Chiral phases of ultra cold fermions in optical lattices

F. Mila
Ecole Polytechnique Fédérale de Lausanne
Switzerland

Collaborators: C. Boos (Erlangen and Lausanne), P. Nataf (Grenoble)
K. Schmidt (Erlangen), M. Lajko (Lausanne), K. Penc (Budapest),
A. Wietek, C. Ganahl, C. Romen, A. Läuchli (Innsbruck)

Scope

- The SU(N) Hubbard model of cold atoms

  → Generic phase diagram
- Heisenberg model with one particle per site

  → color order, VBS, algebraic liquid
- Chiral spin liquids

  → several particles per site

  → multi-site interactions
- Conclusions

Fermions in optical lattice

N-flavour fermions: ^{87}Sr,...

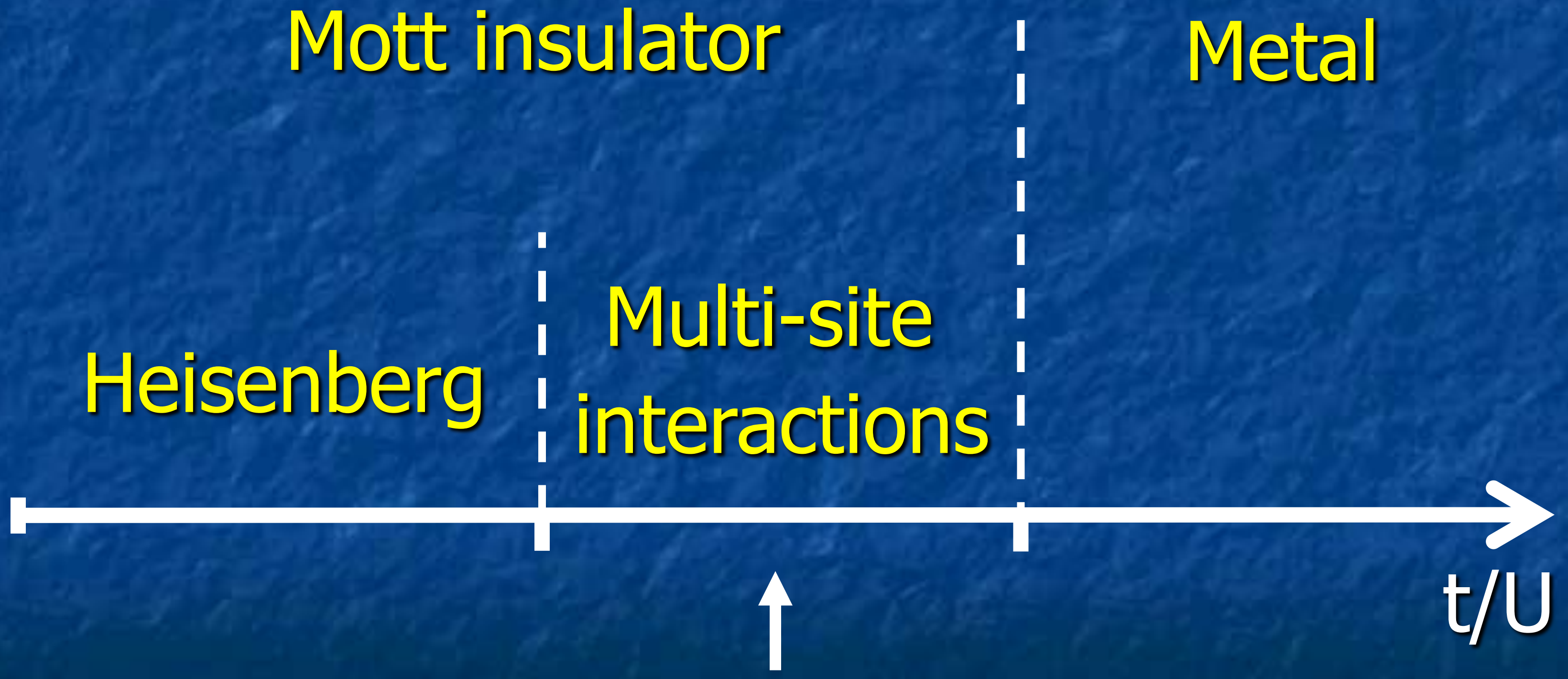
N=2I+1, I=nuclear spin, N up to 10

N-flavour Hubbard model

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} \sum_{\alpha=1}^{N} (c_{i,\alpha}^{\dagger} c_{j,\alpha} + h.c.) + U \sum_{i} \sum_{(\alpha,\beta)} n_{i,\alpha} n_{i,\beta}$$

m fermions per site (m up to N)

Generic phase diagram for m particles per site (m integer)



Intermediate phase?

SU(2) on square lattice

- Metal-insulator transition as soon as U>0 because of perfect nesting
- Néel antiferromagnetic order from weak to strong coupling

→ No evidence of an intermediate phase

→ Possible modification of the dispersion of spin waves

SU(2) case on triangular lattice

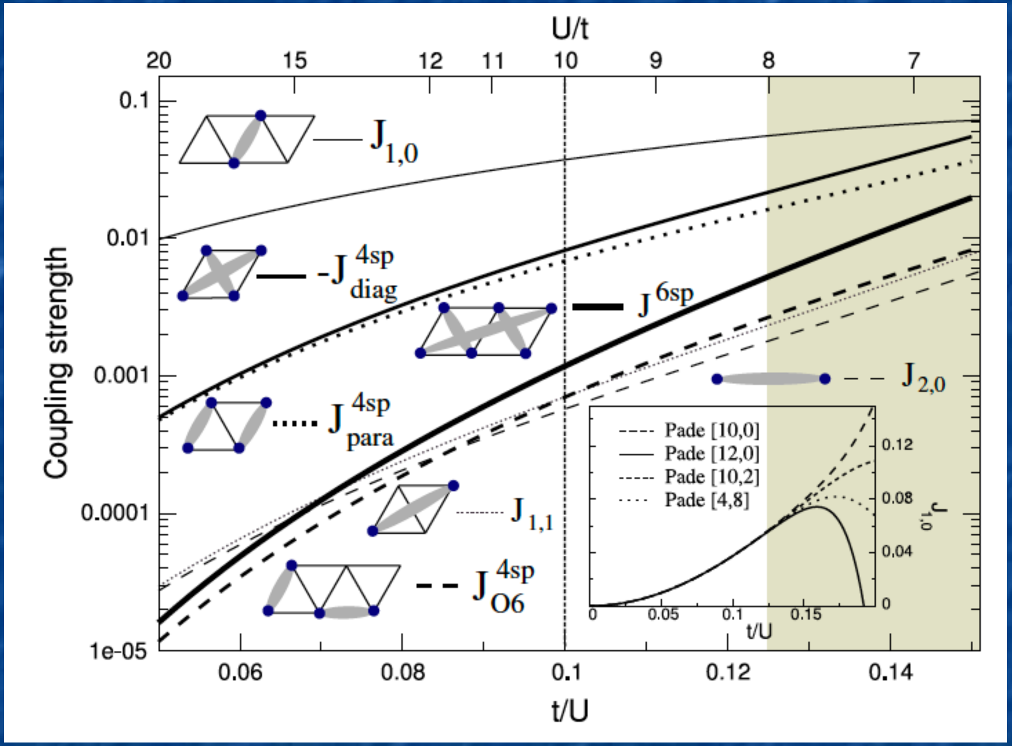
- DMRG on Hubbard model
  → Chiral intermediate phase

  Szasz, Motruk, Zaletel, Moore, 2018
- Effective model
  → Four-site interactions dominate

  Yang, Laeuchli, FM, Schmidt, 2010
- Early study of four-site interaction
  → Spinon Fermi surface

  Motrunich, 2005; Sheng, Motrunich, Fischer, 2009

Effective model (triangular lattice)



Yang, Laeuchli, FM, Schmidt, PRL 2010

Important parameters

- N of SU(N): 2 in Mott insulators with electrons, up to 10 in cold atoms
- m: number of fermions per site
- t/U: simple Heisenberg or more complicated interactions
- Phase of hopping integral t

  → artificial gauge fields if flux ≠ 0
- Lattice geometry: square, triangular, kagome,…

SU(N) Heisenberg model

$$H = \sum_{\langle i,j \rangle} S_\alpha^\beta(i) S_\beta^\alpha(j)$$

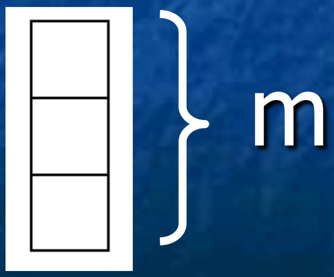
S_α^β generators of SU(N)

$$[S_\alpha^\beta, S_\mu^\nu] = \delta_{\beta\mu} S_\alpha^\nu - \delta_{\alpha\nu} S_\mu^\beta$$

Antisymmetric irrep with m boxes

→ fermionic representation

$$S_\alpha^\beta = c_\alpha^\dagger c_\beta - \frac{m}{N}\delta_{\alpha\beta}$$



One particle per site

- Fundamental representation
- Hilbert space = {l $\sigma_1\ \sigma_2\ \ldots\ \sigma_L$ >}

σ_i = 1,2, …,N or σ_i = A, B, C,… or , , …

$$\mathcal{H} = \frac{2t^2}{U} \sum_{\langle i,j \rangle} P_{ij}$$

$$P_{ij} |\sigma_1 \ldots \sigma_i \ldots \sigma_j \ldots \sigma_N\rangle = |\sigma_1 \ldots \sigma_j \ldots \sigma_i \ldots \sigma_N\rangle$$

→ Quantum permutation

General properties

- Soluble in 1D with Bethe Ansatz
  → algebraic correlations with periodicity $2\pi/N$
  Sutherland, 1974

- Equivalent of SU(2) dimer singlet: N sites

  $| S > = (1/\sqrt{N!}) \sum_P (-1)^P | \sigma_{P(1)} \sigma_{P(2)} \ldots \sigma_{P(N)} >$

  with $\{\sigma_1 \sigma_2 \ldots \sigma_N\}=\{1\ 2 \ldots N\}$

  Li, Ma, Shi, Zhang, PRL'98

Real space mean-field

$$|\psi\rangle = \prod_i |\varphi_i\rangle$$

$$\langle\varphi_1\varphi_2|P_{12}|\varphi_1\varphi_2\rangle = \langle\varphi_1\varphi_2|\varphi_2\varphi_1\rangle = |\langle\varphi_1|\varphi_2\rangle|^2$$

→ on 2 sites, energy minimal if $\langle\varphi_1|\varphi_2\rangle = 0$

→ on a lattice, MF energy certainly minimal if colors on nearest neighbors are different

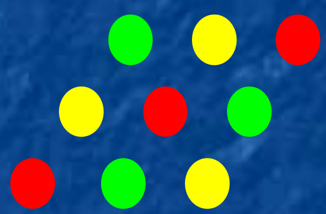
Typical phases

- Long-range color order
  → SU(3) on triangular and square lattice
- Plaquette order
  → SU(3) on kagome, honeycomb,…
- Algebraic order
  → SU(4) on honeycomb

Long-range color order

SU(3) Triangular lattice

3-sublattice order



Tsunetsugu, Arikawa, JPSJ 2006

A. Läuchli, FM, K. Penc, PRL 2006

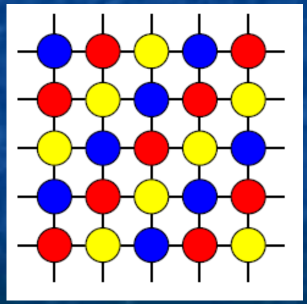
SU(3) Square lattice

A B A B

B A B A

A B A B

Selection by zero-point fluctuations →



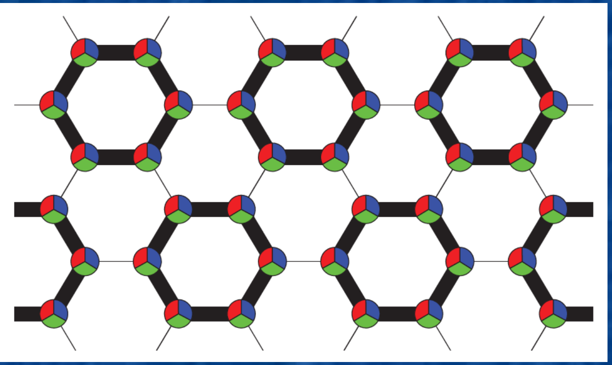
Infinite degeneracy

A, B → C

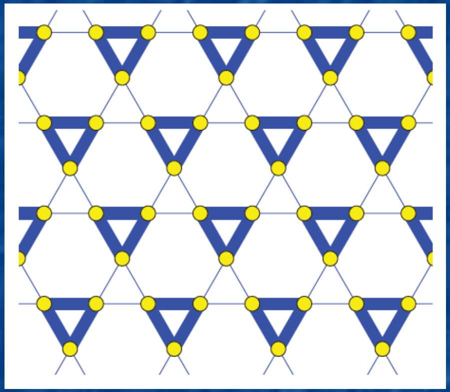
T. Toth, A. Läuchli, FM, K. Penc, PRL 2010

Plaquette order

SU(3)

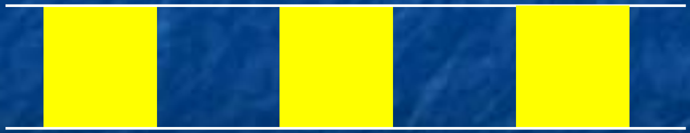


P. Corboz, M. Lajko, K. Penc, FM, A. Läuchli, PRB 2013

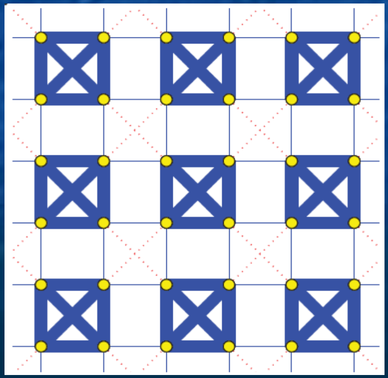


D.Arovas, PRB 2008; P. Corboz, K. Penc, FM, A. Läuchli, PRB 2012

SU(4)



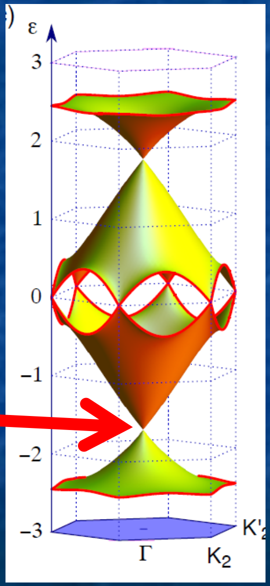
M. Van Den Bossche, P. Azaria, P. Lecheminant, FM, PRL 2001



SU(4) on honeycomb lattice

- No color order, no bond-energy order
- Good variational wave function with π-flux per plaquette
  → algebraic quantum liquid

Dirac point at quarter filling



P. Corboz, M. Lajko, A. Läuchli, K. Penc, FM, PRX 2012

Chiral spin liquids

- $N \rightarrow +\infty$, $m \rightarrow +\infty$, $k=N/m$ fixed
- Fermionic mean-field theory exact
- Chiral phase: Hofstadter spectrum for flux $2\pi/k$ per plaquette
- On square lattice, chiral phase for $k \geq 5$

Hermele, Gurarie, Rey, PRL 2009

What about finite N?

Numerical approaches for finite N

- Gutzwiller projected fermionic wave-functions
  → exact number of particles/site: variational
  → comparison of plaquette and chiral phases

- Exact diagonalizations taking full advantage of SU(N) symmetry   P. Nataf, FM, PRL 2014
  → basis of standard Young tableaus
  → large values of N for clusters with 20-30 sites

Hubbard model on triangular lattice

- Effective model up to order t^3/U^2

$$\mathcal{H} = J \sum_{\langle i,j \rangle} P_{ij} + \sum_{\langle i,j,k \rangle} \left(K\, P_{ijk} + \mathrm{h.c.} \right)$$

$$J = 2t^2/U - 12\cos(\Phi)\, t^3/U^2 \qquad K = -6\mathrm{e}^{i\Phi} t^3/U^2$$

- True for any N>2

Φ: flux per triangular plaquette (hopping term of Hubbard model)

Purely imaginary 3-site term

- $\Phi = \pi/2$ per triangular plaquette in Hubbard model
  → K purely imaginary: $K = i K_3$

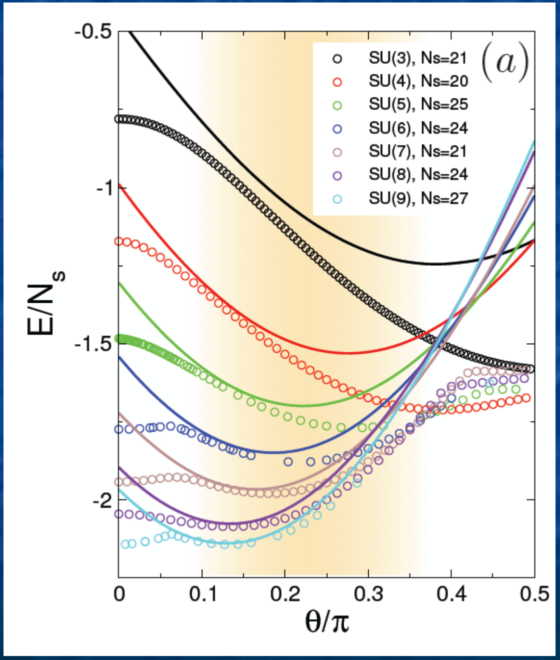
$$H = J \sum_{\langle i,j \rangle} P_{ij} + K_3 \sum_{(i,j,k)} (iP_{ijk} + h.c.)$$

- Time reversal symmetry explicitly broken
  → N low-lying singlets if chiral phase

Chiral phase for intermediate K_3

$$H = J \sum_{\langle i,j \rangle} P_{ij} + K_3 \sum_{(i,j,k)} (i P_{ijk} + h.c.)$$

$$J = \cos\theta, \quad K_3 = \sin\theta$$

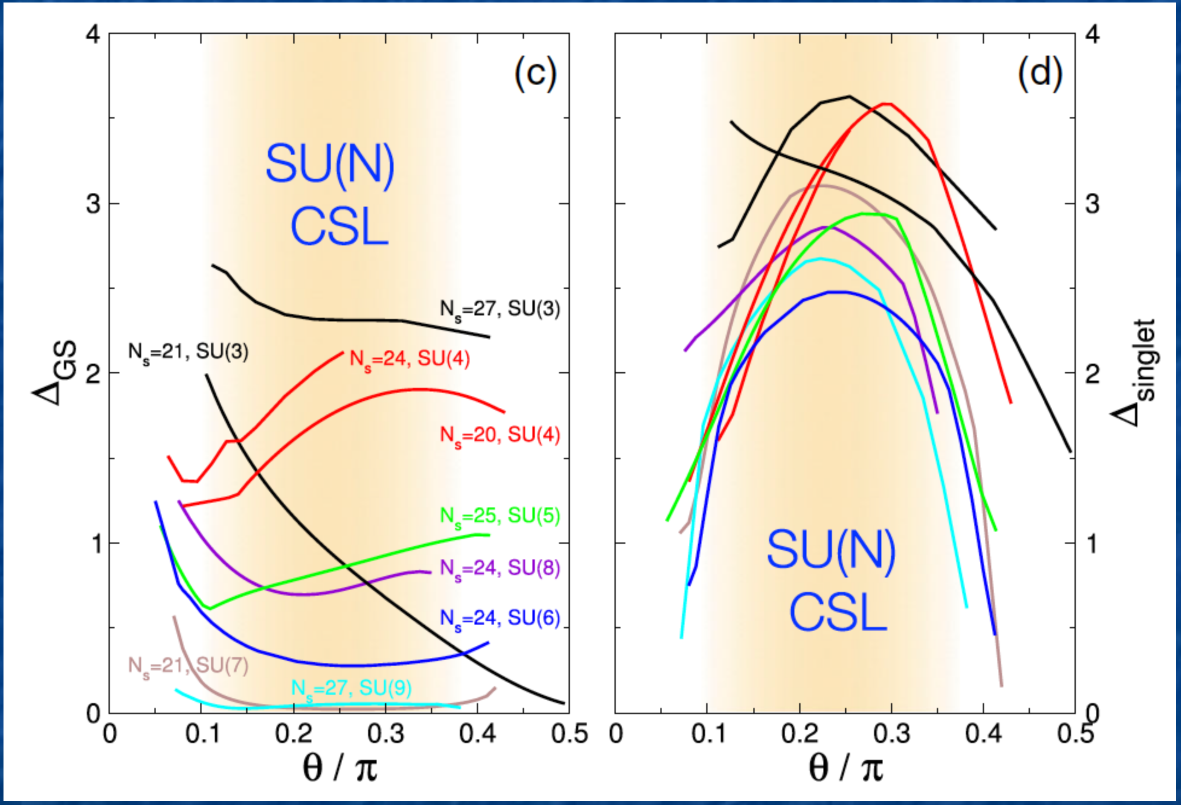


Excellent agreement between Gutzwiller projected Hofstadter wave function and ED for a range of K_3

→ Chiral liquid

P. Nataf, M. Lajko, A. Wietek, K. Penc, FM, A. Läuchli, PRL 2016

Ground-state quasi-degeneracy



N low-lying singlets on a torus

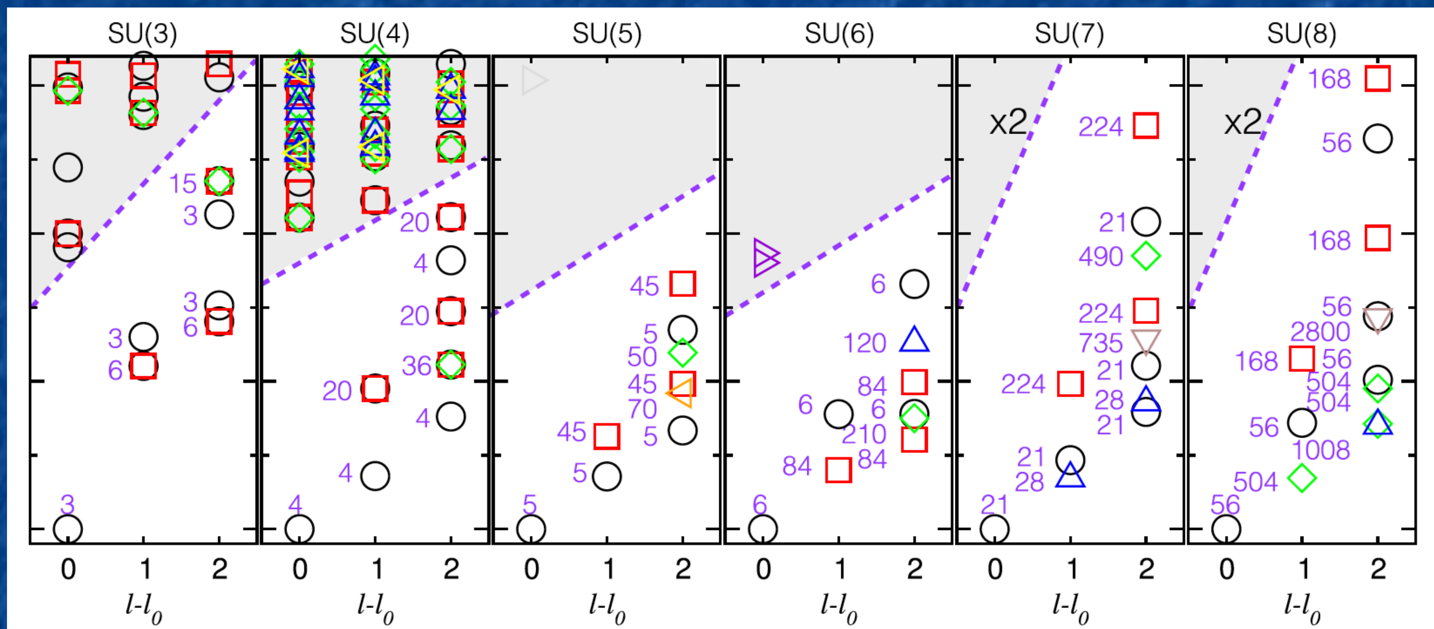
SU(N) Chern-Simons theory

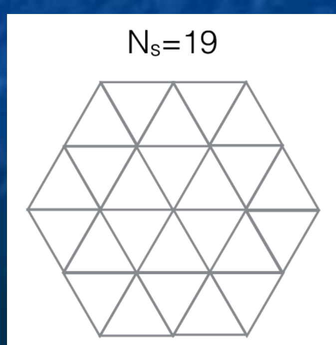
$$\mathcal{L}_{CS} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho} tr \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right)$$

$$A_\mu \in \text{Lie algebra of SU(N)}$$

- Antisymmetric irrep
  → k=1 (abelian chiral spin liquid)
- Degeneracy on a torus: N
- Edge states on a droplet: chiral $SU(N)_1$ WZW

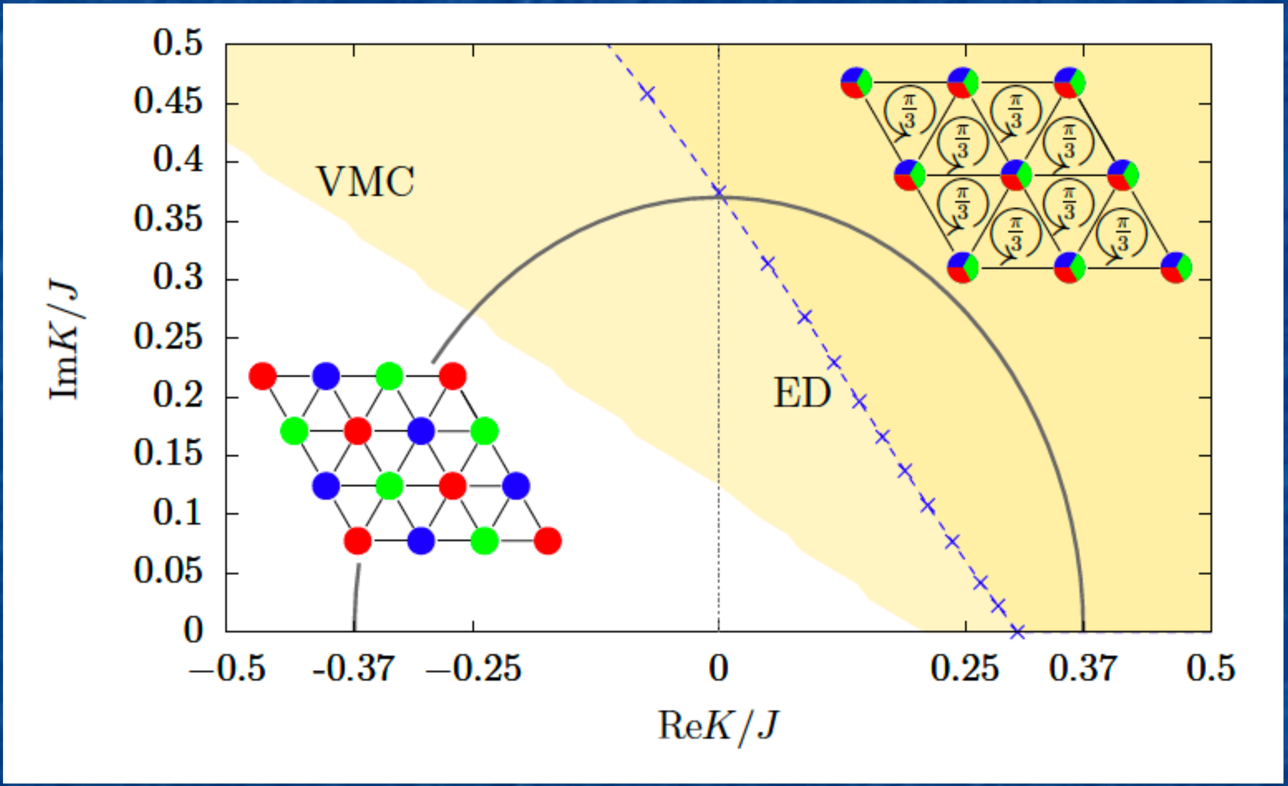
Edge states





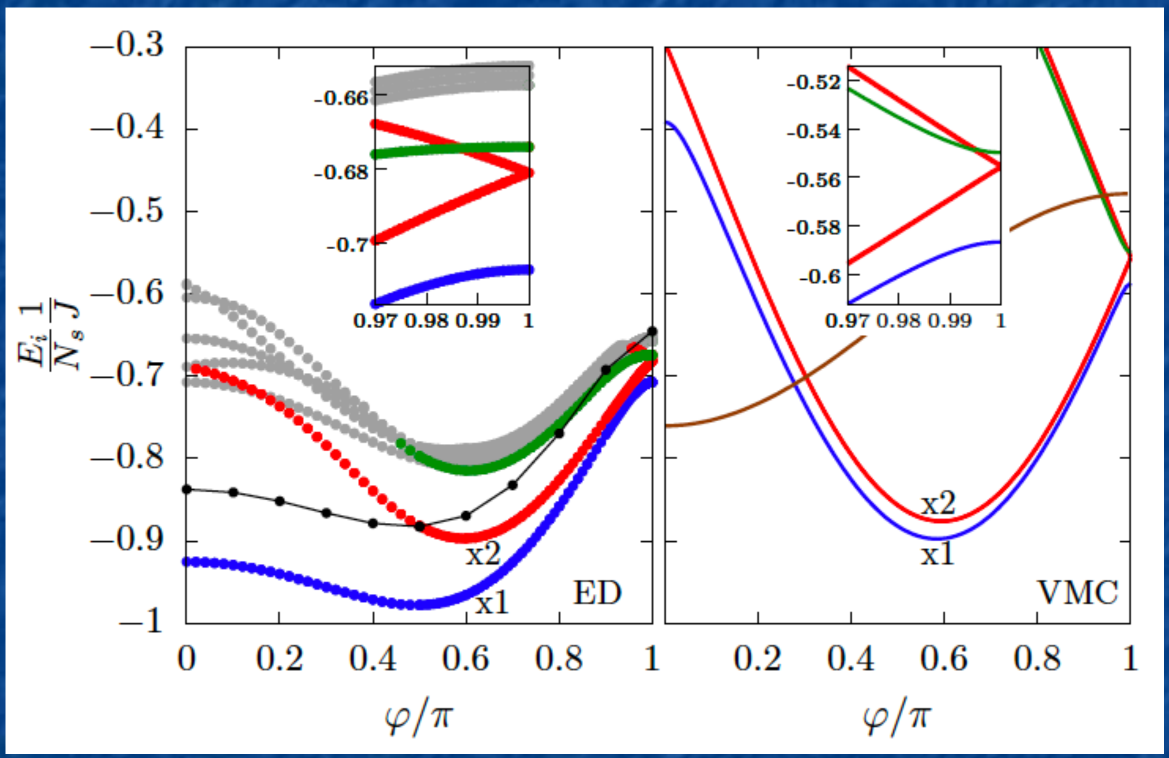
Tower of states predicted by conformal field theory

General K for SU(3)



C. Boos, M. Lajko, P. Nataf, K. Penc, K. Schmidt, FM, arXiv:1802.03179

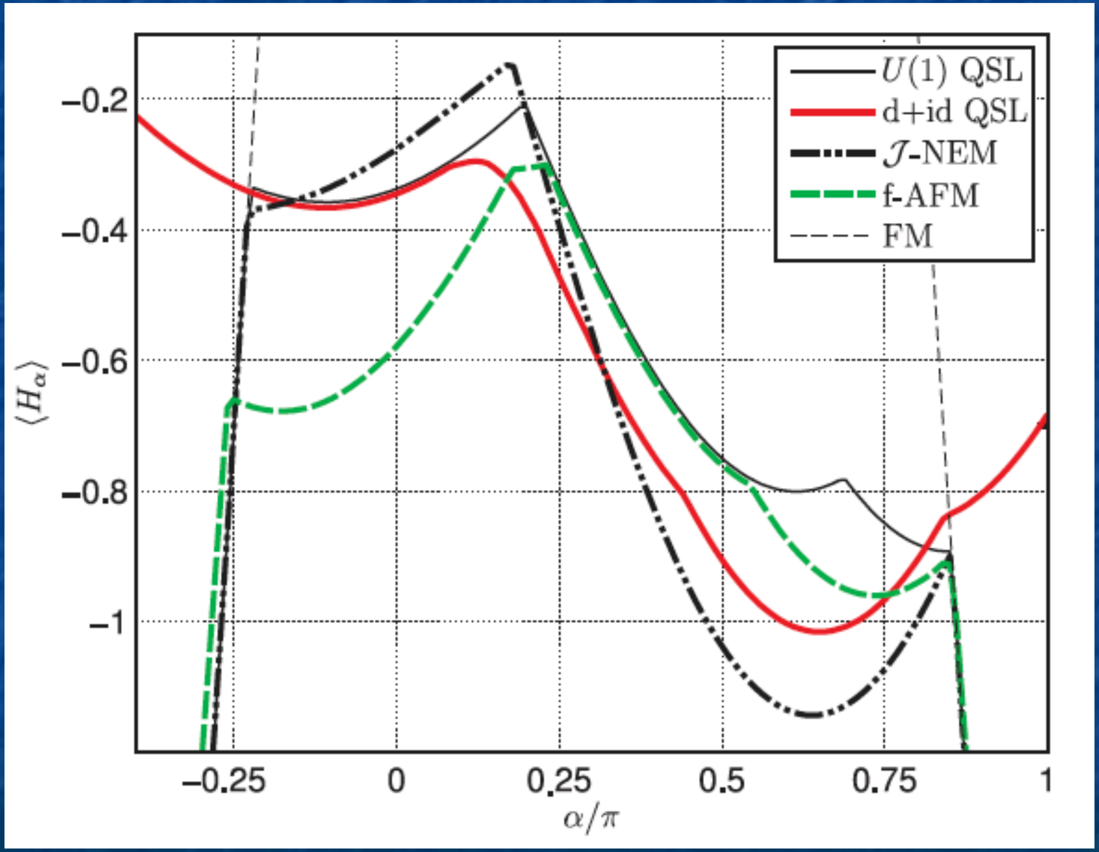
Spectrum of SU(3) along semi-circle



6 low-lying singlets at $\varphi=\pi$

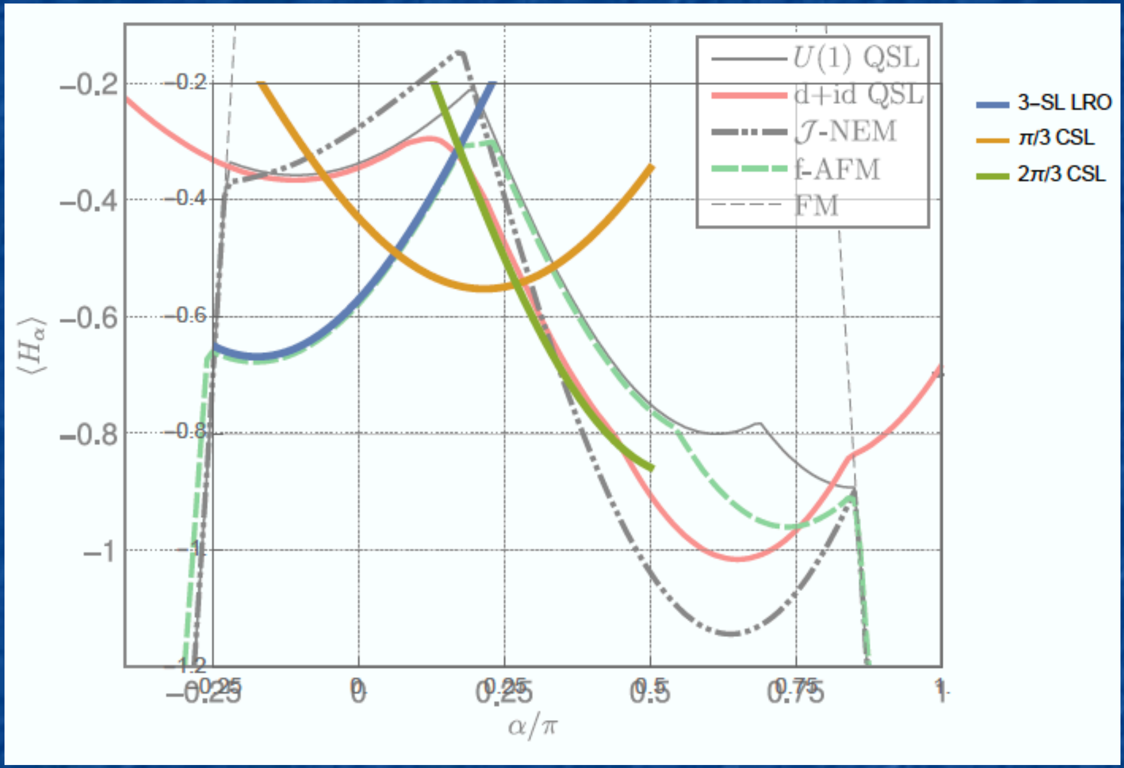
$K/J = 0.37\ e^{i\varphi}$

Real three-site term



Bieri, Serbyn, Senthil, Lee, 2012

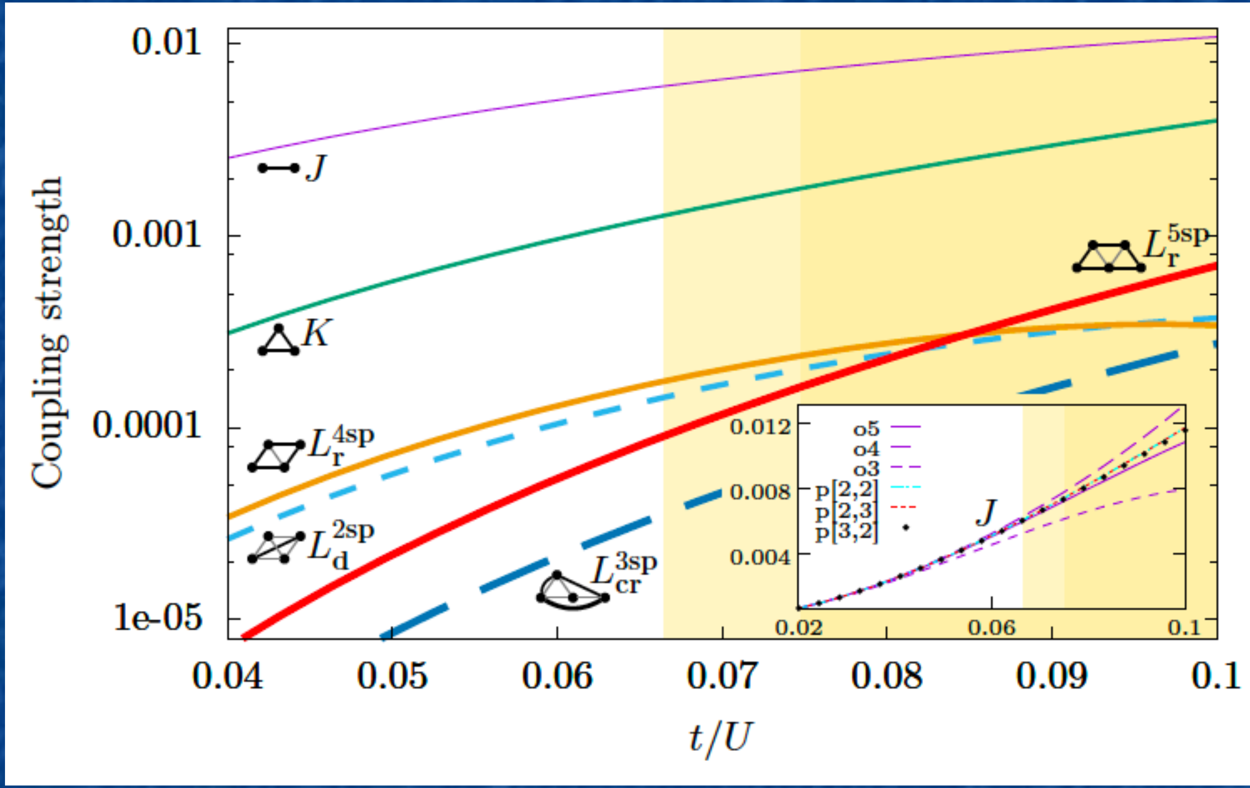
Comparison with chiral phases



Two chiral phases first proposed by Lai, 2013

M. Lajko et al, unpublished

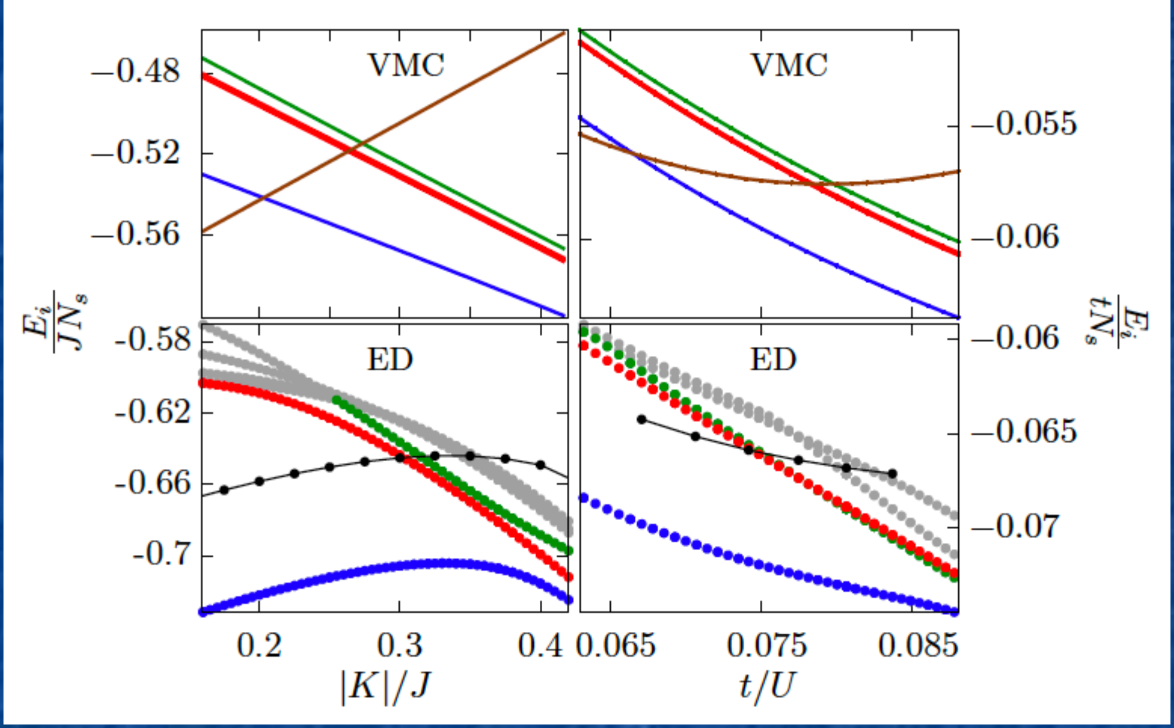
Connection to Hubbard



t/U = 0.07 <=> K/J = 0.25

J-K and higher order effective model on 21 sites

J-K

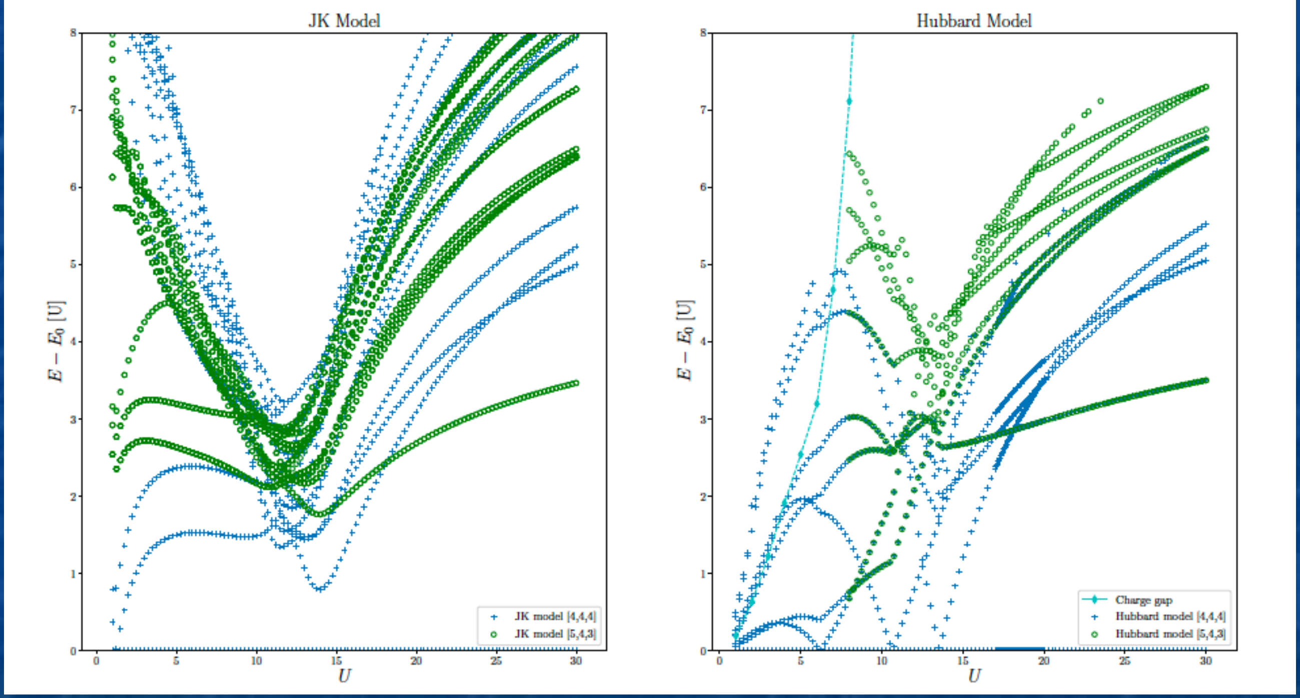


Higher order

6 low lying singlets for K/J not too small
→ spontaneous chiral symmetry breaking

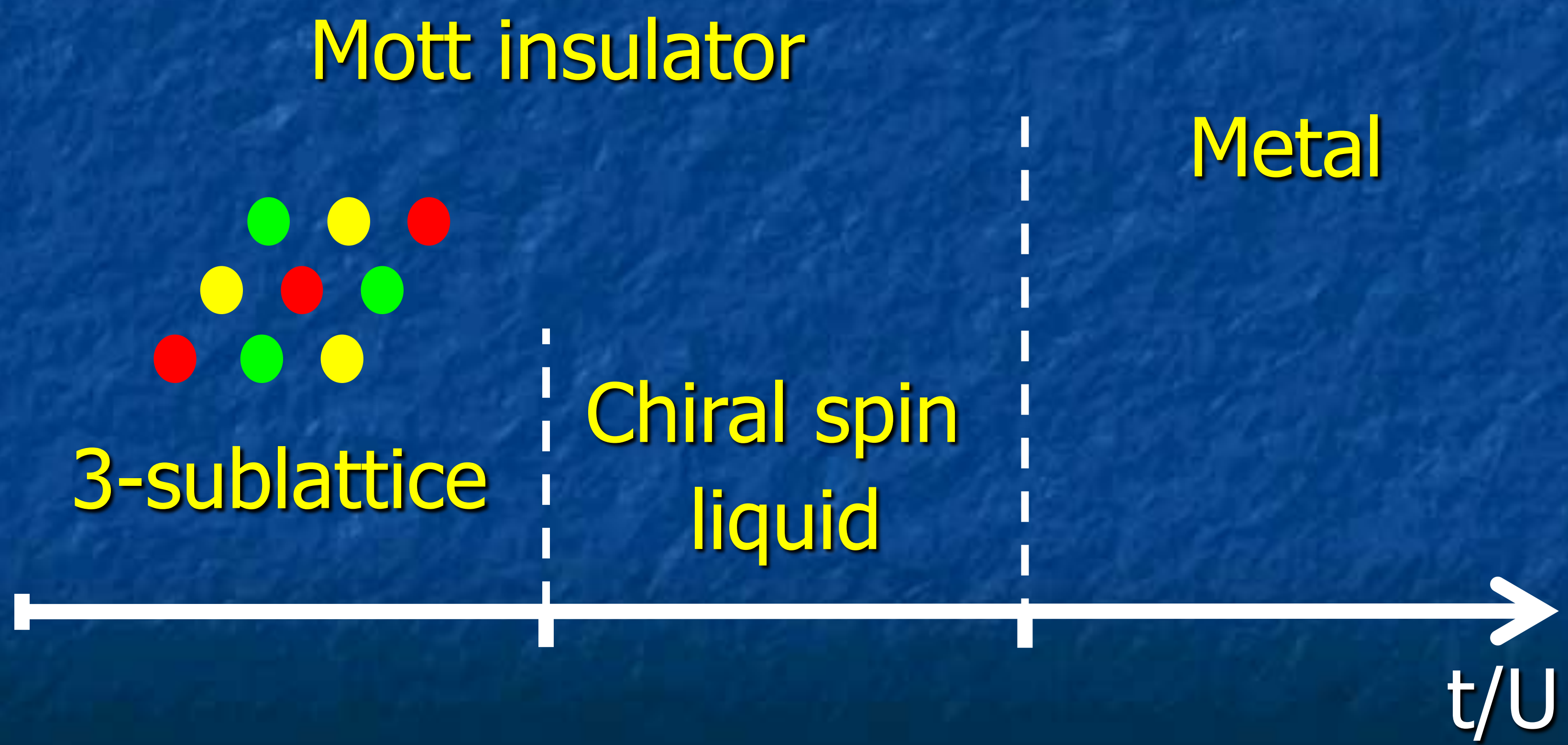
Comparison of J-K and Hubbard

12 sites



A. Läuchli et al, unpublished

Phase diagram of SU(3) Hubbard on triangular lattice



Conclusions

- SU(N) on simple 2D lattices

  → color order

  → singlet plaquettes

  → algebraic order

  → chiral liquid (with artificial gauge field)

  → spontaneous chiral symmetry breaking

for SU(3) Hubbard on triangular lattice