The physics and quantum computational supremacy of multi-photon correlation interference with single-photon states of arbitrary distinguishability

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Two-photon interference at a beam splitter


“It is not the photons that interfere physically, it is their probability amplitudes that interfere – and probability amplitudes can be defined equally well for arbitrary numbers of photons”

ONE HUNDRED YEARS OF LIGHT QUANTA Nobel Lecture, December 8, 2005 by Roy J. Glauber
Interference of probability amplitudes

$$G^{(2)}(t_1', t_2') = \frac{1}{2} \left| + A_1(t_2') A_2(t_1') - A_1(t_1') A_2(t_2') \right|^2$$

$$= \frac{1}{2} \left| \text{perm} \begin{pmatrix} -A_1(t_1') & A_2(t_1') \\ A_1(t_2') & A_2(t_2') \end{pmatrix} \right|^2$$
Three-photon interference

\[3! = 6 \text{ interfering 3-photon probability amplitudes}\]
Multiboson Correlation Interferometry with Arbitrary Single-Photon Pure States

Vincenzo Tamma* and Simon Laibacher

N! interfering N-photon probability amplitudes:
N! simultaneous computational tasks!
Quantum Computational Supremacy

- Zooming in on arbitrary N-Photon State Evolutions
- N-Photon Entanglement Generation
- Multi-Boson Computational Speed-Up
Quantum Computational Supremacy

- Zooming in on arbitrary N-Photon State Evolutions
Multi-Photon Correlation Landscapes

\[ |S\rangle := \bigotimes_{s=1}^{N} |1[\xi_s]\rangle_s, \]

\[ G_{\{t_d,p_d\}}^{(N)} = \left| \text{perm} \mathcal{T}_{\{t_d,p_d\}} \right|^2 \quad \text{with} \quad \mathcal{T}_{\{t_d,p_d\}} := [\mathcal{U}_{d,s} (p_d \cdot \chi_s(t_d))]_{d=1,\ldots,N}^{s=1,\ldots,N}, \]

\[ \chi_s(t) := \mathcal{F}[\xi_s](t - \Delta t) \]
Quantum interference with identical photons

Identical photons, perm $U = 0$ → Destructive Quantum Interference
Photons of different colors: no time-resolved detections

Different colors:
\[ \omega_s - \omega_{s'} \gg \Delta \omega \ \forall s \neq s' \]  
No multiphoton interference
Photons of different colors: time-resolved detections

Detection integration time:

\[ T_I \ll \left| \omega_s - \omega_{s'} \right|^{-1} \]

Three-Photon “Dip”
Quantum Beats

Zooming in on arbitrary N-photon state evolution

Quantum Computational Supremacy

- Zooming in on arbitrary N-photon State Evolutions
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The Computational Complexity of Linear Optics

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2M-port linear interferometer

$N$ single photons in input config. $S$

$\mathcal{U}^{(\mathcal{D}, S)}$

$N$ single photons detected in the $N$-port sample $\mathcal{D}$

$\mathcal{U}(\mathcal{D}, S) := [\mathcal{U}_{d,s}]_{d \in \mathcal{D}, s \in S}$

$P(\mathcal{D}; S) = \left| \text{per} \mathcal{U}(\mathcal{D}, S) \right|^2$

- Identical photons
- Random unitary transformation $\mathcal{U}$
- $M >> N \geq 30$
- Sampling measurements (no time-resolved detections)

Boson sampling with identical bosons hard to simulate classically
Partialy distinguishable input photon states

\[
|S\rangle := \bigotimes_{s=1}^{N} |1, \xi_s\rangle_s.
\]

Interference-type matrices:

\[
A^{(D,S)}_\rho := \left[ U^*_d,s U_{d,s,\rho(s)} \right]_{d \in D, s \in S}
\]

N-boson indistinguishability factors:

\[
f_\rho(S) := \prod_{s \in S} g(s, \rho(s)) \quad \text{with} \quad g(s, s') = \int_{-\infty}^{\infty} d\omega \, \xi_s(\omega) \cdot \xi_{s'}(\omega)
\]

Pairwise state distinguishability

From the Physics to the Computational Complexity of Multiboson Correlation Interference

Simon Laibacher and Vincenzo Tamma

Multiboson Correlation Sampling:

- Arbitrary single-photon pure states  \[ |S\rangle := \bigotimes_{s \in S} |1[\xi_s]\rangle_s \bigotimes_{s \notin S} |0\rangle_s \]
- Sampling measurements based on time and polarization-resolving detections
For simplicity, we consider input photon spectra in the narrow bandwidth approximation and a polarization-configuration.

\[ \mathcal{U}(\mathcal{D}, \mathcal{S}) \]

Correlated detections \( (\mathcal{D}, \{t_d, \vec{p}_d\}) \) at an \( N \)-port sample \( \mathcal{D} \)

\[
G^{(\mathcal{D}, \mathcal{S})}_{\{t_d, \vec{p}_d\}} = \left| \text{perm} \mathcal{T}^{(\mathcal{D}, \mathcal{S})}_{\{t_d, \vec{p}_d\}} \right|^2
\]

with

\[
\mathcal{T}^{(\mathcal{D}, \mathcal{S})}_{\{t_d, \vec{p}_d\}} := \left[ \mathcal{U}_{d,s} \left( \vec{p}_d \cdot \chi_s(t_d) \right) \right]_{d \in \mathcal{D}, s \in \mathcal{S}}
\]

\[
\chi_s(t) := \mathcal{F}[\xi_s](t - \Delta t)
\]
Photons of different colors

Different colors: \( \omega_s - \omega_{s'} \gg \Delta \omega \forall s \neq s' \)

➤ Boson Sampling Trivial
Different colors: \( \omega_s - \omega_{s'} \gg \Delta\omega \ \forall s \neq s' \)

Detection integration time: \( T_I \ll |\omega_s - \omega_{s'}|^{-1} \)

N-photon interference at any detection time:

\[
G_{\{t_d, p_d\}}^{(D, S)} \propto \left| \text{perm} \left( [U_{d,s}^{(D, S)} e^{i\omega_s t_d}]_{d\in D, s\in S} \right) \right|^2
\]

- Multi-Boson Correlation Sampling Hard even in the Approximate case
Summary

- Zooming in on N-photon state evolution


- N-photon Entanglement Generation

- Multi-Boson Computational Speed-Up