University of Zurich[UZH]

Titus Neupert

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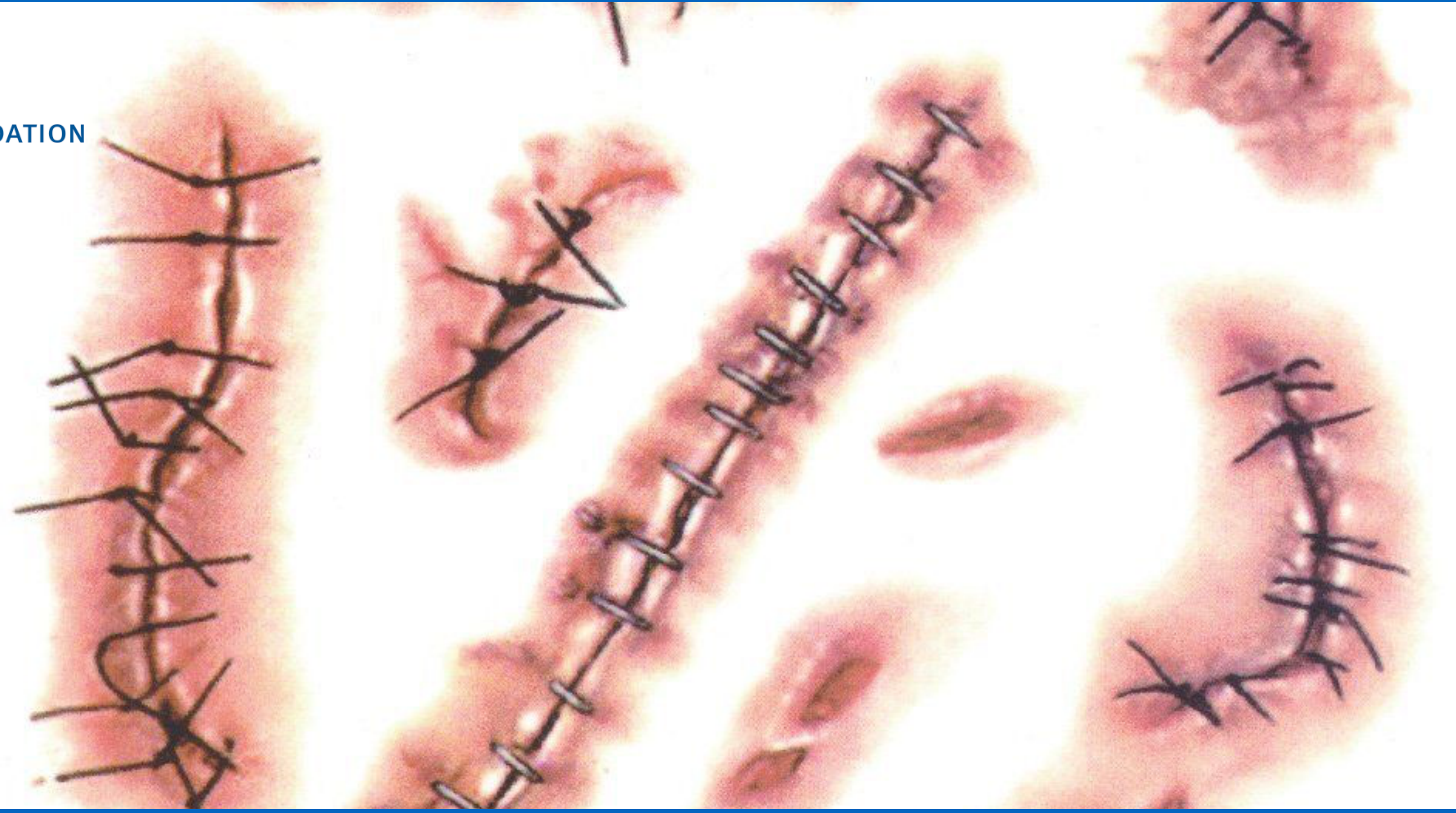
FNSNF Swiss National Science Foundation

erc European Research Council



Topological Scars

Content

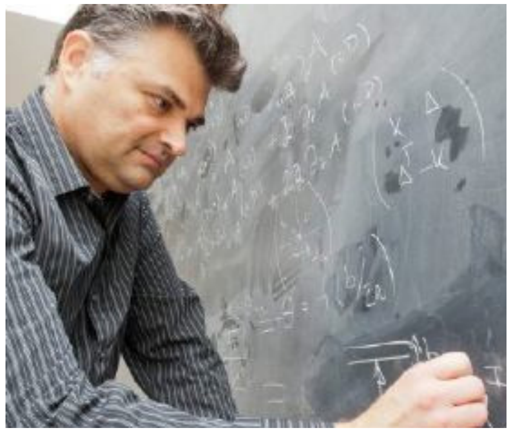
1. Scar states and ETH violation
2. Non-topological scar state in 1D
3. General construction
4. Topological scars in 1D, 2D, 3D

Collaboration


Seulgi Ok


Kenny Choo


Claudio Castelnovo

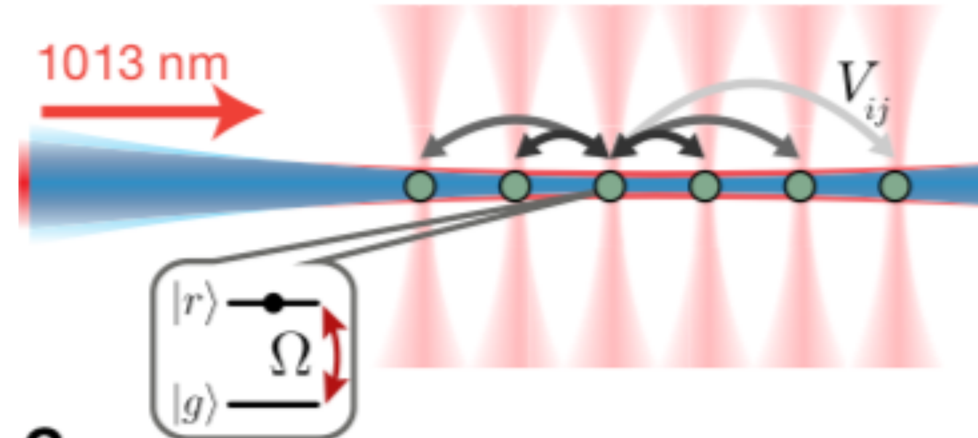

Claudio Chamon

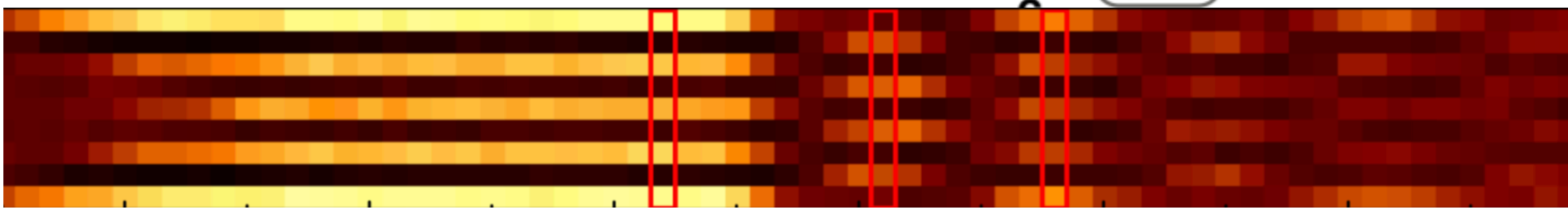

Christopher Mudry

arxiv:1901.01260

Scars

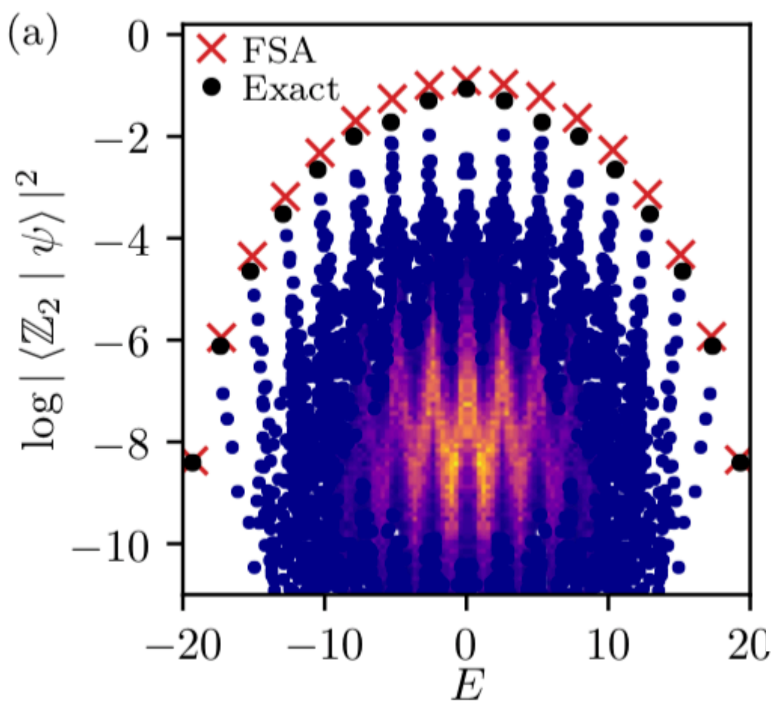
Rydberg atom experiments: Emergent oscillations in many-body dynamics after sudden quench

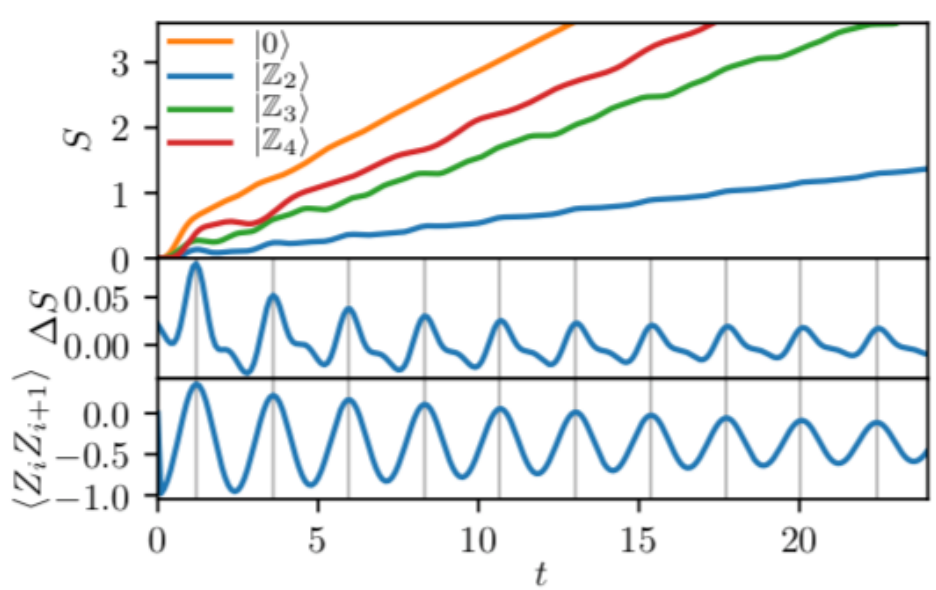




[Lukin group, Nature 551, 579-584 (2017)]

Theoretical explanation: special “scar” states in the middle of the spectrum





[Turner et. al, Nature Physics (2018)]

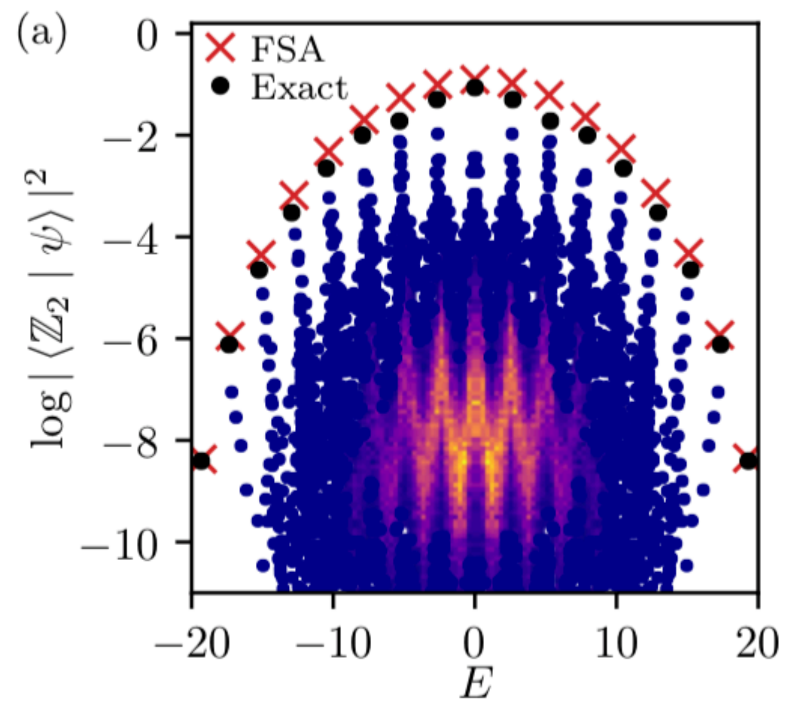
Scar characteristics

Non-integrable model

Low (sub-volume law) entanglement entropy states

No disorder (distinct from many-body localization)

“Few” scar states distributed over spectrum



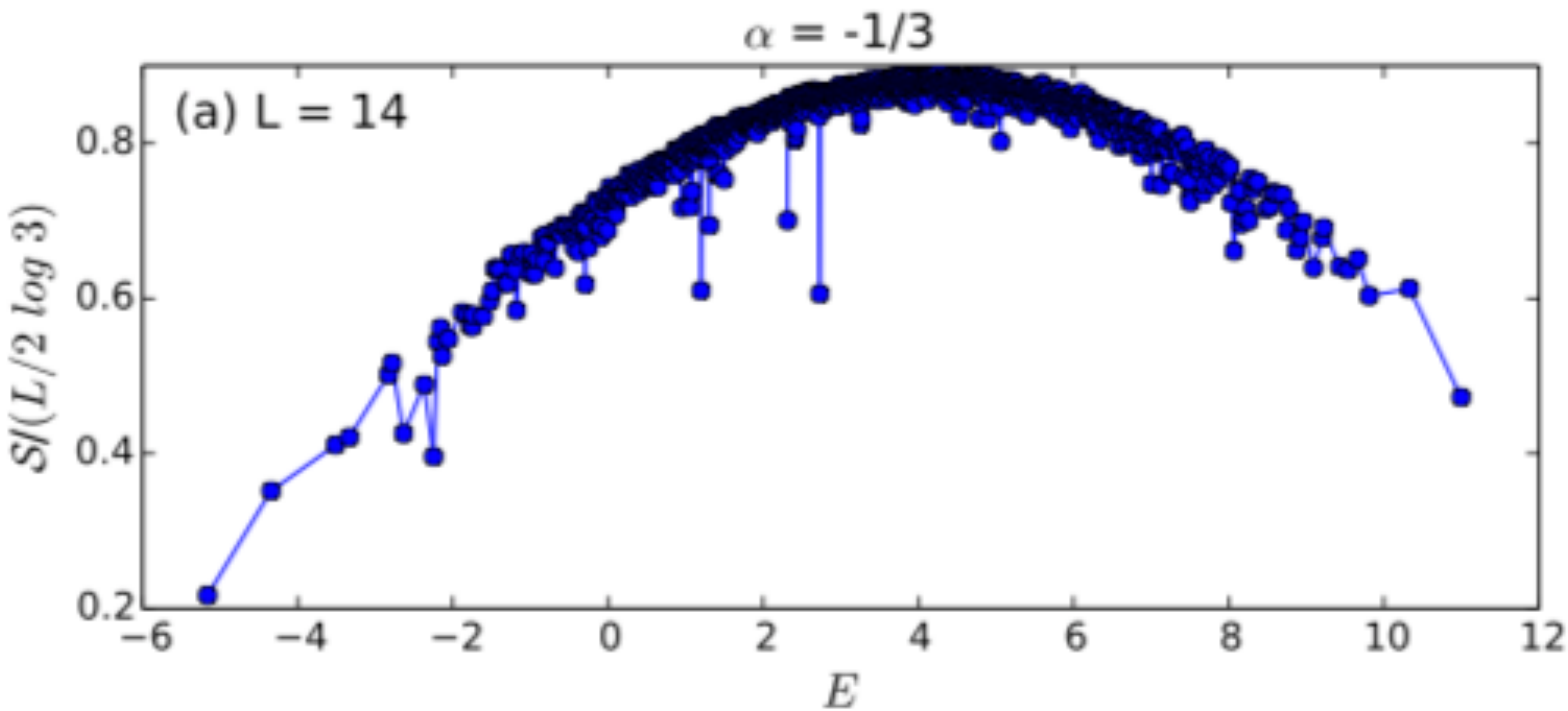
[Turner et. al, Nature Physics (2018)]

Analytical scars

[S. Moudgalya, et al., PRB 2018, 2X]

Infinite series of low-entropy states in AKLT chain

MPS based construction



Eigenstate thermalization hypothesis

Classical thermalization: equivalence of time average and ensemble average

Quantum thermalization:
Assume system behaves thermally

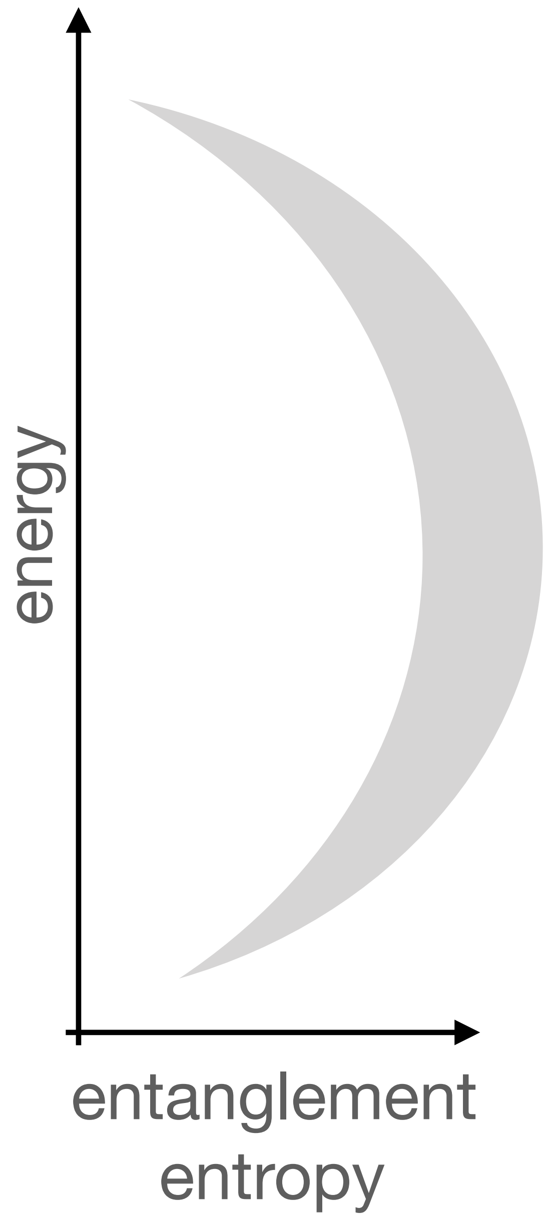
Unitary evolution cannot construct thermal state

Thermal behavior in quantum systems occurs on the level of individual eigenstates

Eigenstate thermalization hypothesis

… and how it fails in many-body quantum systems

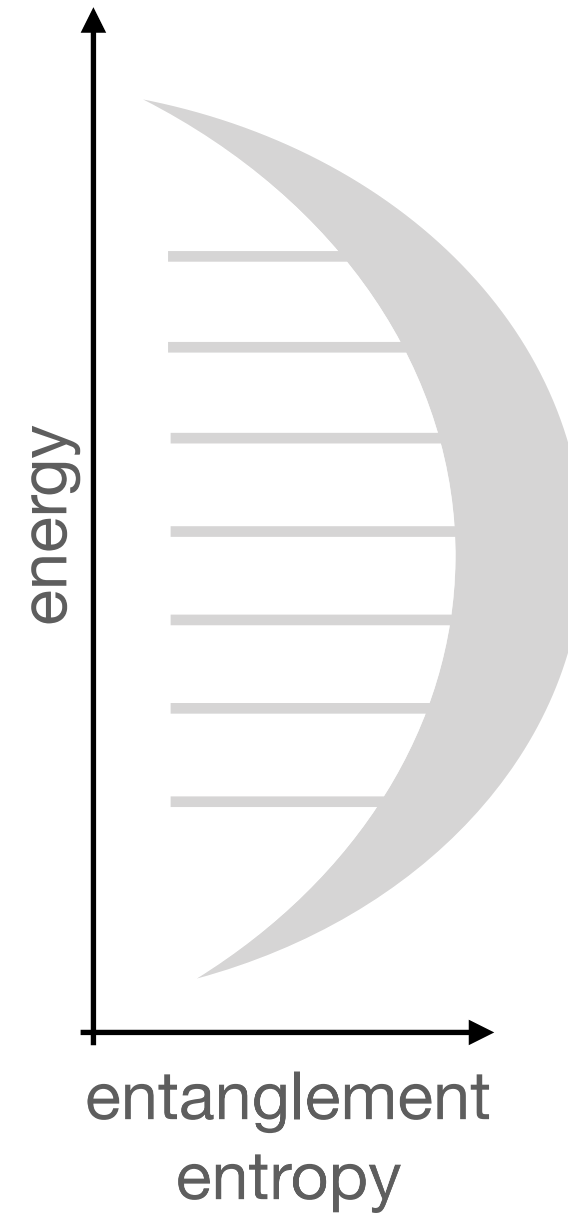
ETH obeying generic non-integrable system



nearby states have same expectation values of any observable

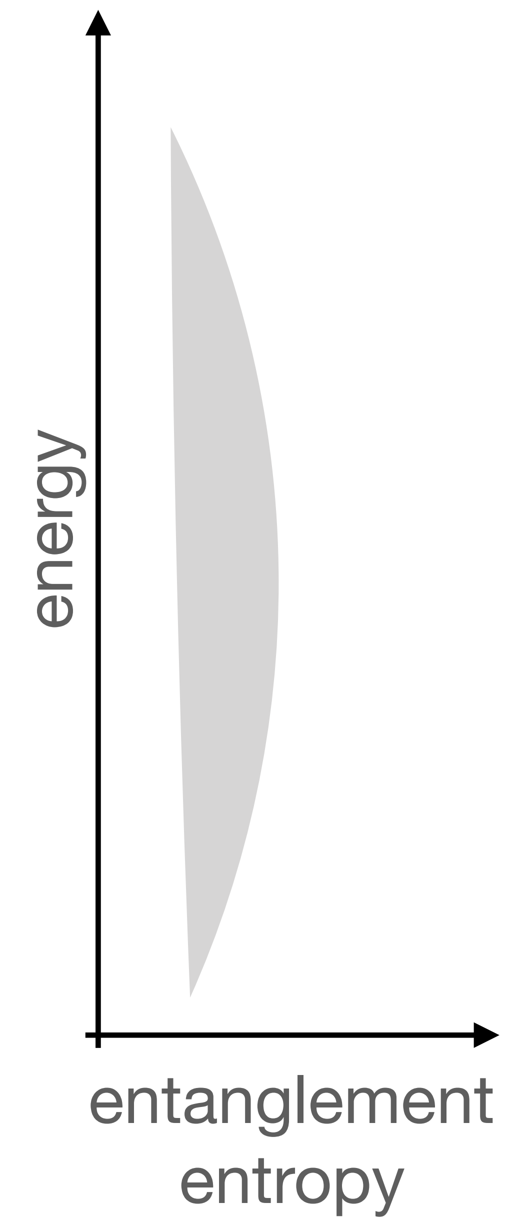
strong ETH

system with scar states



weak ETH

many-body localized



no ETH

What we do

Recipe for analytical construction of scar states

In systems of any dimension (1D, 2D, 3D)

In systems with topological character (SPT/topological order), inherited by the scar states

Area law entanglement scar states

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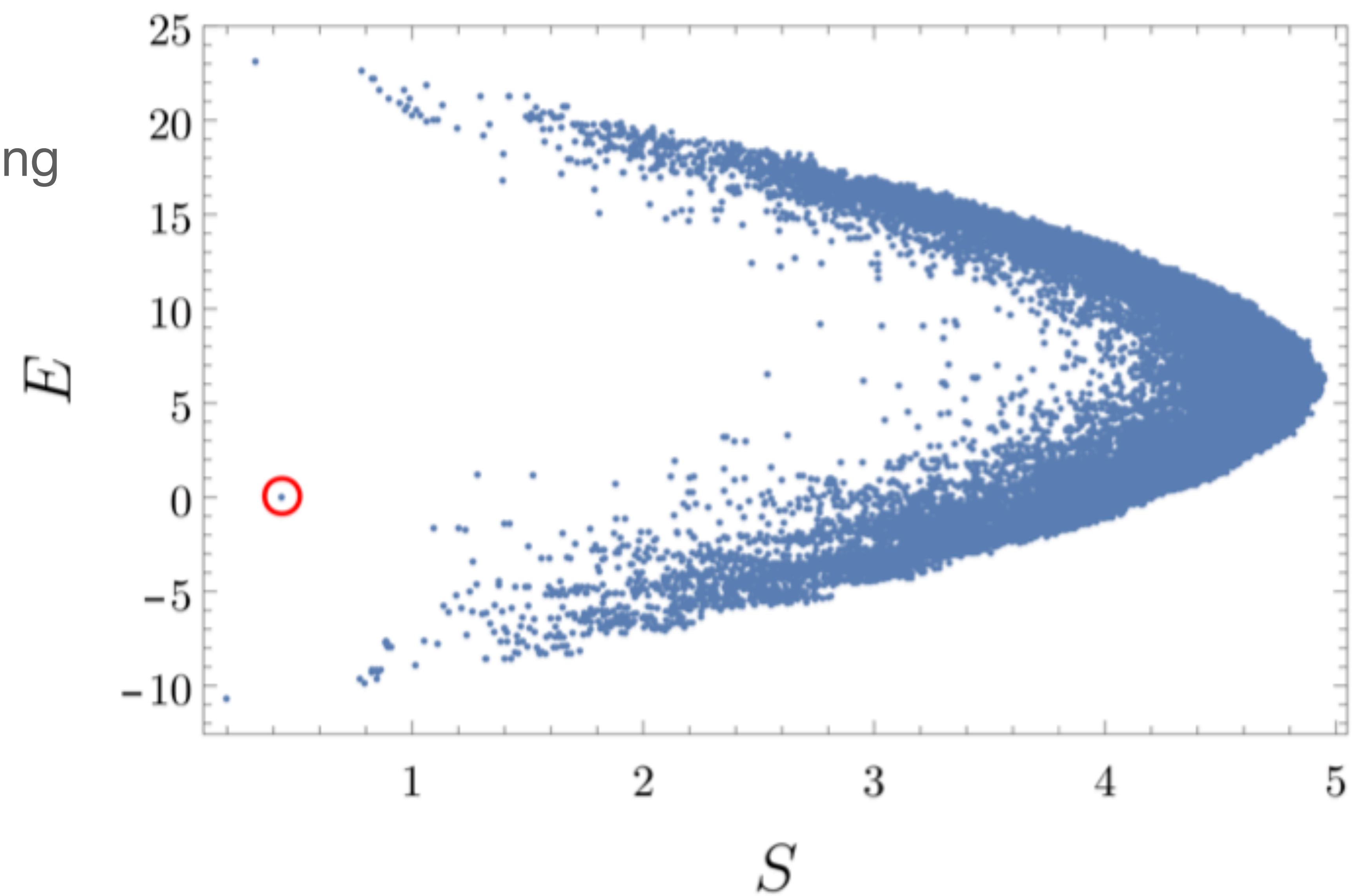
Simple (non-topological) example

1D spin-1/2 chain $$H(\beta) := \sum_i \alpha_i\, Q_i(\beta),$$

$$\alpha_i := \alpha + (-1)^i, \qquad Q_i(\beta) := e^{-\beta(Z_{i-1}Z_i + Z_i Z_{i+1})} - X_i$$

Transverse field Ising model
+ NNN coupling
+ sign-alternating coupling



Simple (non-topological) example

Proof of non-integrability: energy level statistics

$$s_n := E_{n+1} - E_n$$

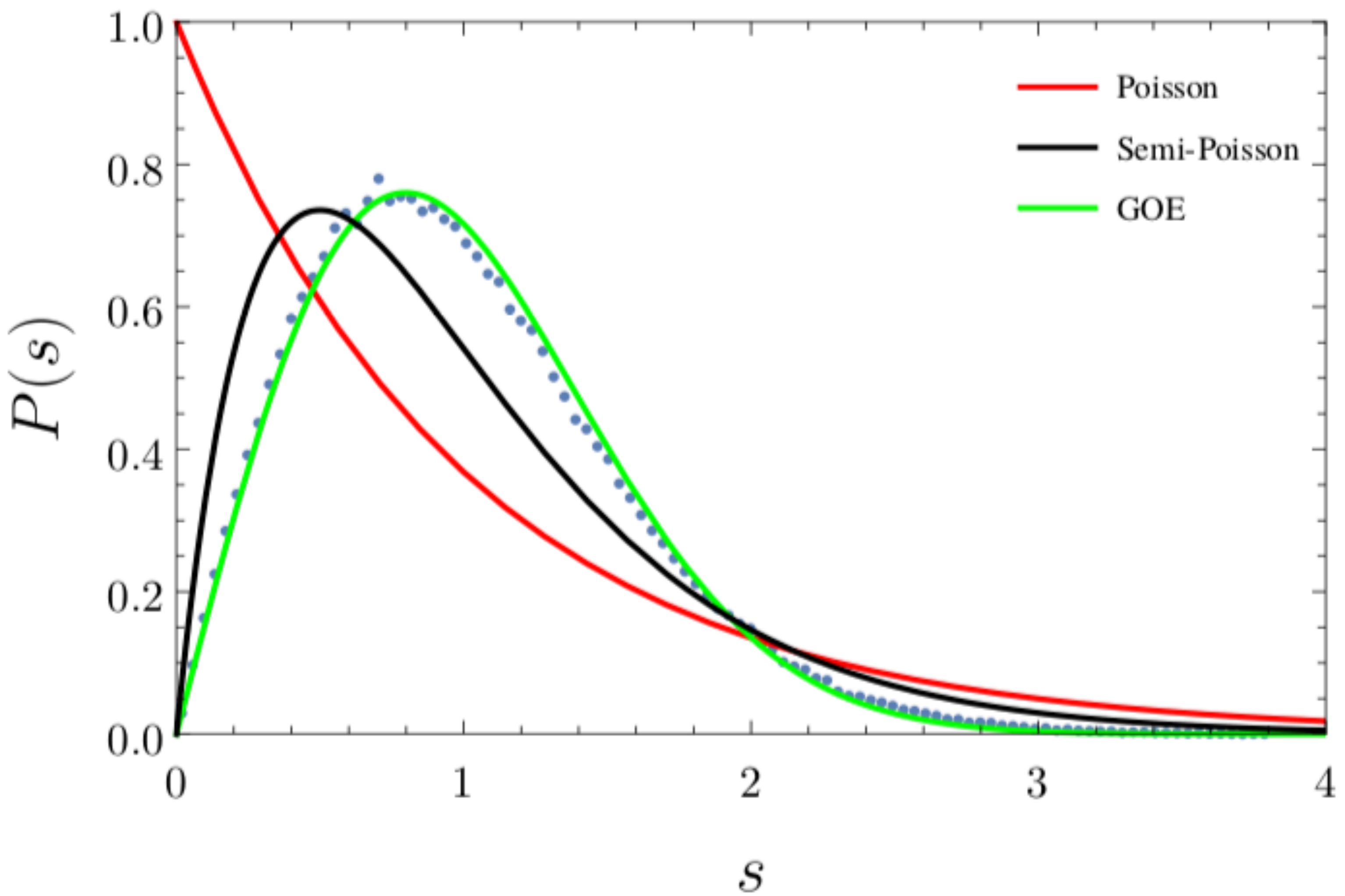
$$r_n := \min(s_n, s_{n-1})/\max(s_n, s_{n-1})$$

in one momentum and inversion symmetry sector

$$\langle r \rangle = 0.531$$

$$r_{\text{GOE}} = 0.5359$$

$$r_{\text{Poisson}} = 0.3863$$



Simple (non-topological) example

Why the scar state?

Consider alternative Hamiltonian

$$\tilde{H}(\beta) := \sum_i Q_i(\beta)$$

$$Q_i(\beta) := e^{-\beta(Z_{i-1}Z_i + Z_i Z_{i+1})} - X_i$$

Each term is positive semidefinite

$$Q_i(\beta)^2 = 2\cosh\left(\beta\left(Z_{i-1}Z_i + Z_i Z_{i+1}\right)\right) Q_i(\beta)$$

State annihilated by all terms
= ground state
area law entanglement

$$|\mathrm{scar}(\beta)\rangle := \exp\left(\frac{\beta}{2}\sum_j Z_j Z_{j+1}\right) \bigotimes_i |+\rangle_i^x$$

Exact eigenstate of
in the middle of the spectrum

$$H(\beta) := \sum_i \alpha_i Q_i(\beta) \qquad \alpha_i := \alpha + (-1)^i$$

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Recipe

Commuting projector model → Scar model (nonintegrable)

Ground state properties → Scar state properties

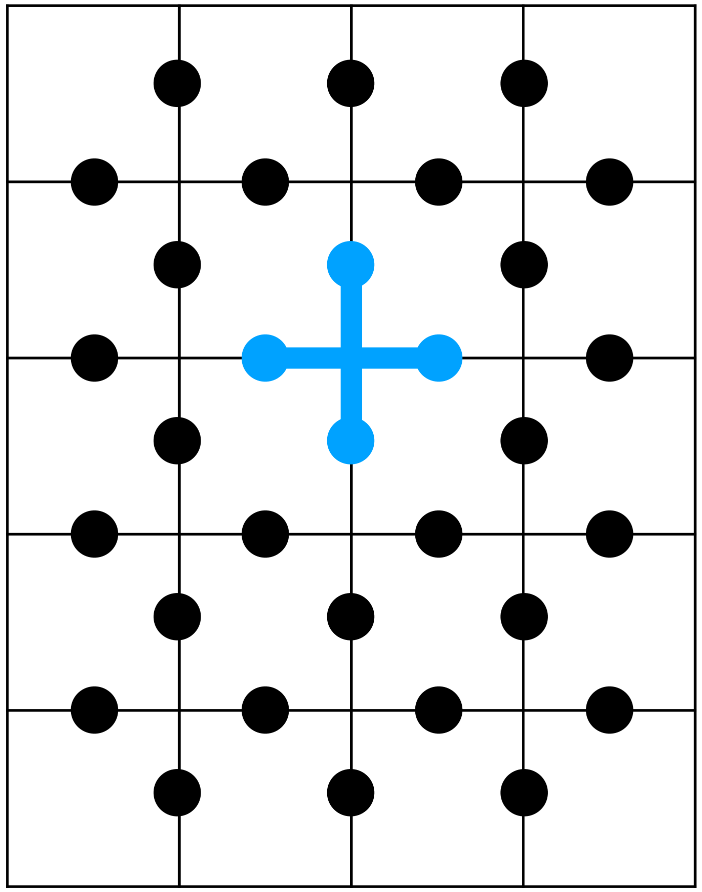
Operators constituting a commuting projector model

$$A_s \qquad A_s^2 = 1 \qquad [A_s, A_{s'}] = 0$$

Extra set of operators we will use to deform the model

$$M = \sum_i O_i \qquad M_s = \sum_{i \in s} O_i$$

$$[O_i, O_j] = 0 \qquad \bar{M}_s = \sum_{i \notin s} O_i$$



Such that

$$\{M_s, A_s\} = 0$$

$$[\bar{M}_s, A_s] = 0$$

Recipe

Deformation:

$$F_s = e^{\beta/2\, M}(1 - A_s)e^{-\beta/2\, M}$$
$$= 1 - e^{\beta\, M_s} A_s$$
$$= e^{\beta\, M_s}(e^{-\beta\, M_s} - A_s) =: e^{\beta\, M_s} Q_s$$

$Q_s = Q_s^\dagger$ is local operator!

Positive semidefinite

$$Q_s^2 = (e^{-\beta M_s} - A_s)(e^{-\beta M_s} - A_s)$$
$$= e^{-2\beta M_s} - (e^{-\beta M_s} + e^{-\beta M_s})A_s + 1$$
$$= 2\cosh(\beta M_s)\, Q_s$$

Recipe

Ground state of commuting projector model

$$|\Psi_0\rangle = \prod_{s'} (1 + A_{s'})|\Omega\rangle$$

$$\begin{aligned}(1 - A_s)|\Psi_0\rangle =&(1 - A_s)\prod_{s'}(1 + A_{s'})|\Omega\rangle \\ =&(1 - A_s)(1 + A_s)\prod_{s'\neq s}(1 + A_{s'})|\Omega\rangle \\ =&0.\end{aligned}$$

Deformed state is annihilated by all Q_s $\quad |\Psi_\beta\rangle = e^{-\beta/2\,M}|\Psi_0\rangle$

$$\begin{aligned}F_s|\Psi_\beta\rangle =&e^{-\beta/2\,M}(1 - A_s)e^{\beta/2\,M}e^{-\beta/2\,M}\prod_{s'}(1 + A_{s'})|\Omega\rangle \\ =&0\end{aligned}$$

$$\Rightarrow Q_s|\Psi_\beta\rangle = 0$$

Recipe

$$|\Psi_\beta\rangle = e^{-\beta/2\, M}|\Psi_0\rangle$$ is ground state of $$H = \sum_s Q_s$$

And **excited eigenstate** of $$H' = \sum_s \alpha_s Q_s$$ if coefficients of different sign

No need for deformed Hamiltonians to be integrable $[Q_s, Q_{s'}] \neq 0$
since O_i may belong to different s

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SPT example

Nontrivial gapped paramagnet $$H_{\mathrm{SPT}} = \sum_i Z_{i-1} X_i Z_{i+1}$$

spin-1/2 chain

Pauli algebra of chain end operators $$Z_1, \quad X_1 Z_2, \quad Y_1 Z_2$$

- commute with Hamiltonian
- double degeneracy of each state (including the ground state)
- anticommute with protecting symmetries (cannot be added as perturbations)

$$\prod_{i \in \mathrm{even}} X_i \qquad \prod_{i \in \mathrm{odd}} X_i$$

or

$$T = K \prod_i X_i$$

$$\mathbb{Z}_2 \times \mathbb{Z}_2 \qquad\qquad \mathbb{Z}_2^T$$

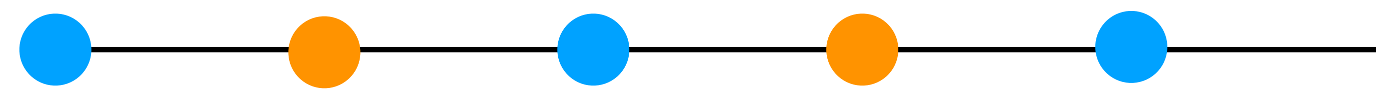
Protect topological degeneracy at each end, stable to symmetry-preserving deformations (unitary spectral evolution)

$\mathbb{Z}_2$ classification

SPT example

Deform SPT paramagnet

$$H_{\mathrm{SPT}} = \sum_i Z_{i-1} X_i Z_{i+1}$$



$$H = H_1 + H_2$$

$$H_a = \sum_{i \in \mathrm{SL}_a} \alpha_{a,i} Q_{a,i}$$

$$\alpha_{a,i} = \alpha + (-1)^{(i-a)/2}$$

$$Q_{a,i} = e^{-\beta_a (X_{i-1} + X_{i+1})} - Z_{i-1} X_i Z_{i+1}$$

Has **scar state**:

$$|\mathrm{scar}\rangle = \exp\left(\frac{\beta_1}{2} \sum_{i \in \mathrm{SL}_1} X_{i-1}\right) \exp\left(\frac{\beta_2}{2} \sum_{i \in \mathrm{SL}_2} X_{i-1}\right) |+, \cdots, +\rangle$$

But: **integral of motion** H_1

SPT example

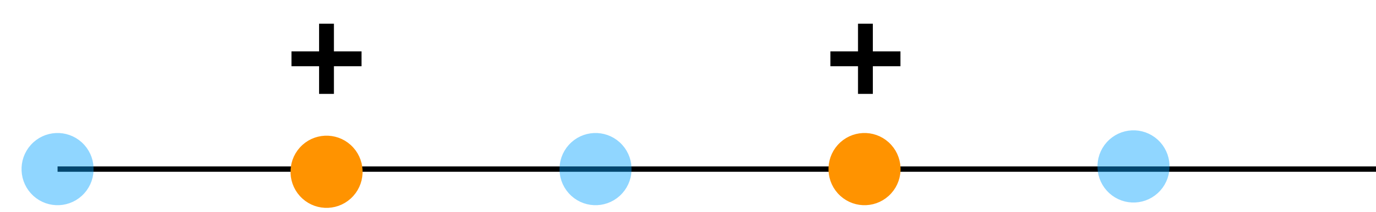
$$H_a = \sum_{i \in \mathrm{SL}_a} \alpha_{a,i} Q_{a,i}$$

Unitary transformation:

$$Q_{a,i} = e^{-\beta_a (X_{i-1} + X_{i+1})} - Z_{i-1} X_i Z_{i+1}$$

$$W := \exp\left(\mathrm{i}\frac{\pi}{4} \sum_{j \in \mathrm{SL}_1} Z_j Z_{j+1} - \mathrm{i}\frac{\pi}{4} \sum_{j \in \mathrm{SL}_2} Z_j Z_{j+1} \right)$$

$$\tilde{Q}_{a,i} = e^{-\beta_a (Z_{i-2} X_{i-1} Z_i + Z_i X_{i+1} Z_{i+2})} - X_i$$



For diagonalization of H_1 alone, fix X state of every other spin (integral of motion)

$$Q_i(\beta) := e^{-\beta (Z_{i-1} Z_i + Z_i Z_{i+1})} - X_i$$

Same as trivial scar Hamiltonian:

- **non-integrable** (as argued before)
- scar state is **not ground state of integral of motion**

Toric code example

$$H = H_1 + H_2$$

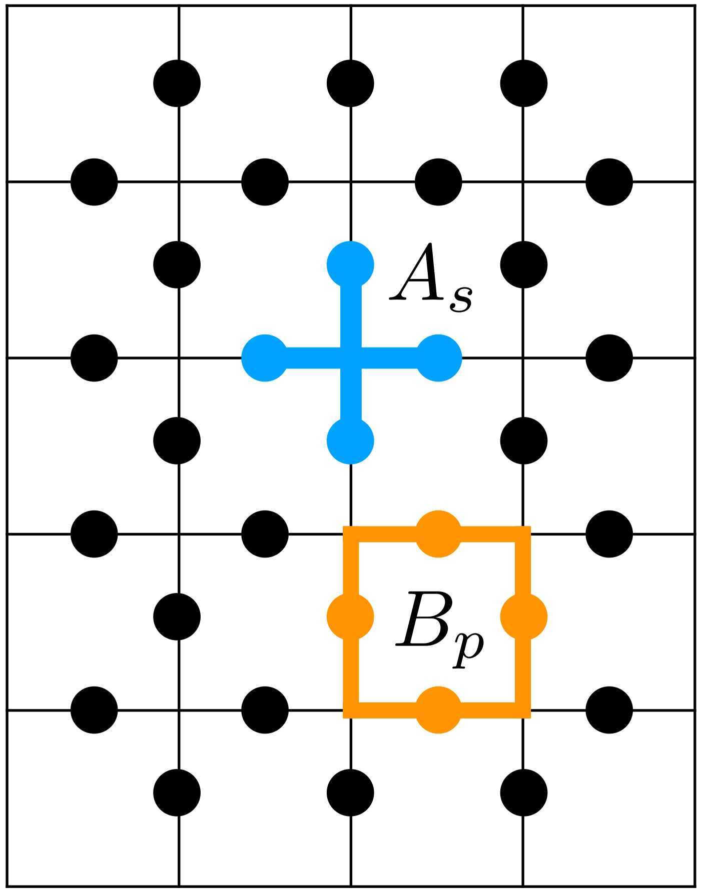
$$H_1 = -\sum_s A_s$$

$$H_2 = -\sum_p B_p$$

$$B_p = \prod_{i \in p} Z_i$$

$$A_s = \prod_{i \in s} X_i$$

4-fold degenerate ground state due to Wilson loops (and every excited state)



deformation

$$H_1 = \sum_s \left[\exp\left(-\beta_1 \sum_{i \in s \cap \mathcal{P}_1} Z_i \right) - A_s \right]$$

$$H_2 = \sum_p \left[\exp\left(-\beta_2 \sum_{i \in p \cap \mathcal{P}_2} X_i \right) - B_p \right]$$

Toric code example

$$H = H_1 + H_2$$

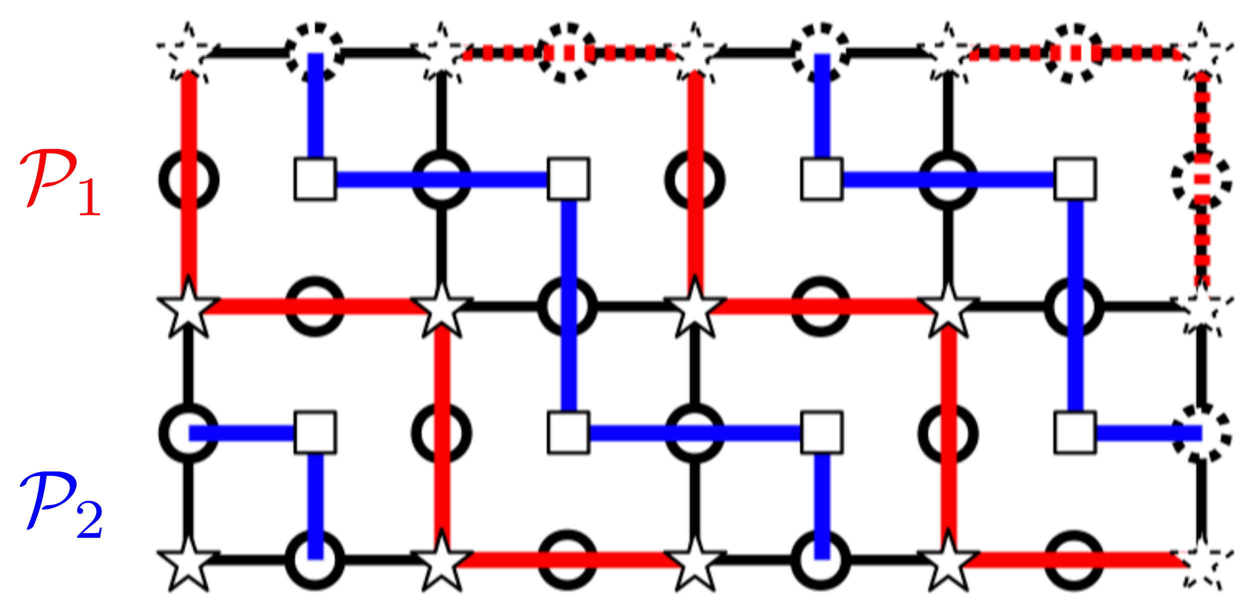
$$H_1 = \sum_s \alpha_s \left[\exp\left(-\beta_1 \sum_{i \in s \cap \mathcal{P}_1} Z_i \right) - A_s \right]$$

$$H_2 = \sum_p \alpha_p \left[\exp\left(-\beta_2 \sum_{i \in p \cap \mathcal{P}_2} X_i \right) - B_p \right]$$

Choice of paths:

- non-intersecting
- cover all sites
- connected

Guarantees no extra integrals of motion

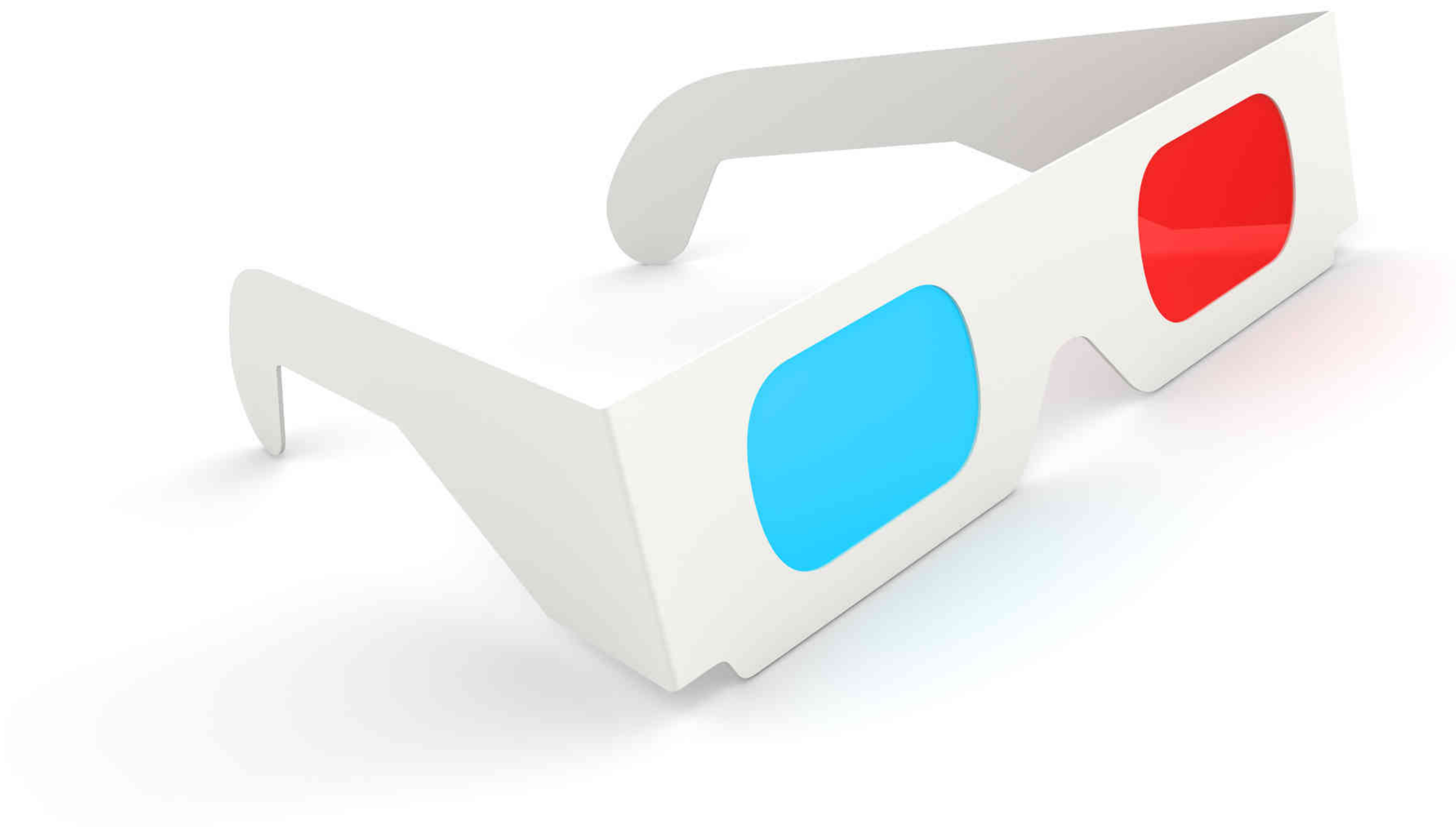


Consider only H_1 : Z eigenvalues for spins not on path are integrals of motion
In each sector, H_1 reduces to **1D scar paramagnet**

Non-integrable, supports scar state in the middle of spectrum

+ 4-fold “topological” degeneracy of scar state

Let's go 3D

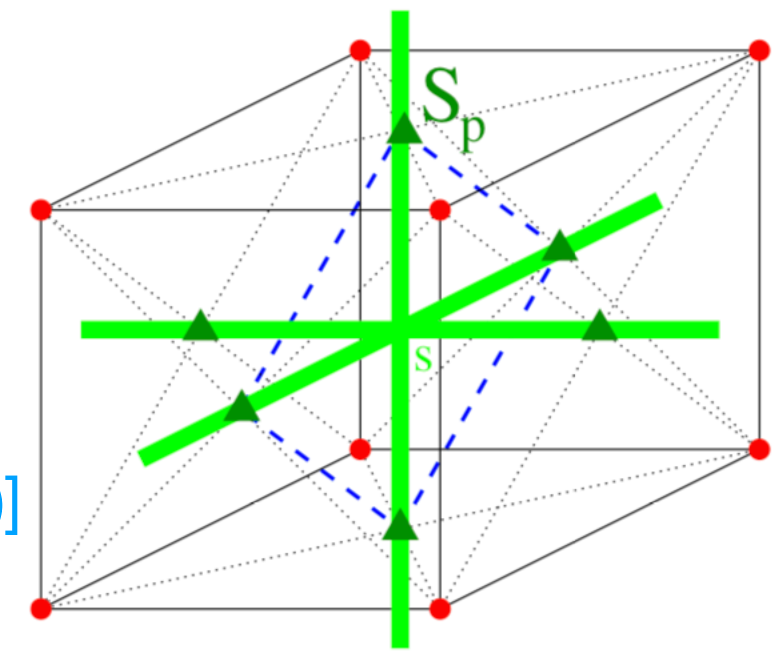


Fracton topological order

Topological order beyond the gauge theory paradigm

- 3D (or higher D)
- (partially) immobile excitations
- **exponential in L** (ground) state degeneracy: **great topological quantum memory**

Motivation: T-stable topological order (did not work)

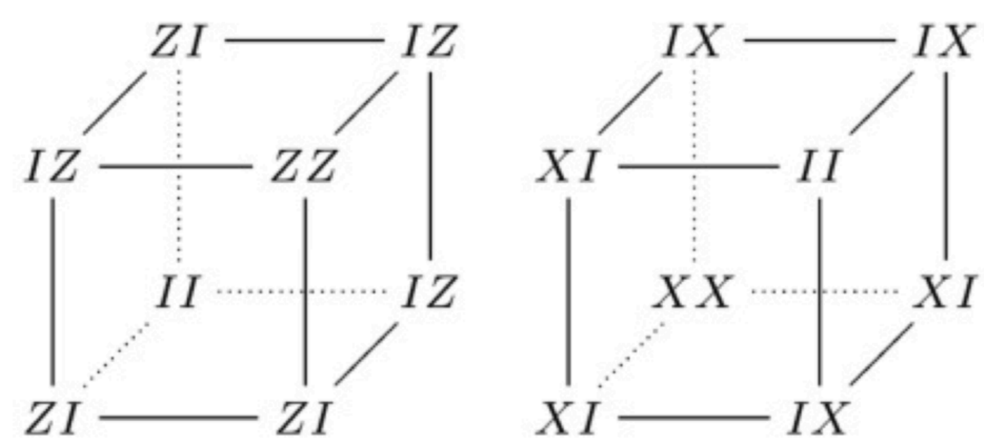


First model

[C. Castelnovo, C. Chamon, and D. Sherrington, PRB **81**, 184303 (2010)]

Haah code

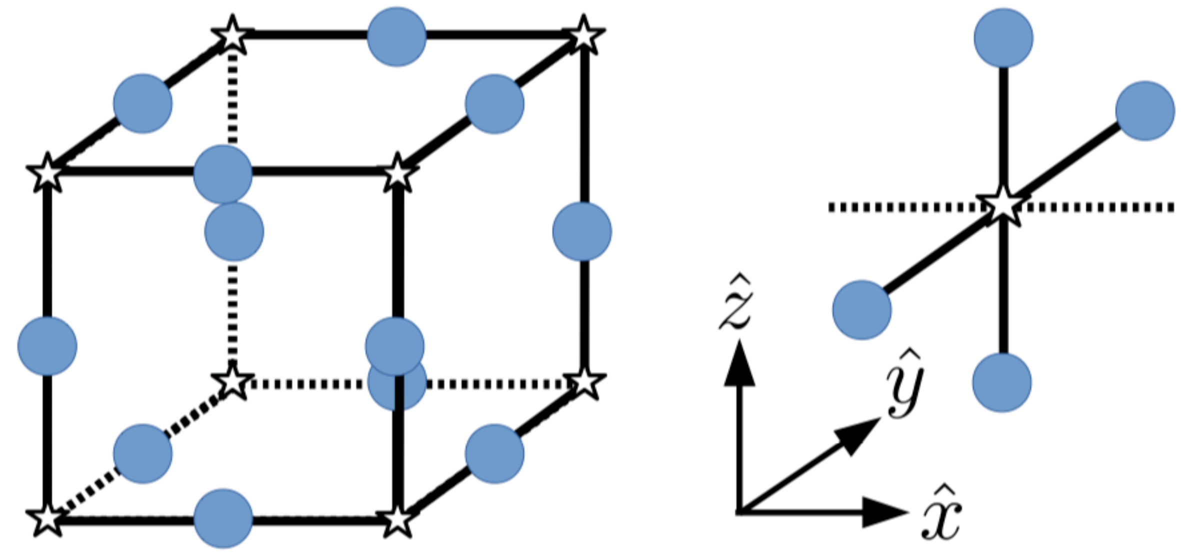
[J. Haah, PRA 83, 042330 (2011)]



X-cube model

spin 1/2 on cubic lattice bonds

$$H = -\sum_c B_c - \sum_s A_s$$



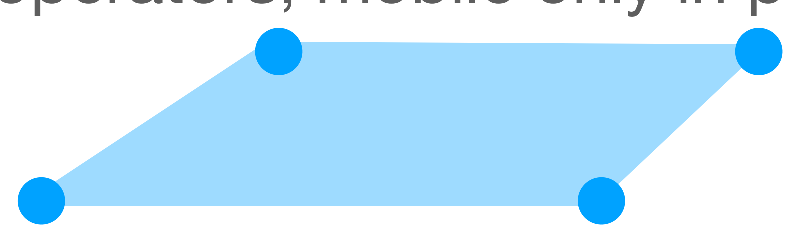
$$B_c = \prod_{i \in c} Z_i \qquad A_s = \prod_{i \in s} X_i$$

cubes

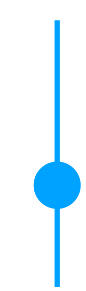
three types of stars

Excitations:

corners of membrane operators, mobile only in pairs



1D confined particles



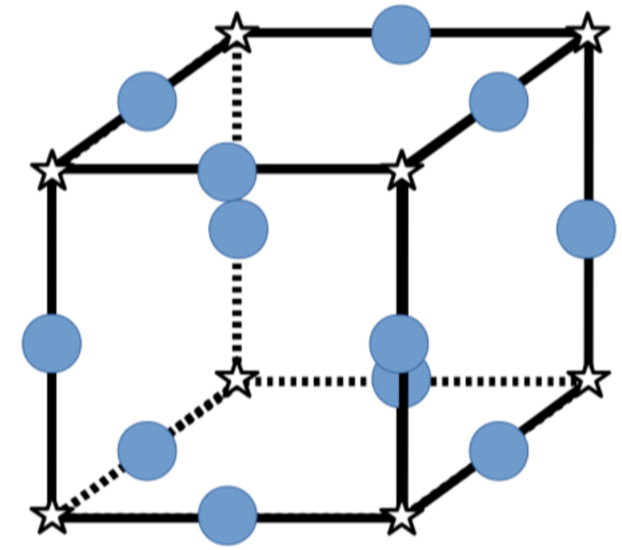
degeneracy on LxLxL system with pbc: **6L-3**

$$|+, \cdots, +; \zeta\rangle \qquad \zeta \in \{-,+\}^{6L-3}$$

eigenvalues of independent stars/cubes

X-cube model

Deformed version $$H = H_1 + H_2$$



$$H_1 = \sum_s \alpha_s \left[\exp\left(-\beta_1 \sum_{i \in s \cap \mathcal{P}_1} Z_i \right) - A_s \right]$$

$$H_2 = \sum_c \alpha_c \left[\exp\left(-\beta_2 \sum_{i \in c \cap \mathcal{P}_2} X_i \right) - B_c \right]$$

- alternating sign coefficients
- same conditions on paths as in 2D

AGAIN:

Consider only H_1 : Z eigenvalues for spins not on path are integrals of motion
In each sector, H_1 reduces to **1D scar paramagnet**

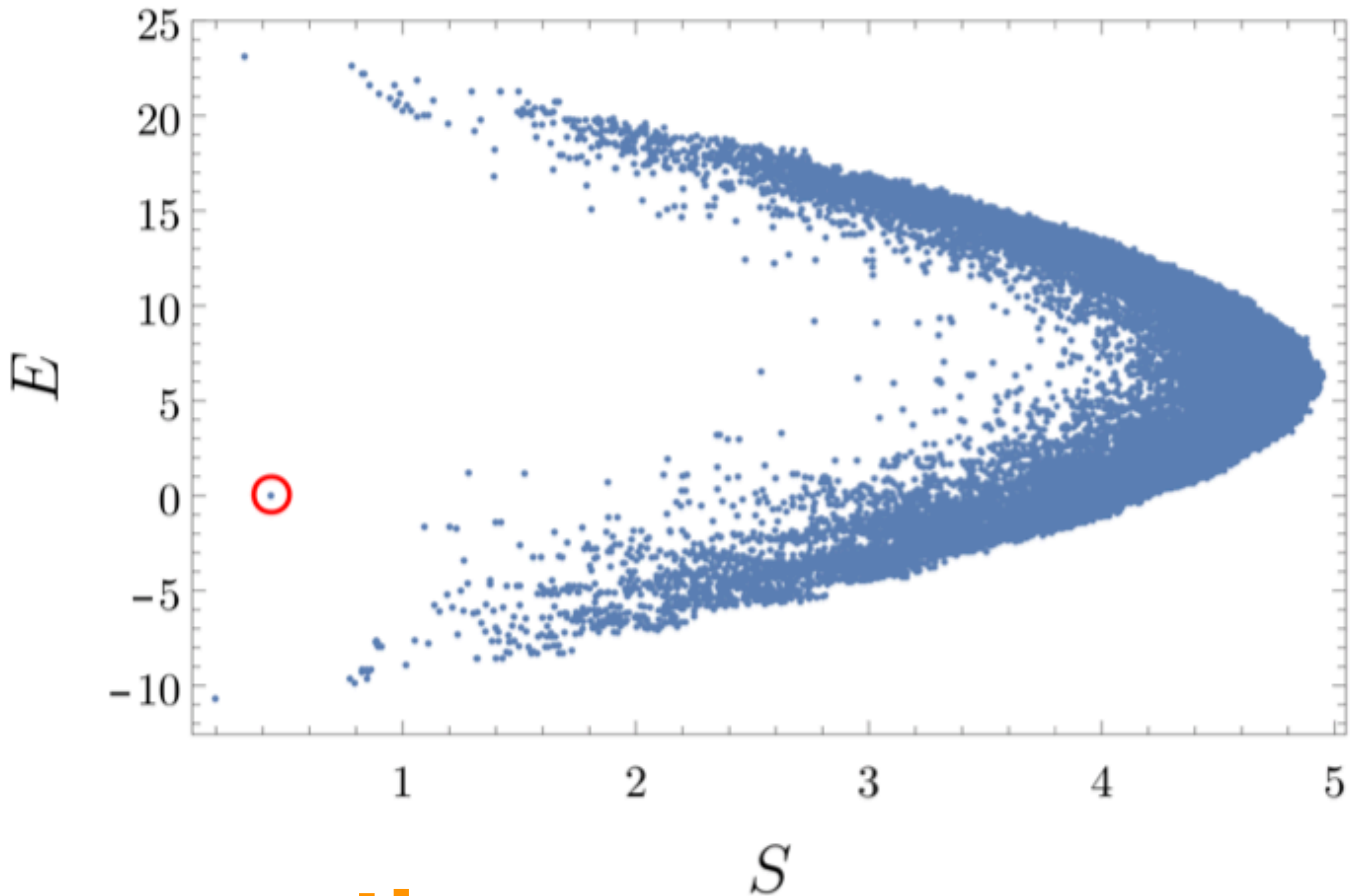
Non-integrable, supports scar state in the middle of spectrum

+ exp(L) "topological" degeneracy of scar state

$$|\text{scar}; \zeta\rangle = \exp\left(\frac{\beta_1}{2} \sum_{i \in \mathcal{P}_1} Z_i \right) \exp\left(\frac{\beta_2}{2} \sum_{i \in \mathcal{P}_2} X_i \right) |+, \cdots, +; \zeta\rangle$$

Conclusions

- new models with scar states **in all dimensions** [only single scar state]
- strong numerical evidence for their non-integrability
- **topological features** of the scar states, in particular exp(L) degeneracy for fracton models in 3D



Open questions

- Is there any sense of **stability** of the “topological” scar state degeneracies?
- Stability of scar states in general?
- Observables/physical consequences of single scar states?

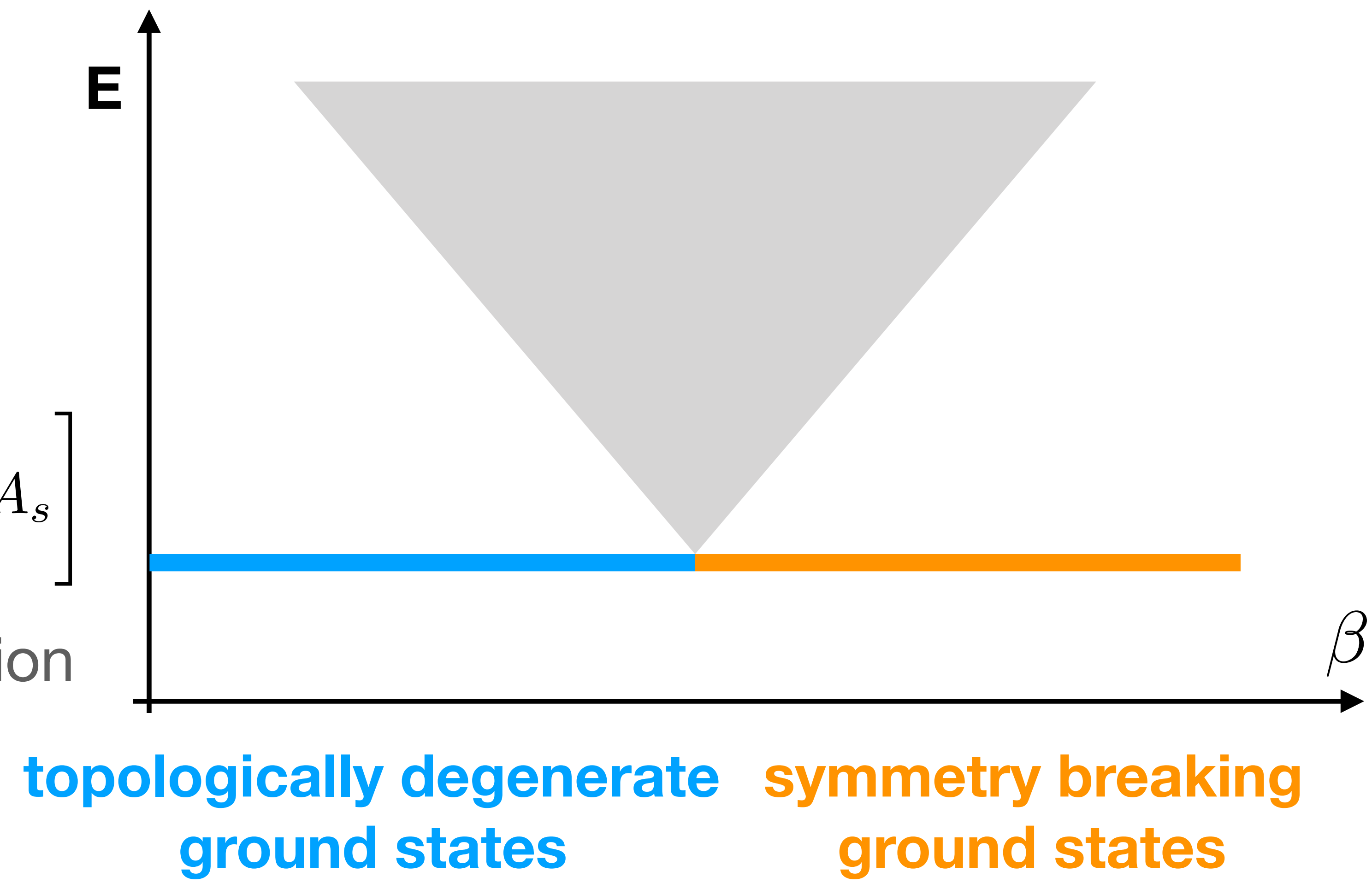
arxiv:1901.01260

Open questions

2D model:
phase transition (in ground state)

$$H_1 = \sum_s \alpha_s \left[\exp\left(-\beta_1 \sum_{i \in s \cap \mathcal{P}_1} Z_i \right) - A_s \right]$$

related to Ising model phase transition



Turn into scar state: **phase transition** in excited states**?**