

Coherent backscattering in the Fock space of ultracold bosonic atoms

Peter Schlagheck



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Coworkers



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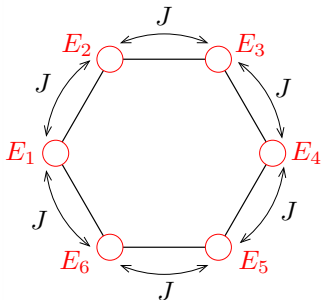
Outline

- Thermalization and localization in bosonic many-body systems
- Weak localization and coherent backscattering
- Coherent backscattering in Fock space
- Verification with ultracold atoms
- Conclusion

Interacting bosonic many-body systems

We consider a one-dimensional disordered Bose-Hubbard ring:

$$\hat{H} = \sum_{l=1}^L \left[E_l \hat{b}_l^\dagger \hat{b}_l - J \left(\hat{b}_l^\dagger \hat{b}_{l-1} + \hat{b}_{l-1}^\dagger \hat{b}_l \right) + \frac{U}{2} \hat{b}_l^\dagger \hat{b}_l^\dagger \hat{b}_l \hat{b}_l \right]$$



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J.M. Deutsch, PRA 43, 2046 (1991); M. Srednicki, PRE 50, 888 (1994);

L. D'Alessio *et al.*, Adv. Phys. 65, 239 (2016)

- all many-body eigenstates are equidistributed within the classical phase space of the system
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- Bose-Einstein distribution of the population of individual one-body eigenmodes

PS & D. Shepelyansky, PRE 93, 012126 (2016)

Interacting bosonic many-body systems

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I.V. Gornyi, A.D. Mirlin & D.G. Polyakov, PRL 95, 206603 (2005);

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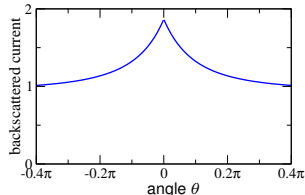
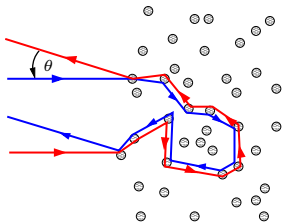
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→ weak localization in the Fock space of many-body systems?

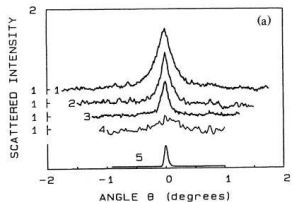
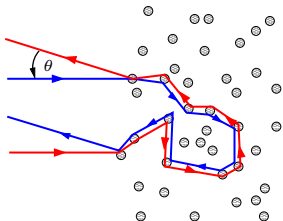
Coherent backscattering in disordered systems

- constructive wave interference between reflected classical paths and their time-reversed counterparts



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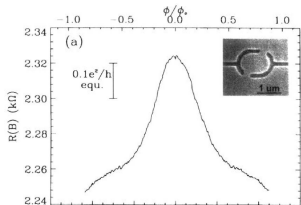
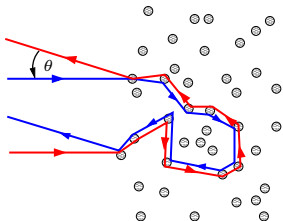
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- coherent backscattering of laser light in disordered media
M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)
P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)

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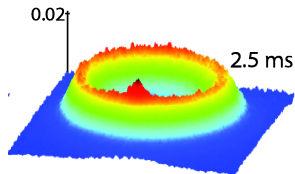
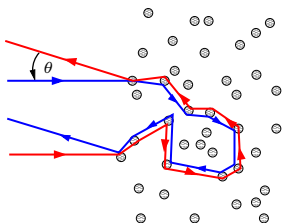


- coherent backscattering of laser light in disordered media
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A. M. Chang *et al.*, PRL 73, 2111 (1994)

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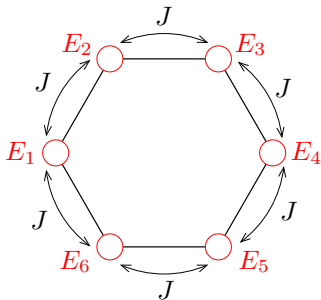
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- coherent backscattering of laser light in disordered media
- magnetoresistance within two-dimensional electron gases
- coherent backscattering of ultracold atoms in 2D disorder
F. Jendrzejewski *et al.*, PRL 109, 195302 (2012)
(see also N. Cherroret *et al.*, PRA 85, 011604(R) (2012))

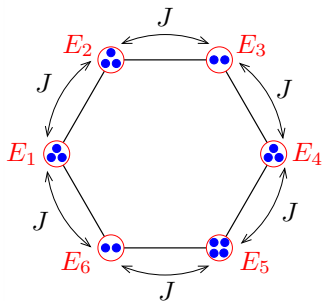
Many-body CBS in disordered Bose-Hubbard rings

$$\hat{H} = \sum_{l=1}^L \left[E_l \hat{b}_l^\dagger \hat{b}_l - J \left(\hat{b}_l^\dagger \hat{b}_{l-1} + \hat{b}_{l-1}^\dagger \hat{b}_l \right) + \frac{U}{2} \hat{b}_l^\dagger \hat{b}_l^\dagger \hat{b}_l \hat{b}_l \right]$$



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Start with a given initial Fock state, e.g. $|\mathbf{n}^i\rangle = |3, 3, 2, 3, 4, 2\rangle$,
and determine the probability to reach a given final Fock state
 $|\mathbf{n}^f\rangle = |n_1^f, \dots, n_L^f\rangle$ at time t under some disorder average

Semiclassical van Vleck-Gutzwiller approach

Represent the quantum transition amplitude

$$\langle \mathbf{n}^f | \hat{U} | \mathbf{n}^i \rangle \equiv \langle \mathbf{n}^f | \exp[-\frac{i}{\hbar} t \hat{H}] | \mathbf{n}^i \rangle = \sum_{\gamma} A_{\gamma} e^{iR_{\gamma}/\hbar}$$

in terms of classical (Gross-Pitaevskii) trajectories γ satisfying

$$i\hbar \frac{\partial}{\partial t} \psi_l(t) = E_l \psi_l(t) - J [\psi_{l+1}(t) + \psi_{l-1}(t)] + U (|\psi_l(t)|^2 - 1) \psi_l(t)$$

with $\psi_l(0) = \sqrt{n_l^i + 0.5} e^{i\theta_l^i}$ and $\psi_l(t) = \sqrt{n_l^f + 0.5} e^{i\theta_l^f}$

for all $l = 1, \dots, L$ with some arbitrary phases $0 \leq \theta_l^{i/f} < 2\pi$

R_{γ} = classical action of the trajectory γ

A_{γ} = stability amplitude (related to Lyapunov exponent) of γ

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Average detection probability of the Fock state $|\mathbf{n}^f\rangle$:

$$\begin{aligned} \overline{|\langle \mathbf{n}^f | \hat{U} | \mathbf{n}^i \rangle|^2} &= \sum_{\gamma, \gamma'} \underbrace{A_{\gamma} A_{\gamma'} e^{i(R_{\gamma} - R_{\gamma'})/\hbar}} \\ &= 0 \text{ if } R_{\gamma} \neq R_{\gamma'} \end{aligned}$$

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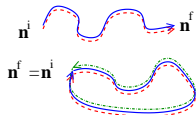
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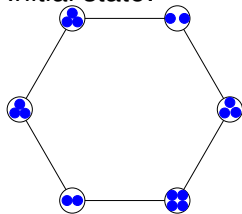
$$\overline{|\langle \mathbf{n}^f | \hat{U} | \mathbf{n}^i \rangle|^2} = \sum_{\gamma} |A_{\gamma}|^2 \text{ if } \mathbf{n}^f \neq \mathbf{n}^i$$

$$\overline{|\langle \mathbf{n}^i | \hat{U} | \mathbf{n}^i \rangle|^2} = 2 \sum_{\gamma} |A_{\gamma}|^2 \text{ due to CBS}$$

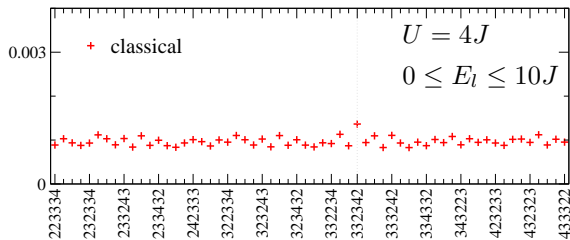


Comparison with numerical data

Initial state:



Detection probability



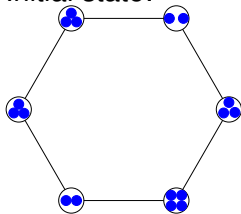
$$\overline{|\langle \mathbf{n}^f | \hat{U} | \mathbf{n}^i \rangle|^2}^{\text{classical}} = \sum_{\gamma} |A_{\gamma}|^2$$
$$= \int_0^{2\pi} \frac{d\theta_2^i}{2\pi} \cdots \int_0^{2\pi} \frac{d\theta_L^i}{2\pi} \prod_{l=2}^L \delta \left(n_l^f + 0.5 - |\psi_l(t; n_1^i, 0, n_2^i, \theta_2^i \dots n_L^i, \theta_L^i)|^2 \right)$$

in the presence of classical chaos and ergodicity

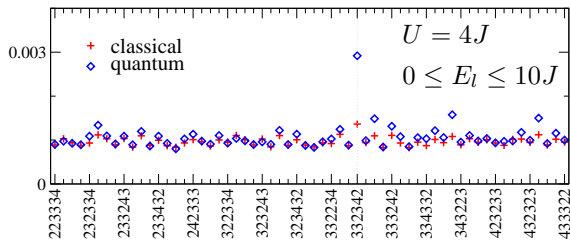
J.H. Hannay & A.M. Ozorio de Almeida, JPA 17, 3429 (1984)

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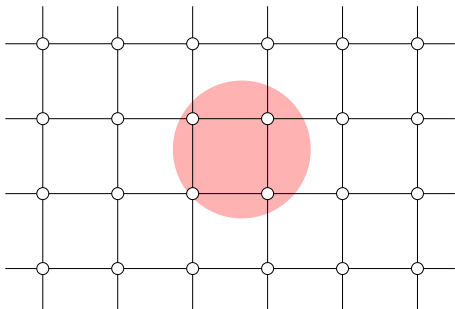
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→ significant deviation from quantum ergodicity

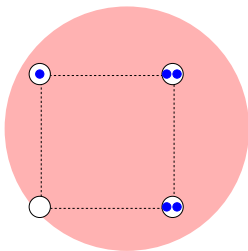
→ observable in a cold-atom experiment?

Proposal for a few-body CBS experiment



Consider an isolated 2D sheet of a 3D optical lattice within which you isolate a single plaquette (e.g. by means of a focused red-detuned laser beam)

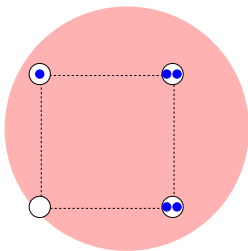
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Experimental procedure:

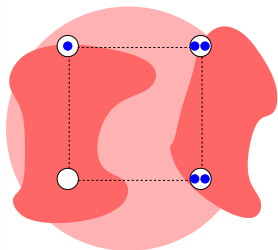
1. Load the plaquette with a well-defined number of bosonic atoms in the deep Mott-insulator regime

Proposal for a few-body CBS experiment



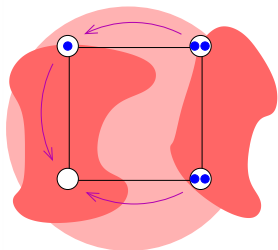
2. Add some disorder (by means of an optical speckle field) and/or randomly displace the focus of the red-detuned laser beam

Proposal for a few-body CBS experiment



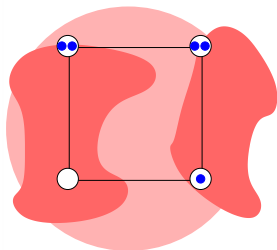
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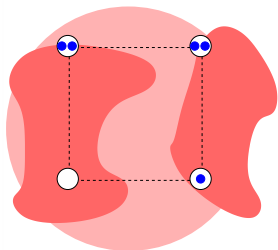
3. Switch on the inter-site hopping and let the atoms move ...

Proposal for a few-body CBS experiment



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Proposal for a few-body CBS experiment



4. Quench back to the Mott regime after a given evolution time and detect the atomic population on each site

W. Bakr *et al.*, *Nature* 462, 74 (2009)

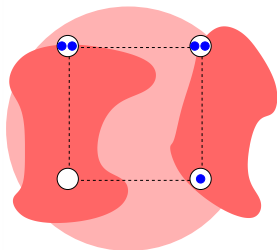
J. Sherson *et al.*, *Nature* 467, 68 (2010)

A.M. Kaufman *et al.*, *Science* 353, 794 (2016)

(→ beyond binary populations)

Proposal for a few-body CBS experiment

2 2 1 0



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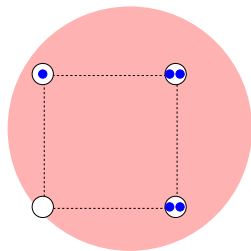
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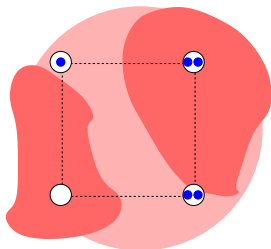


2 2 1 0

5. Repeat the experiment with the same initial state but for a different disorder configuration

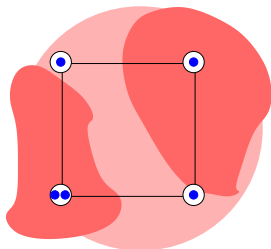
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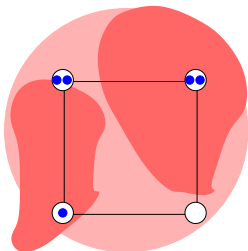
Proposal for a few-body CBS experiment



2	2	1	0
1	1	1	2

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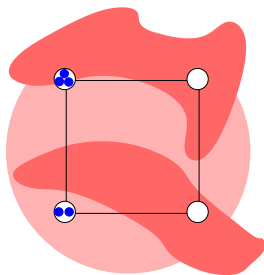
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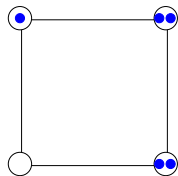
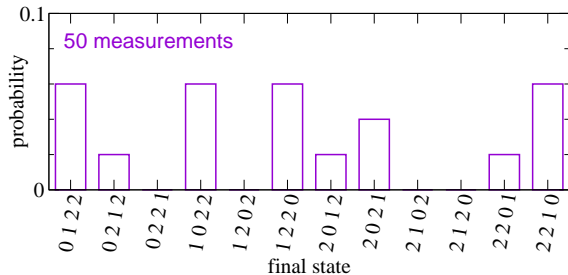


2	2	1	0
1	1	1	2
2	2	0	1
3	0	0	2

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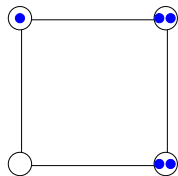
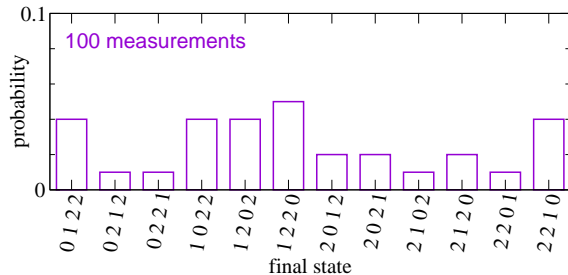
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$



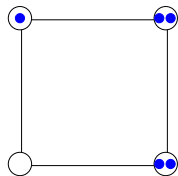
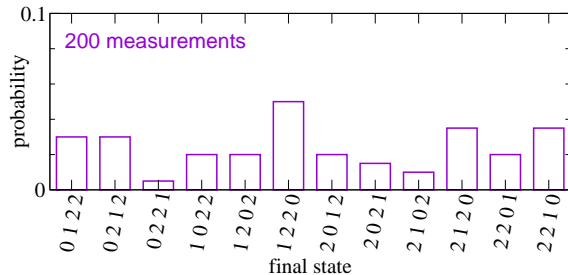
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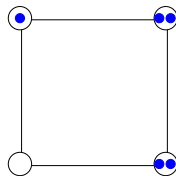
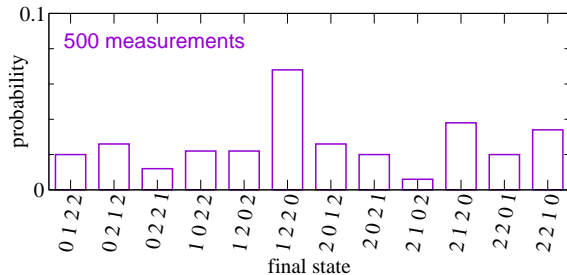
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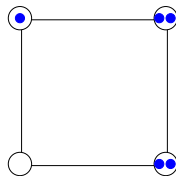
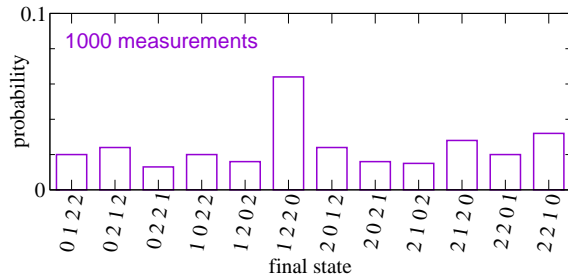
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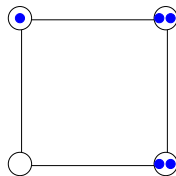
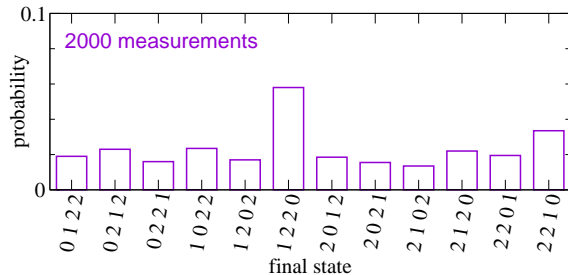
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$



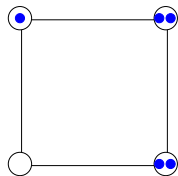
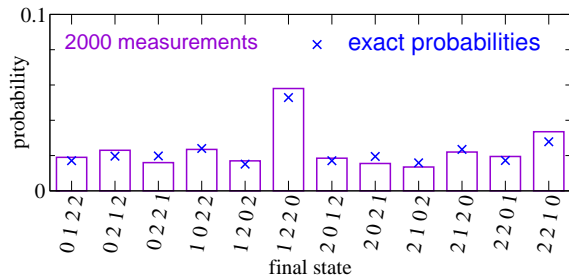
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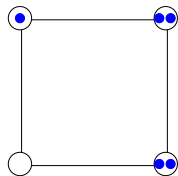
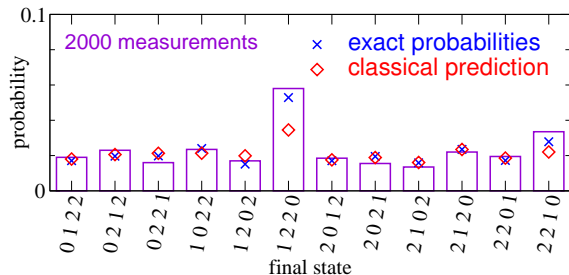
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→ some 1000 repetitions of the experiments are required in order to see the signature of coherent backscattering

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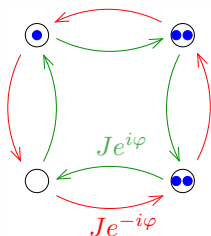
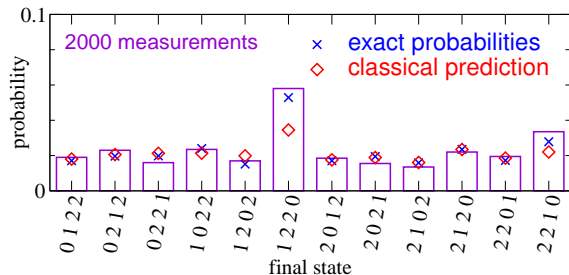
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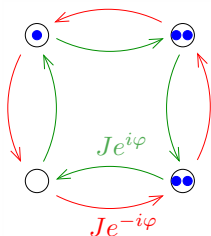
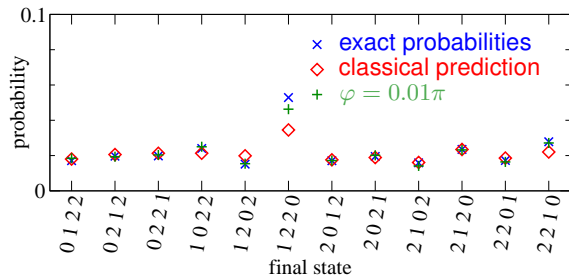
→ induce a synthetic gauge field in order to break time-reversal invariance

Y.J. Lin *et al.*, Nature 462, 628 (2009)

N. Goldman *et al.*, PRL 105, 255302 (2010)

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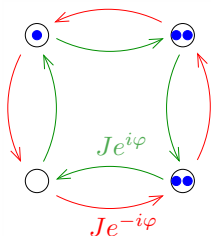
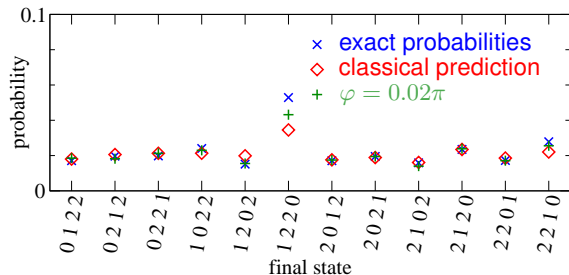
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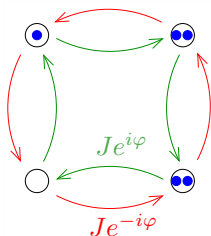
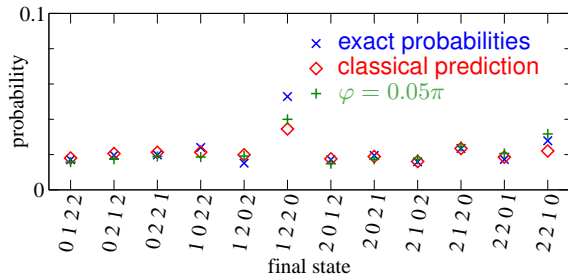
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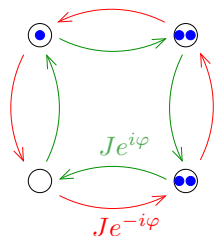
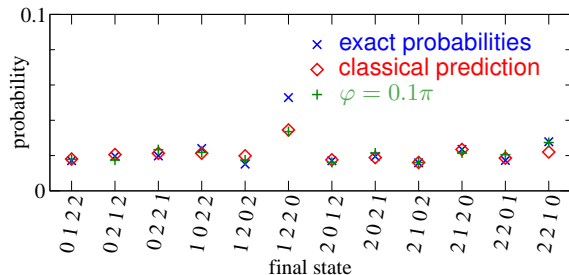
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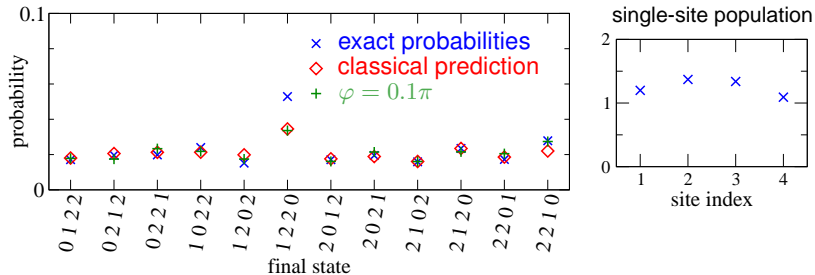
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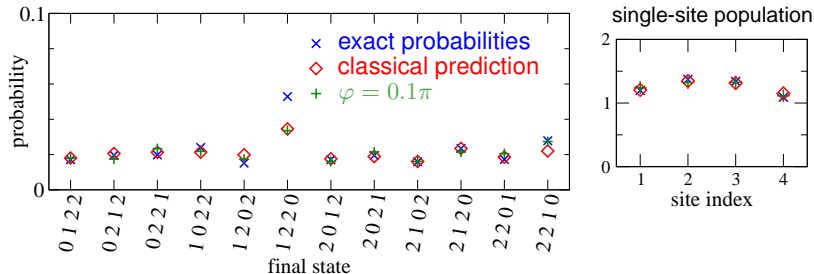
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→ slightly inhomogeneous single-site populations

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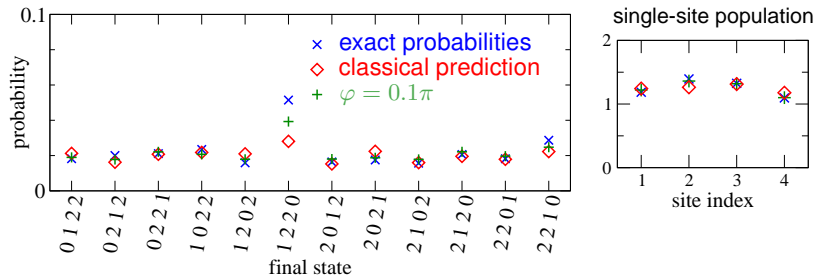
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→ slightly inhomogeneous single-site populations
due to limited classical ergodicity at $t = 10J/\hbar$

Proposal for a few-body CBS experiment

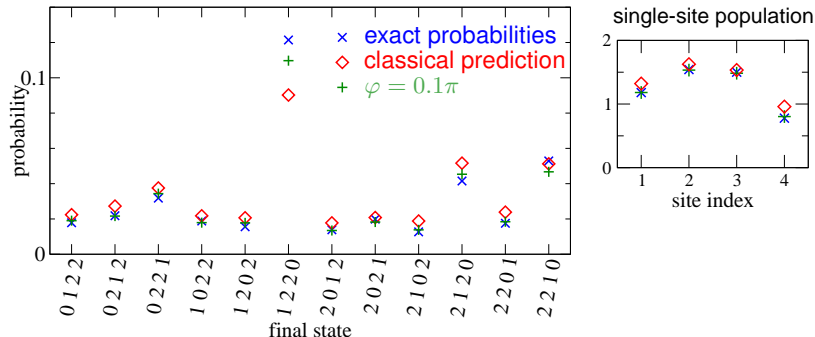
Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 100\hbar/J$



→ slightly inhomogeneous single-site populations
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due to quantum (few-body) localization at $t = 100\hbar/J > \tau_H$

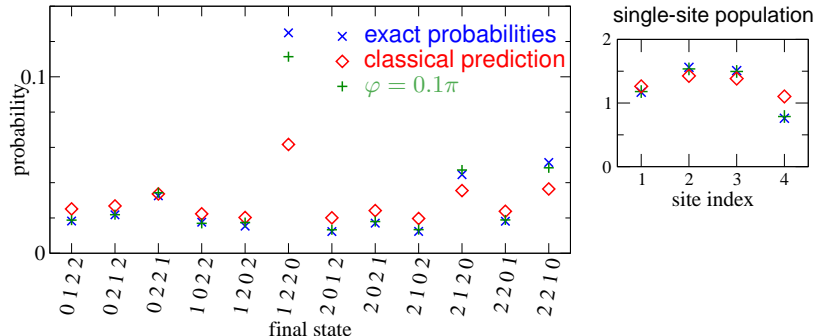
Proposal for a few-body CBS experiment

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Coherent backscattering in Fock space

- privileges, on average, the initial Fock state as compared to other states with comparable energy;
 - significantly affects quantum ergodicity in finite systems, in a regime where the classical dynamics is fully ergodic;
 - can be experimentally detected using ultracold atoms in optical lattices;
 - relies on time-reversal invariance and can therefore be switched off with a synthetic gauge field;
 - can be generalized to fermions [T. Engl *et al.*, arXiv:1409.5684](#) ;
- may act as a precursor to “strong” many-body localization?
[D. Vollhardt & P. Wölfle, PRL 48, 699 \(1982\)](#)