Quasi-electrons in the FQHE

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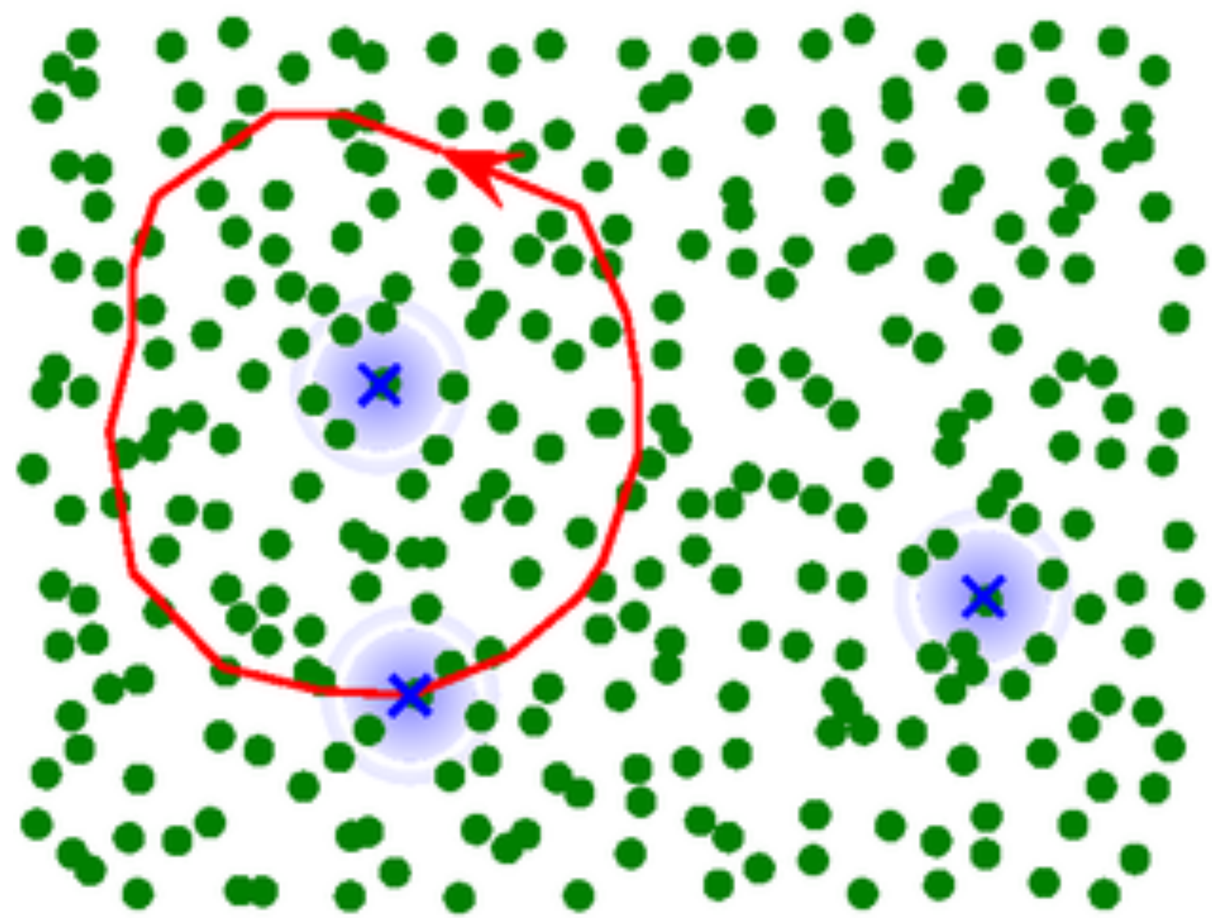
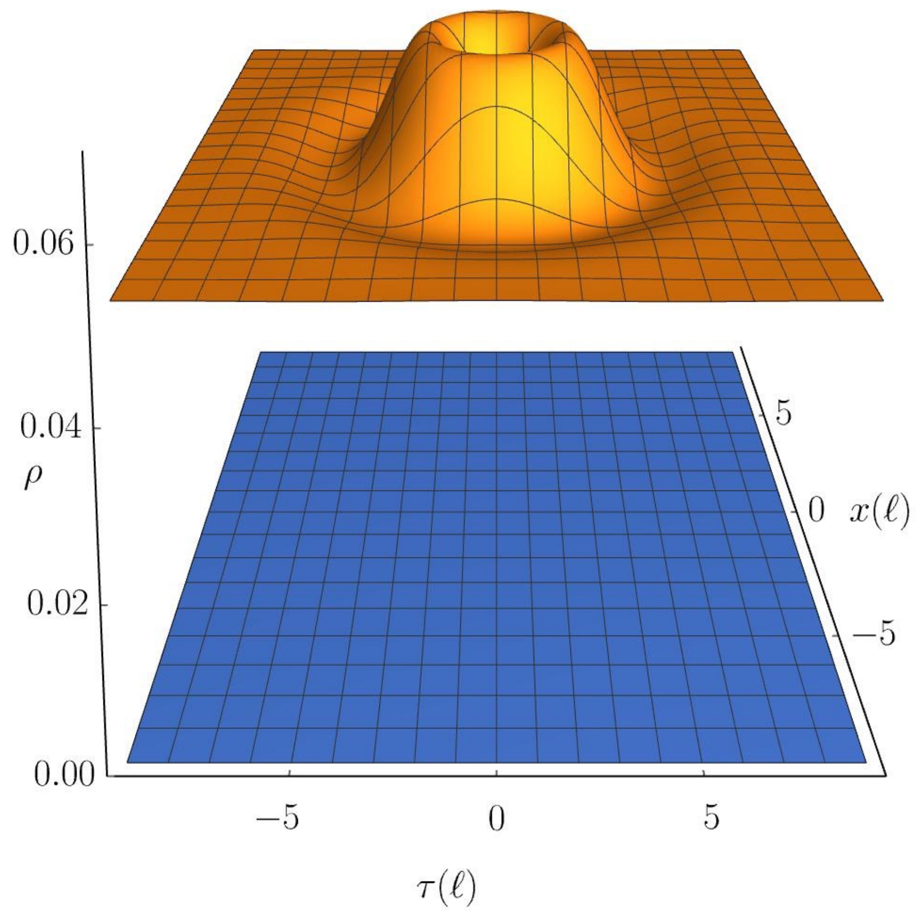
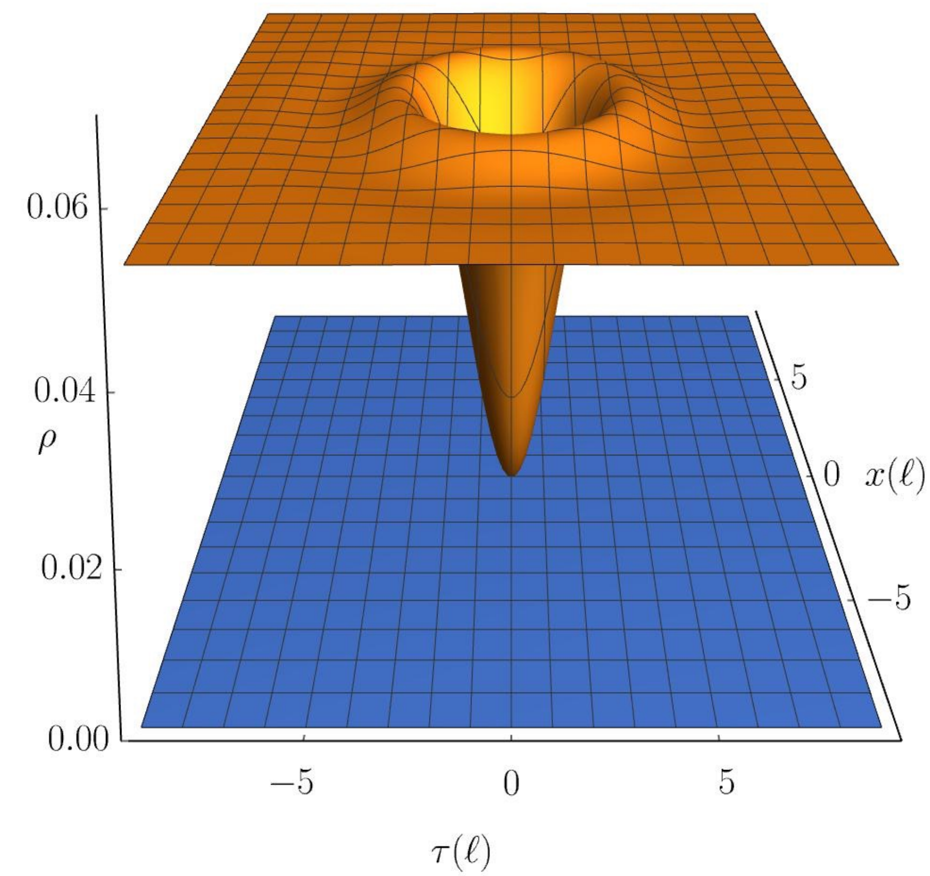


How many electrons in the picture?



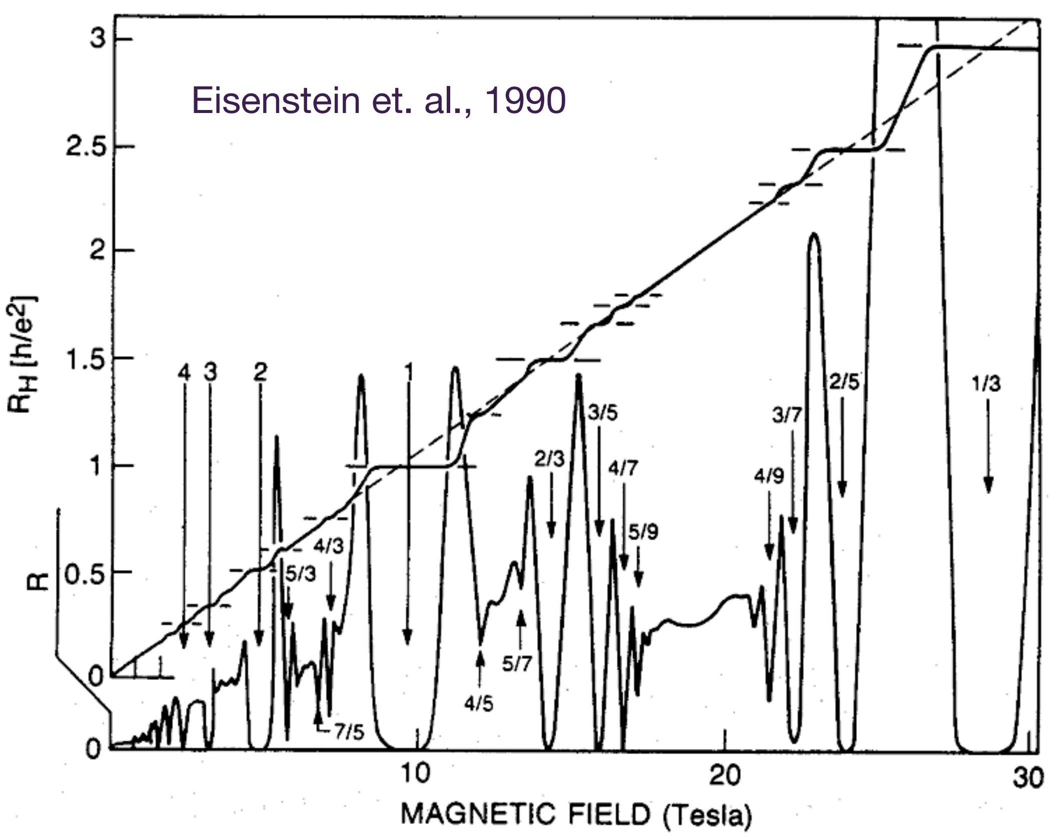


How many electrons are needed for a quasi-particle?

Outline and Results

- Ultra-short intro to FQH and anyons
- FQH model wave functions
- Quasiholes vs. quasi-electrons
- Problem with quasi-electrons: *screening*
- Some answers and a proposal

FQHE

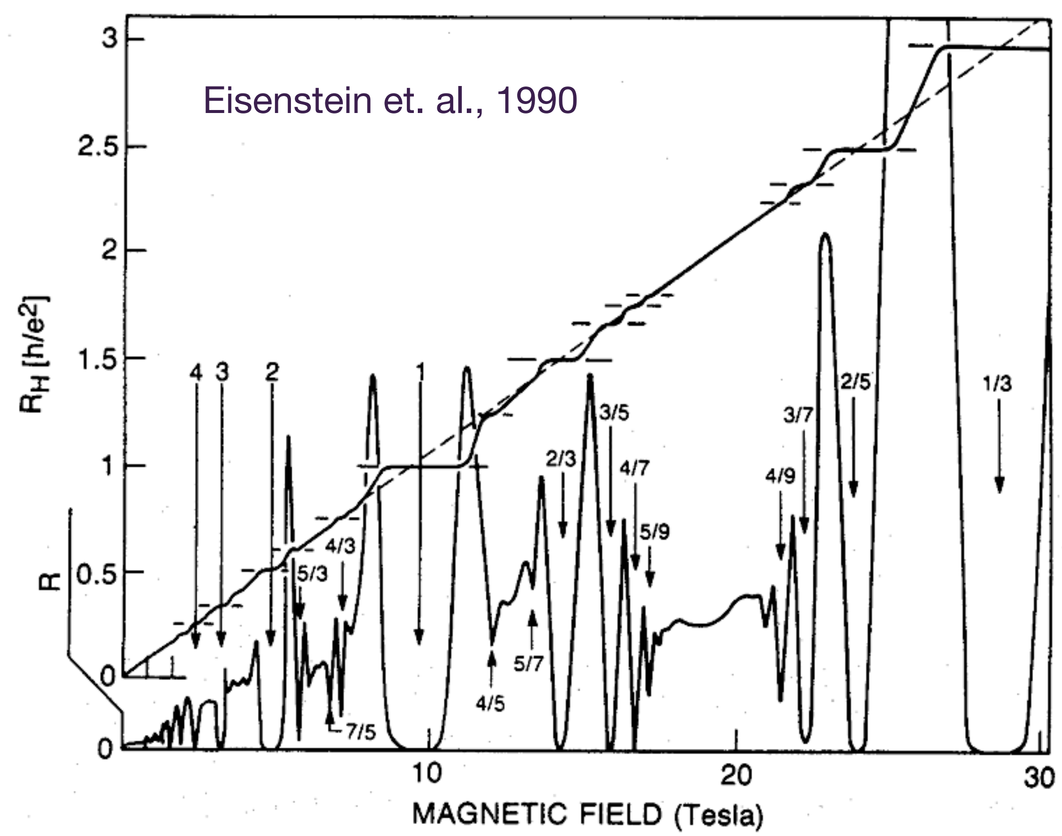


2D electron gas
low T
strong magnetic field

experimental signatures:

- fractional Hall conductance
- fractional electric charge
- (fractional statistics) controversial!

FQHE



2D electron gas
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experimental signatures:

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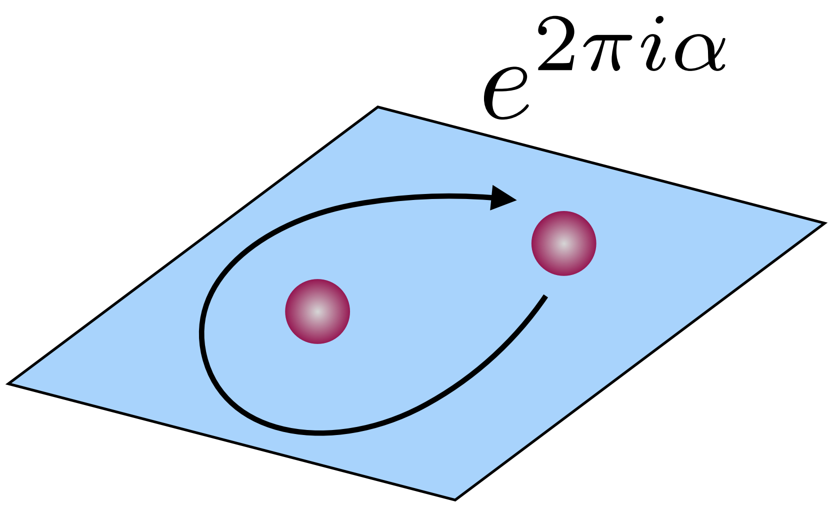
only established experimental system
with anyonic quasi-particles

Anyons

Leinaas & Myrheim (1977), Wilczek (1982)

identical (point) particles in 3D: bosons & fermions
2D: any exchange statistics allowed



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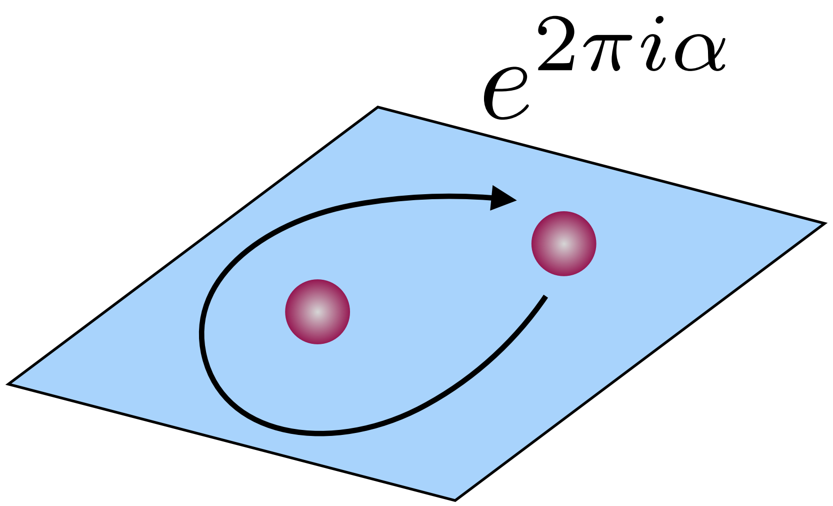
anyons are local quasi-particles:

braiding phase is path independent for sufficiently large distances between anyons: deviations exponentially small in the *distance between the anyons*



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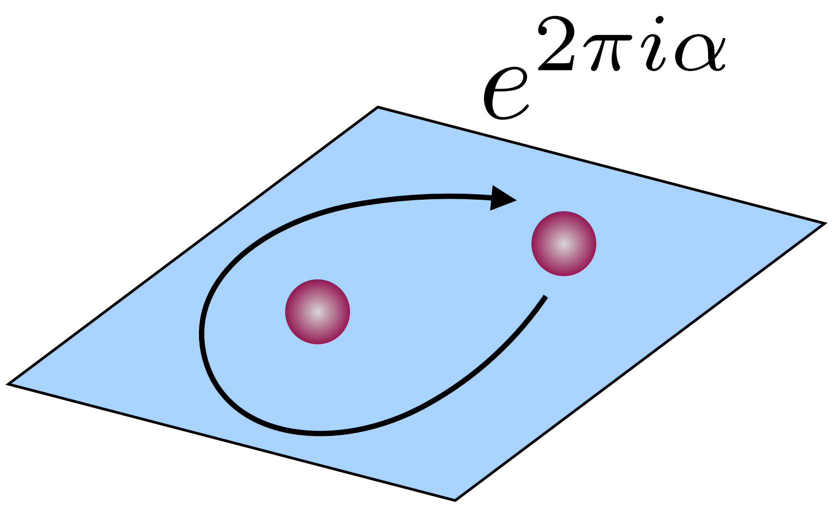

generalizations:

non-abelian anyons
interplay of anyons and symmetry
topological quantum computing

⋮

FQHE and model wave functions

$$\psi_{\frac{1}{3}} = \prod_{i<j} (z_i - z_j)^3 \, e^{-\sum_j |z_j|^2/(4\ell^2)}$$

Laughlin (1983)

incompressible quantum liquid (gapped, homogeneous)
fractional charge quasi-particles

plasma analogy

$$|\psi_{\frac{1}{3}}|^2 \sim Z_{ocp}$$

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'guess' model wave functions, for which we 'know' the properties
verify their relevance by numerical simulations
model wave function ↔ topological quantum field theory

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systematic way of guessing model wave functions:

- hierarchy schemes Haldane (1983), Halperin (1983)
- composite fermions Jain (1989)
- conformal field theory Moore & Read (1999)
- clustering conditions Bernevig & Haldane (2007)
- patterns of zeroes Wen (2012)

⋮

Model wave functions and CFT

Laughlin wave function "looks like" conformal field theory correlator (Moore & Read 1991)

$$\psi_{qh} = \langle \prod_\alpha H(\omega_\alpha) \prod_j V(z_j) \mathcal{O}_{bg} \rangle$$

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neutralizing background charge

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Moore-Read conjecture:
QH wave functions are conformal blocks of unitary, rational CFTs
topological properties manifest when using ‘minimal’ CFT
statistical phase = monodromy
Berry phase trivial (Aharonov-Bohm phase)

bulk CFT ↔ edge CFT (via 2+1 D TQFT)

CFT provides efficient matrix-product-state representation of model wave functions (Zaletel, Mong 2012) [numerical simulations on the cylinder]

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From now on, focus on quasi-particles of Laughlin wave function

Quasi-holes

unique definition / well-defined:

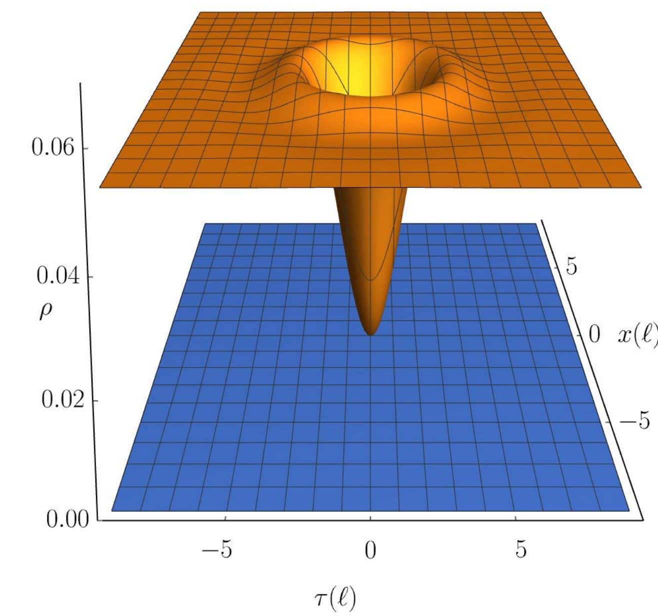
zero mode of model hamiltonian

correct position

correct topological properties

- sharp fractional charge
- path-independent, fractional braiding statistics

Laughlin quasi-hole



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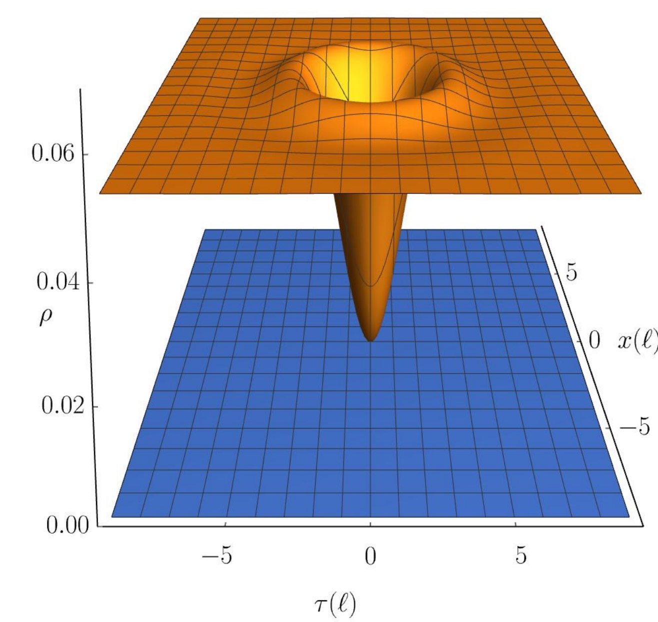

analytical: Arovas, Schrieffer, Wilczek (1984)
generalization by Gurarie & Nayak (1997), Read (2009)

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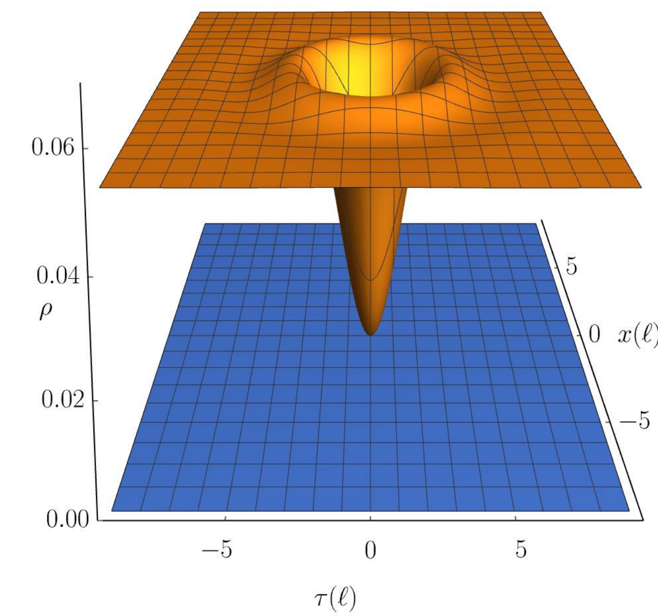


analytical: Arovas, Schrieffer, Wilczek (1984)
generalization by Gurarie & Nayak (1997), Read (2009)

numerical: Kjønsberg & Myrheim (1999)

Quasi-electrons

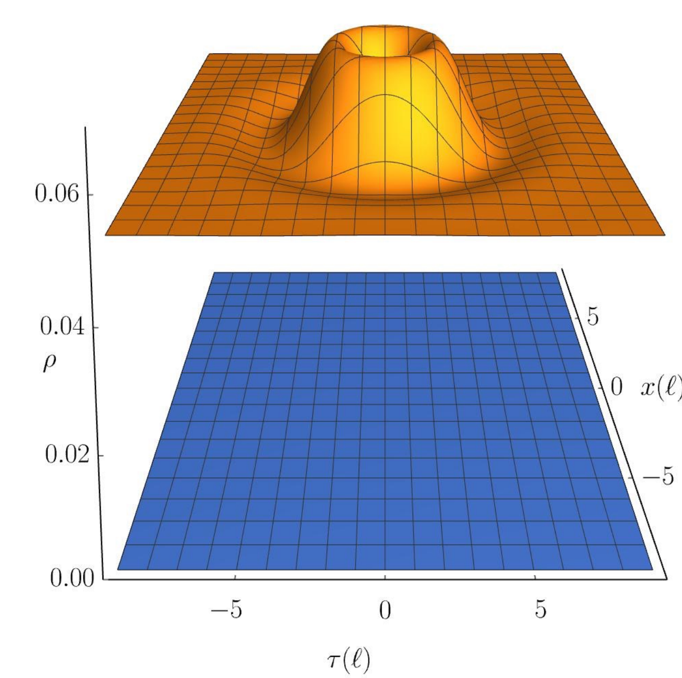
Statistics properties: qe = qh^{-1}

definition not unique (not zero-modes!)

- Laughlin Laughlin (1983)
- composite fermions Jain (2003)
- CFT Hansson, Hermanns, Viefers (2009)
- clustering conditions Bernevig & Haldane (2009)

all give different proposals

CFT quasi-electron



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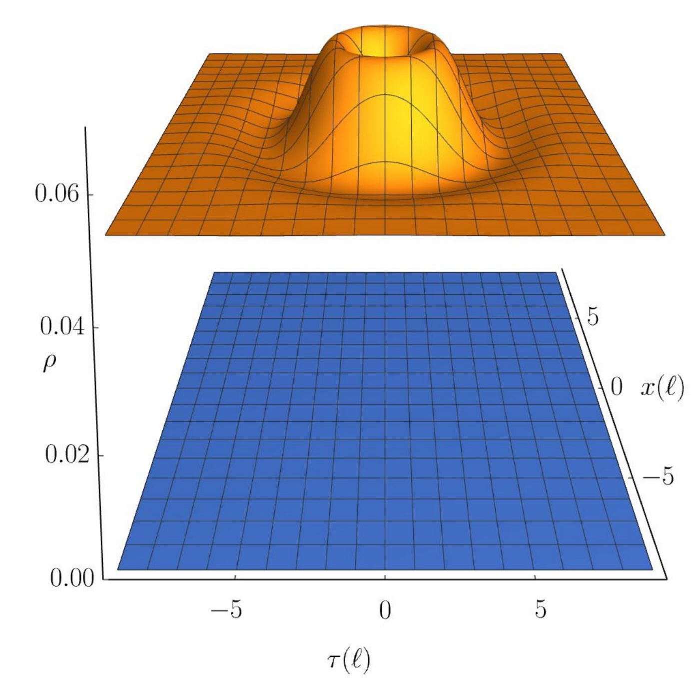
all give different proposals

sharp fractional charge ✓

none of these theoretical proposals has the correct (topological) properties:

- braiding ✗
- correct position ✗
- qe $\neq$ qh^{-1} ✗

CFT quasi-electron



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Laughlin qe $$\psi_{qe} = \prod_i (2\partial_{z_i} - \bar{\omega}) \prod_{i<j} (z_i - z_j)^3 e^{-\sum_j |z_j|^2/(4\ell^2)}$$

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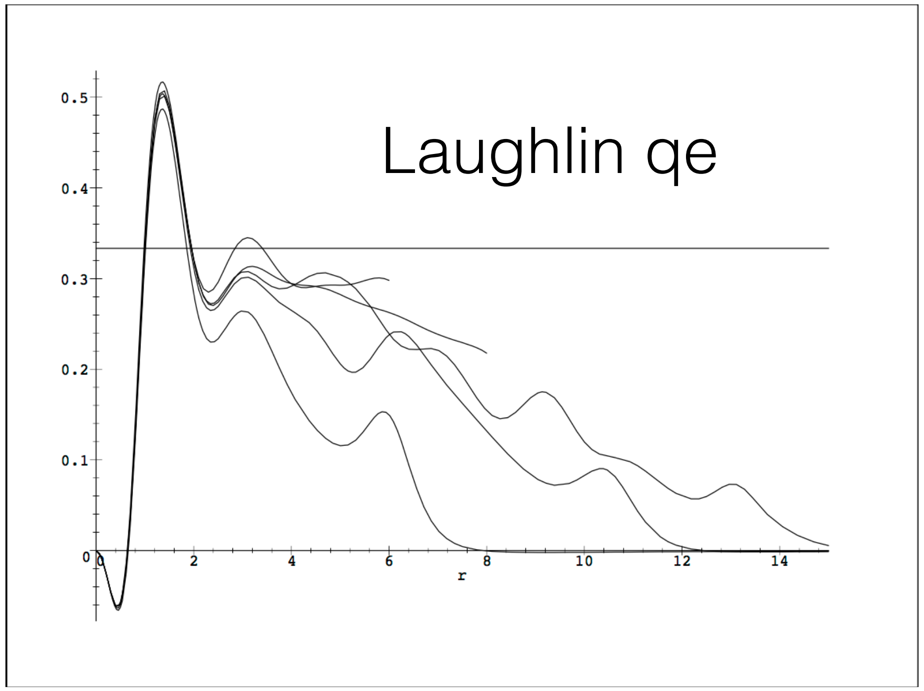

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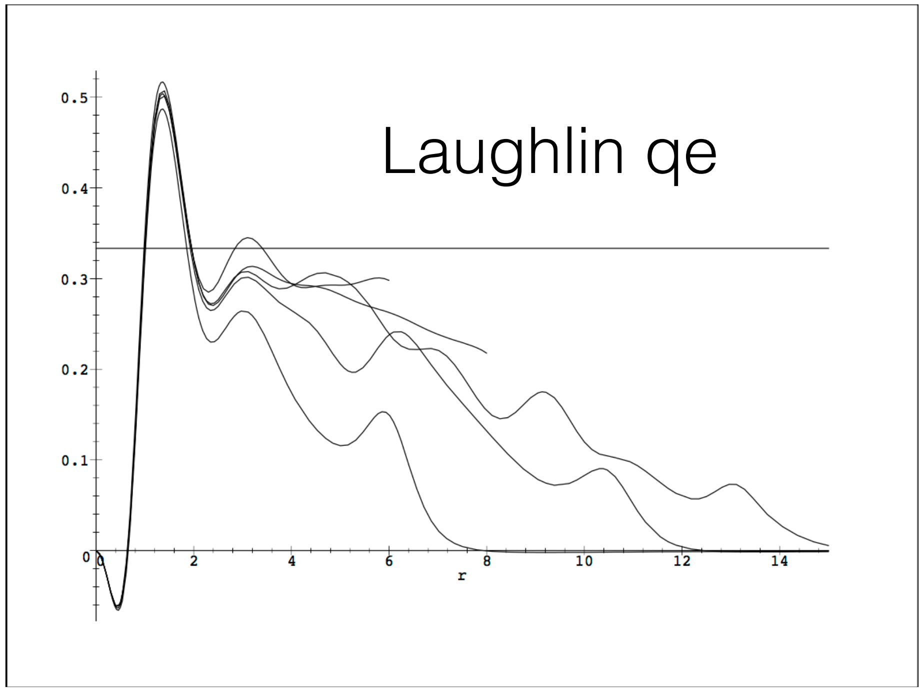

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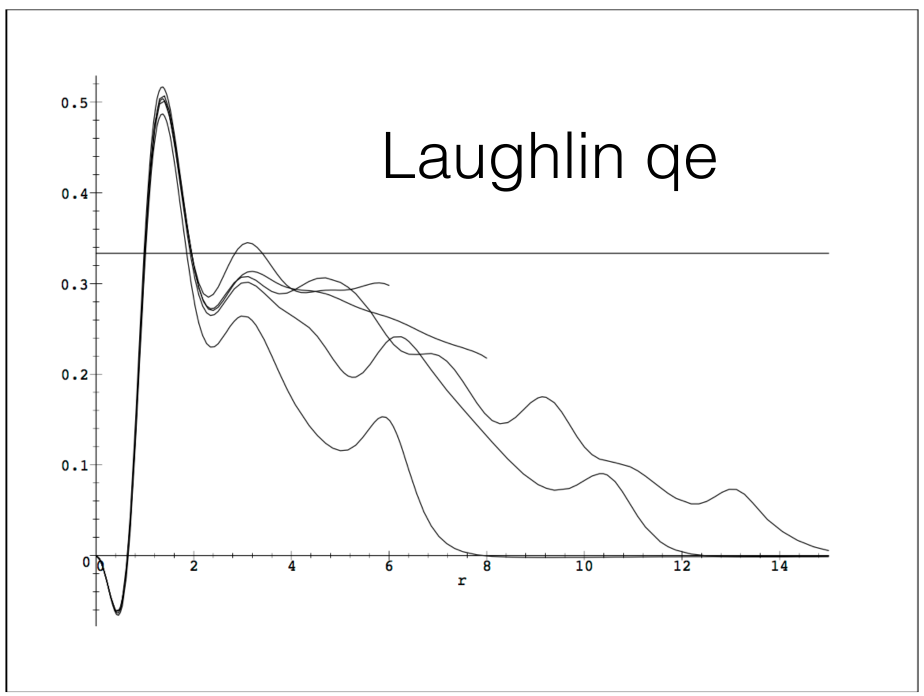

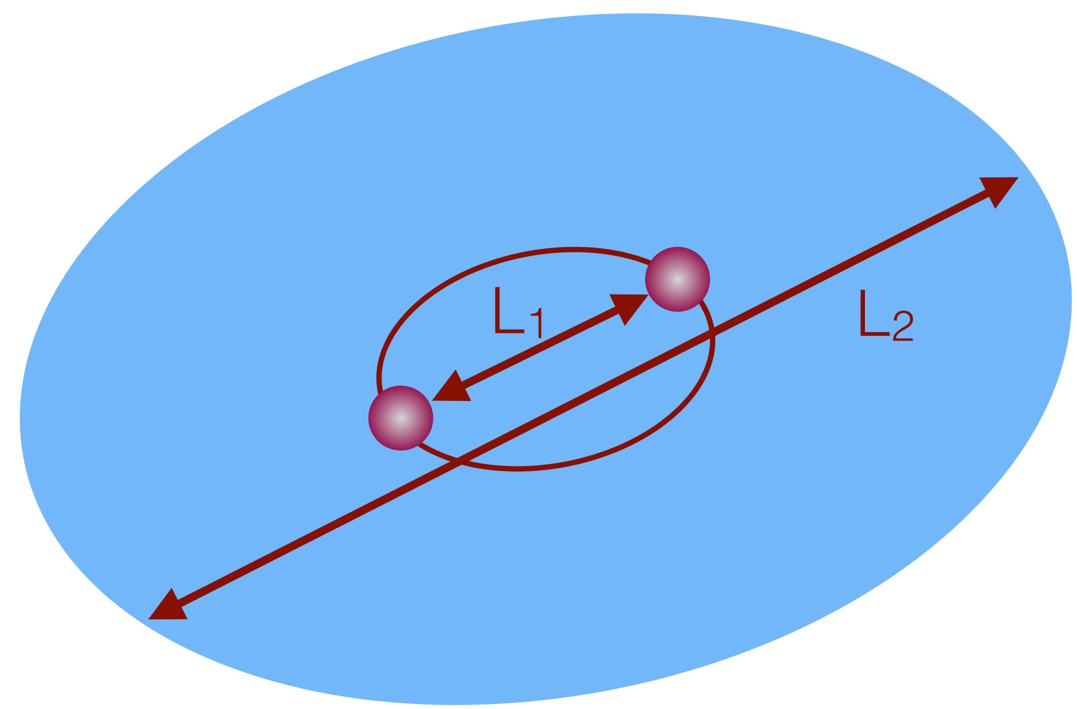

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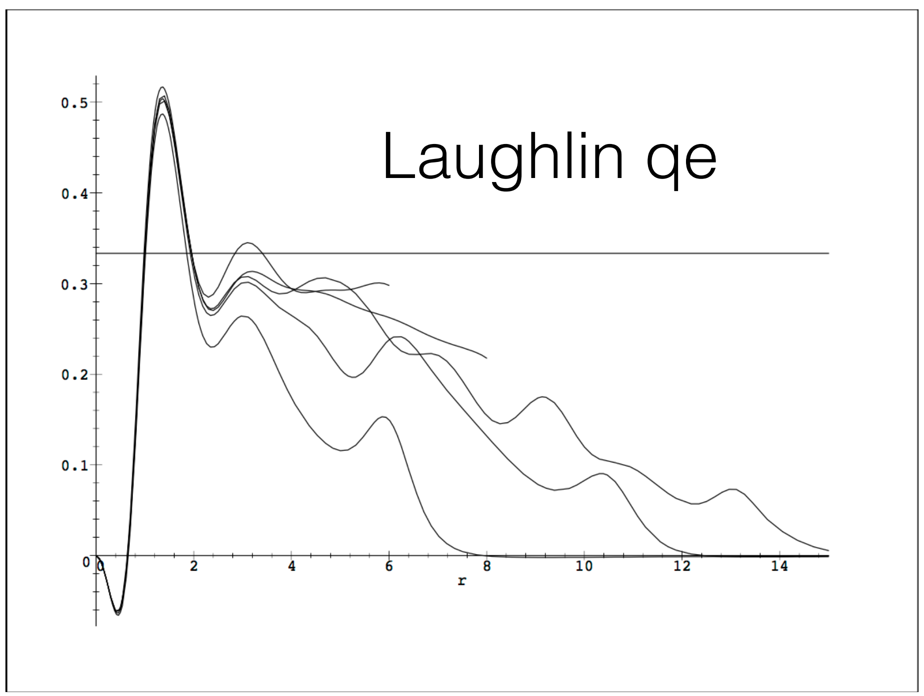

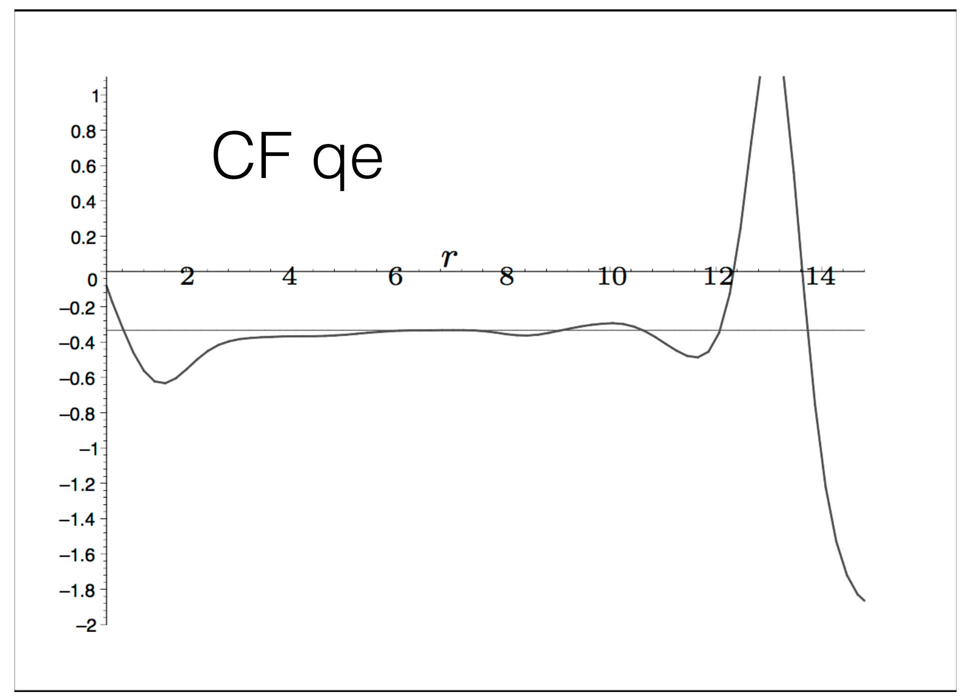

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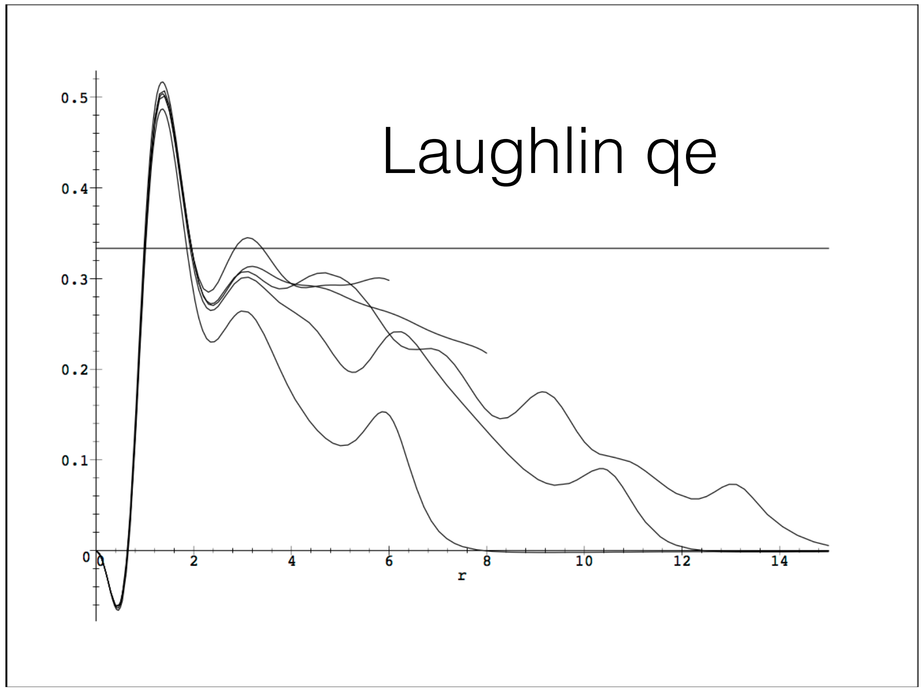

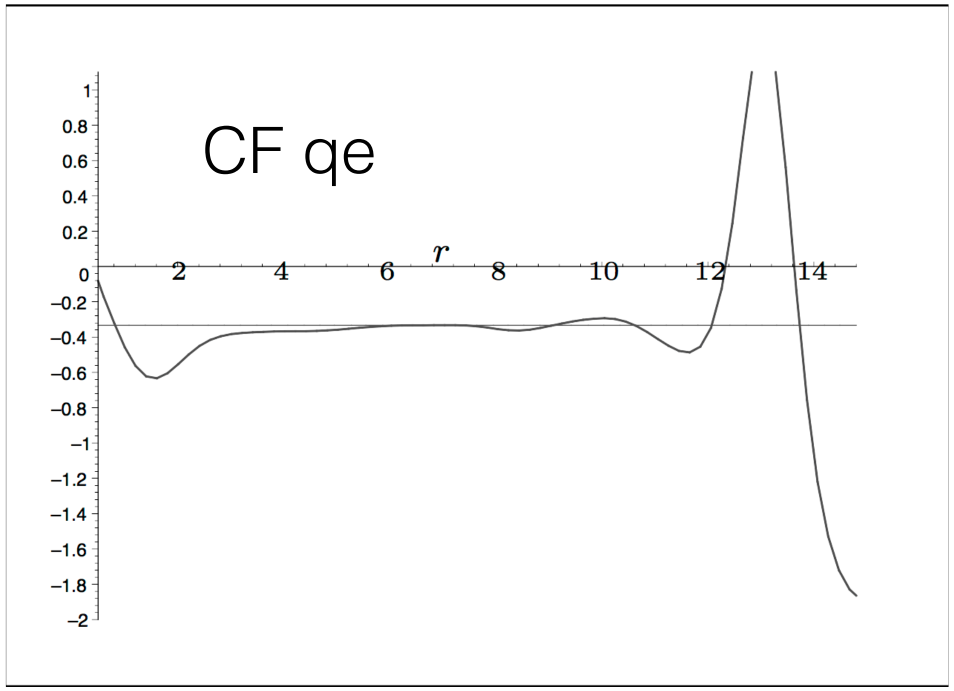

braiding phase not path independent

wrong sign?

Quasi-electrons

CF / CFT qe located at wrong position Jeon, Graham, Jain (2004)
not finite-size effect, but systematic shift Kjäll, Ardonne, Dwivedi, Hermanns, Hansson (2018)

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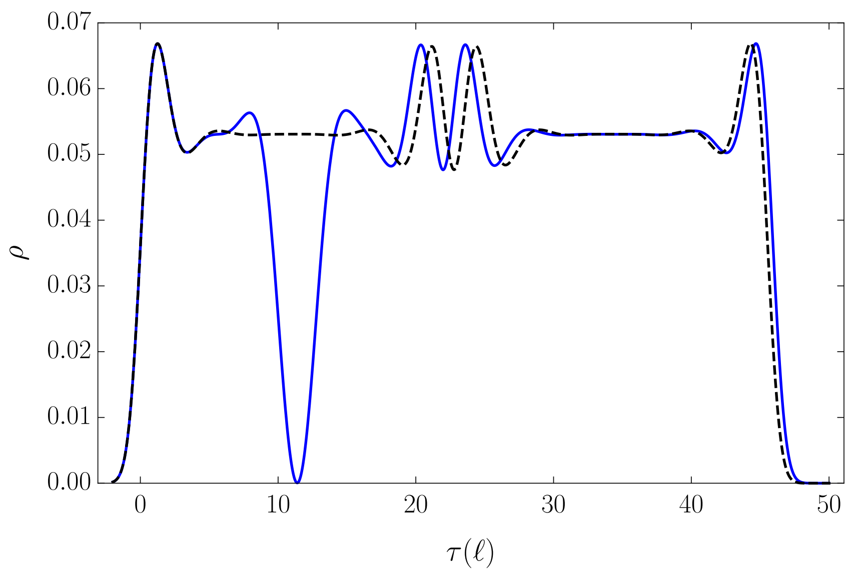
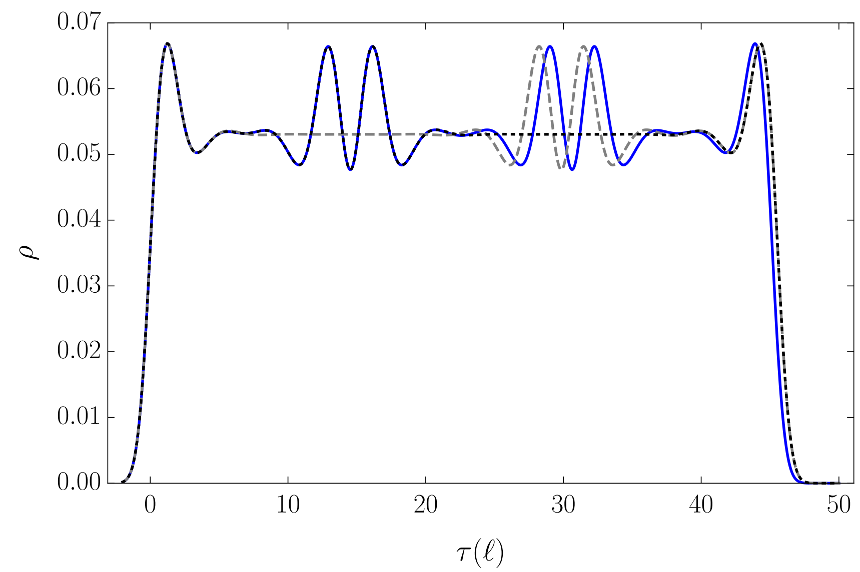


shift in presence of qh



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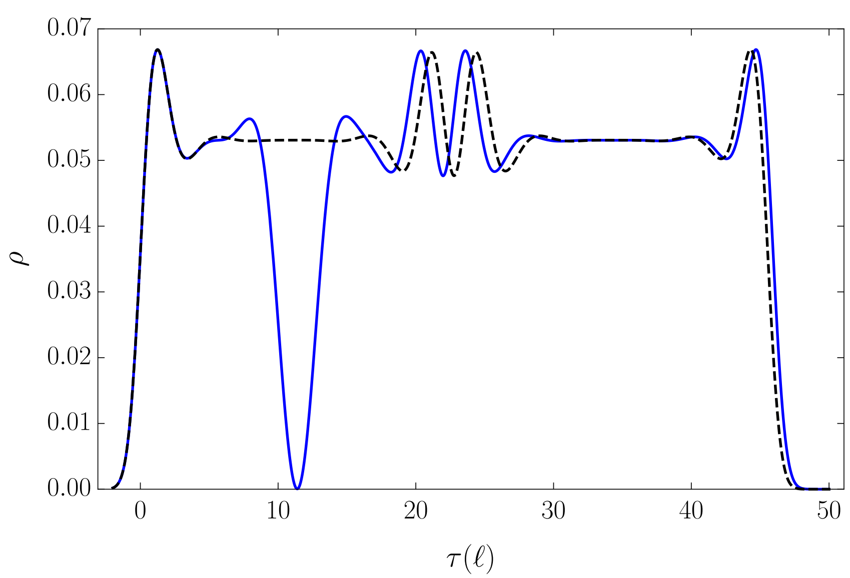

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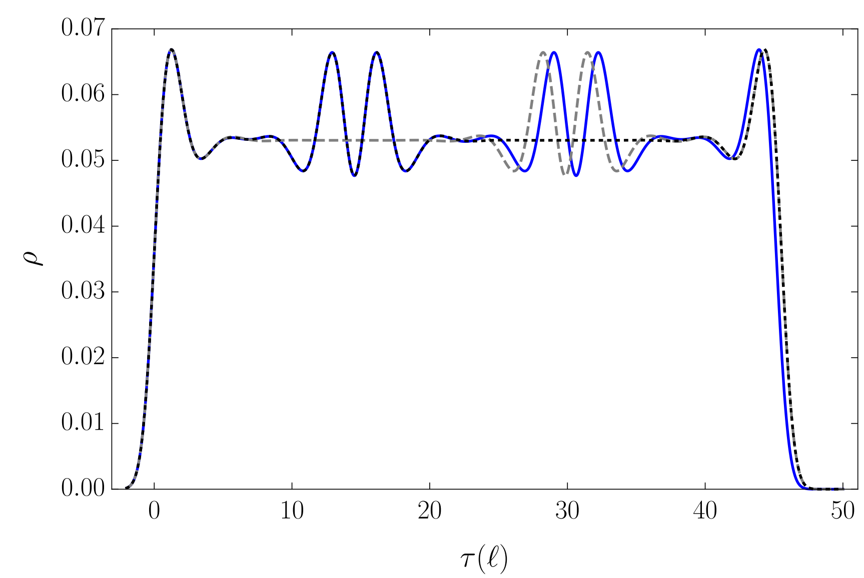

shift in presence of qe

qe’s are not properly screened!

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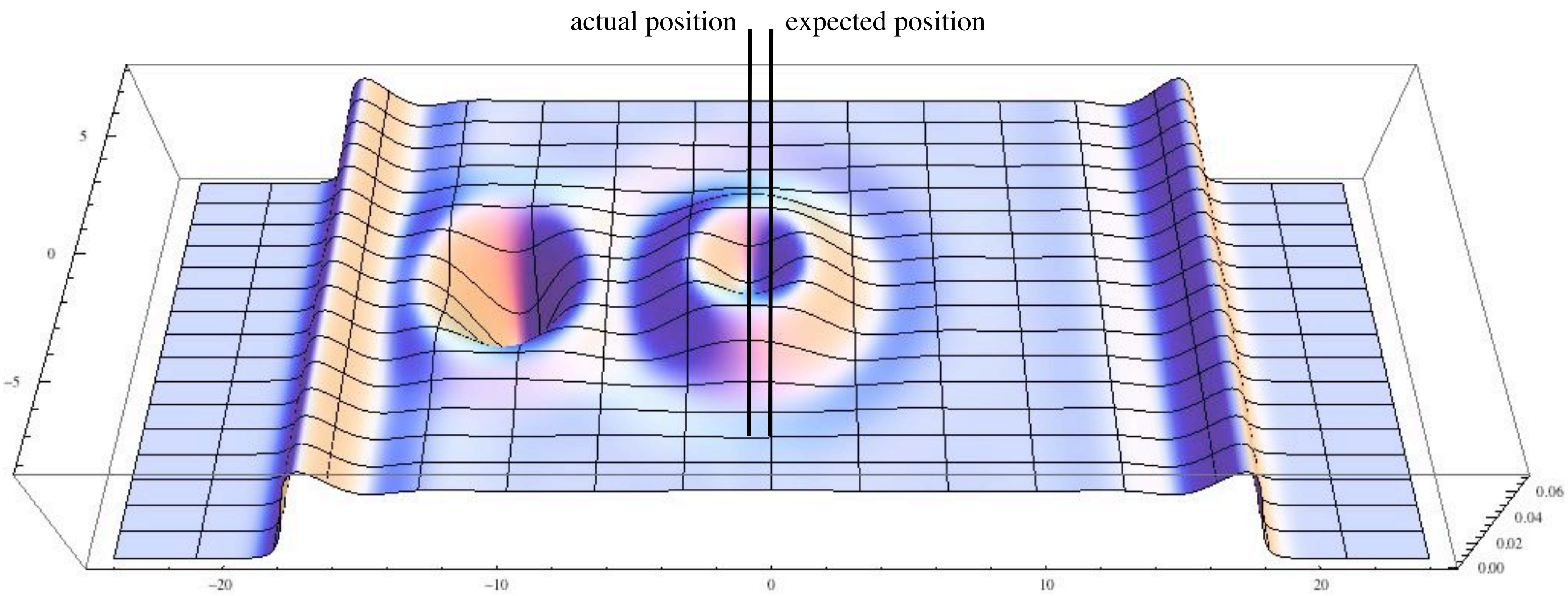

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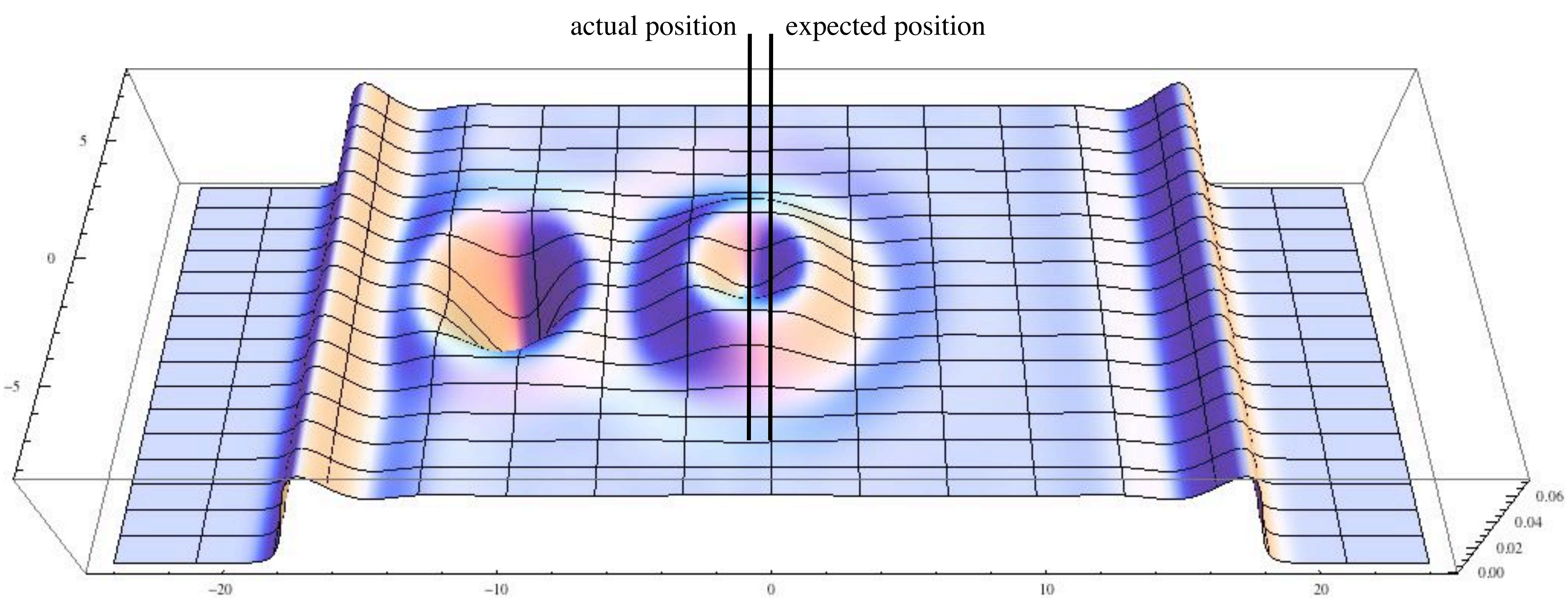

qe's are not properly screened!

Why are quasi-holes screened? plasma is screening
fluctuating, itinerant charges

Constructing quasi-electrons

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Constructing quasi-electrons

Why are quasi-holes screened? plasma is screening
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quasi-electrons: glue (fuse) inverse hole to an electron (shrink correlation hole):

$$V(z) = e^{i\sqrt{3}\varphi(z)} \quad H(\omega) = e^{i/\sqrt{3}\varphi(z)+i2/\sqrt{6}\tilde{\varphi}(z)} \quad \tilde{V}(z) = \partial_z e^{i2/\sqrt{3}\varphi(z)-i2/\sqrt{6}\tilde{\varphi}(z)}$$

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locate using exponential kernel (lowest Landau level projector!):

$$K(\xi, z) = \sum_k \bar{\phi}_k(\xi)\phi_k(z) = \frac{1}{2\pi q\ell^2} e^{-\frac{1}{4q\ell^2}\left(|z-\xi|^2+(\xi\bar{z}-\bar{\xi}z)\right)}$$

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it is the charge of $\tilde{\varphi}(z)$ that is not properly screened!

Screening quasi-electrons

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solution: remove the charge of $\tilde{\varphi}(z)$, that is, drop the zero mode!

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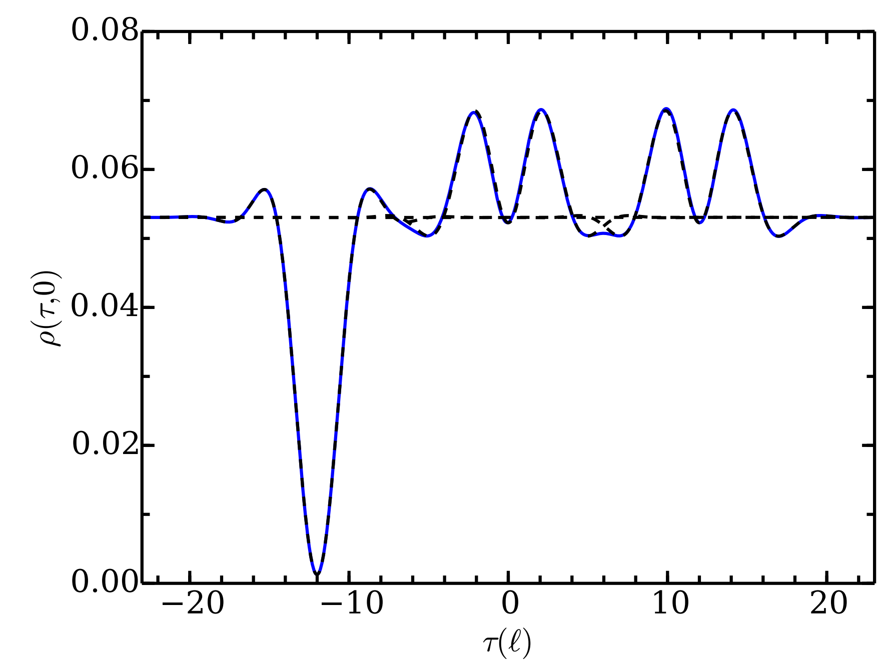
correct localization

correct statistics properties

good density profile for qe's that are far apart

for small distances, we need an additional ad-hoc change of the phases in MPS description



Screening quasi-electrons

Why are quasi-holes screened? plasma is screening fluctuating, itinerant charges

quasi-electrons: glue (fuse) inverse hole to an electron (shrink correlation hole):

$$V(z) = e^{i\sqrt{3}\varphi(z)} \quad H(\omega) = e^{i/\sqrt{3}\varphi(z)+i2/\sqrt{6}\tilde{\varphi}(z)} \quad \tilde{V}(z) = e^{i2/\sqrt{3}\varphi(z)-i2/\sqrt{6}\tilde{\varphi}(z)}$$

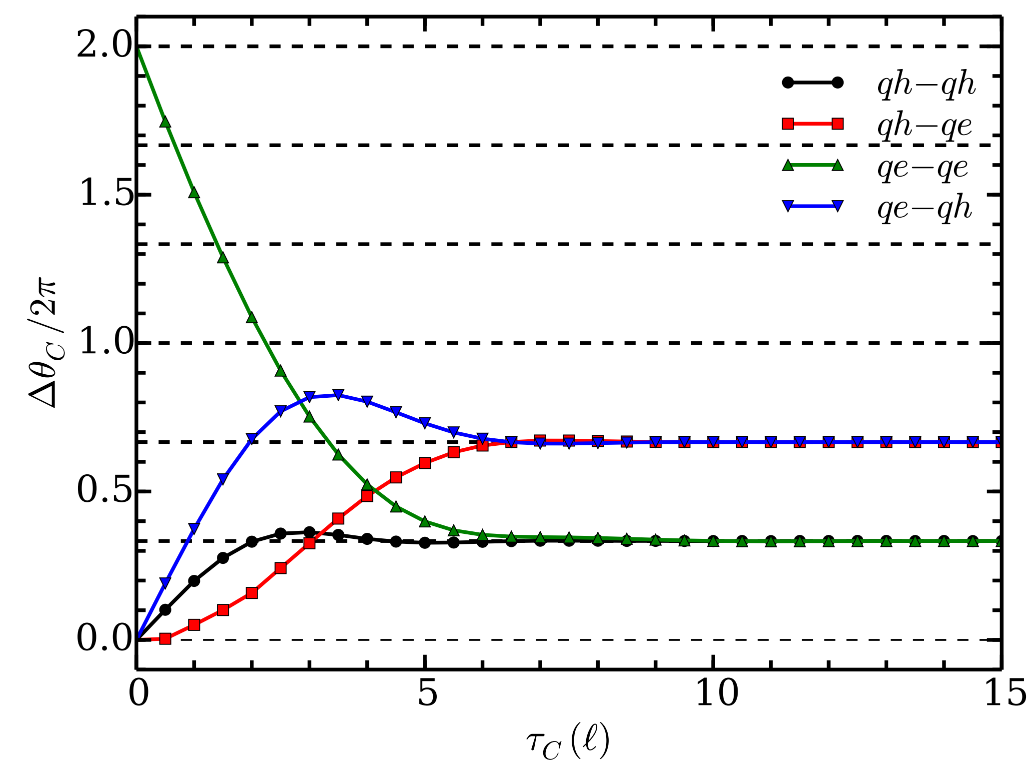
solution: remove the charge of $\tilde{\varphi}(z)$, that is, drop the zero mode!

correct localization

correct statistics properties

good density profile for qe's that are far apart

for small distances, we need an additional ad-hoc change of the phases in MPS description



Quasi-electrons – a proposal

‘Laughlin-like’ quasi-electron using conformal field theory
starting point: modified Laughlin state Girvin & Jach (1984)

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$$= \mathcal{P}_{LLL} \langle \mathcal{O}_{bg} \prod_{i=1}^{N} V(z_i, \bar{z}_i) \rangle \quad \text{with} \quad V(z_i, \bar{z}_i) = e^{i\sqrt{q+1}\varphi(z)} e^{i\bar{\chi}(\bar{z})}$$

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local quasi-electron operator, all charges are screened: $P_{qe} = e^{i\bar{\chi}(\bar{\xi})}$

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correct braiding properties?

Summary and outlook

MPS description of quasi-electrons

- deficiency of CF/CFT quasi-electrons
- remedied by properly screening the operators

Kjäll, Ardonne, Dwivedi, Hermanns, Hansson, J. Stat. Mech. (2018)

CFT description of Laughlin-like quasi-electron

- explicit model wave functions for quasi-electrons
- LLL projection using MPS (✓)

N. Regnault (private communication)

- analytical understanding (?)

Comparison to lattice realizations A. Nielsen et al.

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You for your attention!