CRC183 ENTANGLED STATES OF MATTER

thp Institute for Theoretical Physics University of Cologne

Field-Driven $U(1)$ Quantum Spin Liquid in Kitaev's Honeycomb Model

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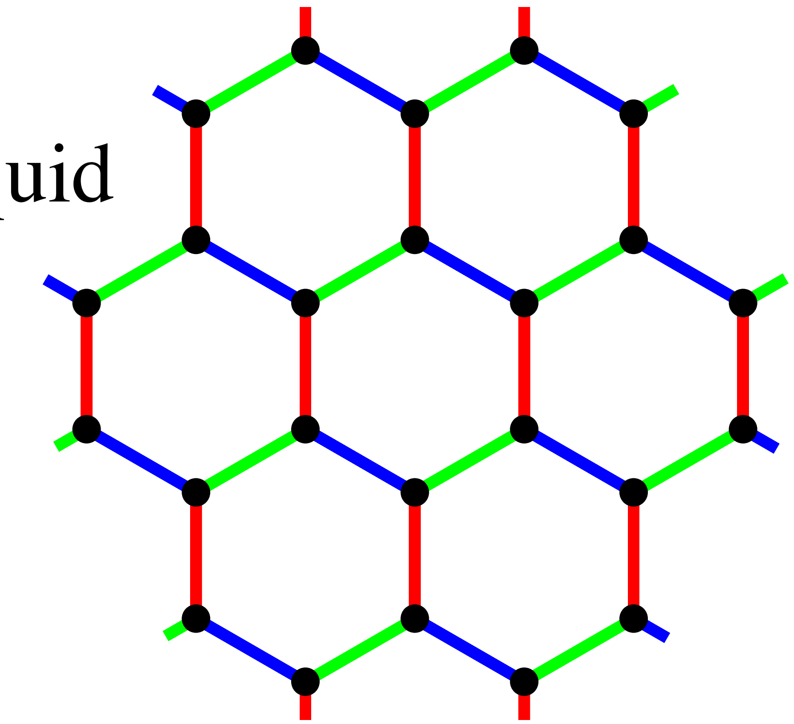
University of Cologne

Fri. 25th January 2019

Anyon19, MPI-PKS

Kitaev Model

$$H = \pm K \left(\sum_{\langle i,j \rangle \in x} S_i^x S_j^x + \sum_{\langle i,j \rangle \in y} S_i^y S_j^y + \sum_{\langle i,j \rangle \in z} S_i^z S_j^z \right)$$



- Exactly solvable model of a Z_2 quantum spin liquid

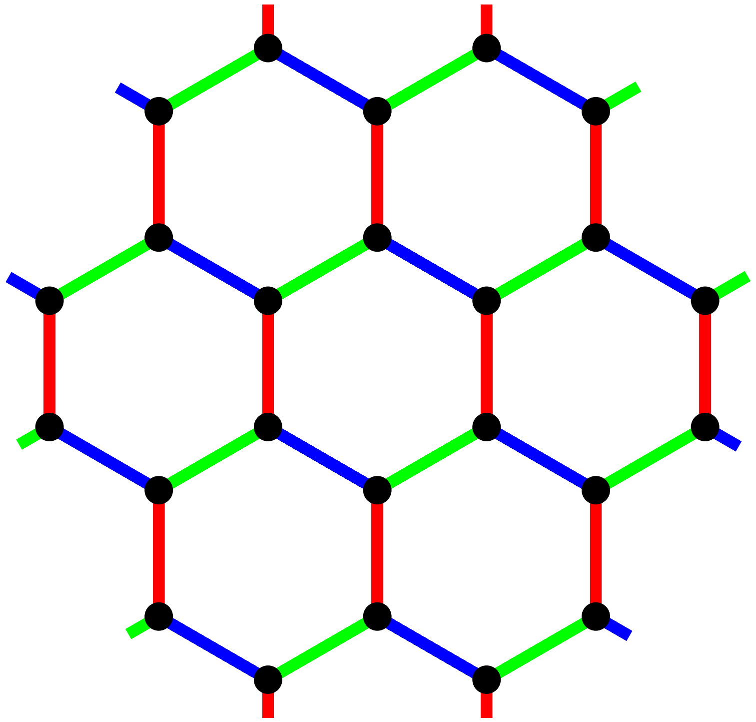
  (with only NN two-spin interactions, which can be realised in materials)

- Rich playground for investigating exact ground state, dynamical and finite temperature properties of Z_2 spin liquids

A. Kitaev, Annals of Physics **321**, 2 (2006)

Kitaev Model

$$H = \pm K \left(\sum_{\langle i,j \rangle \in x} S_i^x S_j^x + \sum_{\langle i,j \rangle \in y} S_i^y S_j^y + \sum_{\langle i,j \rangle \in z} S_i^z S_j^z \right)$$



Kitaev Model
+
Magnetic Field $\approx$???

A. Kitaev, Annals of Physics **321**, 2 (2006)

Outline

- Introduction
- The Kitaev Model in a Magnetic Field
  - Q. Nature of the KSL?
  - Q. Critical Field FM vs AFM?
  - Q. Nature of the GSL?
- Discussion & Conclusion

CH, S. Trebst, arXiv:1805.05953 (Nat. Comm., in press)

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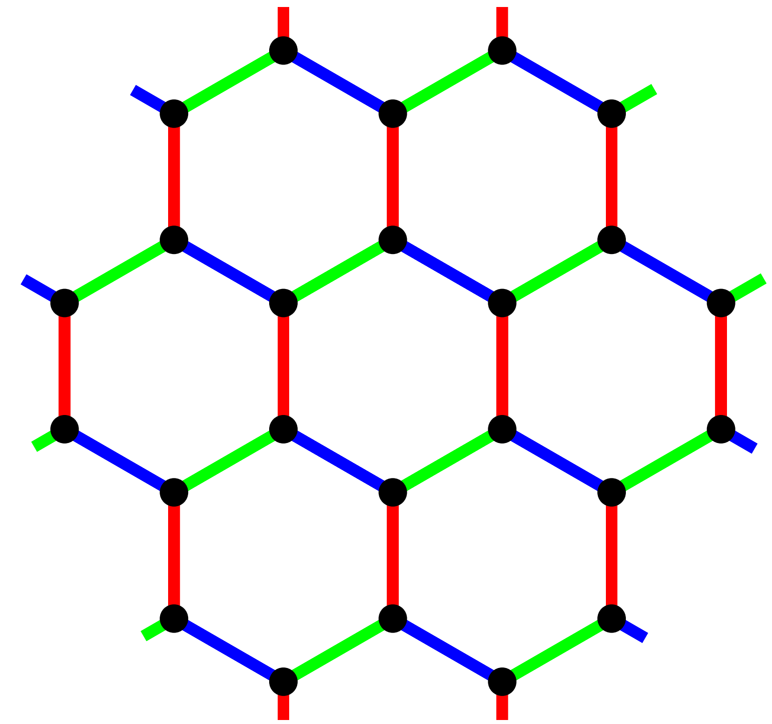
Kitaev Model

$$H = \pm K \left(\sum_{\langle i,j \rangle \in x} S_i^x S_j^x + \sum_{\langle i,j \rangle \in y} S_i^y S_j^y + \sum_{\langle i,j \rangle \in z} S_i^z S_j^z \right)$$

$$H = \pm K \frac{i}{2} \sum_{\langle i,j \rangle} \hat{u}_{ij} c_i c_j$$



Single, free Majorana fermion coupled to static Z_2 gauge field!!!

Kitaev Model

$$H = \pm K \left(\sum_{\langle i,j \rangle \in x} S_i^x S_j^x + \sum_{\langle i,j \rangle \in y} S_i^y S_j^y + \sum_{\langle i,j \rangle \in z} S_i^z S_j^z \right)$$

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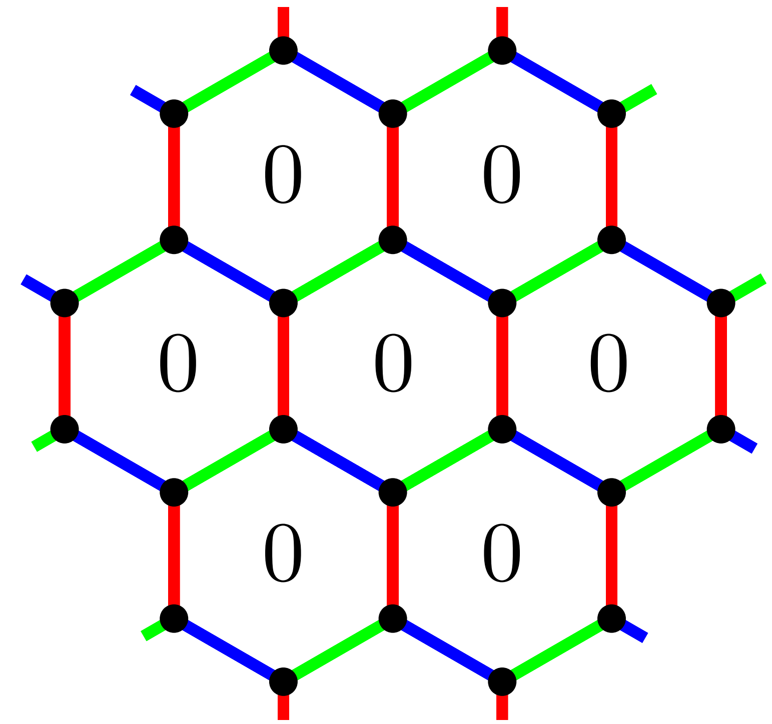
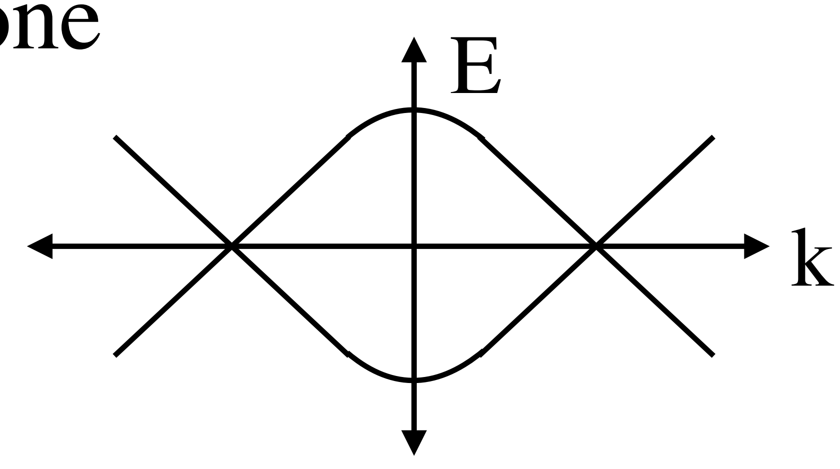


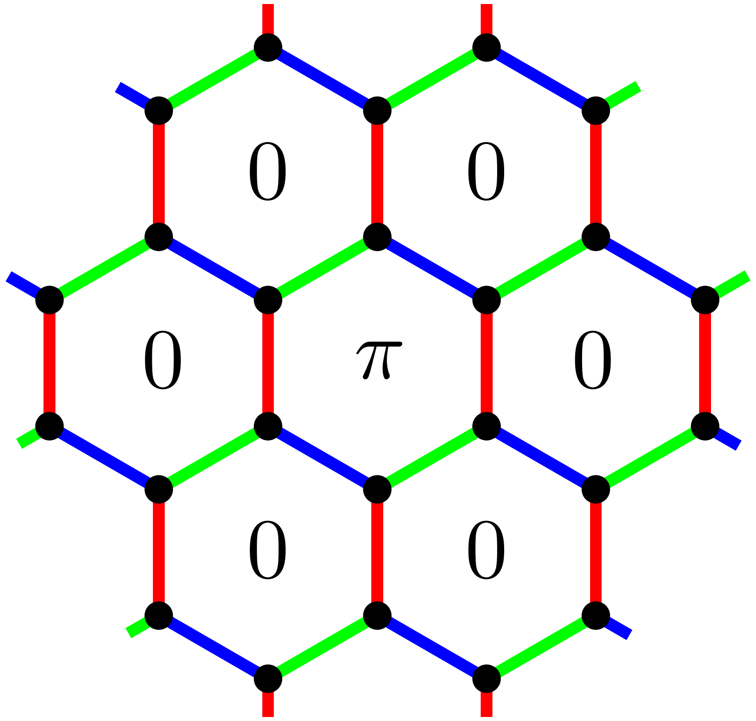
- Ground State:
  - Majorana fermions form a Dirac cone
  - All plaquette fluxes = 0



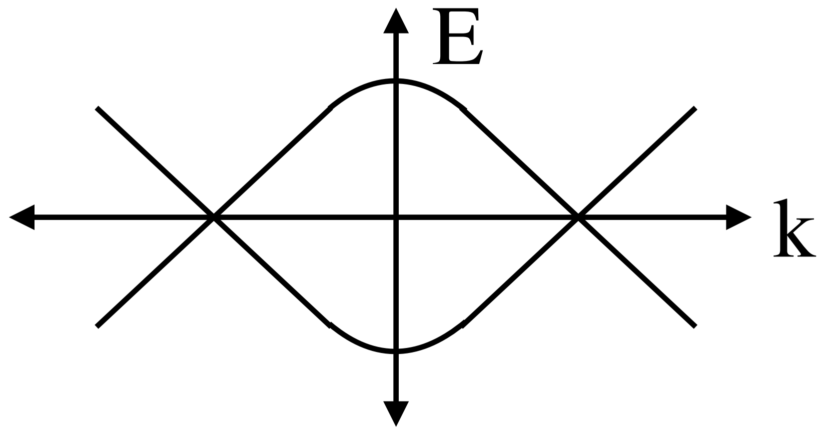
Kitaev Model

$$H = \pm K \left(\sum_{\langle i,j \rangle \in x} S_i^x S_j^x + \sum_{\langle i,j \rangle \in y} S_i^y S_j^y + \sum_{\langle i,j \rangle \in z} S_i^z S_j^z \right)$$

$$H = \pm K \frac{i}{2} \sum_{\langle i,j \rangle} \hat{u}_{ij} c_i c_j$$



- Two kinds of excitations:
    - Gapless Majorana fermions
    - Gapped flux excitations (visons)



Kitaev Materials

- Kitaev interactions can actually be realized in materials

Ingredients:

d-orbitals
+
Crystal field splitting
+
Spin-orbit coupling
+
Interactions

→ $j_{eff} = 1/2$

$|\uparrow\rangle_{eff}$

$|\downarrow\rangle_{eff}$

$$|\uparrow\rangle_{eff} \sim i\,|zx,\downarrow\rangle + |yz,\downarrow\rangle + |xy,\uparrow\rangle$$

$$|\downarrow\rangle_{eff} \sim -i\,|zx,\uparrow\rangle + |yz,\uparrow\rangle - |xy,\downarrow\rangle$$

G. Jackeli and G. Khaliullin, PRL **102**, 017205 (2009)

Kitaev Materials

- Kitaev interactions can actually be realized in materials

Ingredients:

d-orbitals
+
Crystal field splitting
+
Spin-orbit coupling
+
Interactions

Spin-orbit entangled Mott Insulators!

$j_{eff} = 1/2$

$|\uparrow\rangle_{eff}$

$|\downarrow\rangle_{eff}$

$$|\uparrow\rangle_{eff} \sim i\,|zx,\downarrow\rangle + |yz,\downarrow\rangle + |xy,\uparrow\rangle$$

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Kitaev Materials

- Kitaev interactions can actually be realized in materials

Ingredients:

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Crystal field splitting
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Spin-orbit coupling
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Interactions

→ $j_{eff} = 1/2$

$|\uparrow\rangle_{eff}$

$|\downarrow\rangle_{eff}$

Na_2IrO_3 Li_2IrO_3 $RuCl_3$ $H_3LiIr_2O_6$ Cu_2IrO_3 K_2IrO_3 ...

Good for neutrons :) ($RuCl_3$)

Bad for neutrons :((Ir)

Kitaev Materials

- Kitaev interactions can actually be realized in materials
- Minimal model includes:

$$H = \pm K \sum_{\langle i,j \rangle \in \alpha} S_i^\alpha S_j^\alpha \pm J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \pm \Gamma \sum_{\langle i,j \rangle \in \alpha} \left(S_i^\beta S_j^\gamma + S_i^\gamma S_j^\beta \right)$$

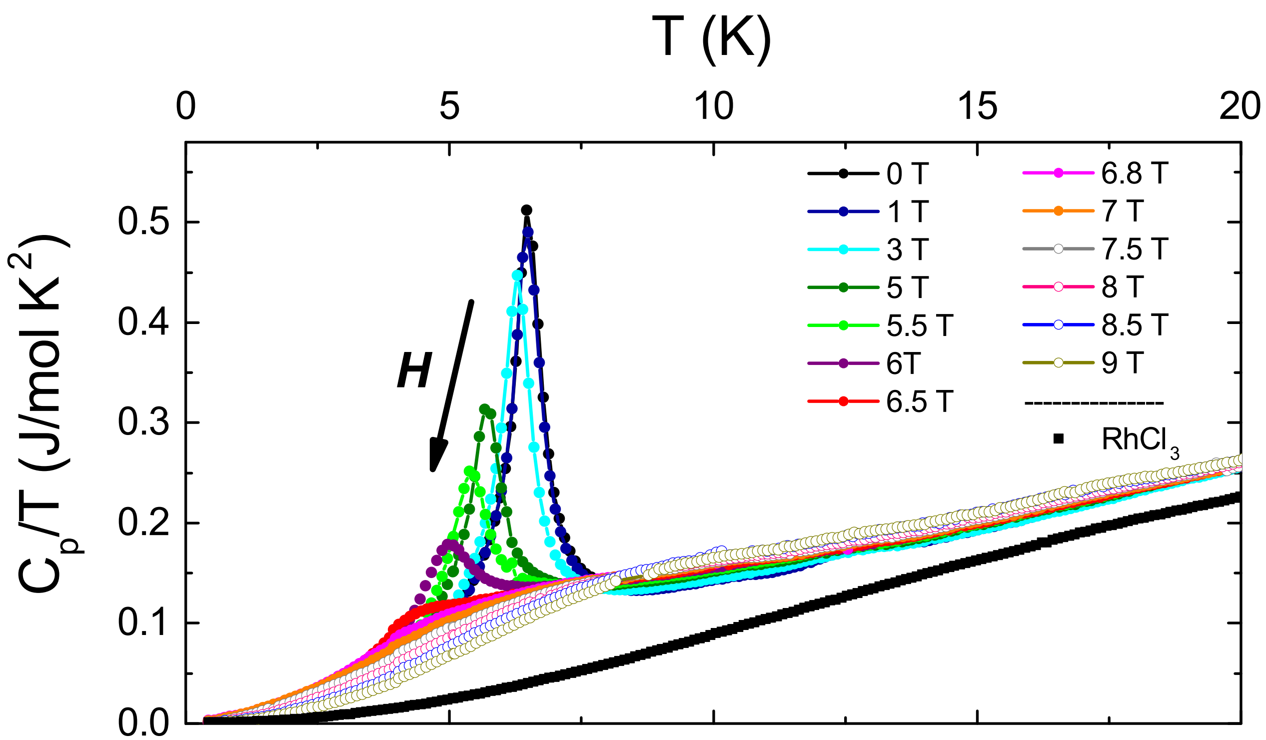
Kitaev　　Heisenberg　　Symmetric Exchange

Na_2IrO_3　Li_2IrO_3　$RuCl_3$　$H_3LiIr_2O_6$　Cu_2IrO_3　K_2IrO_3　$\ldots$

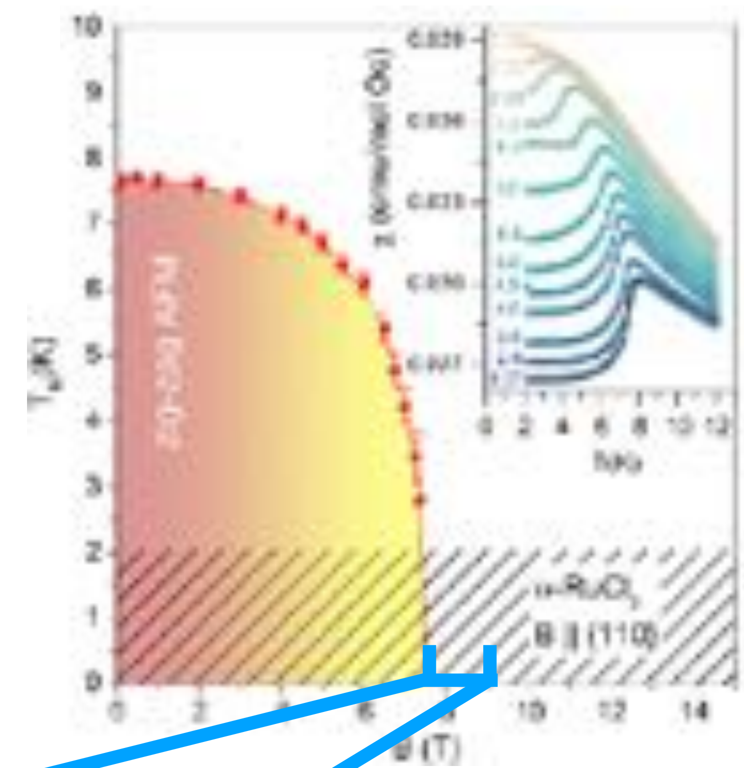
$RuCl_3$: Good for neutrons :)

Cu_2IrO_3: Bad for neutrons :(

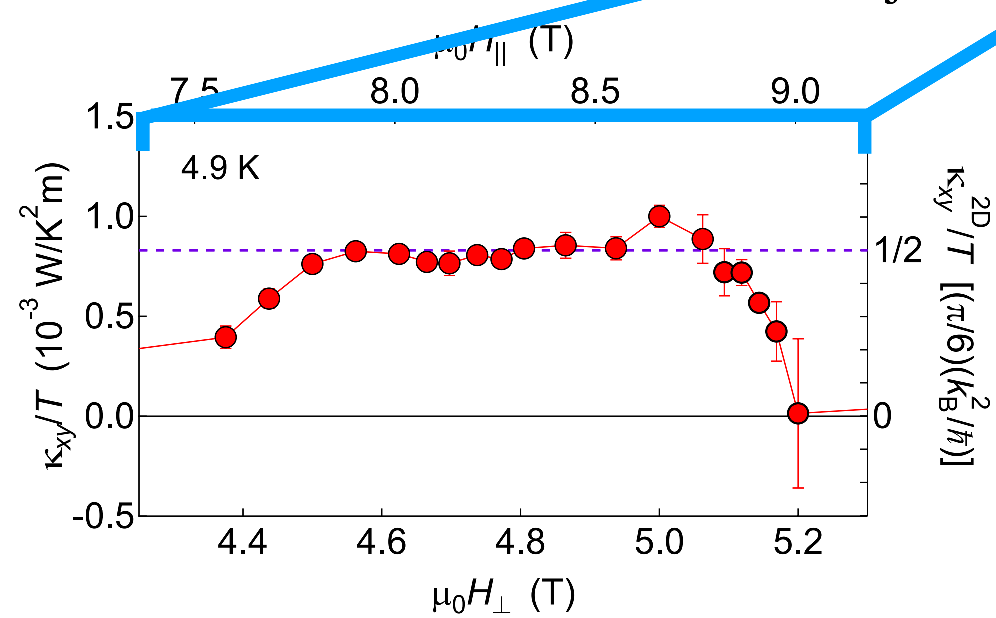
$RuCl_3$ in Field



A. U. B. Wolter et al., PRB **96**, 041405(R) (2017)



A. Banerjee et al., npj Quantum Mater. **3**, 8 (2018)



Y. Kasahara et al., Nature **559**, 227-231 (2018)

Na_2IrO_3 Li_2IrO_3 $RuCl_3$ $H_3LiIr_2O_6$ Cu_2IrO_3 K_2IrO_3 ...

Motivation

1. Before we try to understand the complicated materials:

First we should try to understand just the pure Kitaev model in a magnetic field

2. From a theory perspective:

Natural to ask what happens to Kitaev's quantum spin liquid in a magnetic field

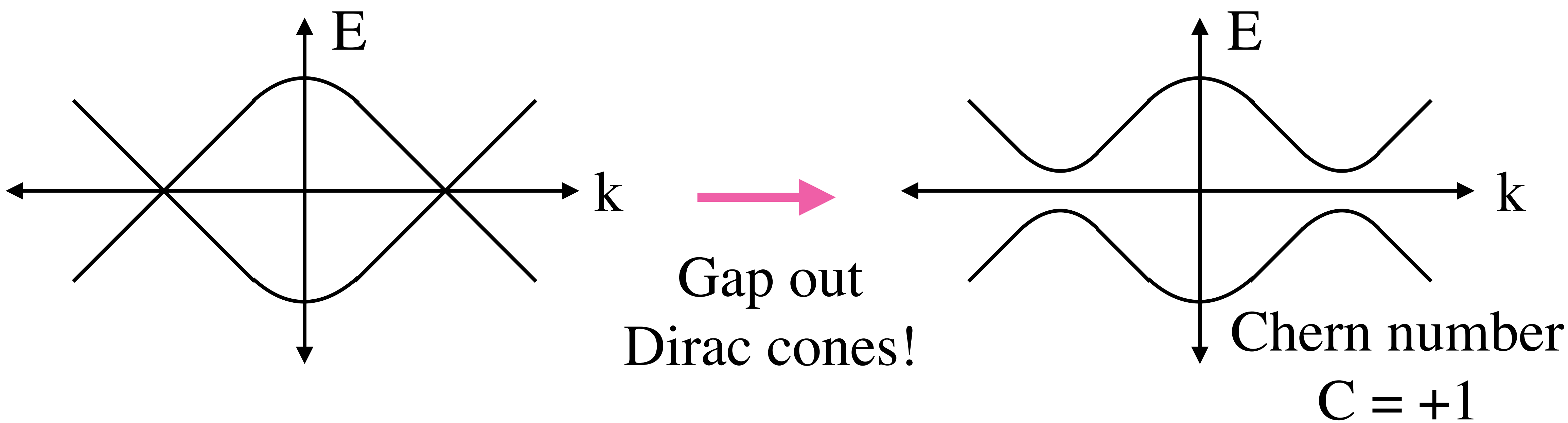
Adding a Magnetic Field

- What happens if we add a magnetic field?

$$H = \pm K \sum_{\langle i,j \rangle \in \gamma} S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$

Kitaev’s Gapless QSL in a [111] Field

- Kitaev showed, using perturbation theory, that a [111] magnetic field can:



- We have a gapped insulator of Majorana’s with C = +1
- Single, chiral Majorana edge mode with half-integer quantised thermal Hall conductance
- In terms of spins we have a gapped quantum spin liquid (KSL)

Adding a Magnetic Field

- What happens if we add a magnetic field?

$$H = \pm K \sum_{\langle i,j \rangle \in \gamma} S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$

FM - Kitaev
Coupling -K → h

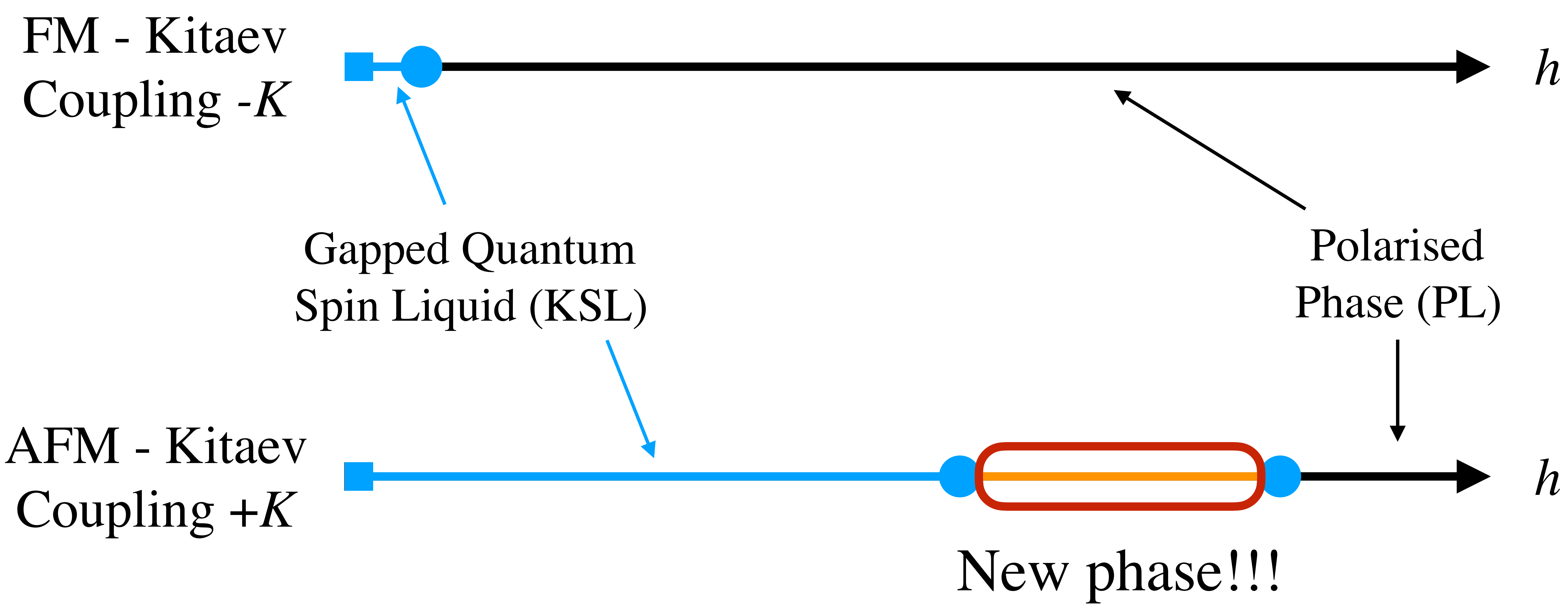
$h << K$		$h >> K$
Gapped Quantum Spin Liquid	???	Polarised along field direction

AFM - Kitaev
Coupling +K → h

Adding a Magnetic Field

- What happens if we add a magnetic field?

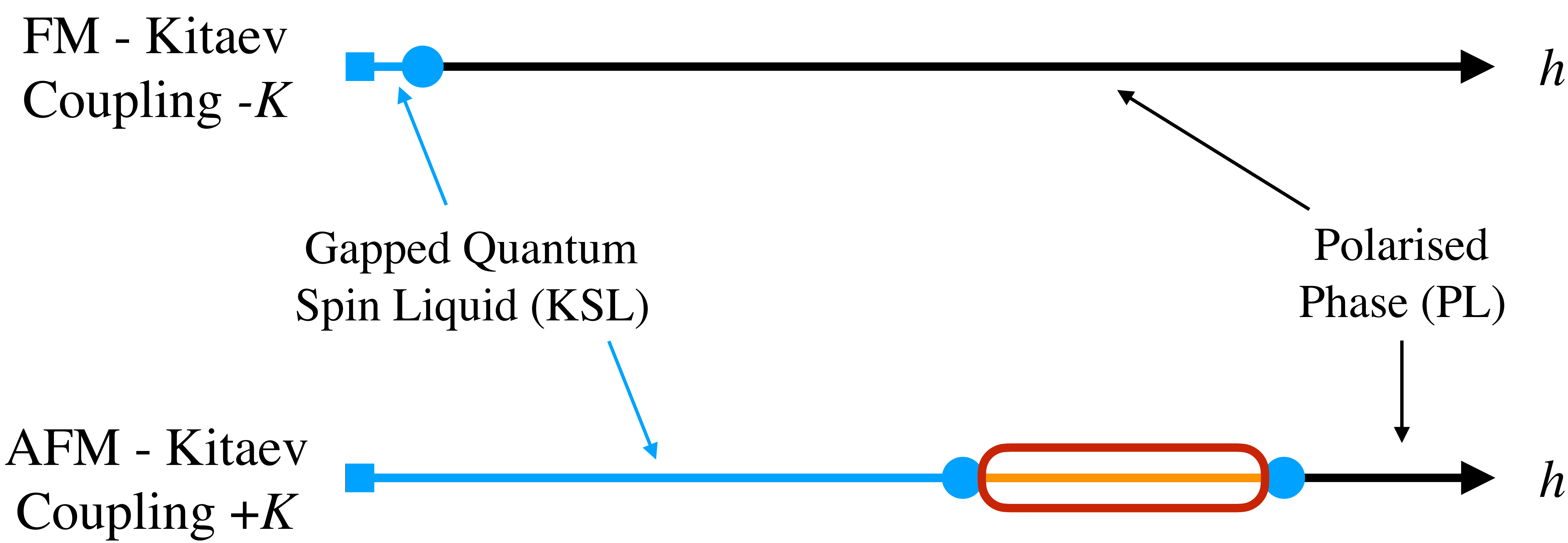
$$H = \pm K \sum_{\langle i,j \rangle \in \gamma} S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$



Adding a Magnetic Field

- What happens if we add a magnetic field?

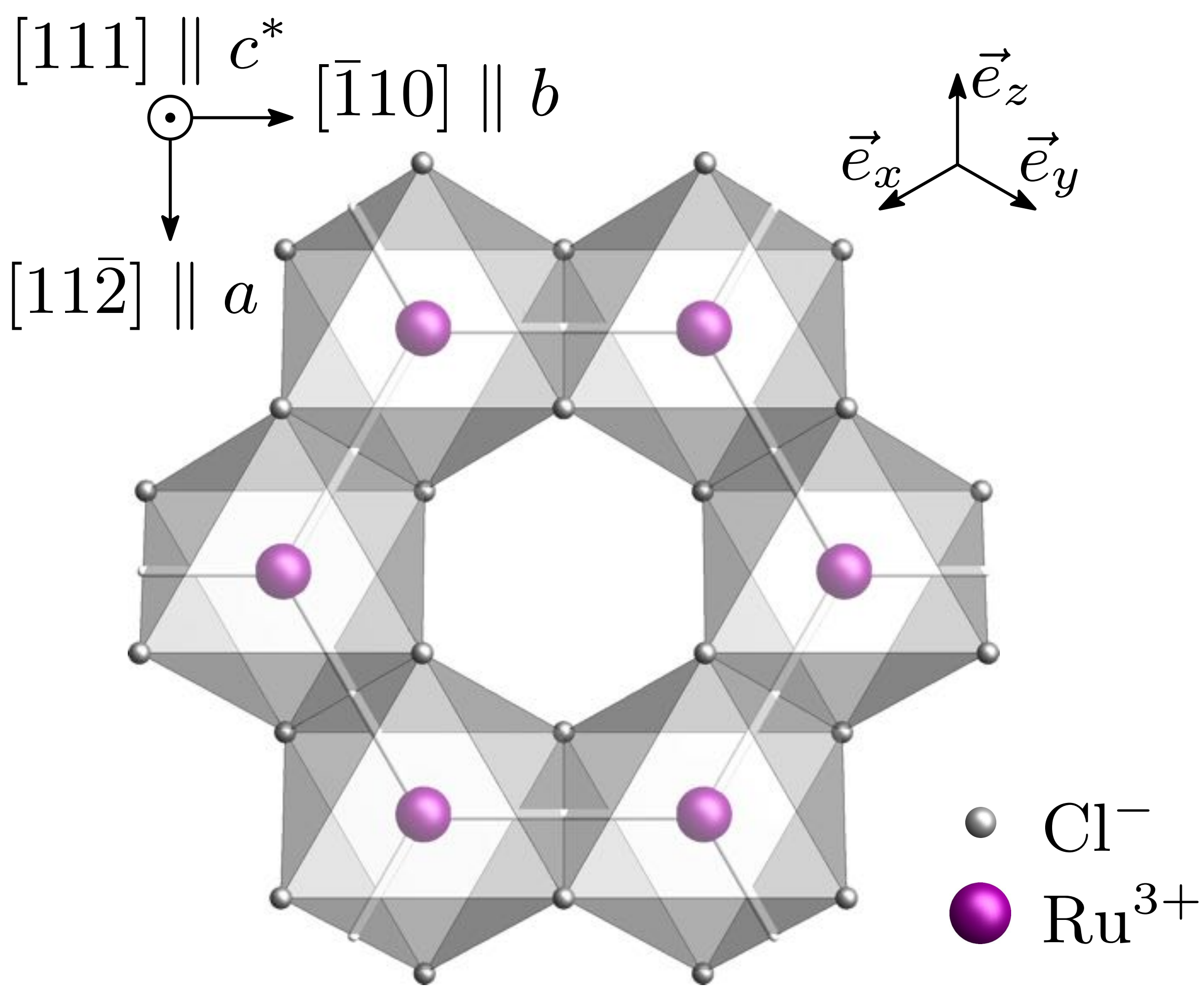
$$H = \pm K \sum_{\langle i,j\rangle \in \gamma} S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$



h || [111] Direction:

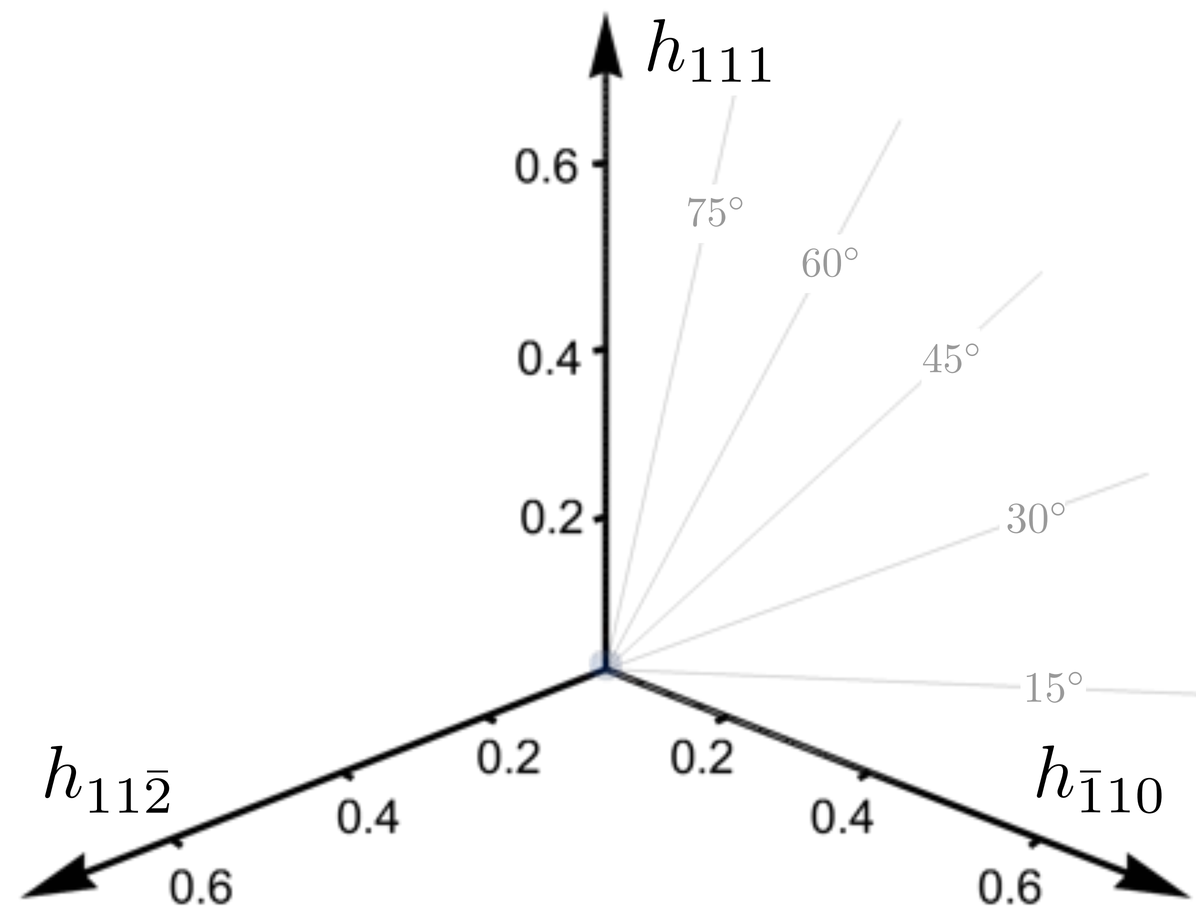
- S. Fey, MSc Thesis (2013)
- Z. Zhu et al., PRB **97**, 241110(R) (2018)
- M. Gohlke et al., PRB **98**, 014418 (2018)

Phase Diagrams in Tilted Magnetic Fields



L. Janssen et al., PRB **96**, 064430 (2017)

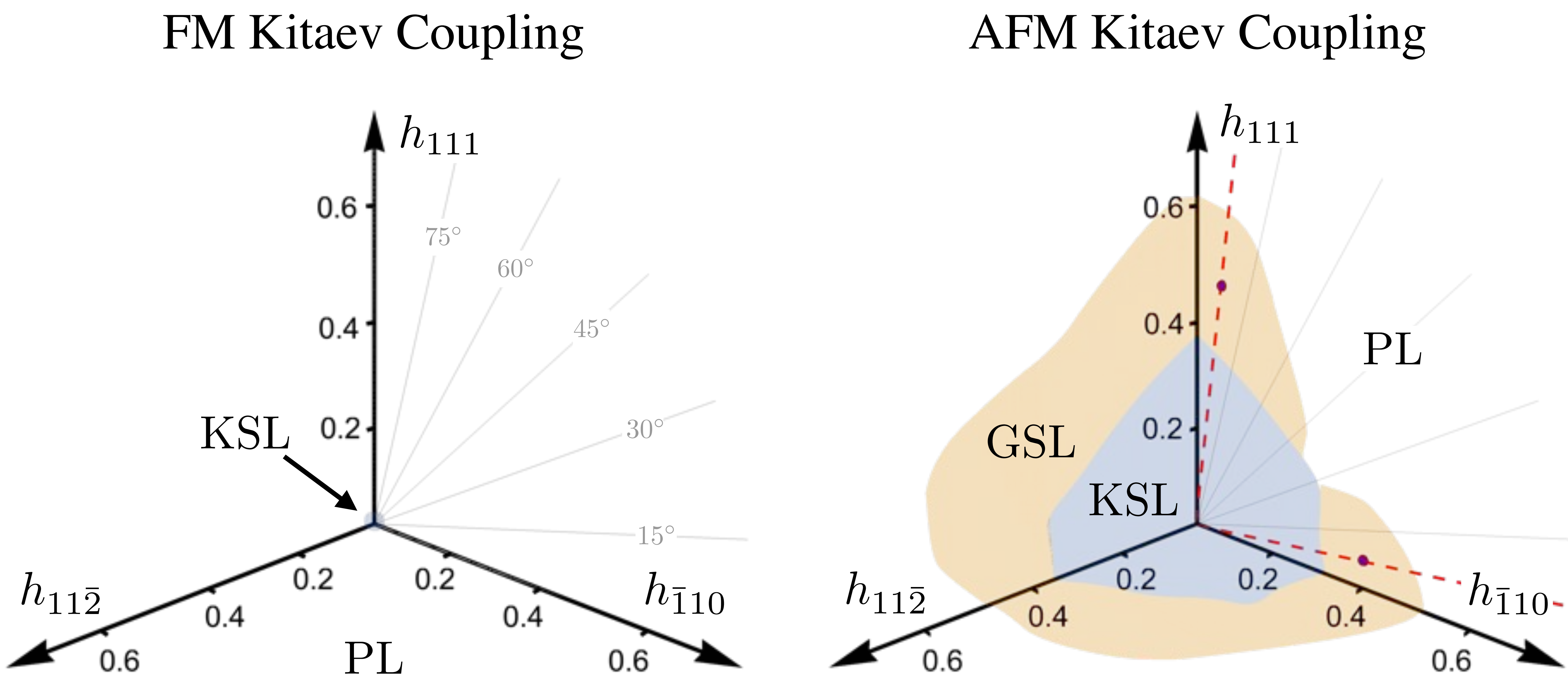
Out-of-plane field direction (c axis)



In-plane field directions (a, b axes)

Phase Diagrams in Tilted Magnetic Fields

Uniform **h**



KSL = Kitaev Spin Liquid

PL = Polarised phase (trivial)

GSL = Gapless Spin Liquid

Phase Diagrams in Tilted Magnetic Fields

Q. What is the nature of the KSL phase in a magnetic field?

Q. Why is the critical field so different?

Q. What is the nature of the intermediate "GSL" phase?

KSL = Kitaev Spin Liquid
PL = Polarised phase (trivial)

GSL = Gapless Spin Liquid

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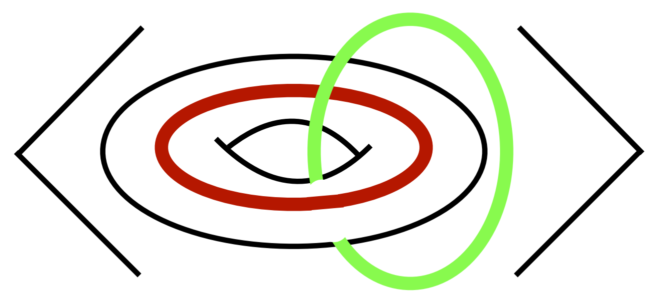
Gapped KSL

- From Kitaev's perturbative arguments we know the phase should exhibit "Ising Anyon Topological Order"

- Can be identified by the modular S-matrix:

$$S_{ab} \sim$$



Info on quantum dimensions

$$S = \begin{pmatrix} 0.50 & 0.71 & 0.50 \\ 0.71 & 0.00 & -0.71 \\ 0.50 & -0.71 & 0.50 \end{pmatrix}$$

Theoretical Result

$$S = \begin{pmatrix} 0.46 & 0.74 & 0.47 \\ 0.71 & 0.04e^{-0.91i} & -0.70 \\ 0.49 & -0.67e^{0.02i} & 0.58e^{-0.13i} \end{pmatrix}$$

Numerical Result

Ising Anyon Topological Order!

Q. What is the nature of the KSL phase in a magnetic field?

A. Gapped Quantum Spin Liquid with "Ising Anyons"

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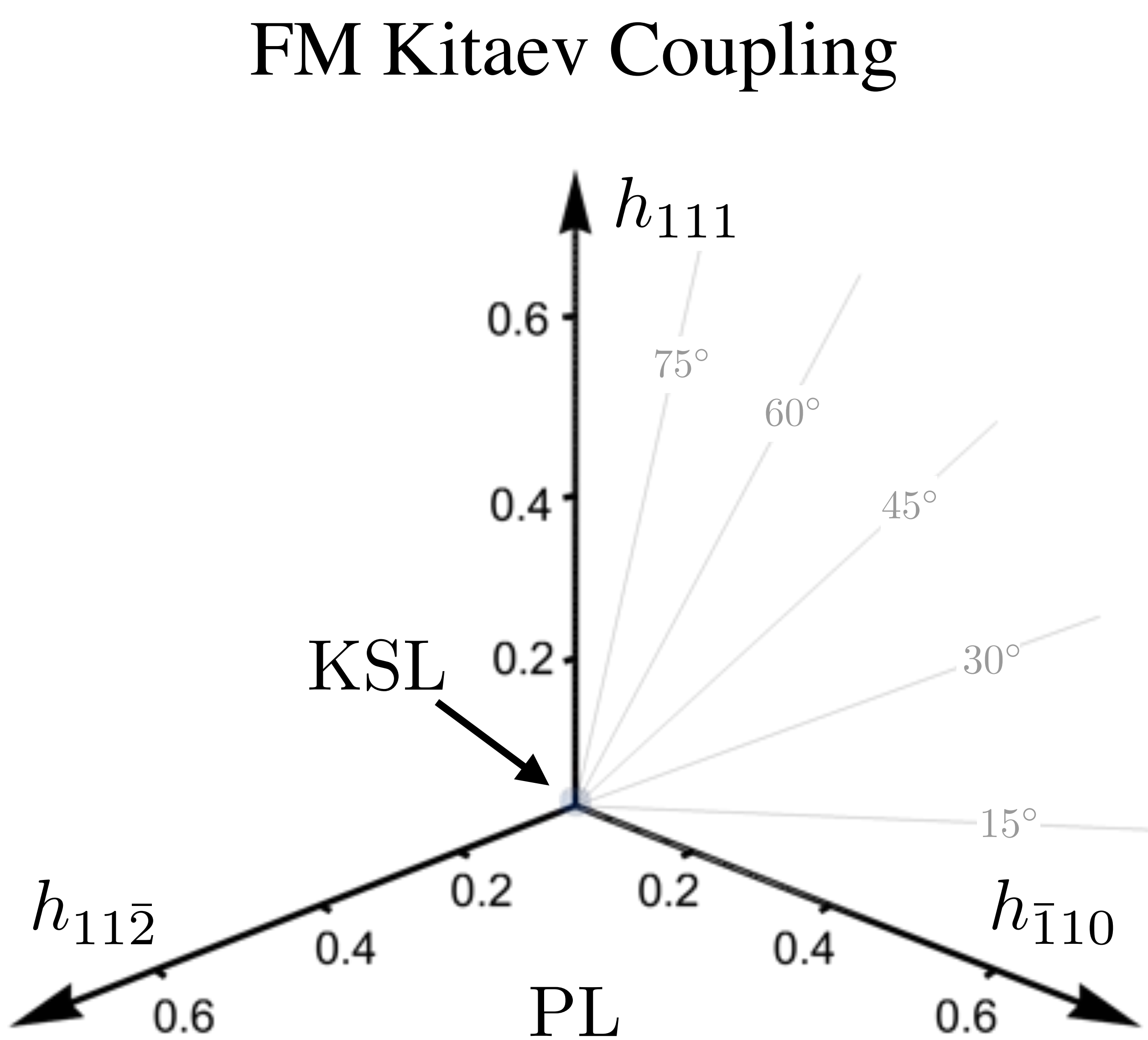
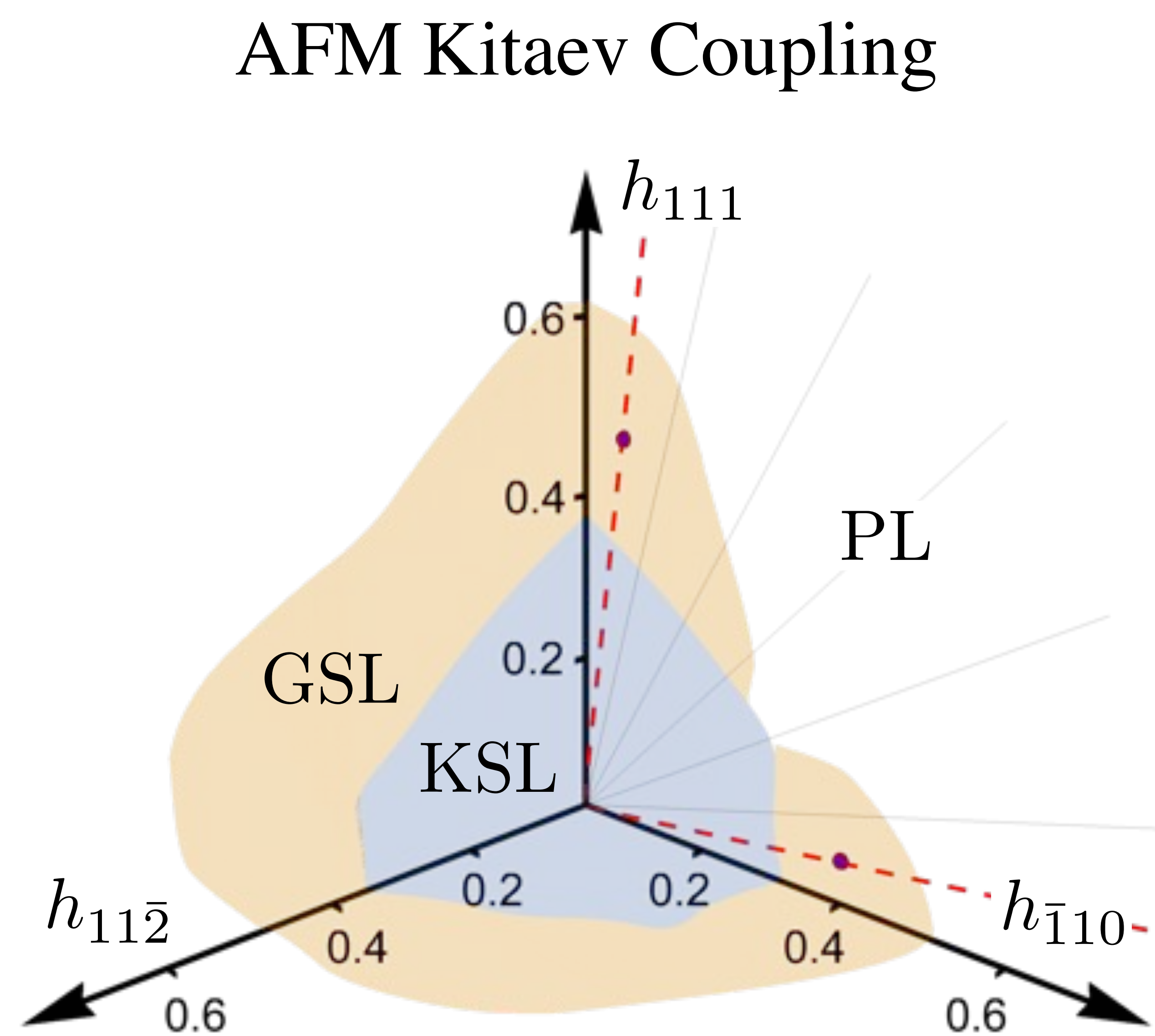
Critical Field for KSL

Uniform $\mathbf{h}$

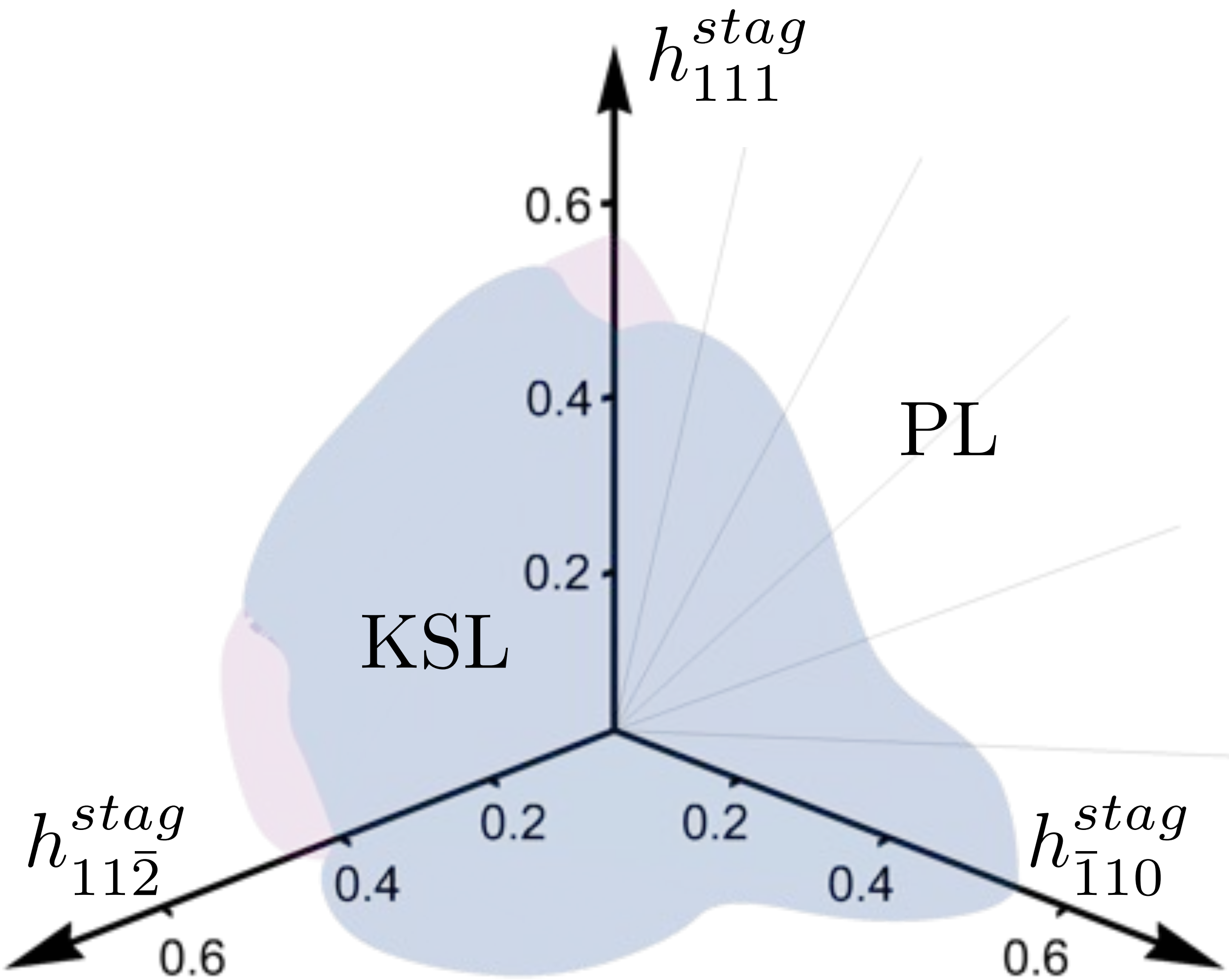




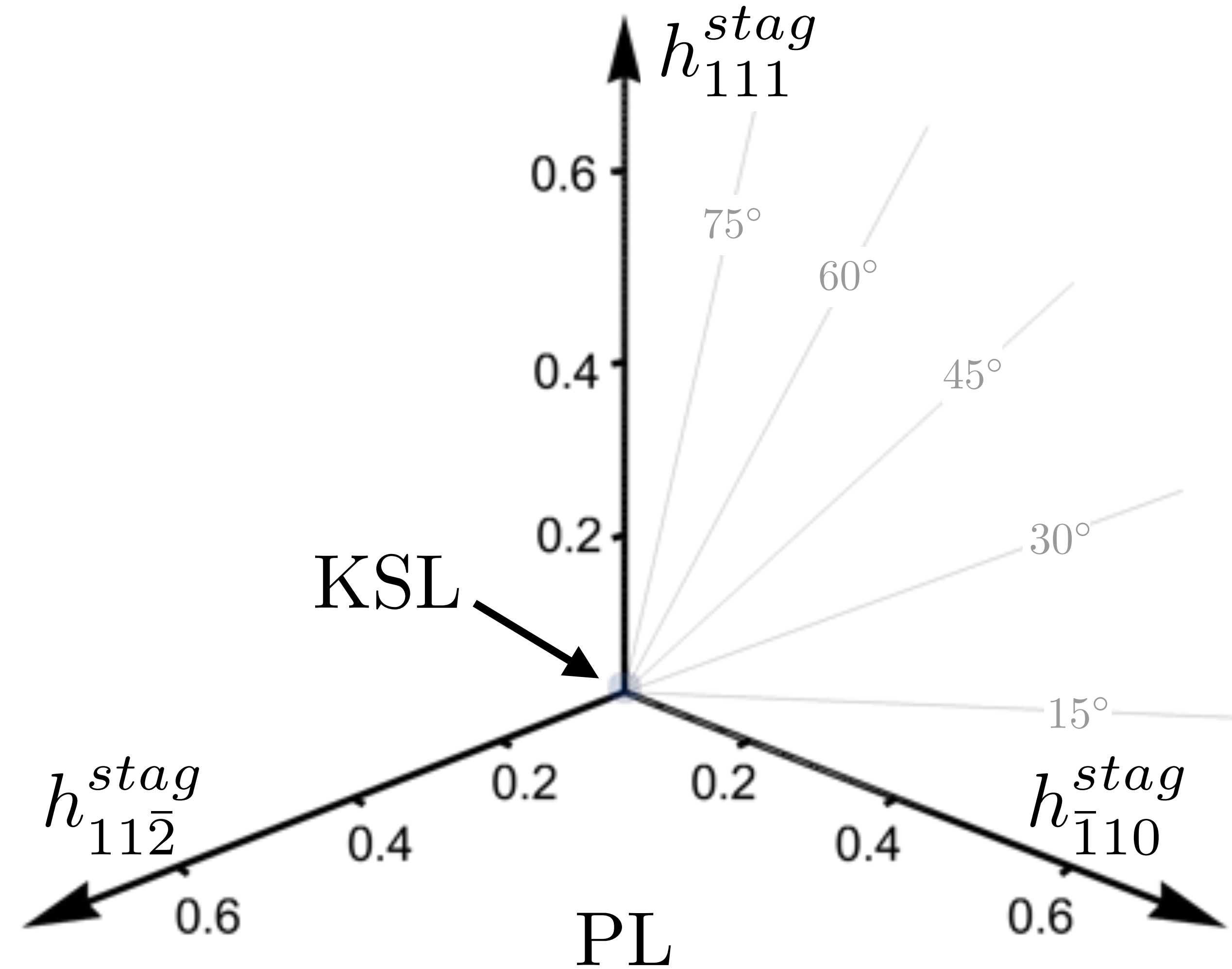
Critical Field for KSL

Staggered **h**

FM Kitaev Coupling

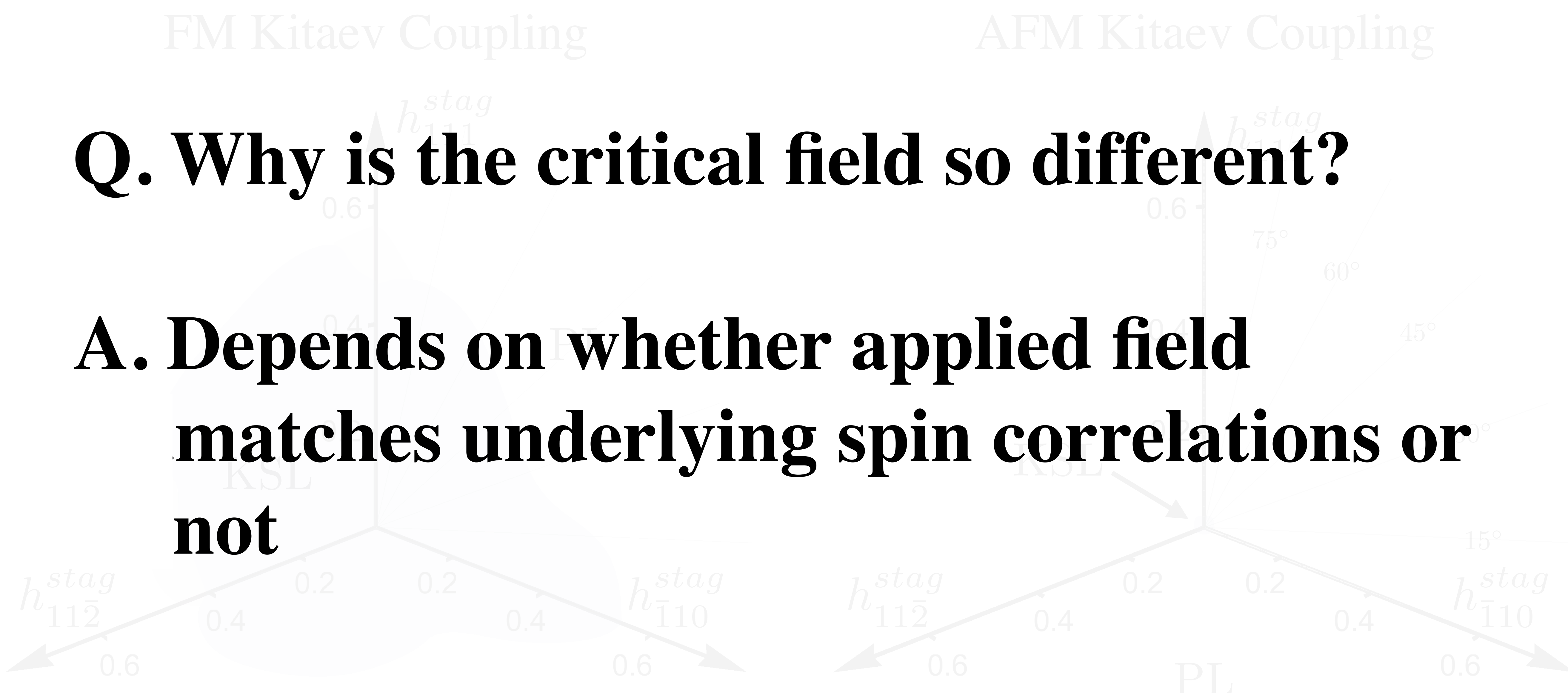


AFM Kitaev Coupling



Critical Field for KSL

Staggered **h**



Q. Why is the critical field so different?

A. Depends on whether applied field matches underlying spin correlations or not

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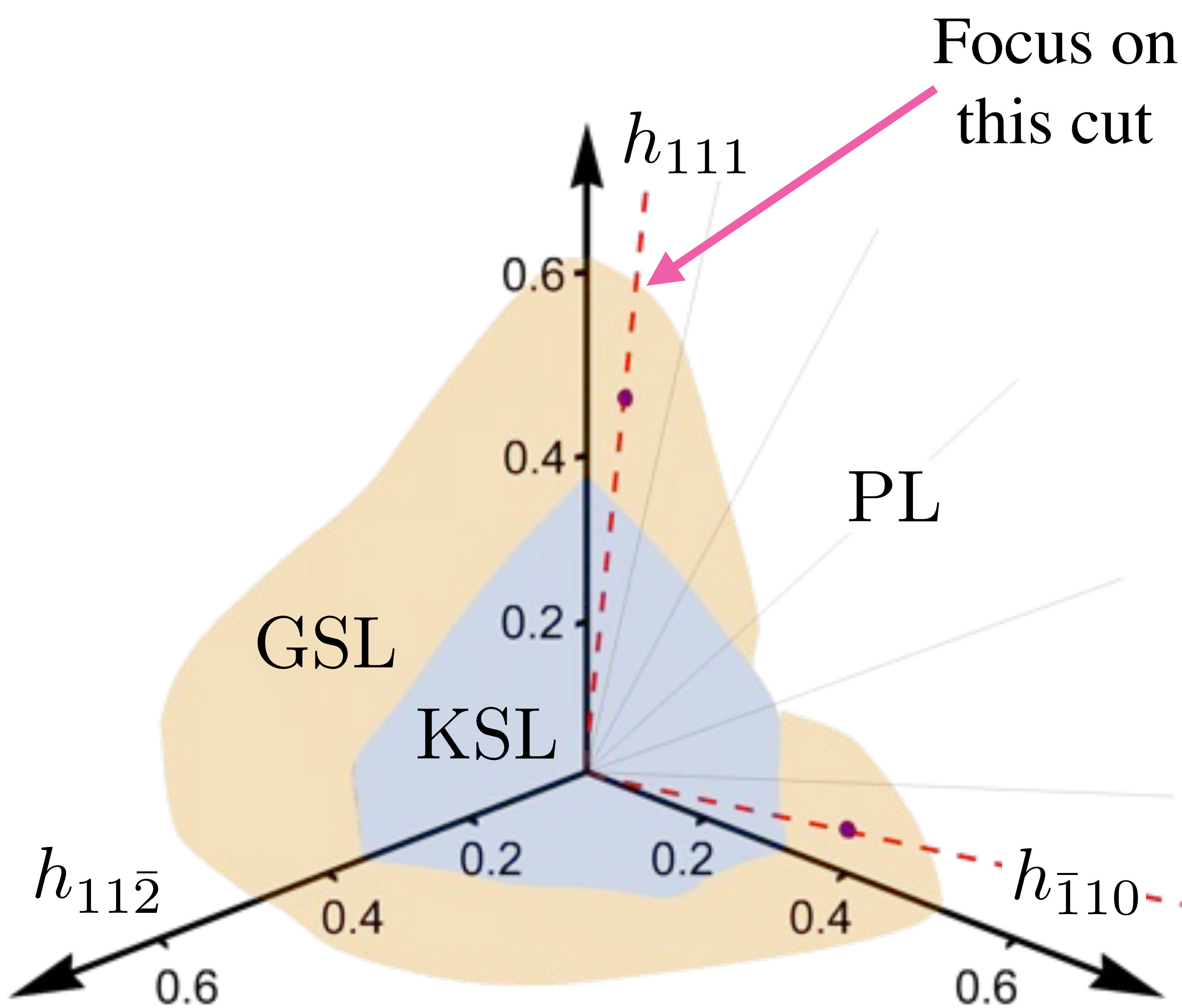
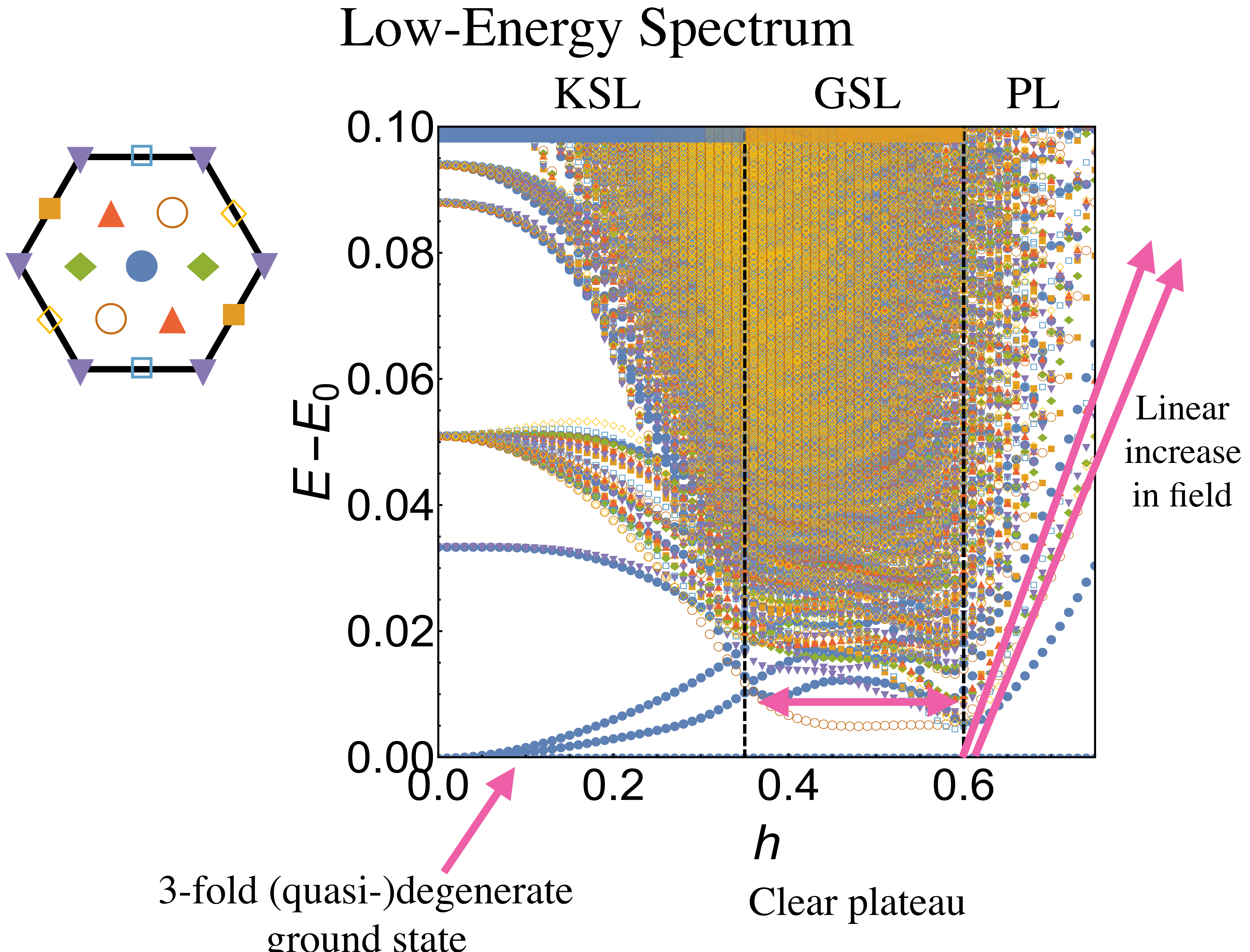
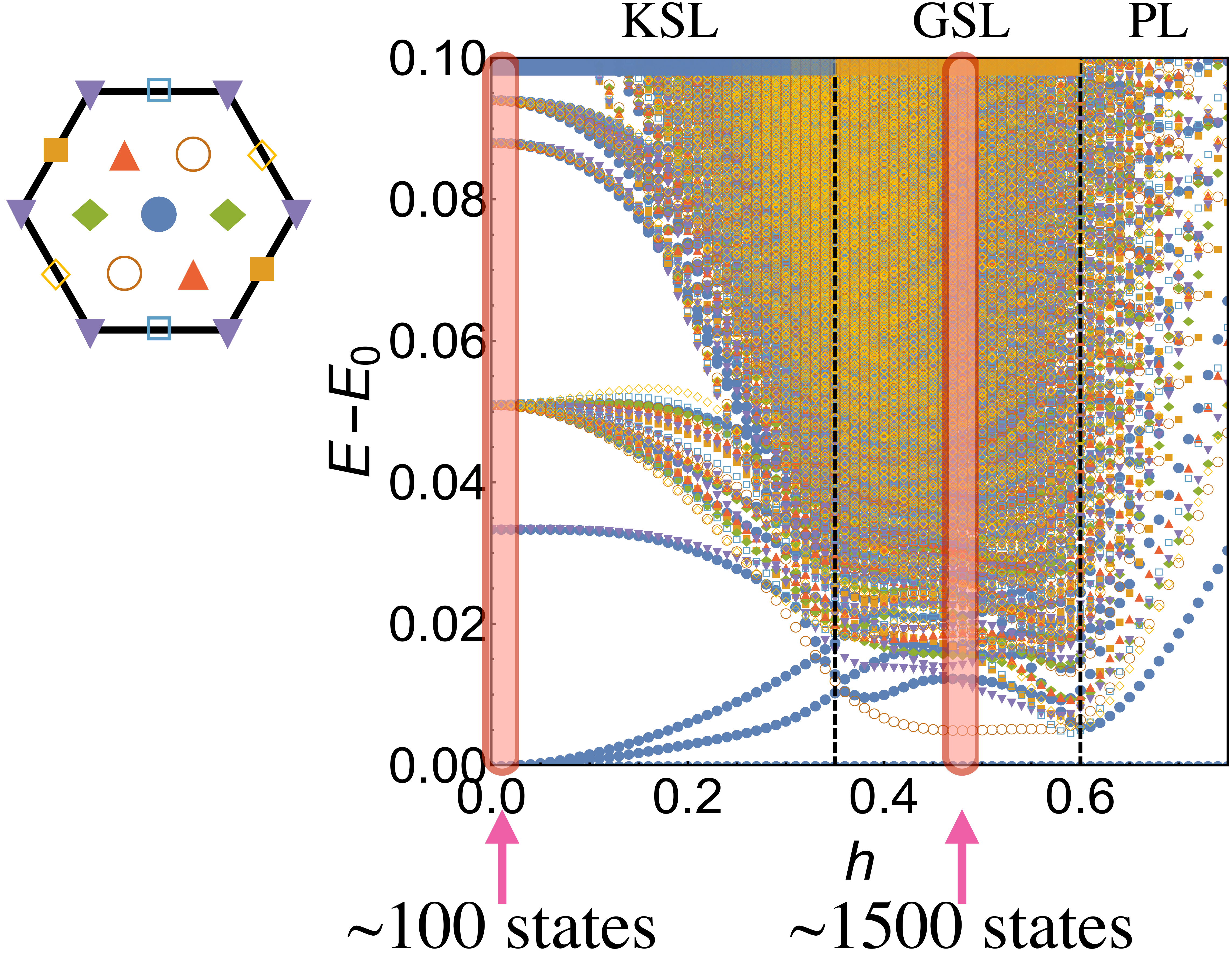
Nature of the GSL?



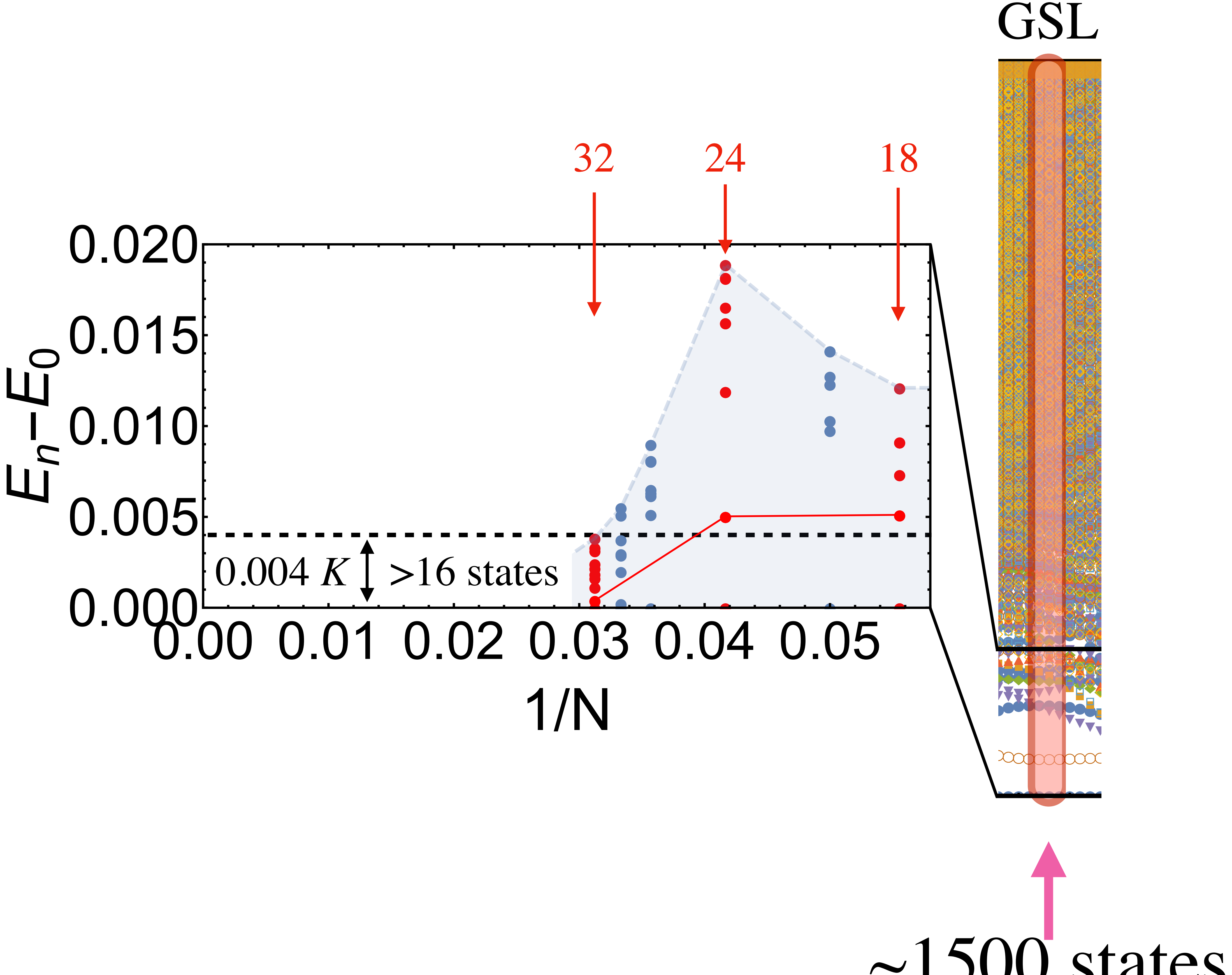


Low-Energy Spectrum
KSL
GSL
PL
0.10
0.08
0.06
0.04
0.02
0.00
$E-E_0$
0.0
0.2
0.4
0.6
h
Linear increase in field
3-fold (quasi-)degenerate ground state
Clear plateau

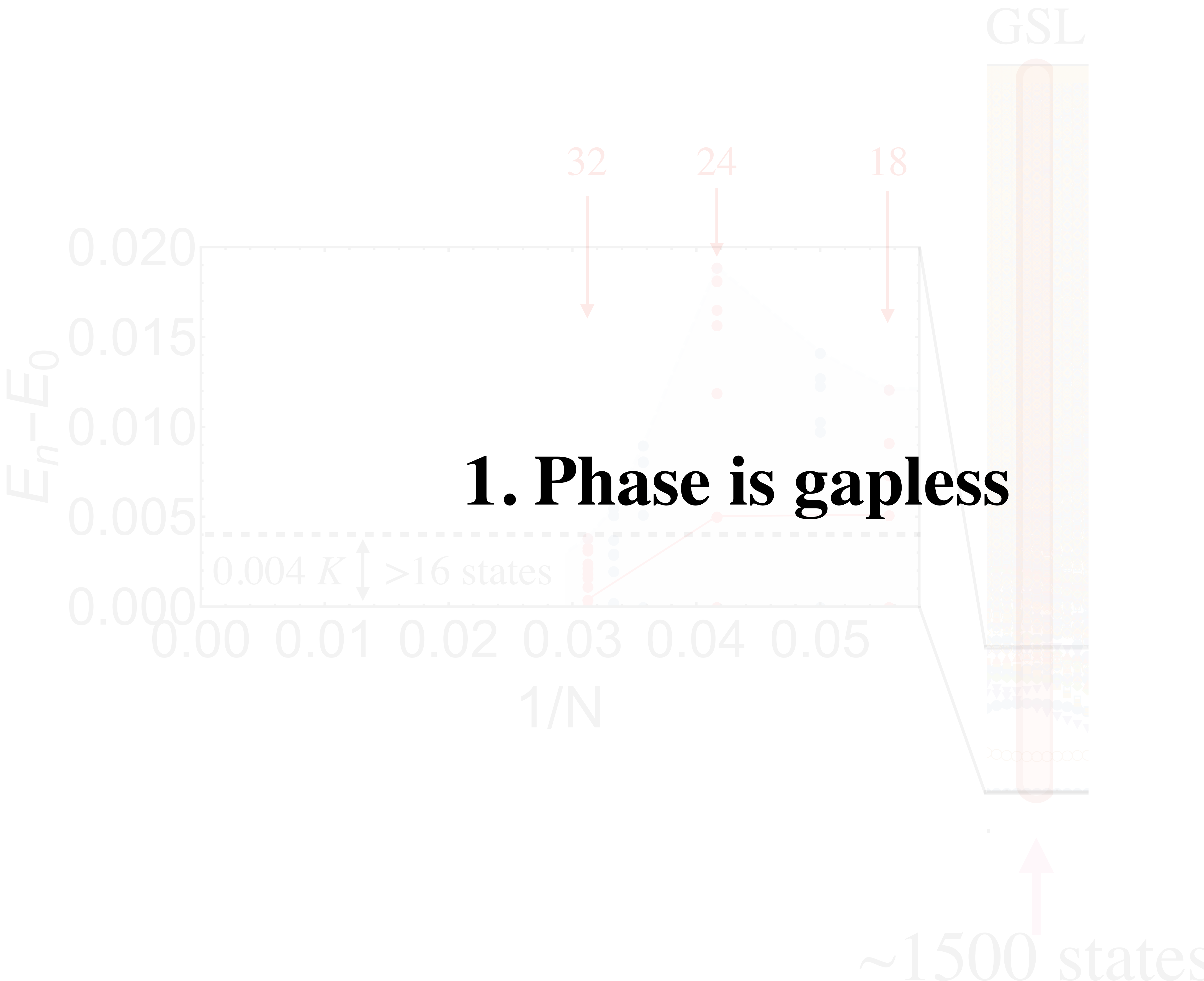
Low-Energy Spectrum



Low-Energy Spectrum



Low-Energy Spectrum



1. Phase is gapless

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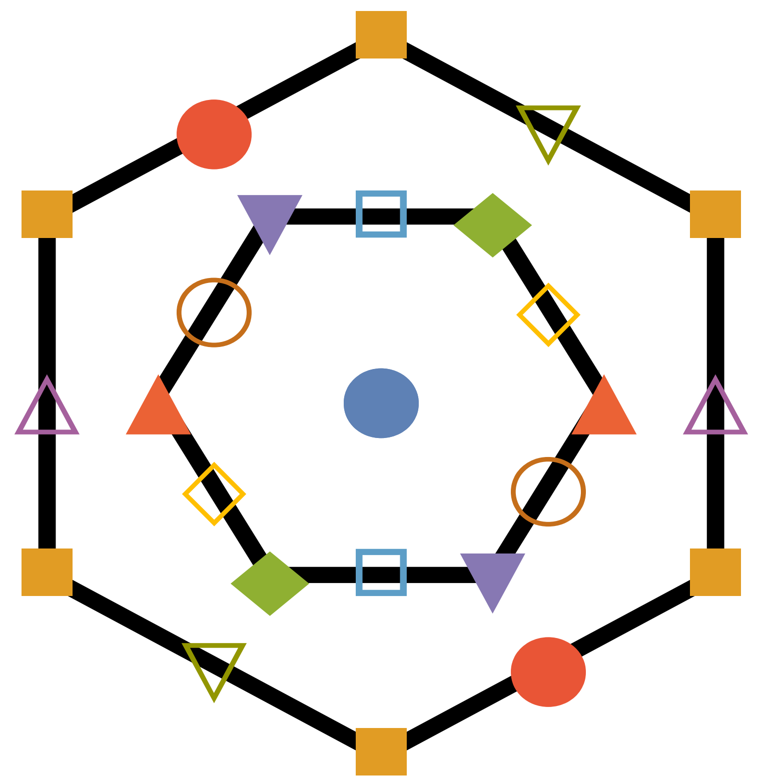
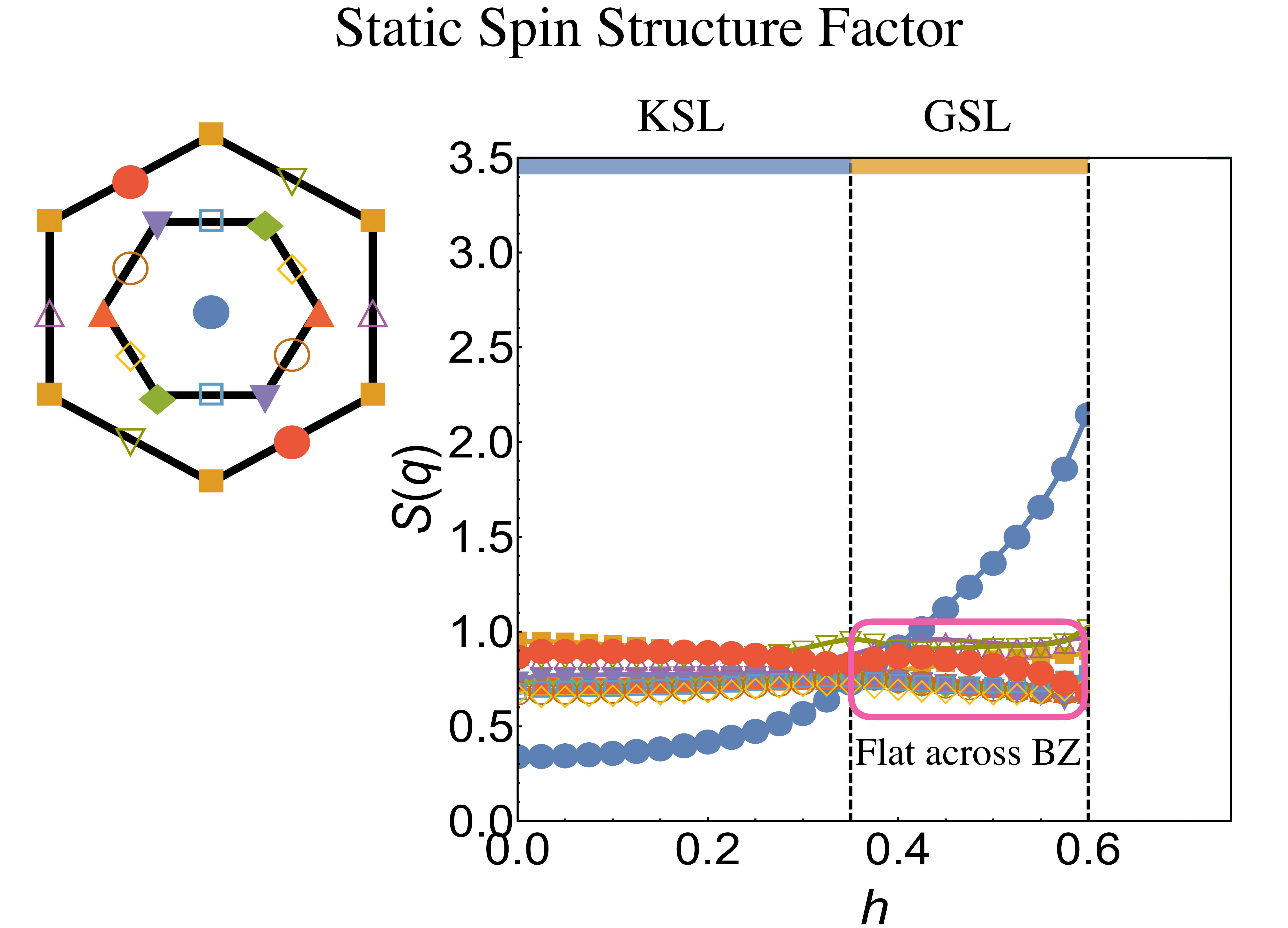
Static Spin Structure Factor

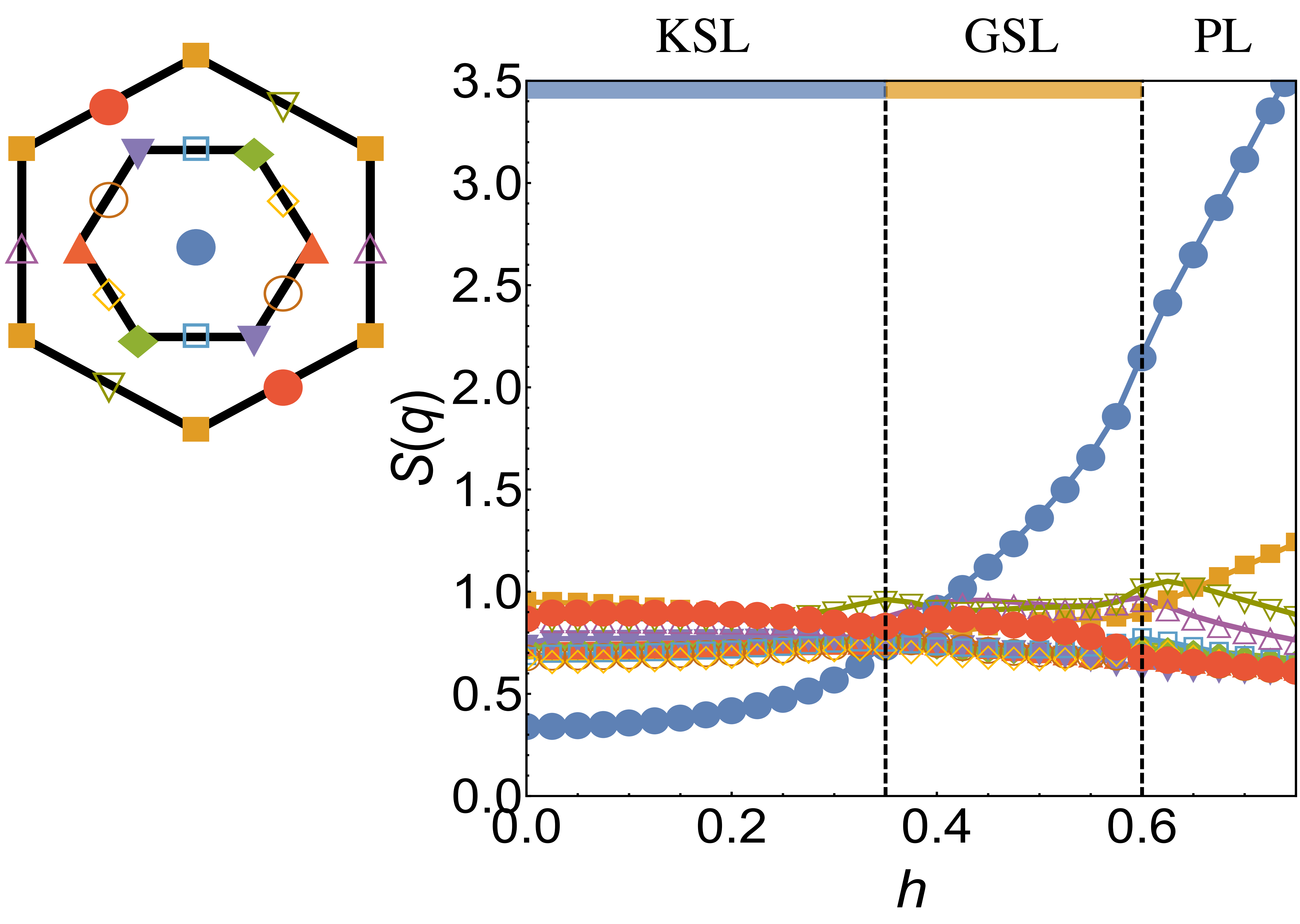
$$S(\mathbf{q}) = \left\langle \sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j \, e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right\rangle$$



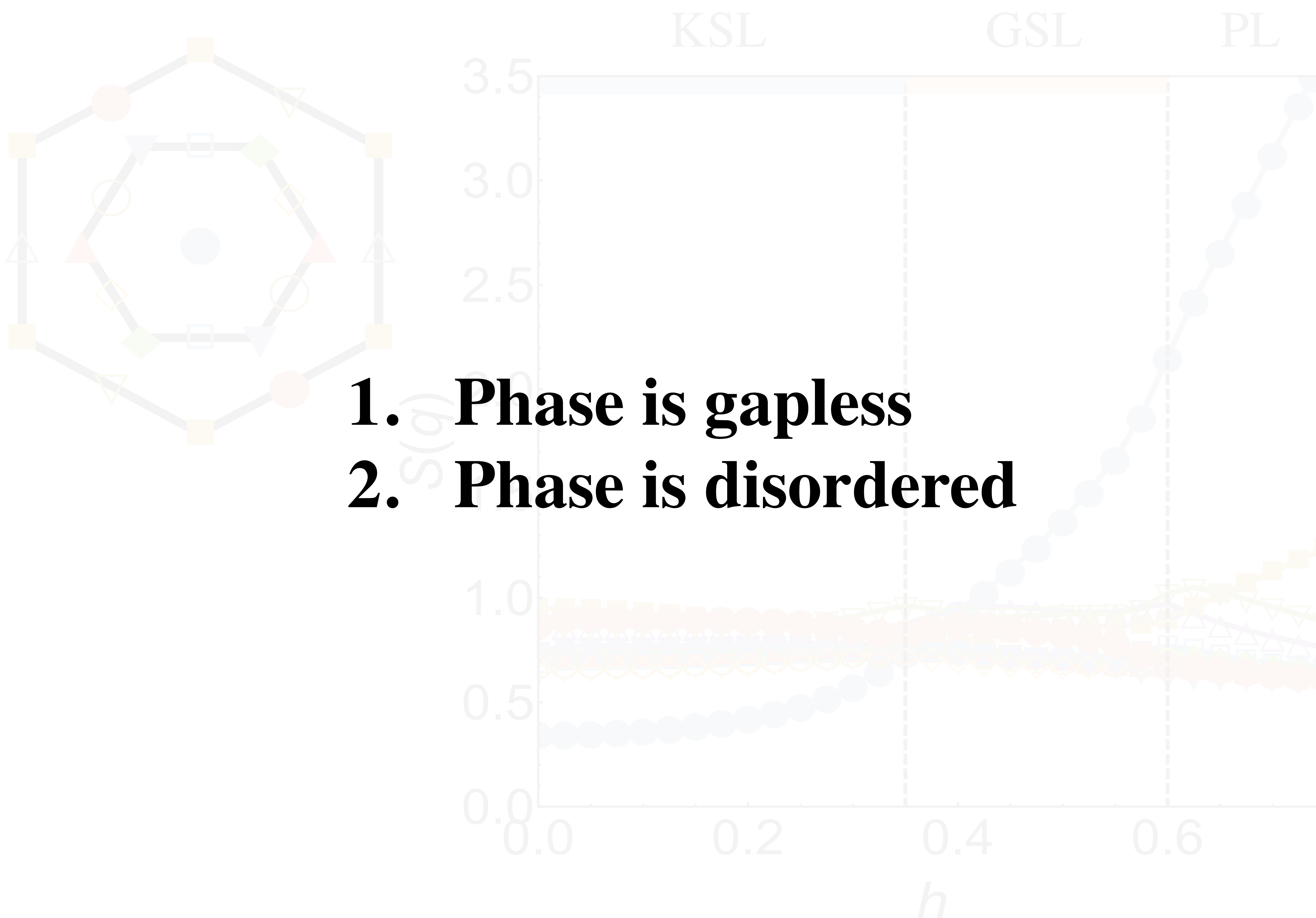
- FT of spin-spin correlation function
- Peaks in the static structure factor indicate magnetic ordering


Static Spin Structure Factor
KSL
GSL
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
S(q)
Flat across BZ
0.0
0.2
0.4
0.6
h

Static Spin Structure Factor



Static Spin Structure Factor



1. **Phase is gapless**
2. **Phase is disordered**

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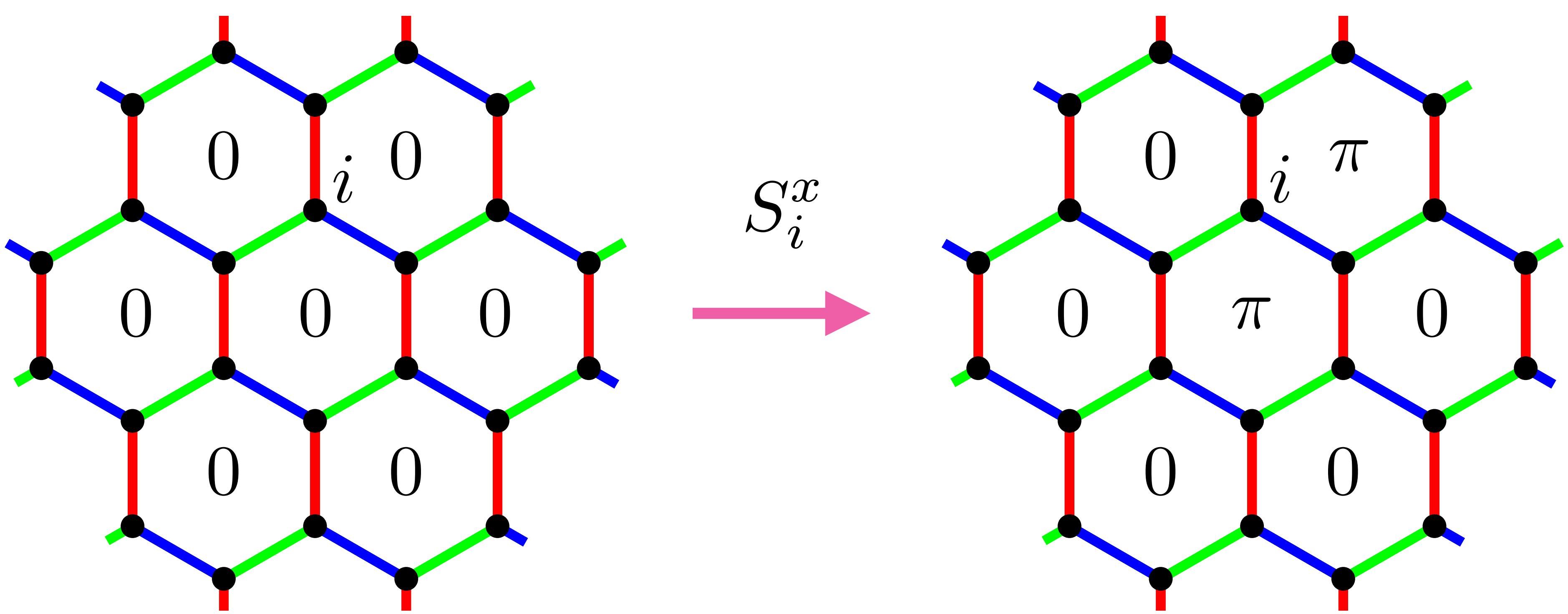
Dynamical Spin Structure Factor

$$S^{\alpha\alpha}(\mathbf{Q},\omega) = \sum_n |\langle n| S^{\alpha}_{\mathbf{Q}} |0\rangle|^2 \delta(\omega - (E_n - E_0))$$

$$\sum_i S_i^{\alpha} e^{i\mathbf{Q}\cdot\mathbf{r}_i} |0\rangle$$

$$S_i^{\alpha} |0\rangle$$



+ c Majorana fermion excitation

Dynamical Spin Structure Factor

$$S^{\alpha\alpha}(\mathbf{Q},\omega) = \sum_n \left| \langle n | S^{\alpha}_{\mathbf{Q}} |0\rangle \right|^2 \delta\left(\omega - (E_n - E_0)\right)$$

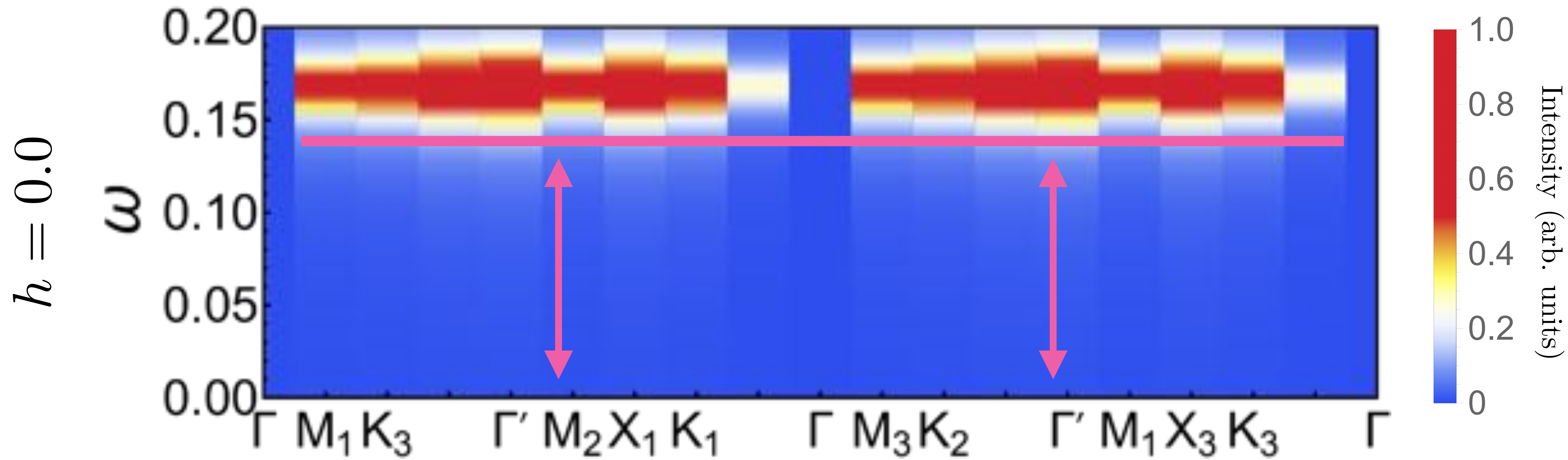
$$\sum_i S_i^{\alpha} e^{i\mathbf{Q}\cdot\mathbf{r}_i} |0\rangle$$

$$S_i^{\alpha} |0\rangle$$

Even though the system is gapless,
the dynamical spin structure factor is gapped!

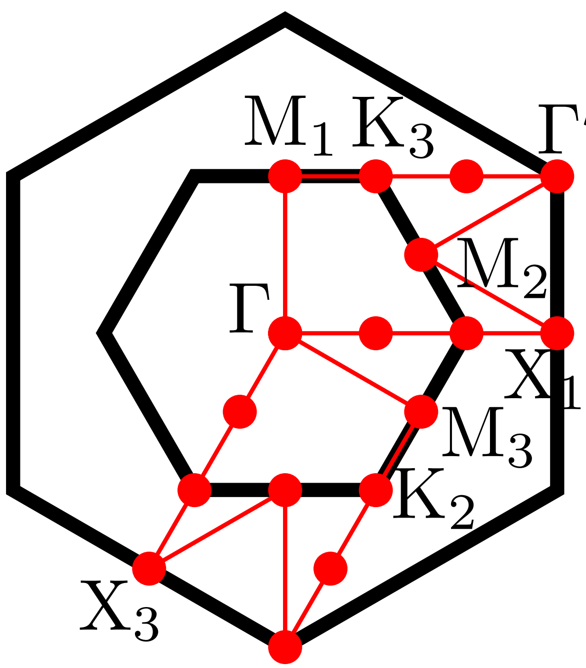
J. Knolle et al., PRL **112**, 207203 (2014)
J. Knolle et al., PRB **92**, 115127 (2015)

Dynamical Spin Structure Factor

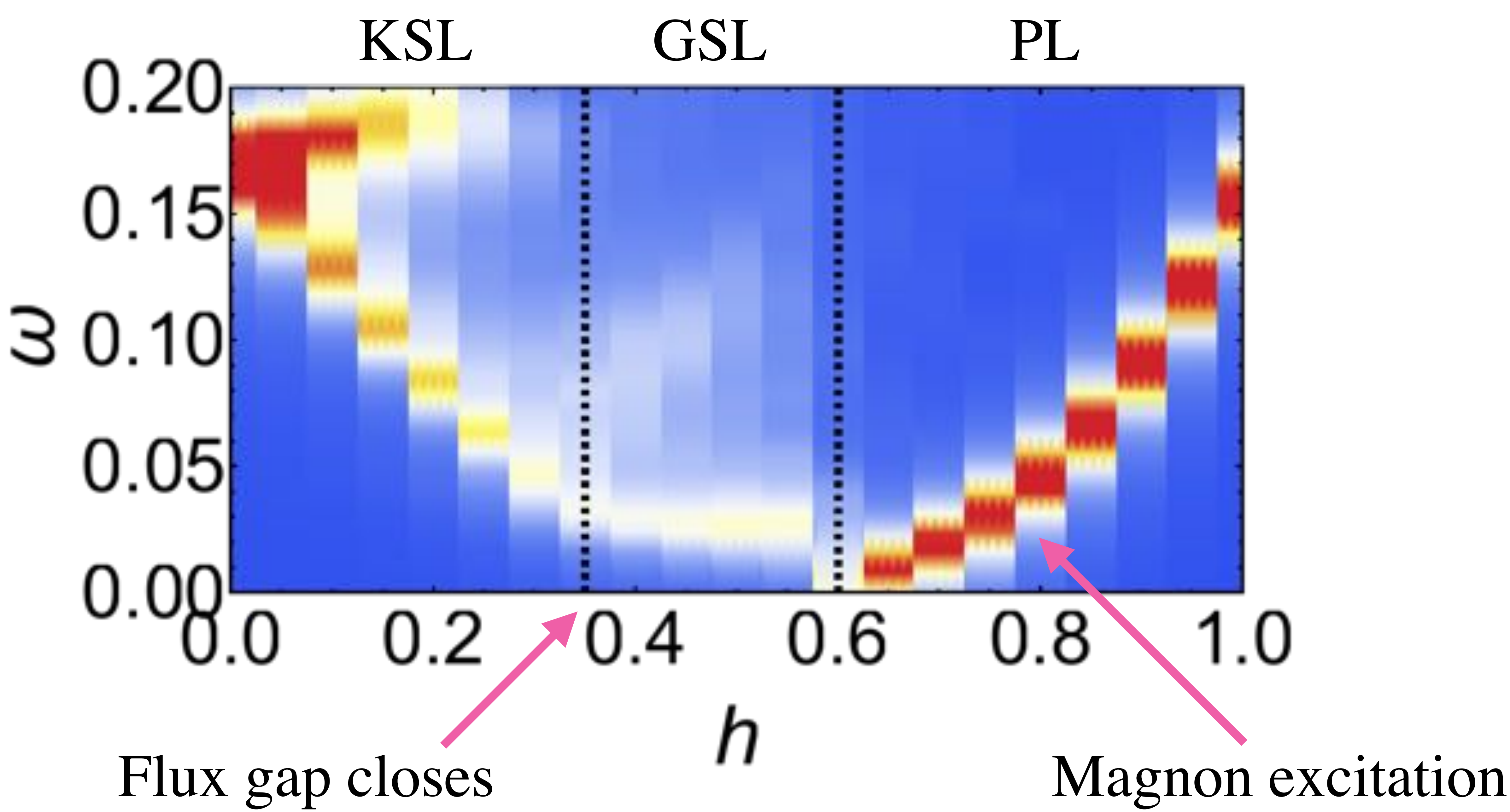


Clear flux gap! (across all momenta)

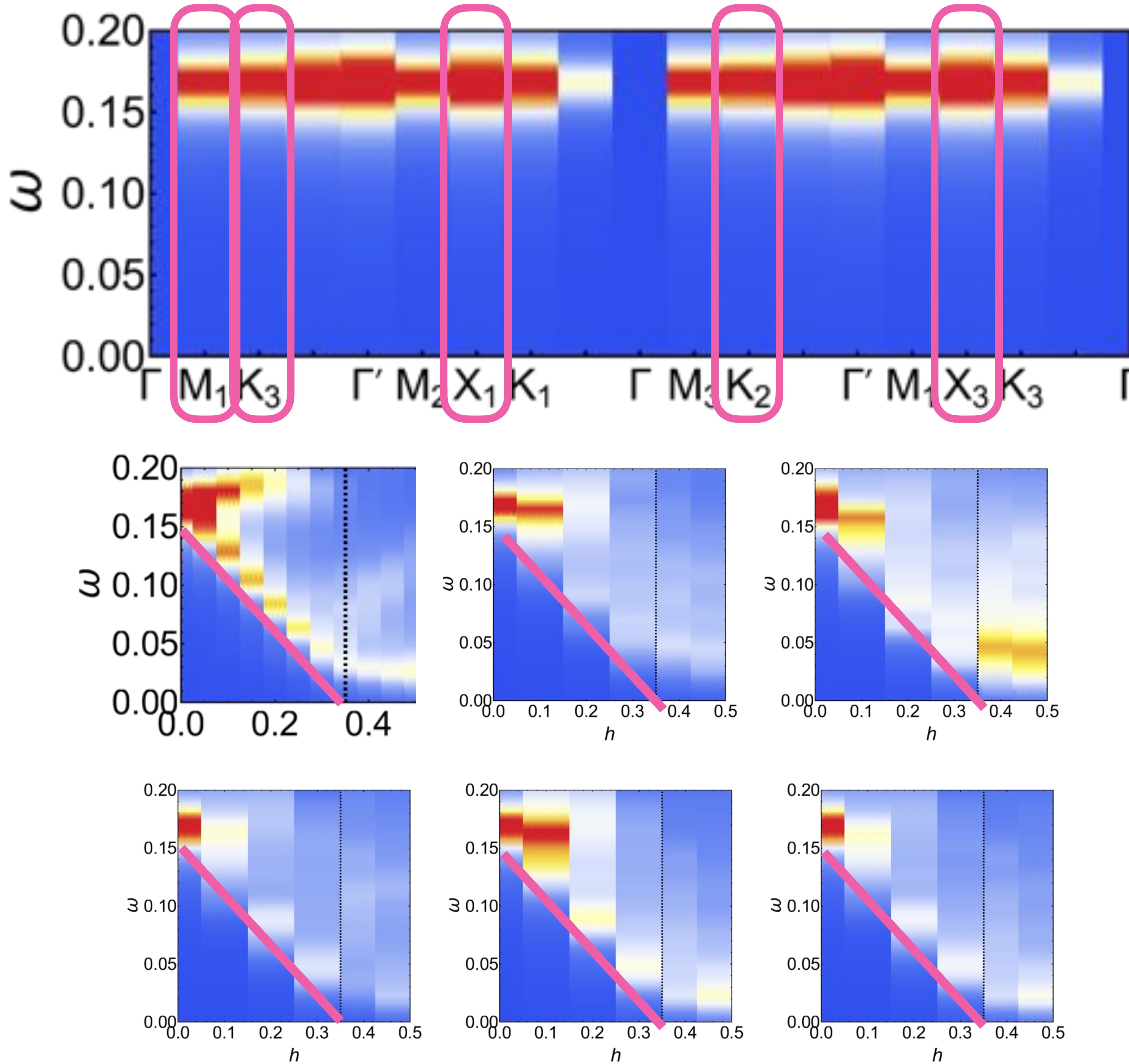
What happens as we increase field?

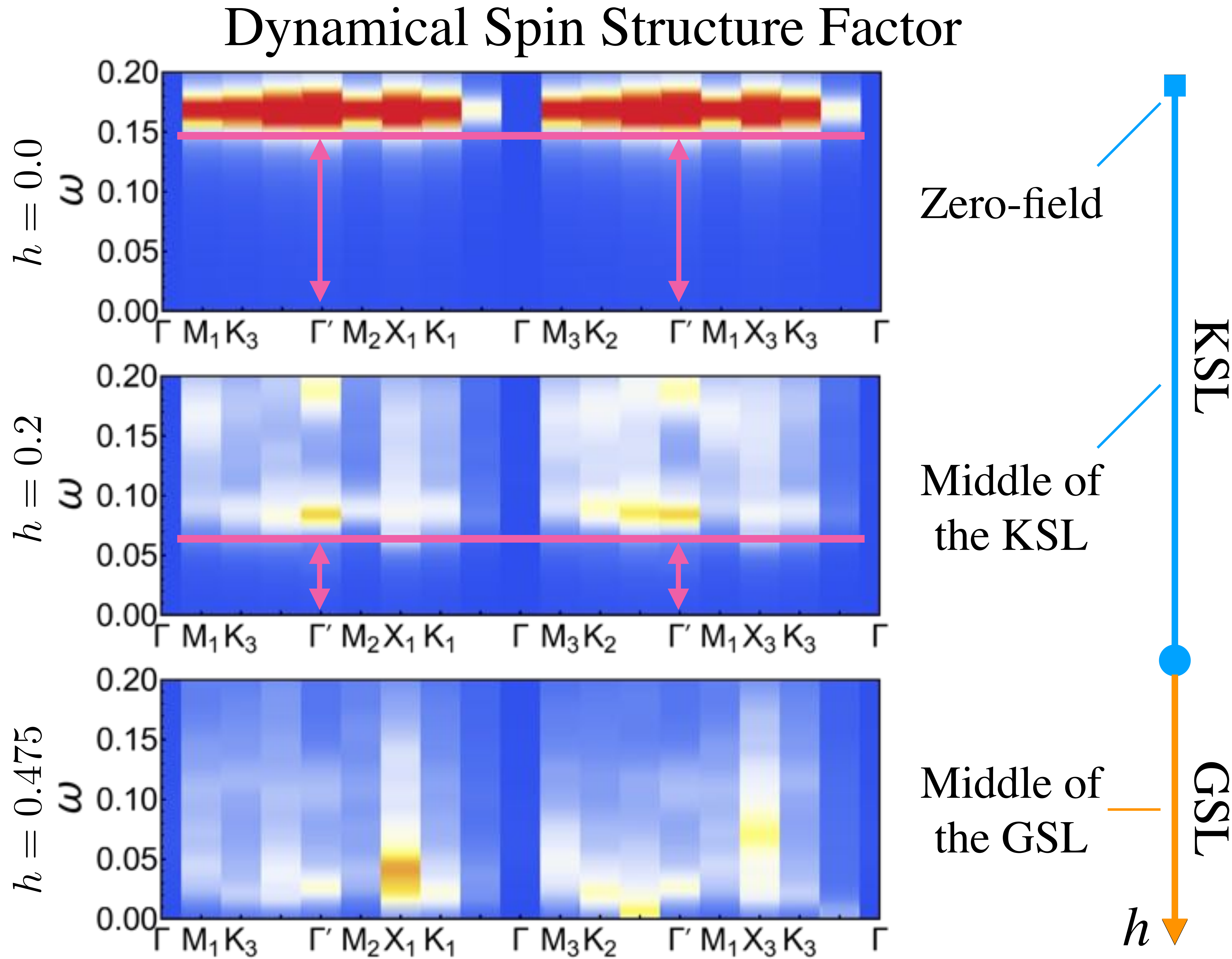


Dynamical Spin Structure Factor

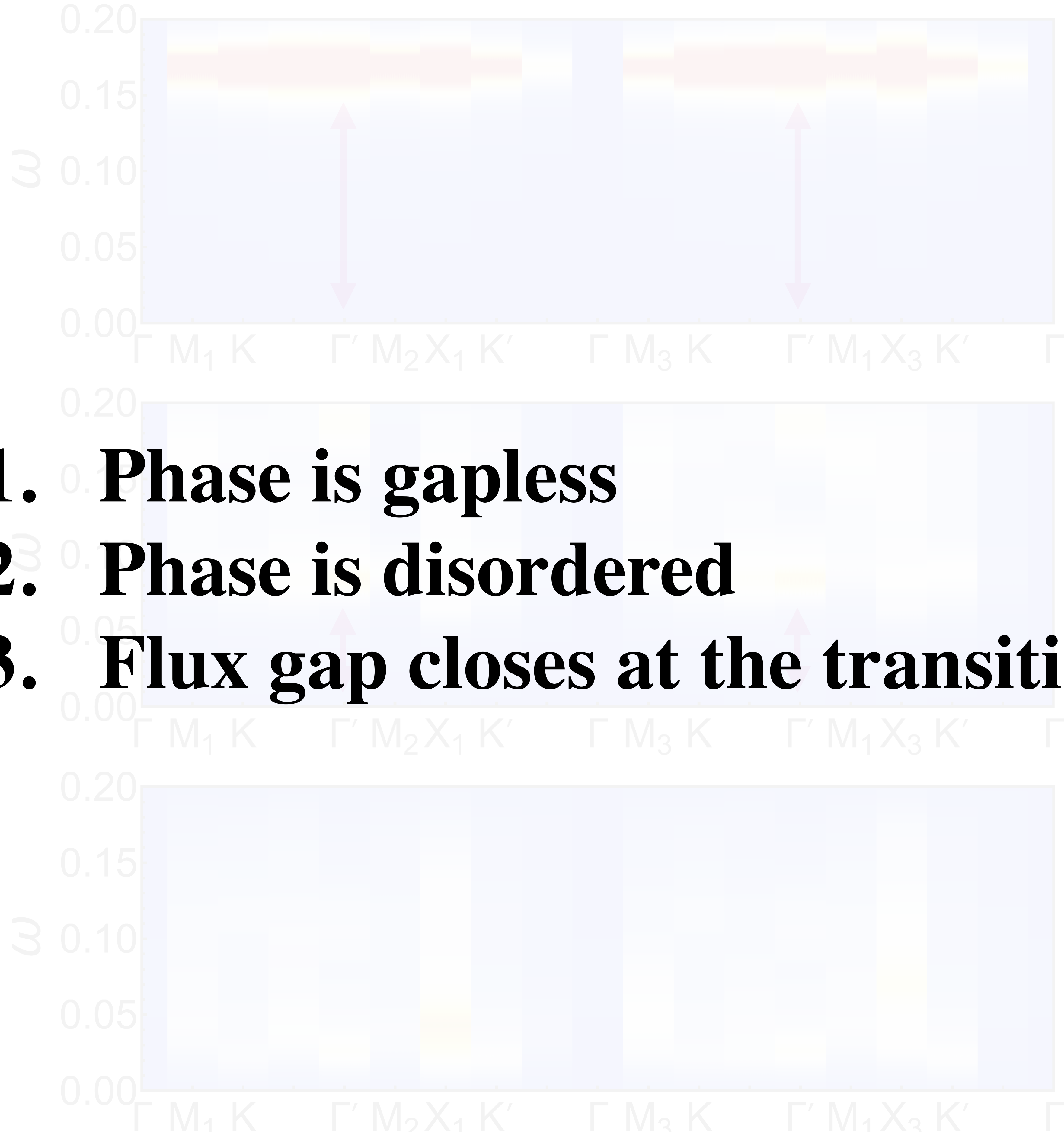


Dynamical Spin Structure Factor





Dynamical Spin Structure Factor
$h = 0.0$
ω
0.20
0.15
0.10
0.05
0.00
Γ M1 K3 Γ′ M2 X1 K1 Γ M3 K2 Γ′ M1 X3 K3 Γ
$h = 0.2$
$h = 0.475$
Zero-field
Middle of the KSL
Middle of the GSL
KSL
GSL
h



1. **Phase is gapless**
2. **Phase is disordered**
3. **Flux gap closes at the transition**

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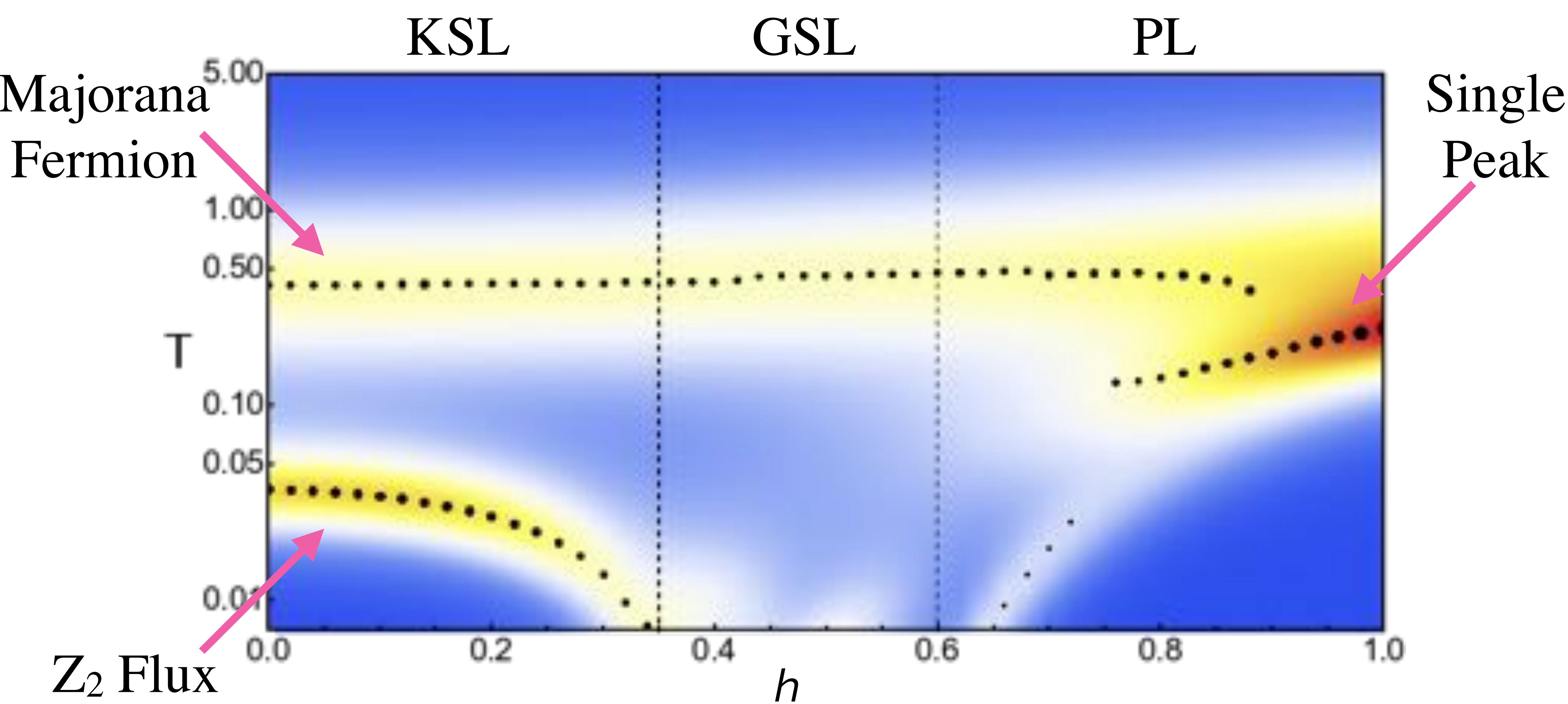
Specific Heat



1. **Phase is gapless**
2. **Phase is disordered**
3. **Flux gap closes at the transition**

4. **Energy scale associated with Z_2 flux decreases as transition is approached**
5. **Fermions seemingly not affected much. Action is occurring mainly in the gauge sector**

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- Introduction

- The Kitaev Model in a Magnetic Field
  - Q. Nature of the KSL?
  - Q. Critical Field FM vs AFM?
  - Q. Nature of the GSL?

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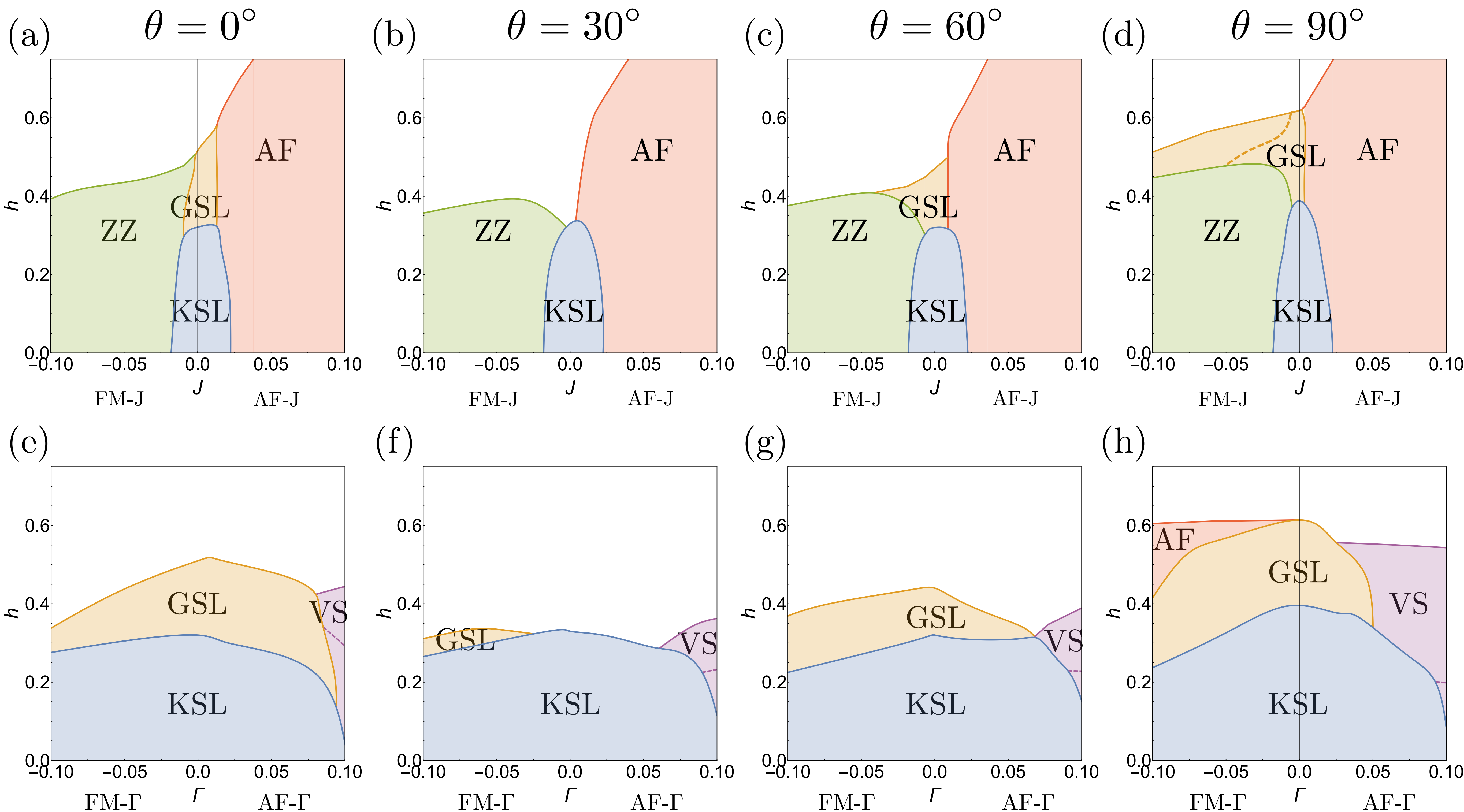
Relevant Pertubations



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Summary of GSL

1. Phase is gapless
2. Phase is disordered
3. Flux gap closes at the transition
4. Energy scale associated with Z_2 flux decreases as transition is approached
5. Fermions seemingly not affected much. Action is occurring mainly in the gauge sector
6. Phase is stable to perturbations

What's going on?

- Rewrite spins in terms of usual (complex) fermions:

$$S_i^\alpha = f_{i,\mu}^\dagger \sigma_{\mu\nu}^\alpha f_{i,\nu}$$

$$f_{i,\mu}^\dagger \to e^{-i\phi_i} f_{i,\mu}^\dagger$$
$$f_{i,\nu} \to e^{i\phi_i} f_{i,\nu}$$

$$S_i^\alpha \to S_i^\alpha$$

U(1) (phase) redundancy!

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Gauge Field: $\mathbb{Z}_2$ (Higgsed)

Fermions: Gapped Topological SC

F. J. Burnell and C. Nayak, PRB **84**, 125125 (2011)

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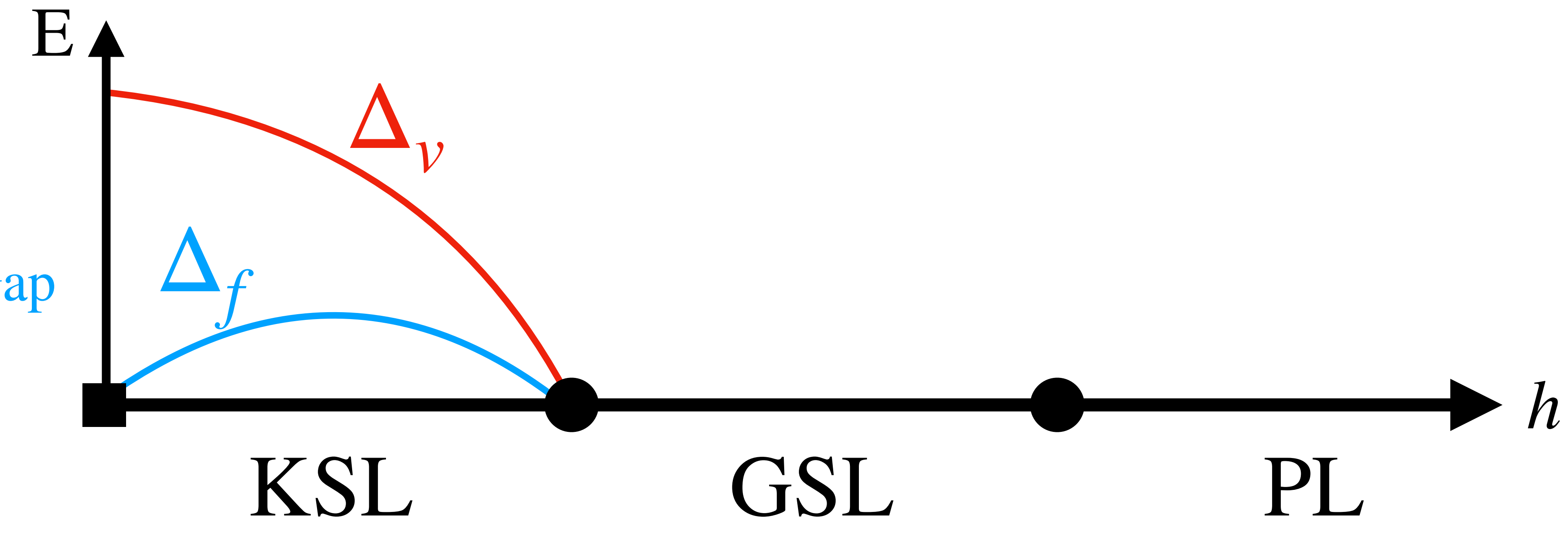
Δ_v = Vison Gap

Δ_f = Fermion Gap



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Gauge Field: $\mathbb{Z}_2$ (Higgsed) $U(1)$

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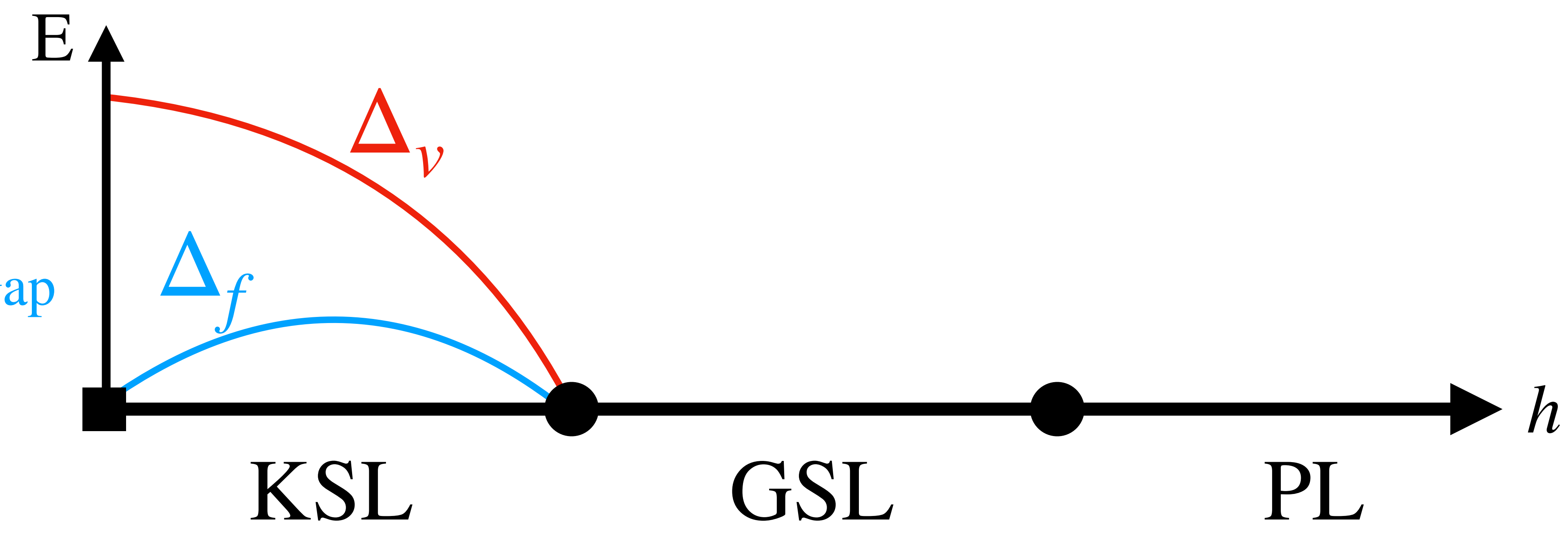

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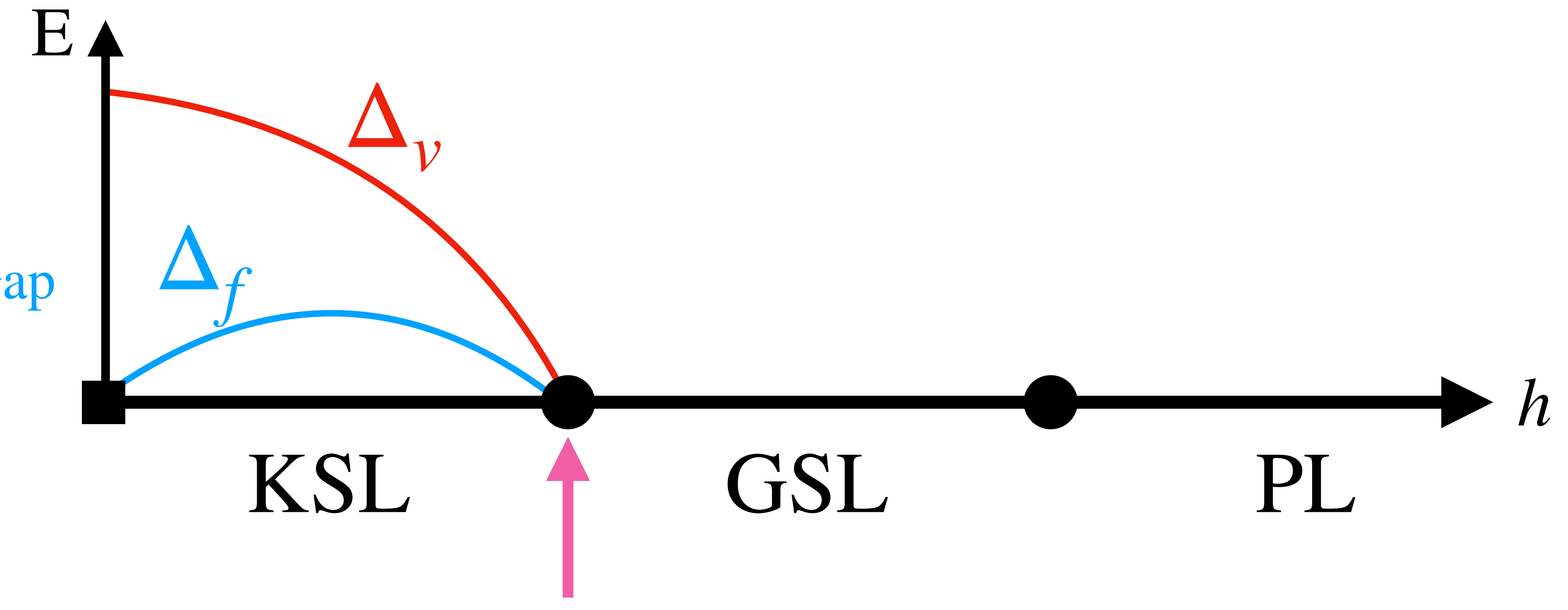

Superconductor - Metal transition (Anderson-Higgs)

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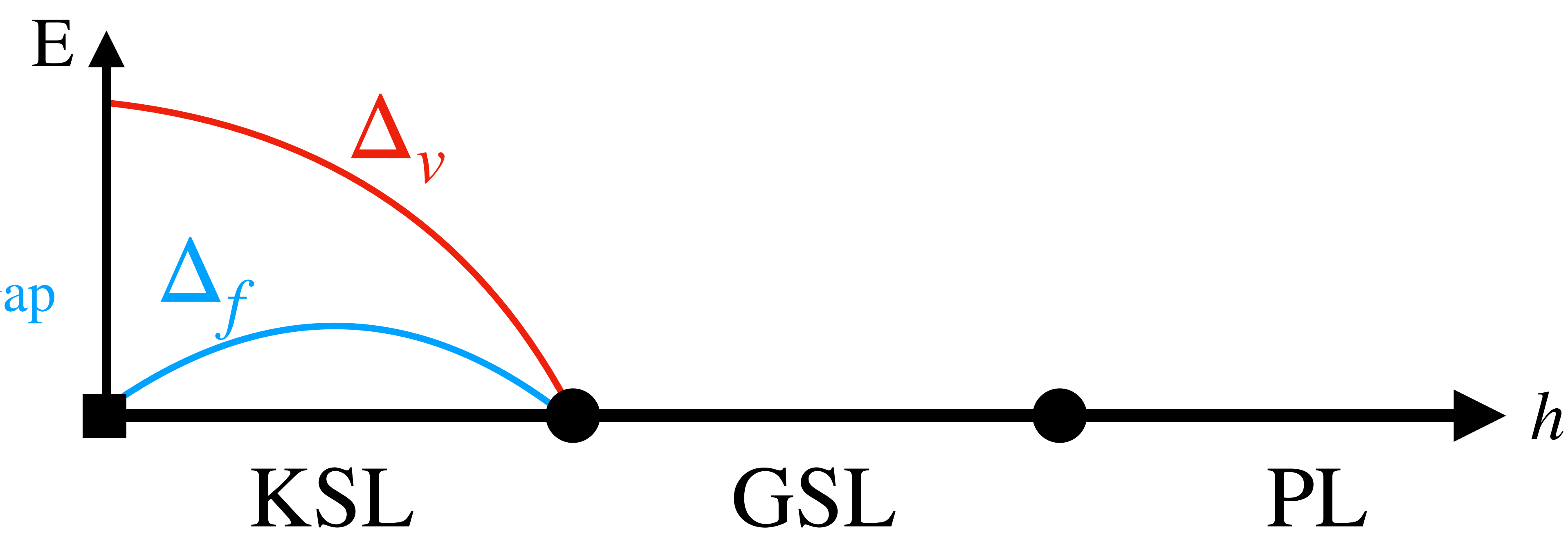

M. Metlitski et al., PRB **91**, 115111 (2015)

…

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- Rewrite spins in terms of usual (complex) fermions:

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Gauge Field:	$\mathbb{Z}_2$ (Higgsed)	$U(1)$	$U(1)$ (confined)
Fermions:	Gapped Topological SC	Gapless FS	Gapped Trivial Insulator

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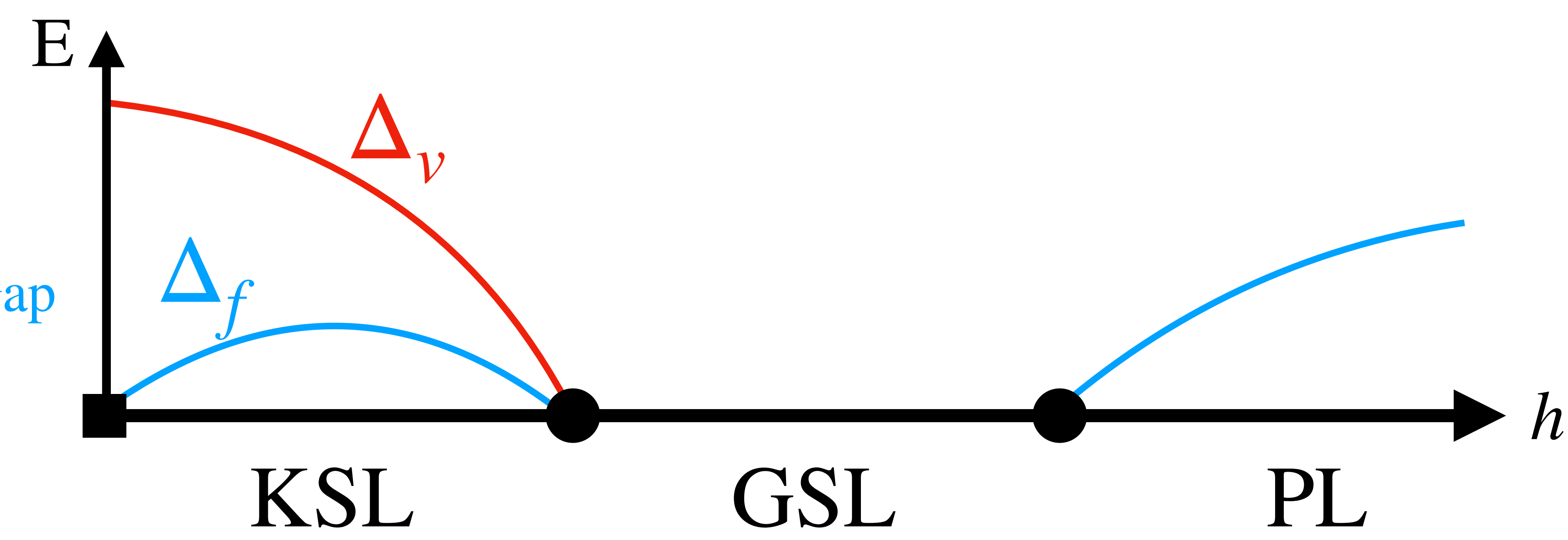

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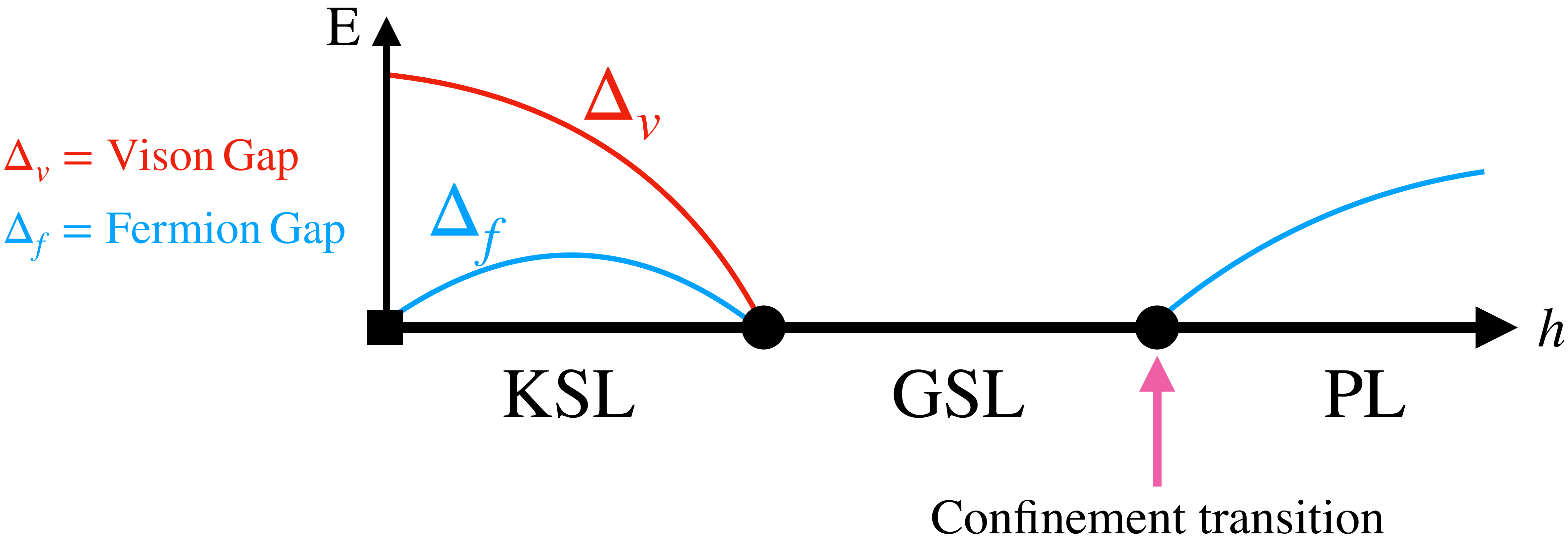

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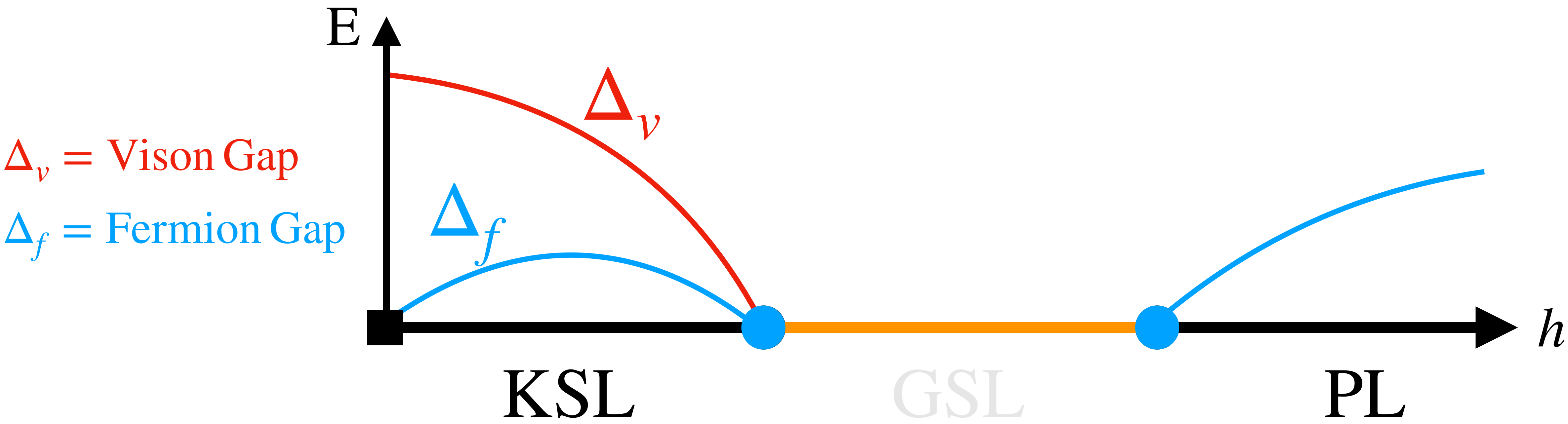
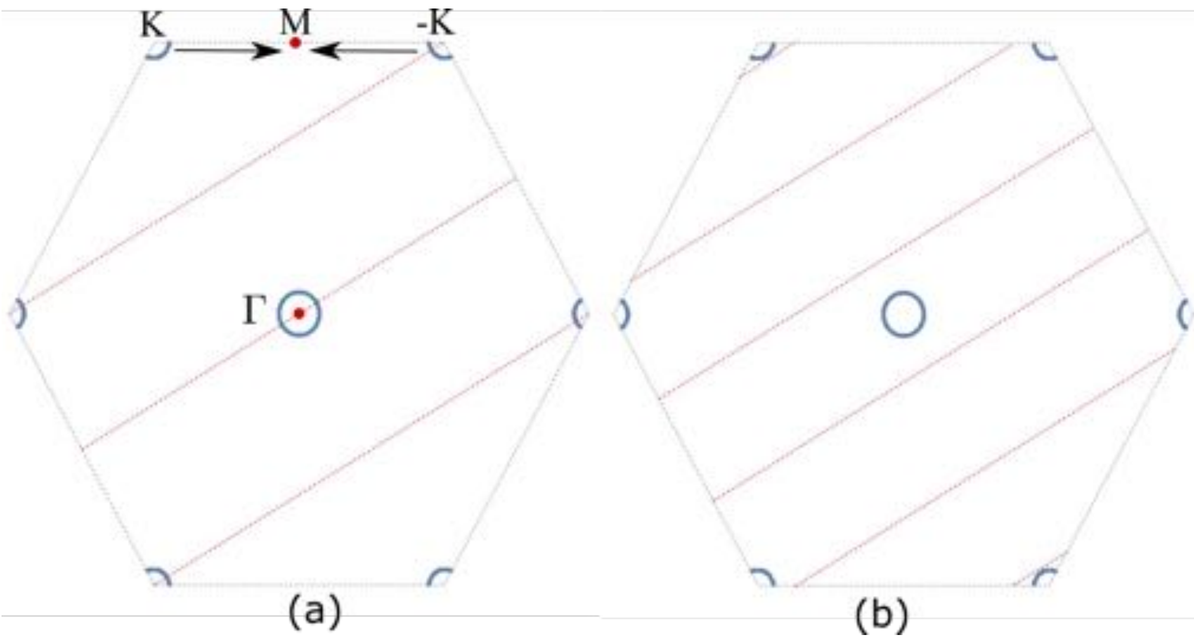


$U(1)$ QSL!

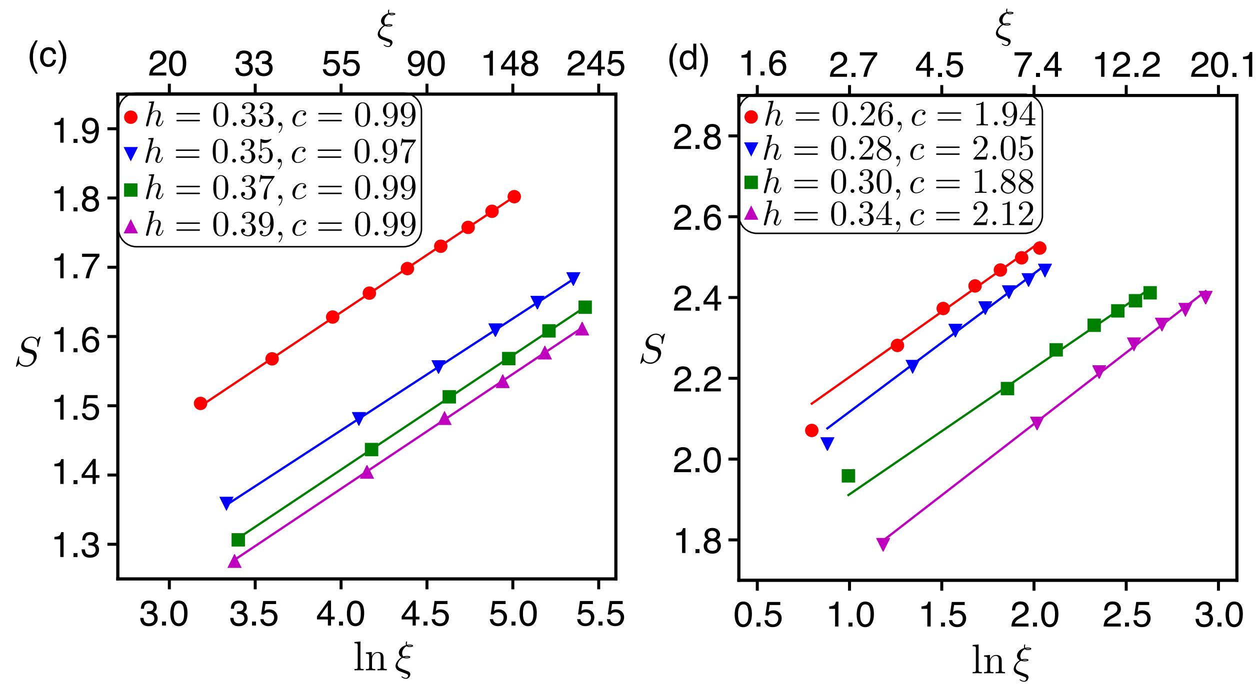
What's going on?

- Central charge consistent with Fermi surface predicted by PSG analysis

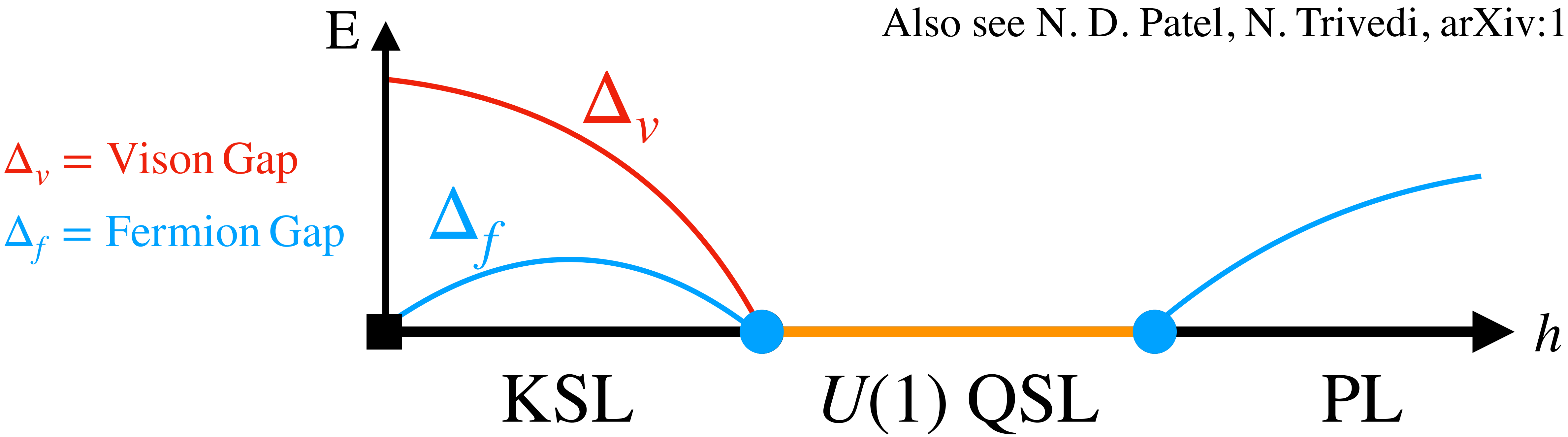
H.-C. Jiang et al., arXiv:1809.08247



L. Zou, Y.-C. He, arXiv:1809.09091



Also see N. D. Patel, N. Trivedi, arXiv:1812.06105



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$$S_i^\alpha \to \begin{array}{c}\text{Gauge field}\\ + \\ \text{Fermions}\end{array}$$

	Fermions	**Gauge Field**
Known "textbook" Kitaev spin liquids	Gapless SC	Gapped Z_2
	Gapped topological SC	Gapped Z_2
	Gapped trivial SC	Gapped Z_2

What's going on?

- Spin is fractionalised:

$$S_i^\alpha \rightarrow \begin{array}{c}\text{Gauge field}\\ +\\ \text{Fermions}\end{array}$$

	Fermions	**Gauge Field**
	Gapless SC	Gapped Z_2
	Gapped topological SC	Gapped Z_2
	Gapped trivial SC	Gapped Z_2
New phase!!!	Gapless Metal	Gapless U(1)

Thank you!