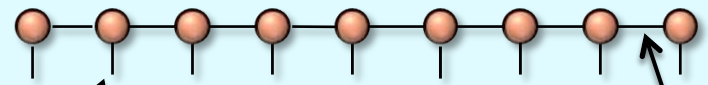


1d

2d, 3d...

Scale-invariant

Matrix Product States (MPS)

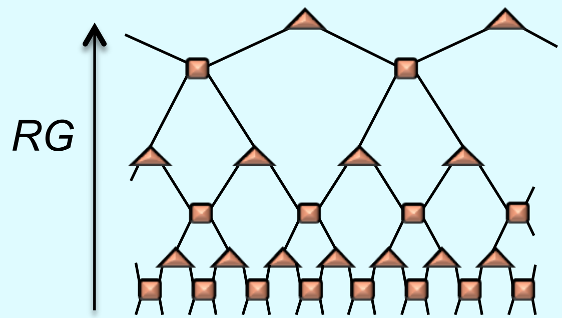


DMRG, PWFRG, TEBD…

physical 1…p

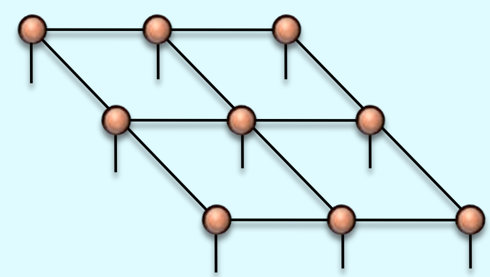
bond 1..D (entanglement)

Multiscale Entanglement Renormalization Ansatz (MERA)



AdS/CFT, Entanglement Renormalization

Projected Entangled Pair States (PEPS), Tensor Product States (TPS)



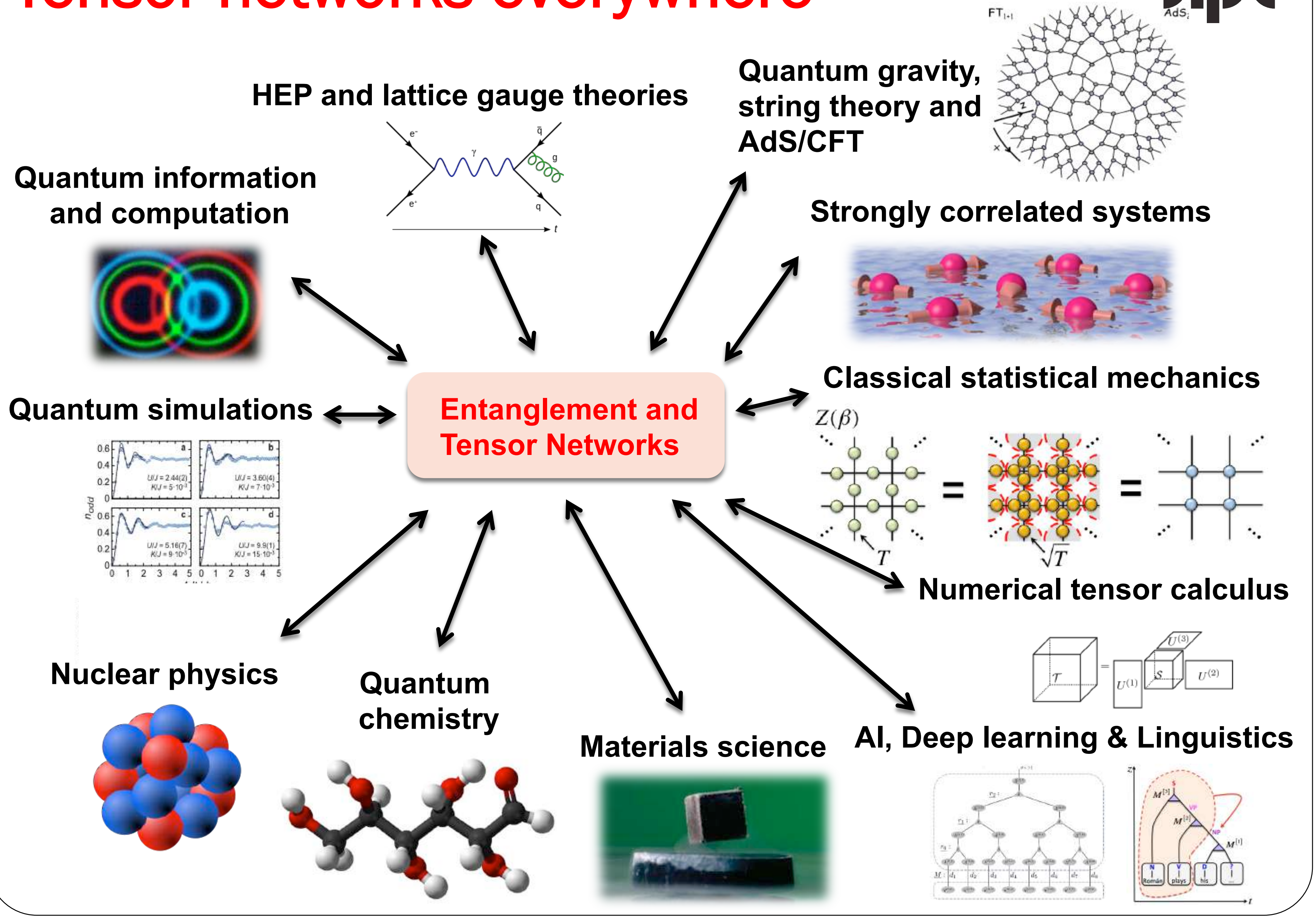
Tensor Product Variational Approach, PEPS & iPEPS algorithms, Tensor-Entanglement Renormalization, TRG/SRG/HOTRG/HOSRG…

Efficient O(poly(N)), satisfy area-law, low-energy eigenstates of local Hamiltonians

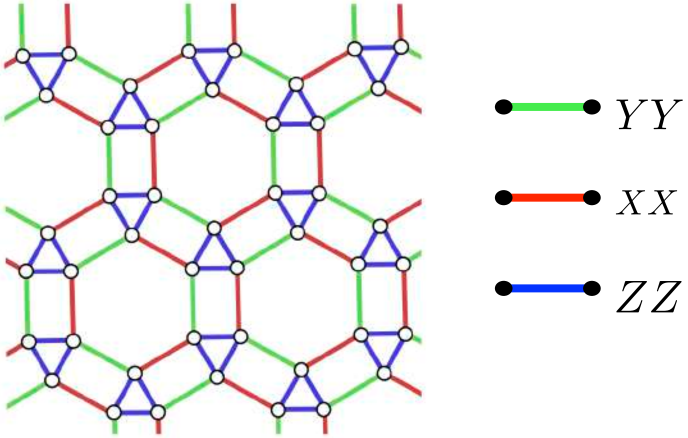
Tensor networks everywhere

Entanglement and Tensor Networks

Tensor networks everywhere



iPEPS for the ruby model (1)



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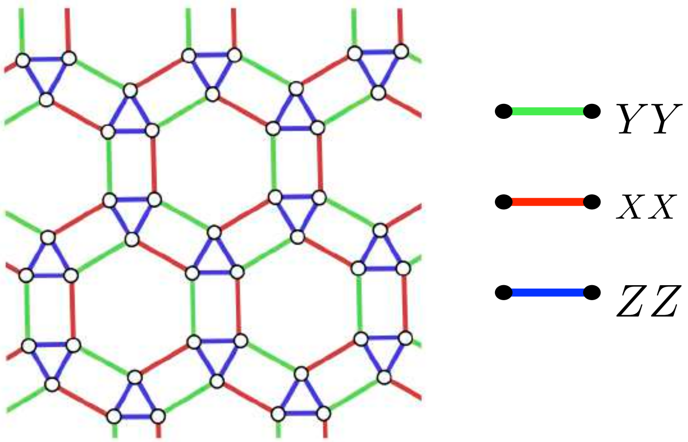

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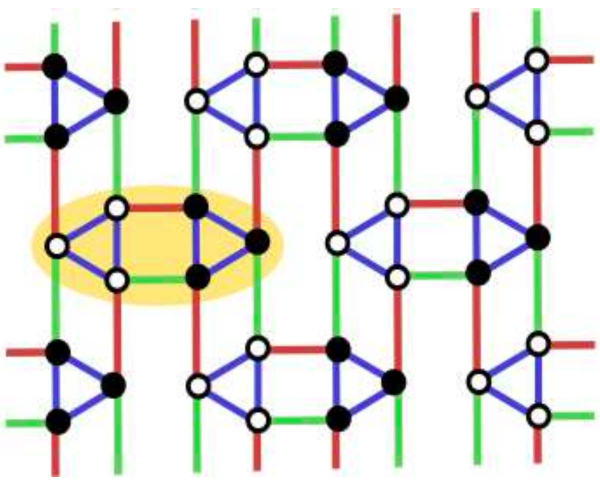
stretch



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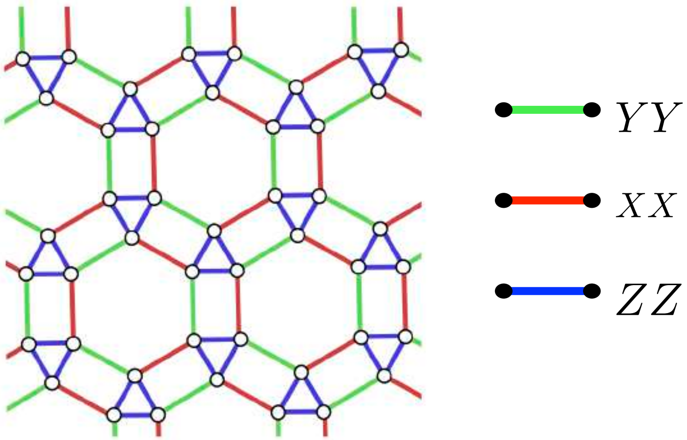

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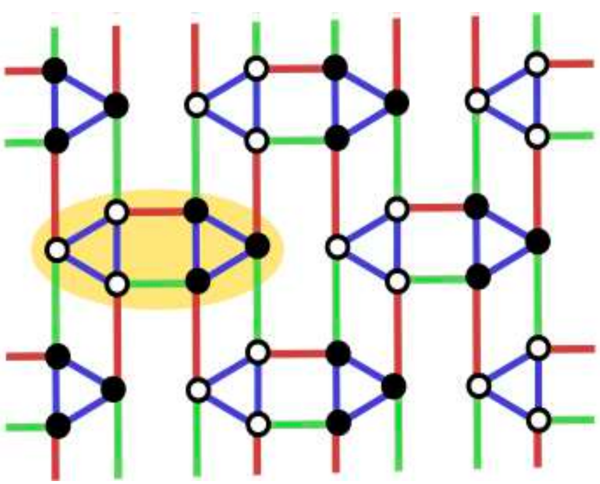
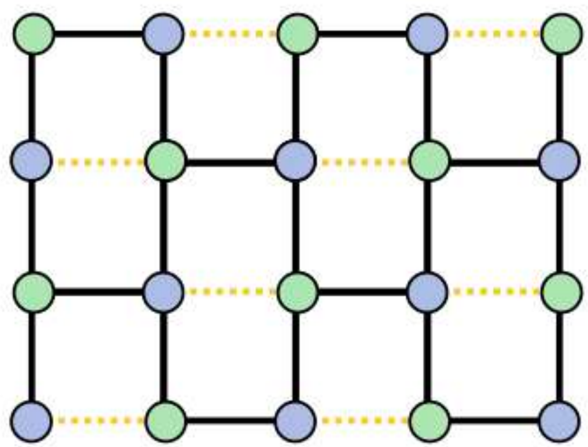
stretch

coarse-grain

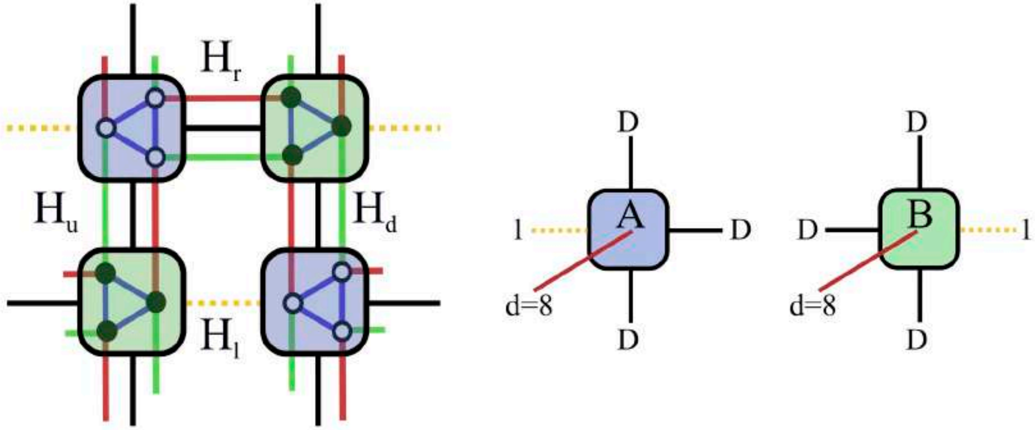




Coarse-graining triangles leads to brickwall lattice

Study with iPEPS

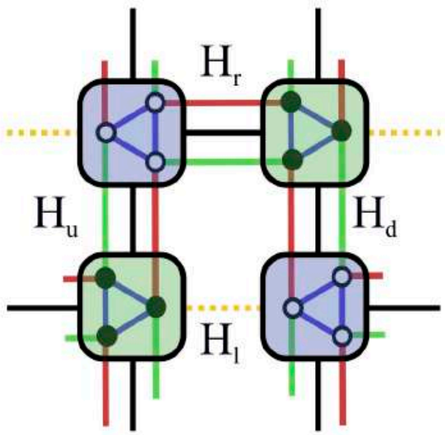
iPEPS for the ruby model (2)

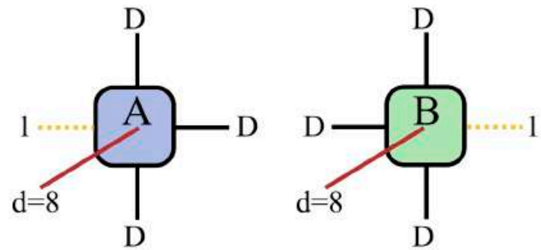


3-site coarse-graining

2 x 1 unit cell

iPEPS for the ruby model (2)





3-site coarse-graining

2 x 1 unit cell

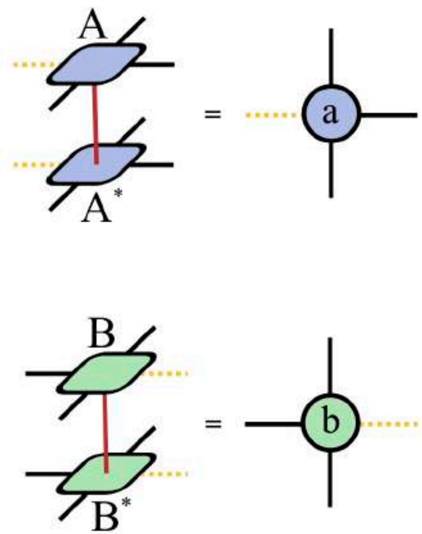
Effective environments computed with Corner Transfer Matrices (CTM)

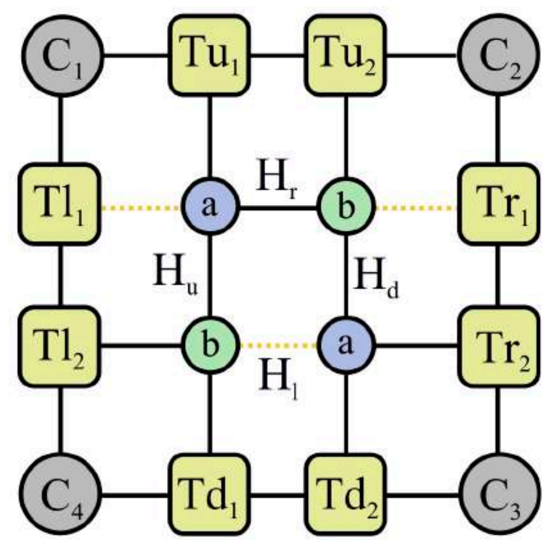
RO, G. Vidal, PRB 80 094403 (2009)

Full update + gauge fixing

H. N. Phien et al, PRB 92, 035142 (2015)

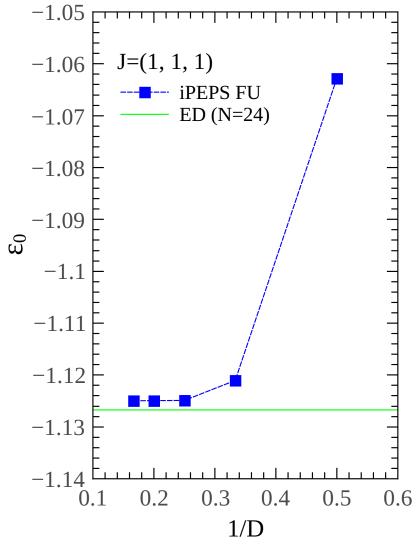
$D = 12, \chi = 100, d = 8$



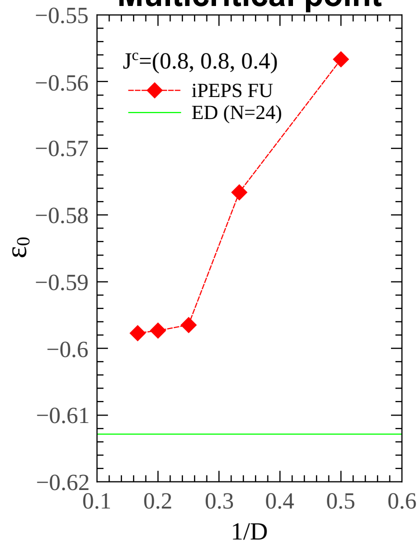


Energies and phase diagram

Ground state energy



Multicritical point



Comparison to exact diag. for 24 sites.

Good convergence. Multicritical point challenging.

3 phases separated by 2nd order QPTs.

QPTs meet at a multicritical point.

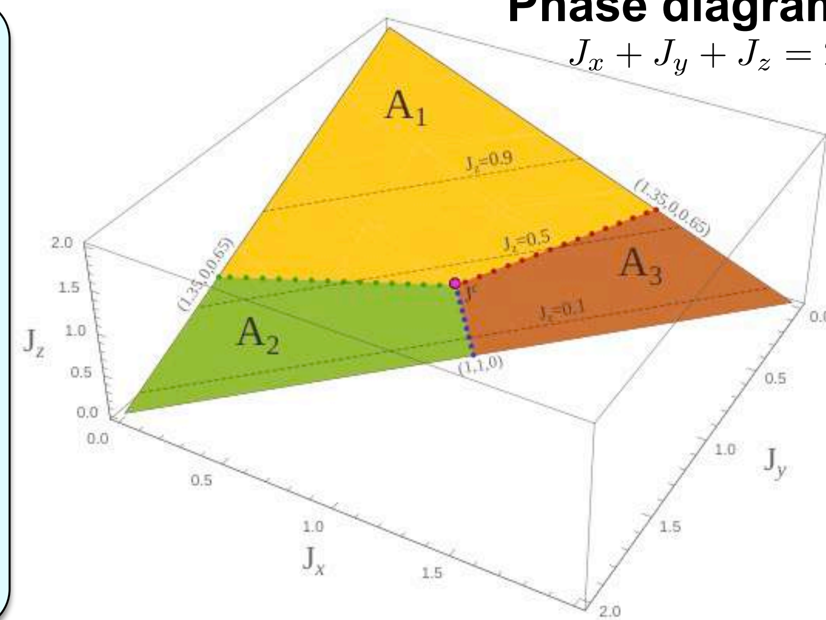
A_1 phase: gapped, Z_2 x Z_2 TO.

A_2 & A_3 phases: gapless. Gapped by a perturbation, colored Ising anyons.

S. S. Jahromi, M. Kargarian, S. F. Masoudi, A. Langari, PRB 94, 125145 (2016)

Phase diagram

$J_x + J_y + J_z = 2$



Fidelity from CTMs (1)

Ground state fidelity per site

H.-Q- Zhou, RO, G. Vidal, PRL 100, 080601 (2008)

$$F(\lambda_1, \lambda_2) = |\langle \Psi(\lambda_2) | \Psi(\lambda_1) \rangle| \sim d(\lambda_1, \lambda_2)^N$$

$$\ln d(\lambda_1, \lambda_2) \equiv \lim_{N \to \infty} \frac{\ln F(\lambda_1, \lambda_2)}{N}$$

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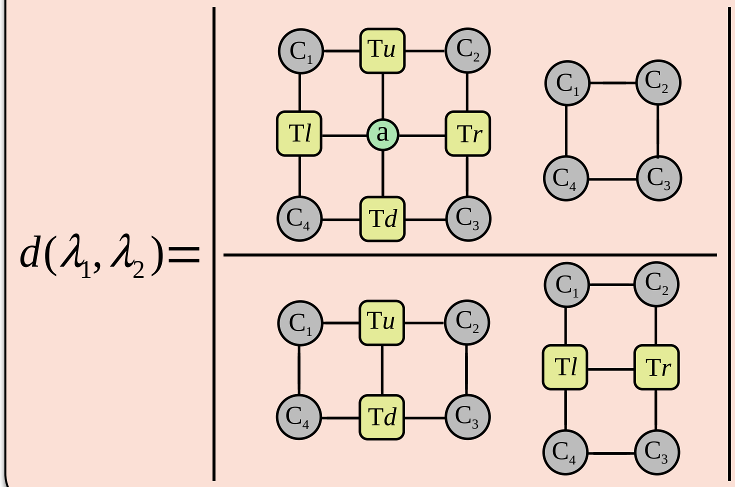
With CTMs



(I'll prove it later)

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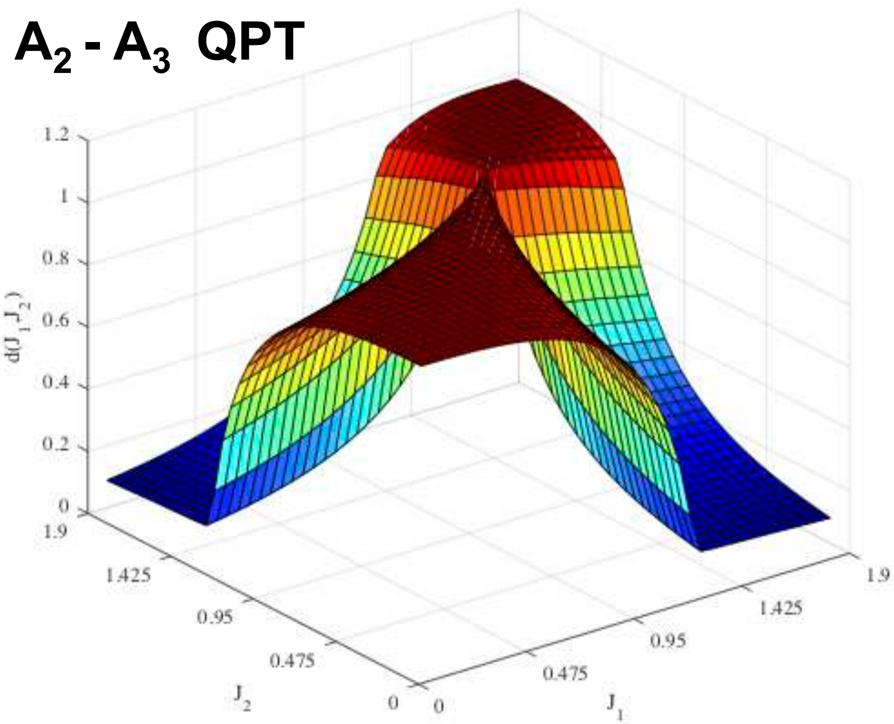
Ground state fidelity per site

H.-Q- Zhou, RO, G. Vidal, PRL 100, 080601 (2008)

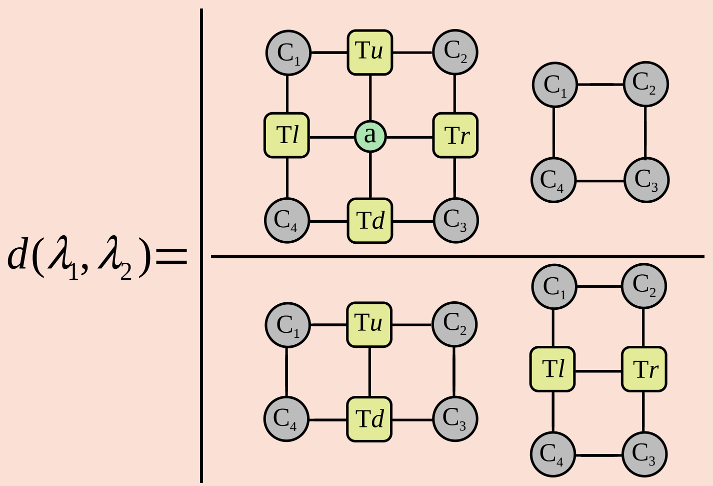
$$F(\lambda_1, \lambda_2) = |\langle \Psi(\lambda_2) | \Psi(\lambda_1) \rangle| \sim d(\lambda_1, \lambda_2)^N$$

$$\ln d(\lambda_1, \lambda_2) \equiv \lim_{N \to \infty} \frac{\ln F(\lambda_1, \lambda_2)}{N}$$

A_2 - A_3 QPT



With CTMs



(I'll prove it later)

Compatible with other observables:

- Entanglement entropy
- 2-point correlators

Fidelity from CTMs (proof of formula)

$$F(\lambda_1, \lambda_2) = |\langle \Psi(\lambda_2) | \Psi(\lambda_1) \rangle| \sim d(\lambda_1, \lambda_2)^N$$

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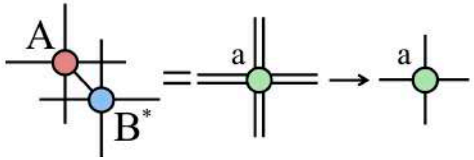

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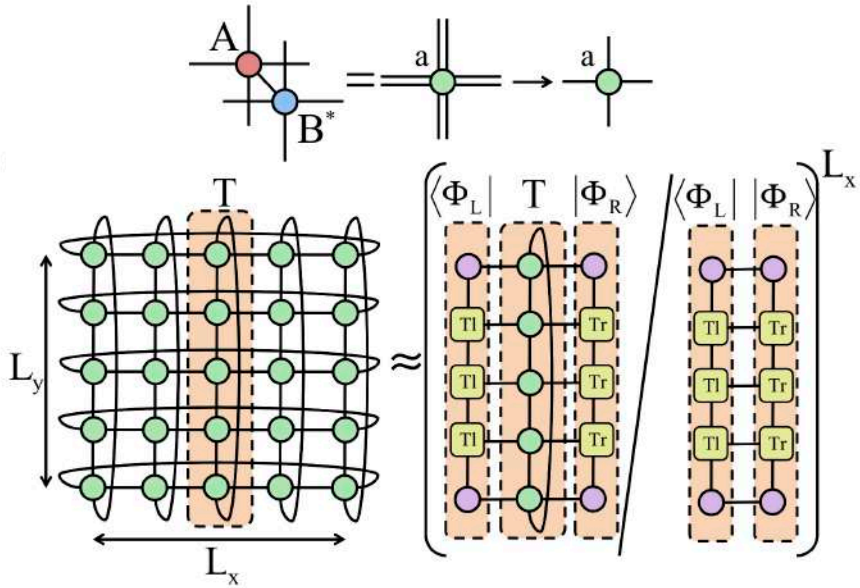

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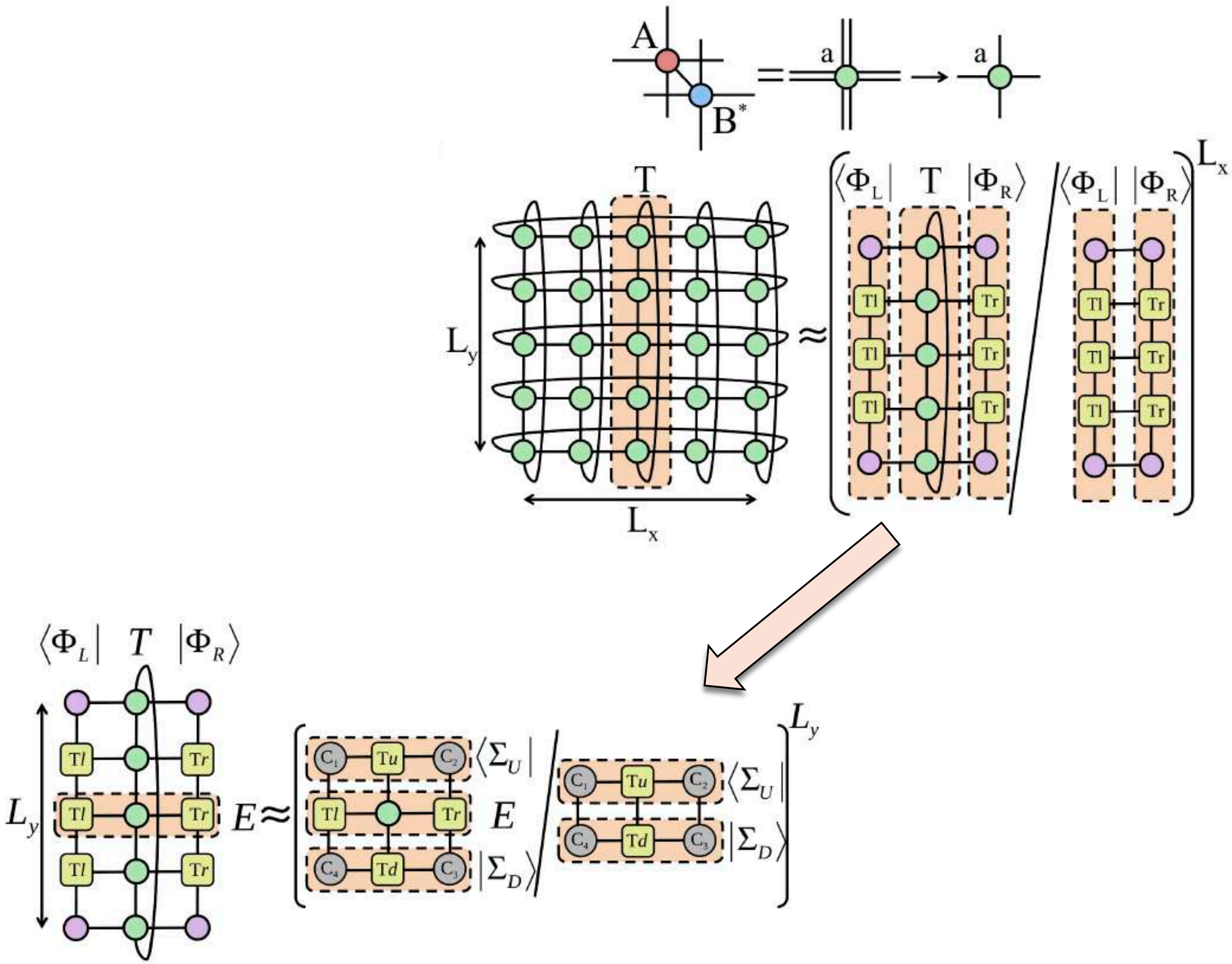
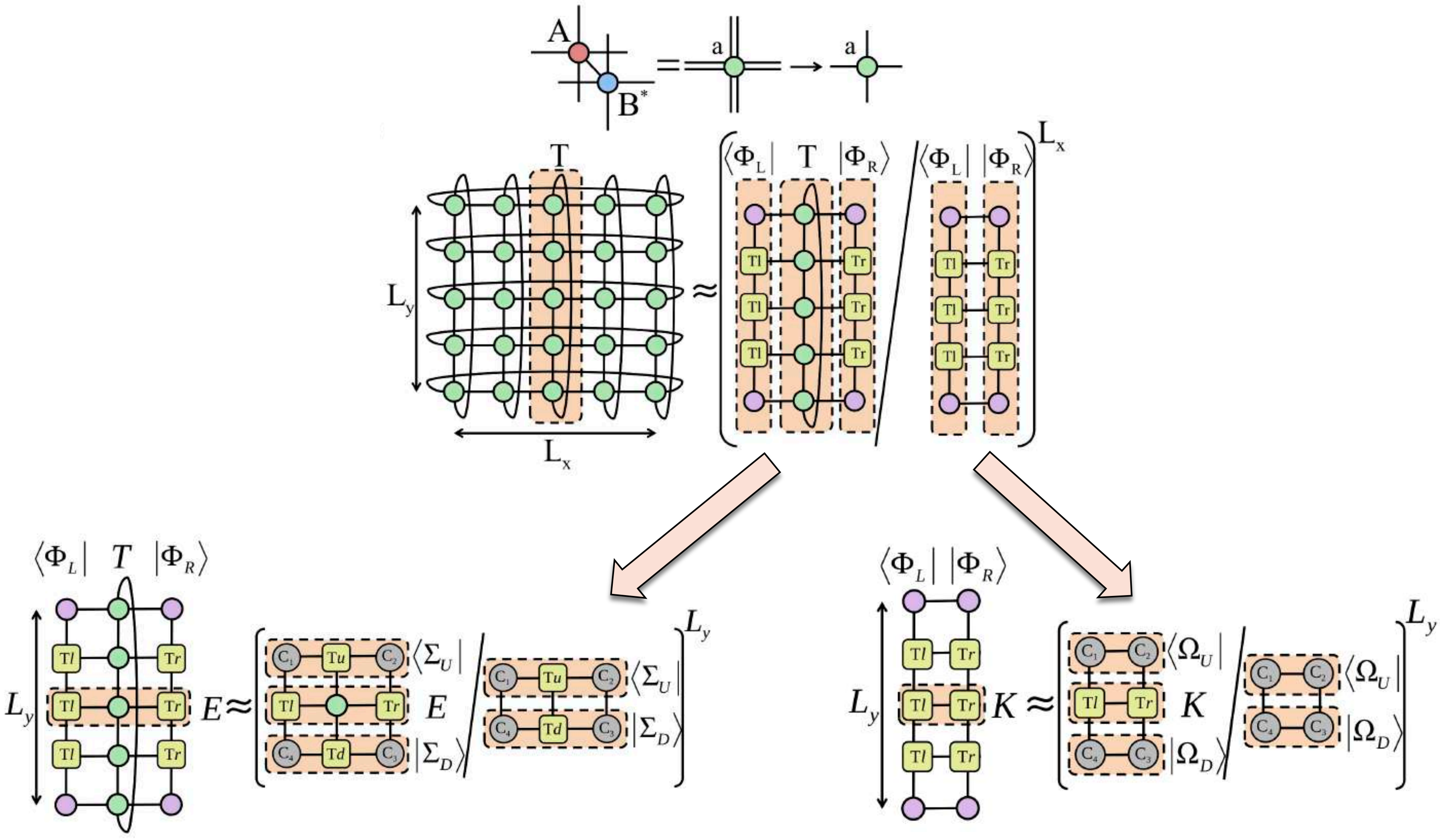


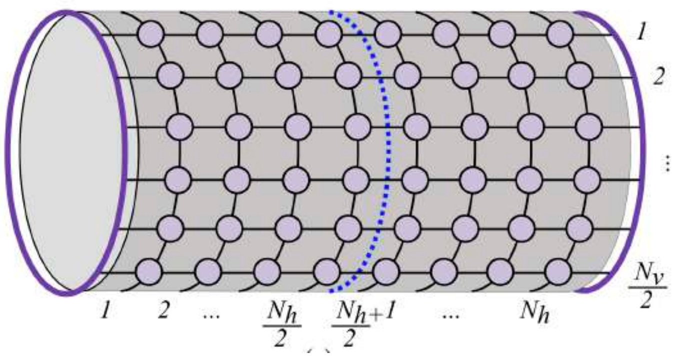
Fidelity from CTMs (proof of formula)

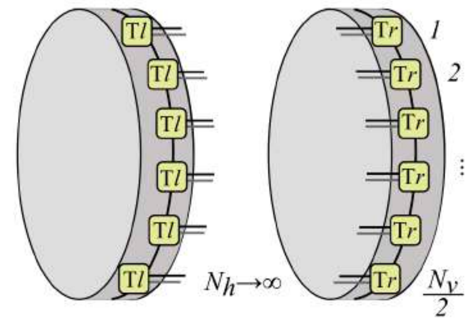
$$F(\lambda_1, \lambda_2) = |\langle \Psi(\lambda_2) | \Psi(\lambda_1) \rangle| \sim d(\lambda_1, \lambda_2)^N$$



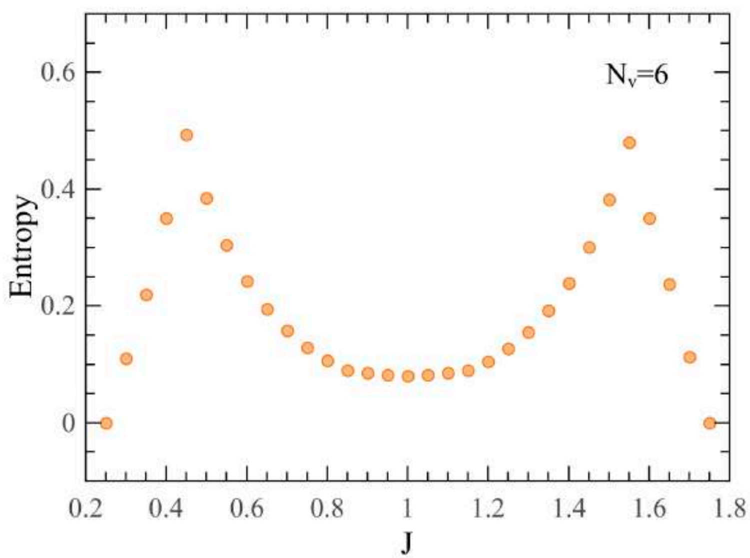
Entanglement entropy on cylinders

J. I. Cirac et al, PRB 83, 245134 (2011)





$$\rho = U\sqrt{\sigma_L^T}\,\sigma_R\sqrt{\sigma_L^T}\,U^\dagger$$ (for half an infinite cylinder)



$A_2 - A_1 - A_3$ transitions

Scaling with N_v: non-zero topological entropy, but hard to identify

Edge Hamiltonian also hard to identify

OUTLOOK

1) Phase diagram and phase transitions of the topological ruby model with iPEPS

2) Ground state energies, local fidelities, entanglement entropy of half-infinite cylinders

3) Open: detailed numerical characterization of topological orders (specially for the non-abelian phases), and the multicritical point. Phase diagram in magnetic field?

4) Open: study via combined approaches? (e.g., pCUT + iPEPS)

S. Dusuel et al, PRL 106, 107203 (2011)

Other recent activities at our group

- Quantum antiferromagnets on the star lattice
  S. S. Jahromi, RO, PRB 98 155108 (2018)
- gPEPS algorithm for any lattice
  S. S. Jahromi, RO, arXiv:1808.00680
- SU(2) tensor network algorithms
  P. Schmoll et al, arXiv:1809.08180
- 2d annealeing with tensor networks
  A. Kshetrimayum et al, arXiv:1809.08258
- Quantum criticality on a chiral ladder
  P. Schmoll et al, arXiv.1812.01311
- Topological order on the Bloch sphere
  R. Liss, T. Mor, RO, arXiv:1812.00671
- Quantum computing for finance
  RO, S. Mugel, E. Lizaso, arXiv: 1807.03890, arXiv:1810.07690









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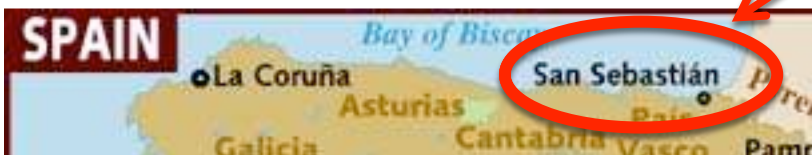
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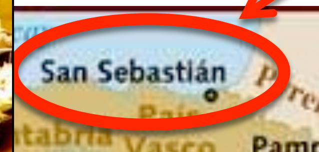

Applications welcomed