Many-body localization characterized by entanglement and occupations of natural orbitals

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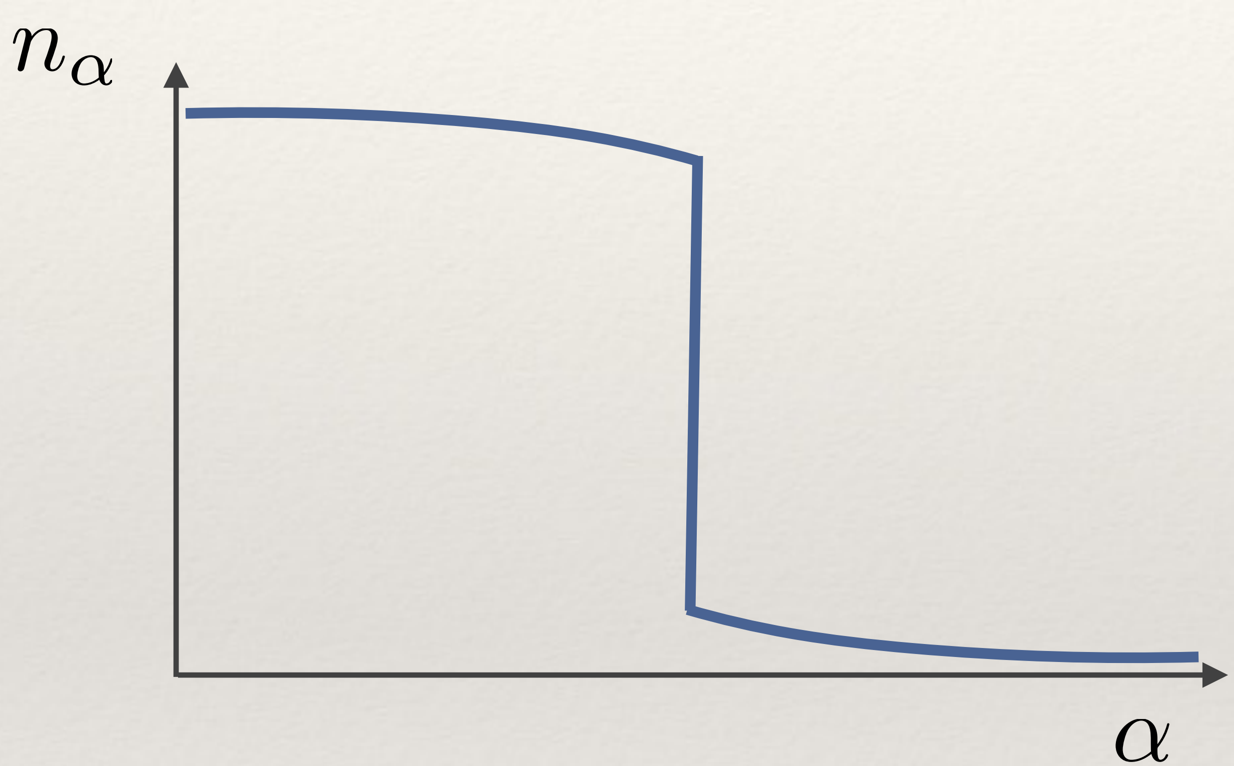
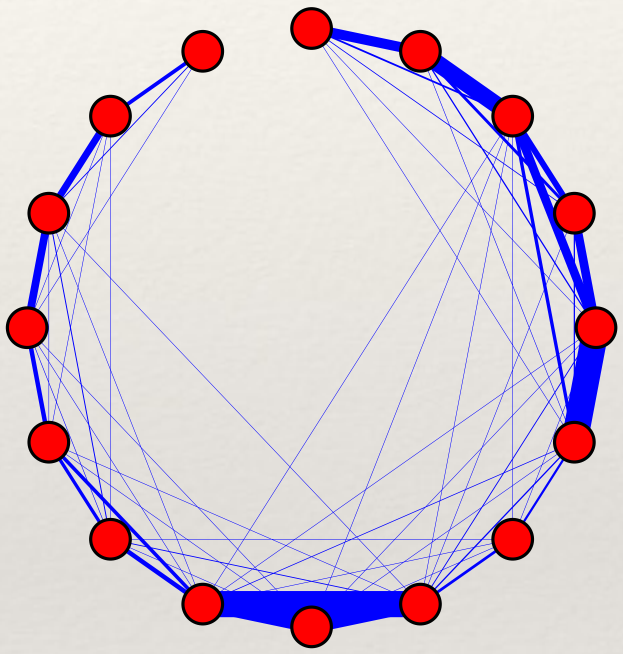
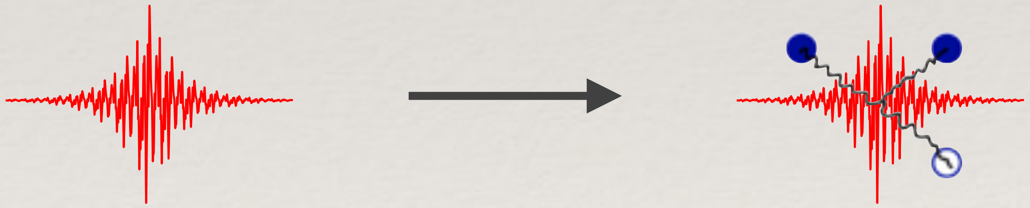


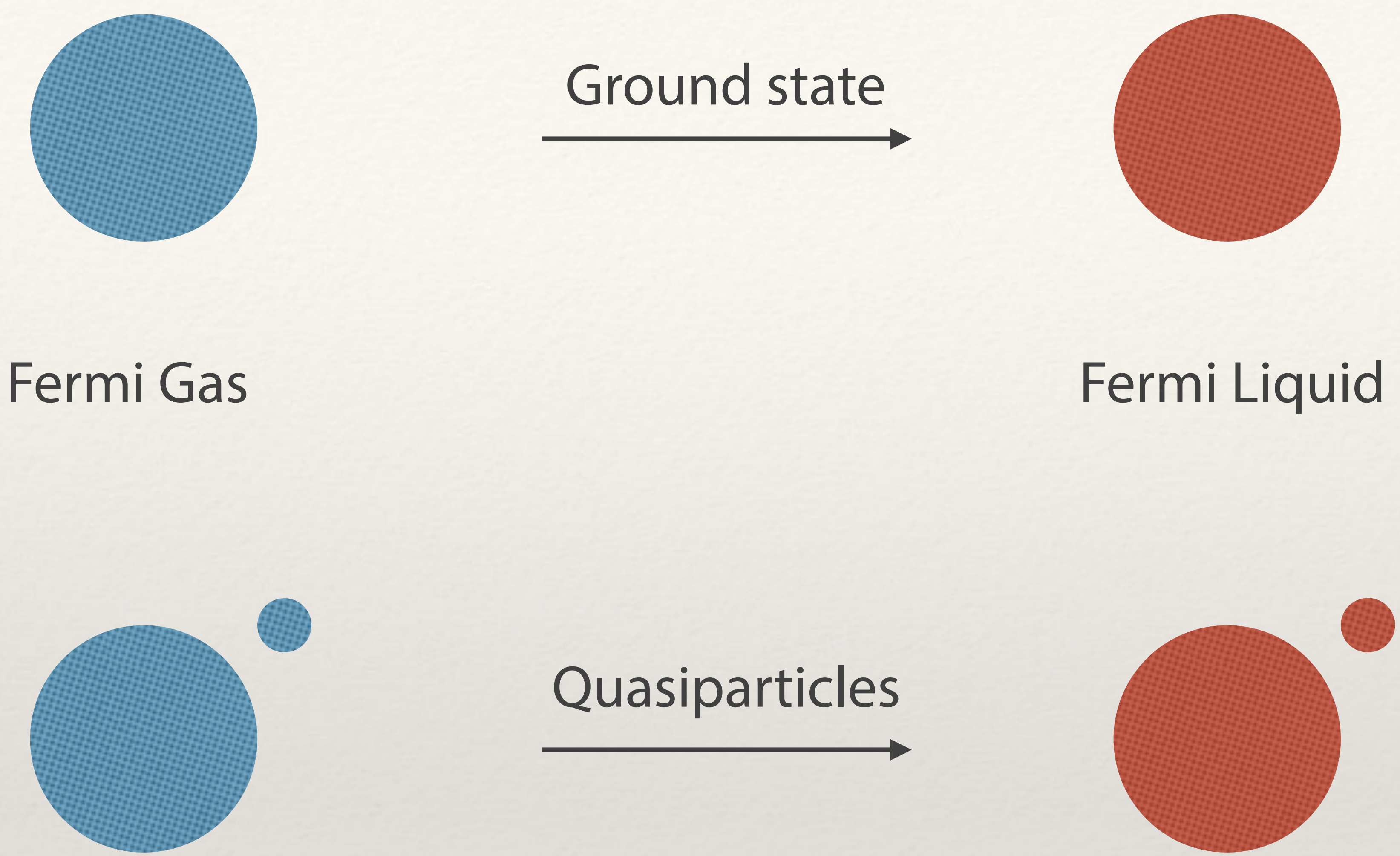


I: Broken symmetry

II: Adiabatic continuity
& renormalization

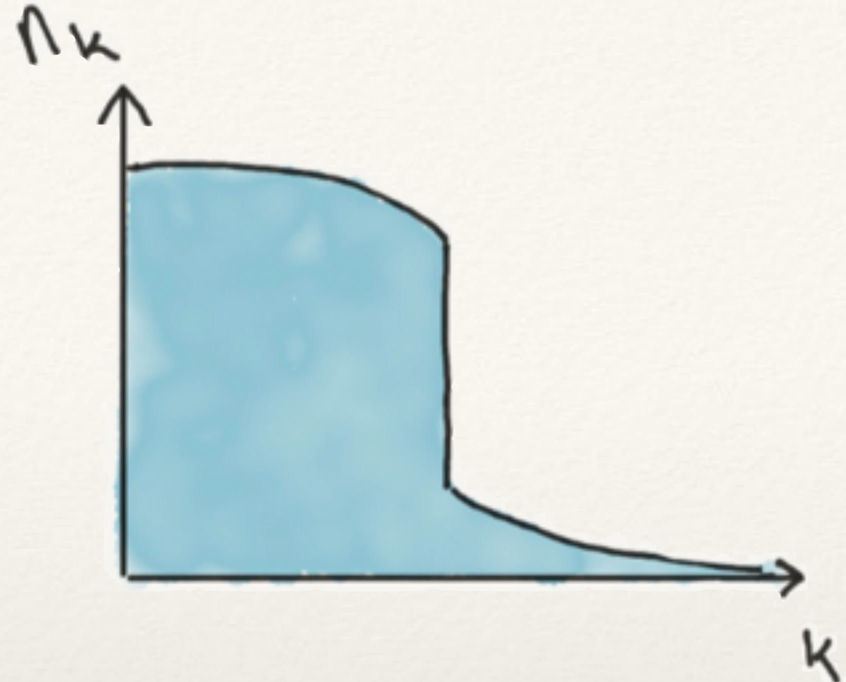


Fermi liquid theory



See e.g. P. Coleman, Introduction to Many-Body Physics

Fermi liquid theory



$$\mathcal{E} = \mathcal{E}_0 + \sum_{\mathbf{k}\sigma} (E^{(0)}_{\mathbf{k}\sigma} - \mu)\delta n_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k}\mathbf{k}',\sigma\sigma'} f_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'} + \dots$$

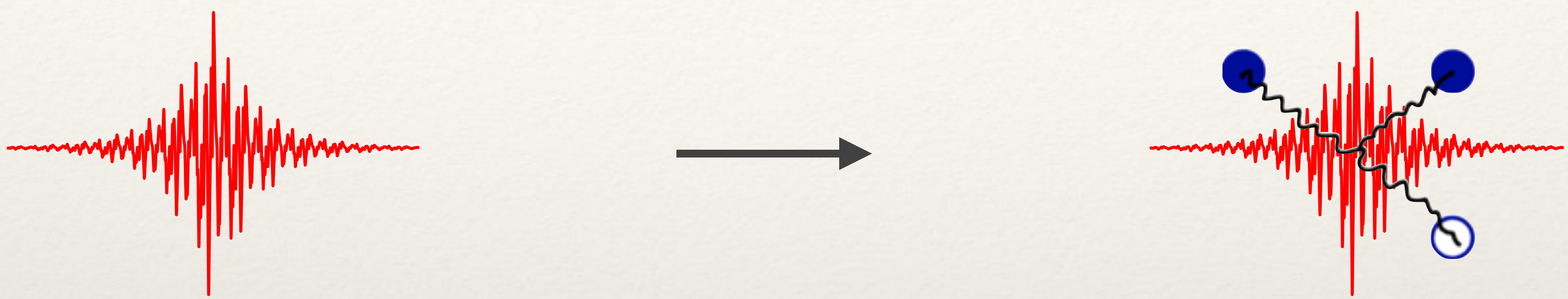
$$c^\dagger_{\mathbf{k}\sigma} = \sqrt{Z_\mathbf{k}} a^\dagger_{\mathbf{k}\sigma} + \sum A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^\dagger_{\mathbf{k}_4\sigma_4} a^\dagger_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \dots$$

See e.g. P. Coleman, Introduction to Many-Body Physics

Many-body localization

Anderson Insulator

Many-body localized insulator



Basko, Aleiner, Altshuler, Ann. Phys 2006

see also: Altshuler, Gefen, Kamenev, Levitov PRL 1997 Gornyi, Mirlin, Polyakov PRL 2005

Anderson insulator (many-body)

$$H_0 = -\frac{1}{2}\sum_{i=1}^{L}\left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} + \epsilon_i n_i\right) = \sum_i \varepsilon_i a_i^\dagger a_i \qquad \epsilon_i \in [-W, W]$$

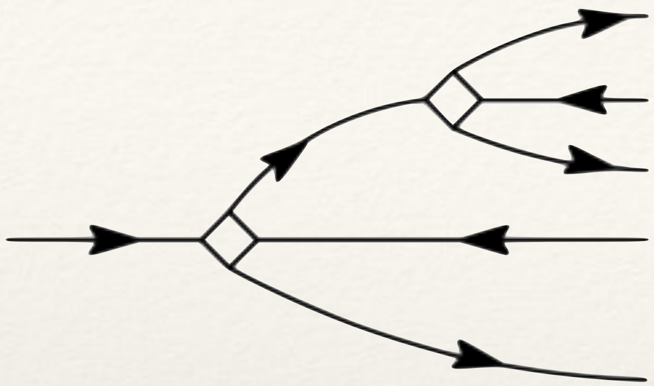
$\tilde{n}_i = a_i^\dagger a_i$ $\qquad [\tilde{n}, H_0] = 0$ $\qquad$ Local integrals of motion

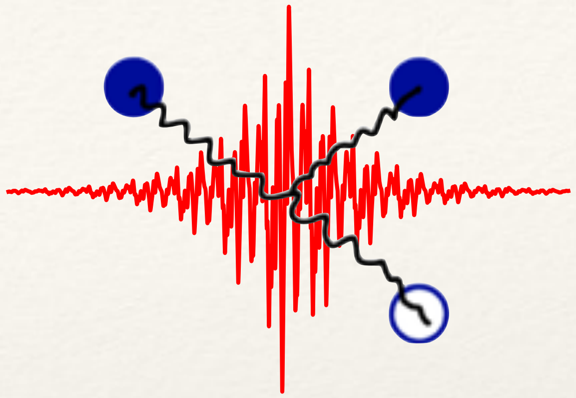
$|\tilde{n}_1, \tilde{n}_2, \ldots \tilde{n}_L\rangle$ $\qquad$ Eigenstates are product states

P. W. Anderson Phys. Rev. 1958

Emergent integrability in many-body localization

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijlk} a_i^\dagger a_j^\dagger a_l a_k$$





$$H = \sum_i h_i n_i + \sum_{ij} J_{ij} n_i n_j + \ldots$$ where $$J_{ij} \sim e^{-|x_i - x_j|/\xi^*}$$

$[n_i, H] = 0$ Local conserved integrals of motion

$|n_1, n_2, \ldots n_L\rangle$ All eigenstates product states

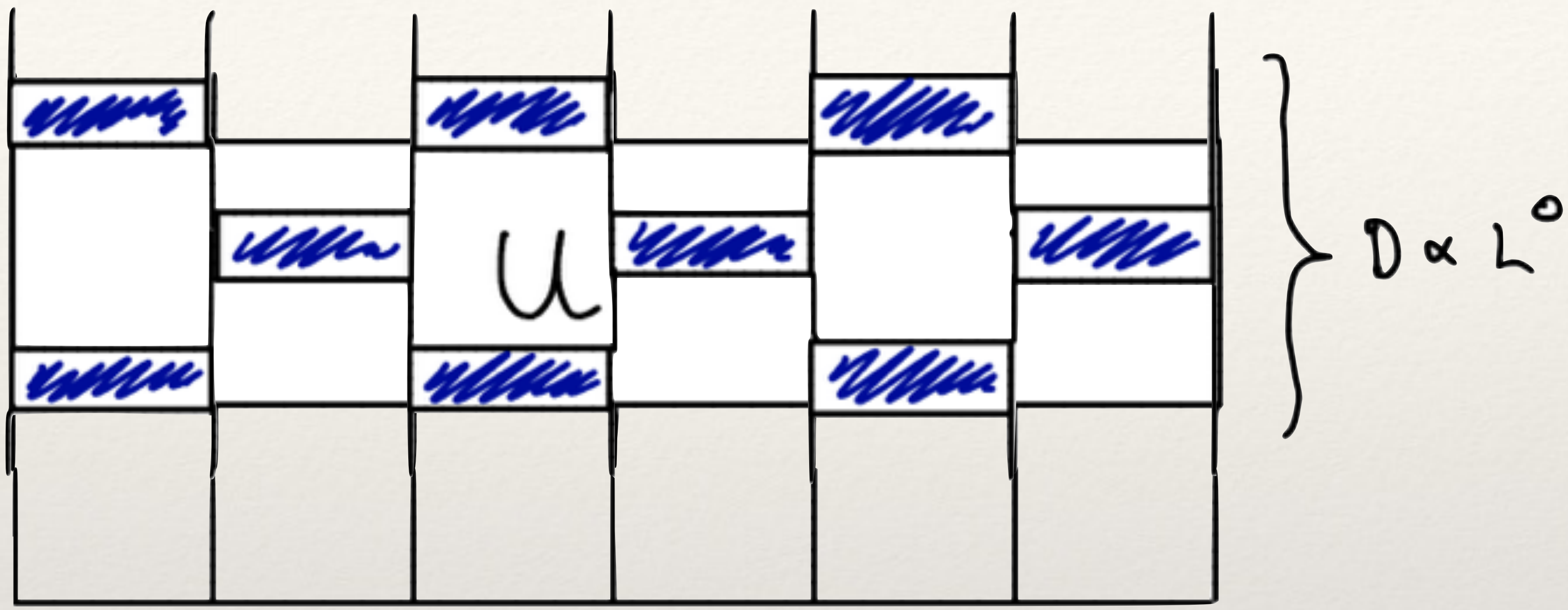
$$c_i = \sum_j Z_j^i a_j + \sum_{jkl} f_{jkl}^i a_j^\dagger a_k a_l + \ldots$$ Quasiparticles

Basko, Aleiner and Altshuler Ann. Phys. 2006; Gornyi, Mirlin, Polyakov, PRL 2005
Serbyn, Papic, and Abanin PRL 2013; Huse, Nandkishore, and Oganesyan PRB 2014; Imbrie J. Stat. Phys. 2016
Ros, Müller, Scardicchio Nucl. Phys. B 2015; Imbrie, Ros, Scardicchio arXiv:1609.08076 (review)

Every MBL eigenstate related to a Anderson insulator eigenstate via a finite depth quantum circuit

U

$|\{n\}\rangle$



Bauer and Nayak, J. Stat. Mech 2013; Pekker & Clark PRB 2017

Many-body localized eigenstates have an area law entanglement

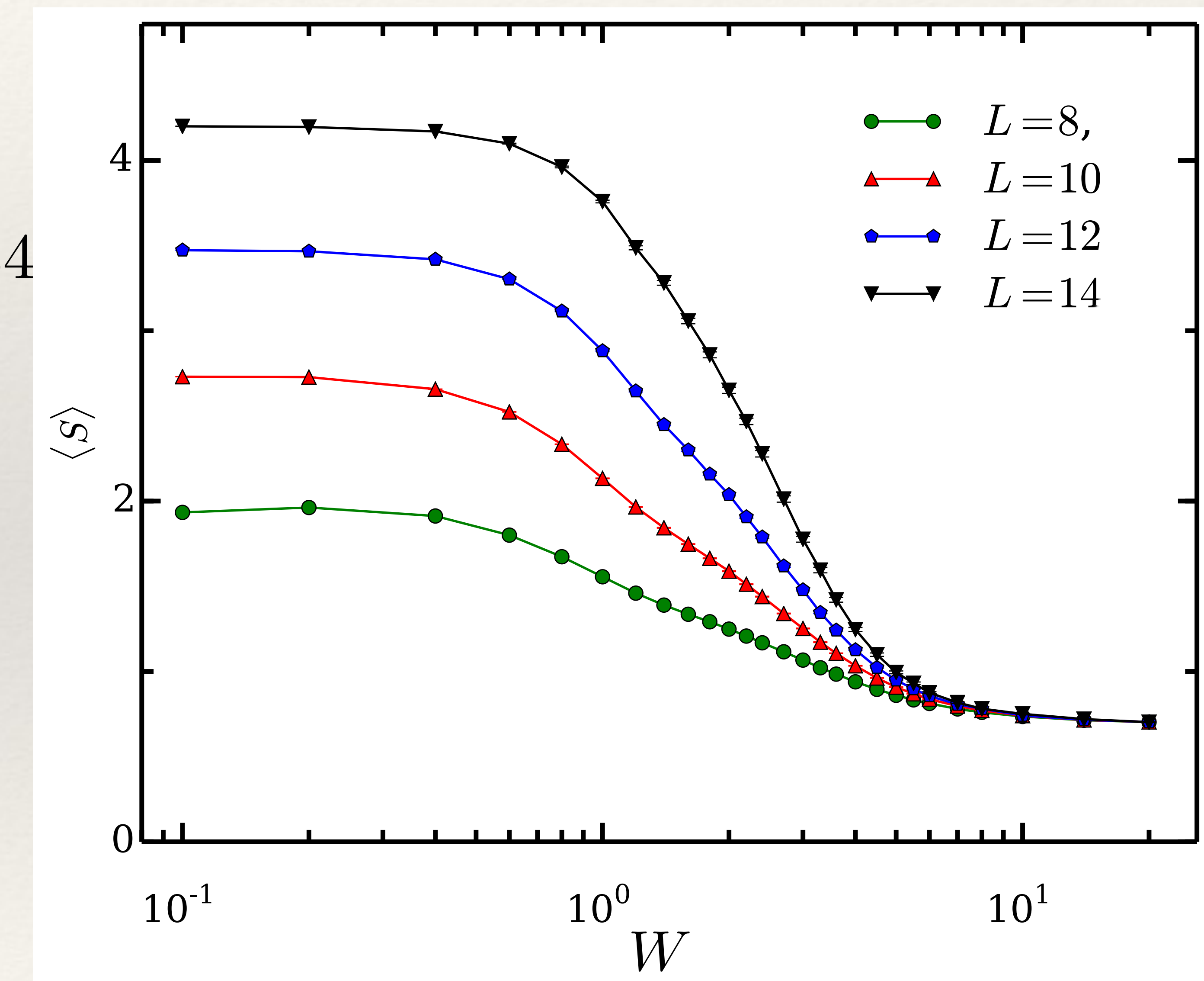
$$S(A) = -\mathrm{Tr}_A \rho_A \log \rho_A \qquad \rho_A = \mathrm{Tr}_{\bar{A}} |\psi\rangle\langle\psi|$$

$$|\psi\rangle = c_1^\dagger c_2^\dagger c_3^\dagger c_4^\dagger c_5^\dagger |0\rangle = |12345\rangle$$

$$\boldsymbol{\rho_R} = |45\rangle\langle 45| \quad |345\rangle\langle 345|$$

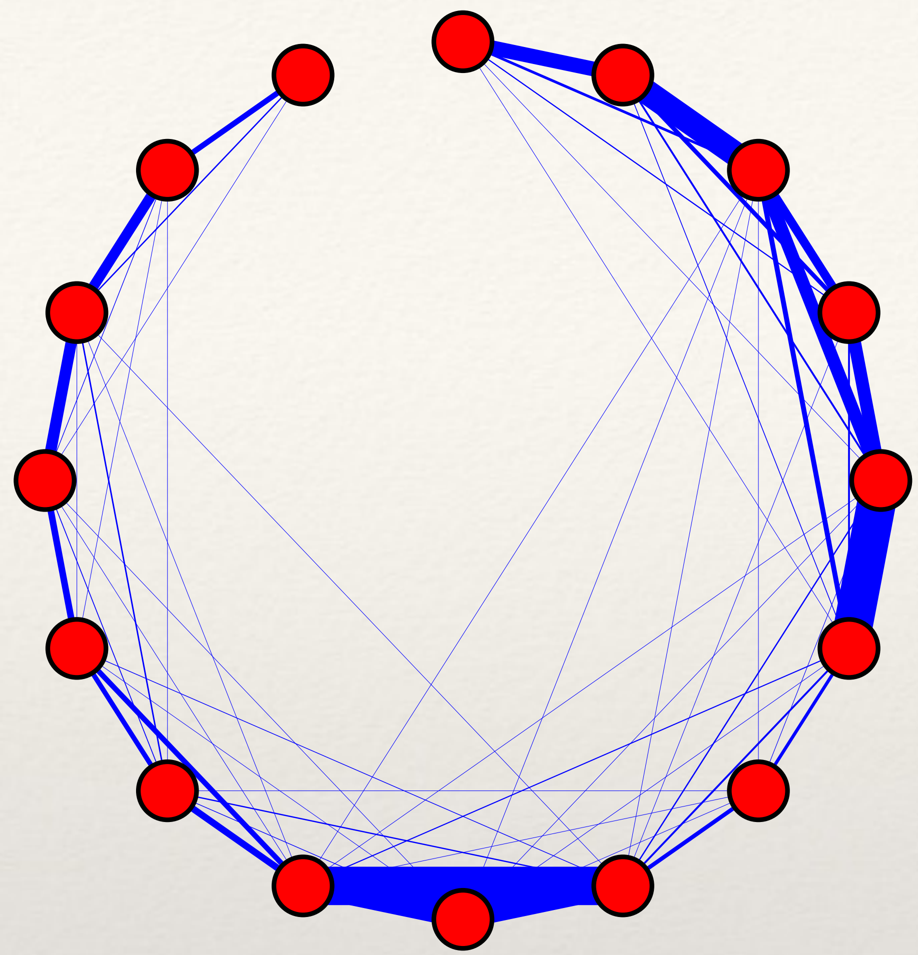
$$S_{\mathrm{MBL}} \sim \xi$$

$$S_{\mathrm{Page}} = \frac{L}{2}\log 2 - \frac{1}{2}$$



Bauer and Nayak, J. Stat. Mech 2013; Kjäll, JHB, Pollmann PRL 2014

Mutual information directly reveals the entanglement structure of eigenstates



$$\mathcal{I}(A, B) = S(A) + S(B) - S(A \cup B)$$

De Tomasi, Bera, JHB, Pollmann PRL 2017

One particle density matrix — definitions

$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$\rho_{ij} = \langle\psi_n|c_i^\dagger c_j|\psi_n\rangle$$

$$\rho|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle$$

$|\phi_\alpha\rangle$ Natural orbitals

n_α Occupations

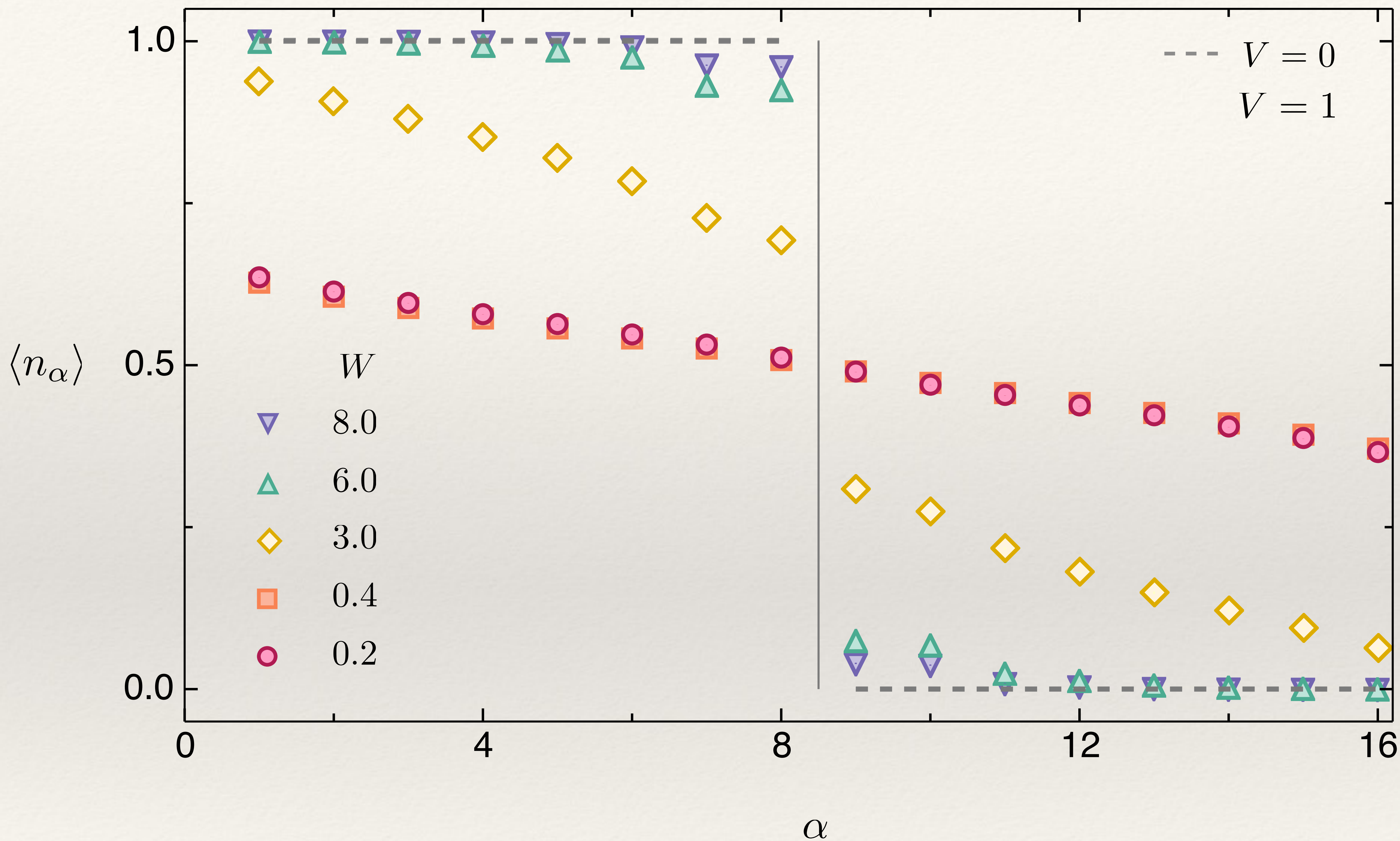
$$n_1 \geq n_2 \geq \ldots \geq n_L$$

$$\mathrm{tr}\ \rho = \sum_{\alpha=1}^{L} n_\alpha = N$$

Penrose & Onsager PR 1956; Koch & Goedecker Solid State Comm. 2001; Gottlieb & Mauser PRL 2005 …
Nessi & Iucci PRA 2011; Gramsch & Rigol PRA 2012

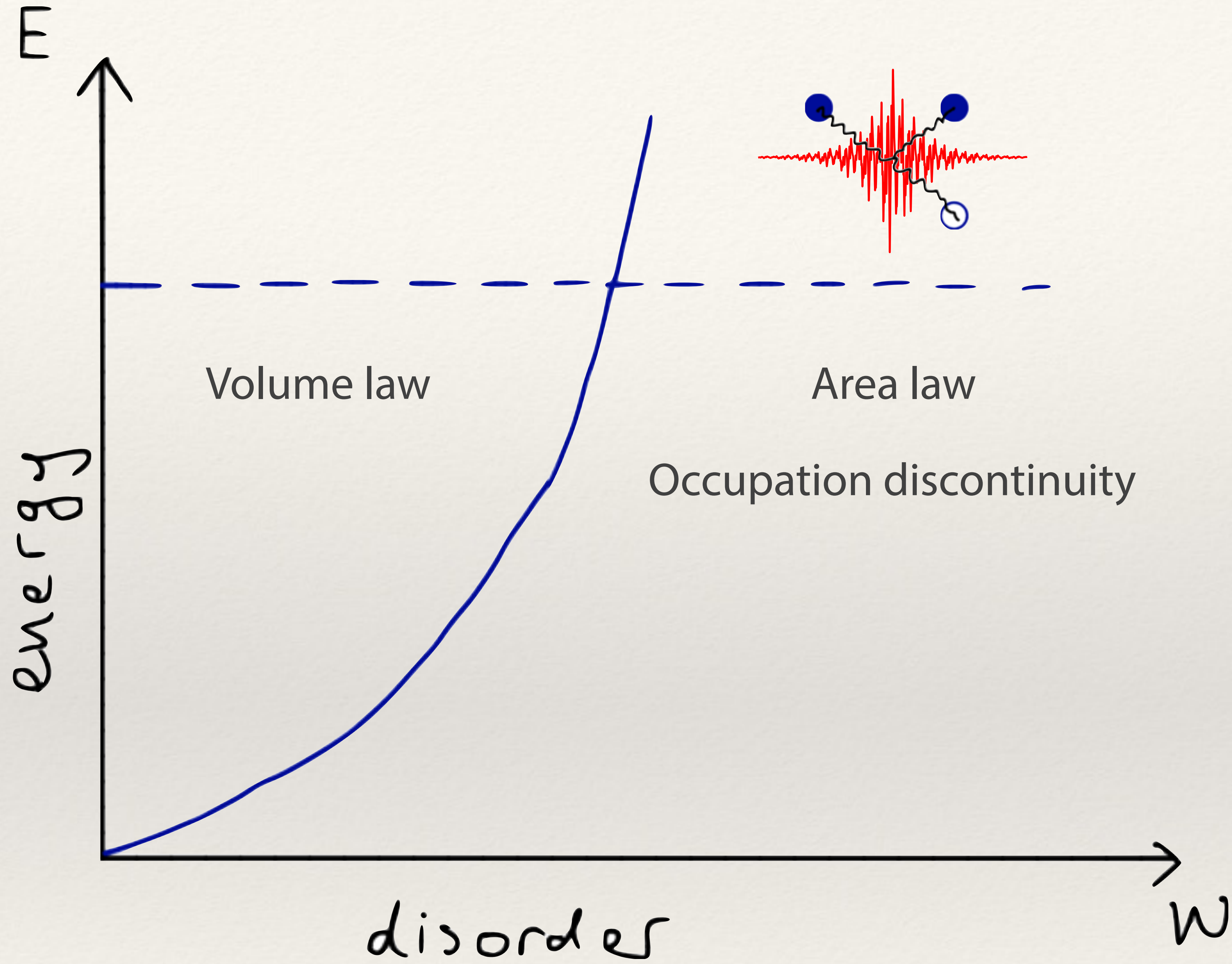
Occupations

$$H = t\sum_{i=1}^{L}\left[-\frac{1}{2}(c_i^\dagger c_{i+1} + \mathrm{h.c.}) + \epsilon_i\left(n_i - \frac{1}{2}\right) + V\left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right)\right]$$



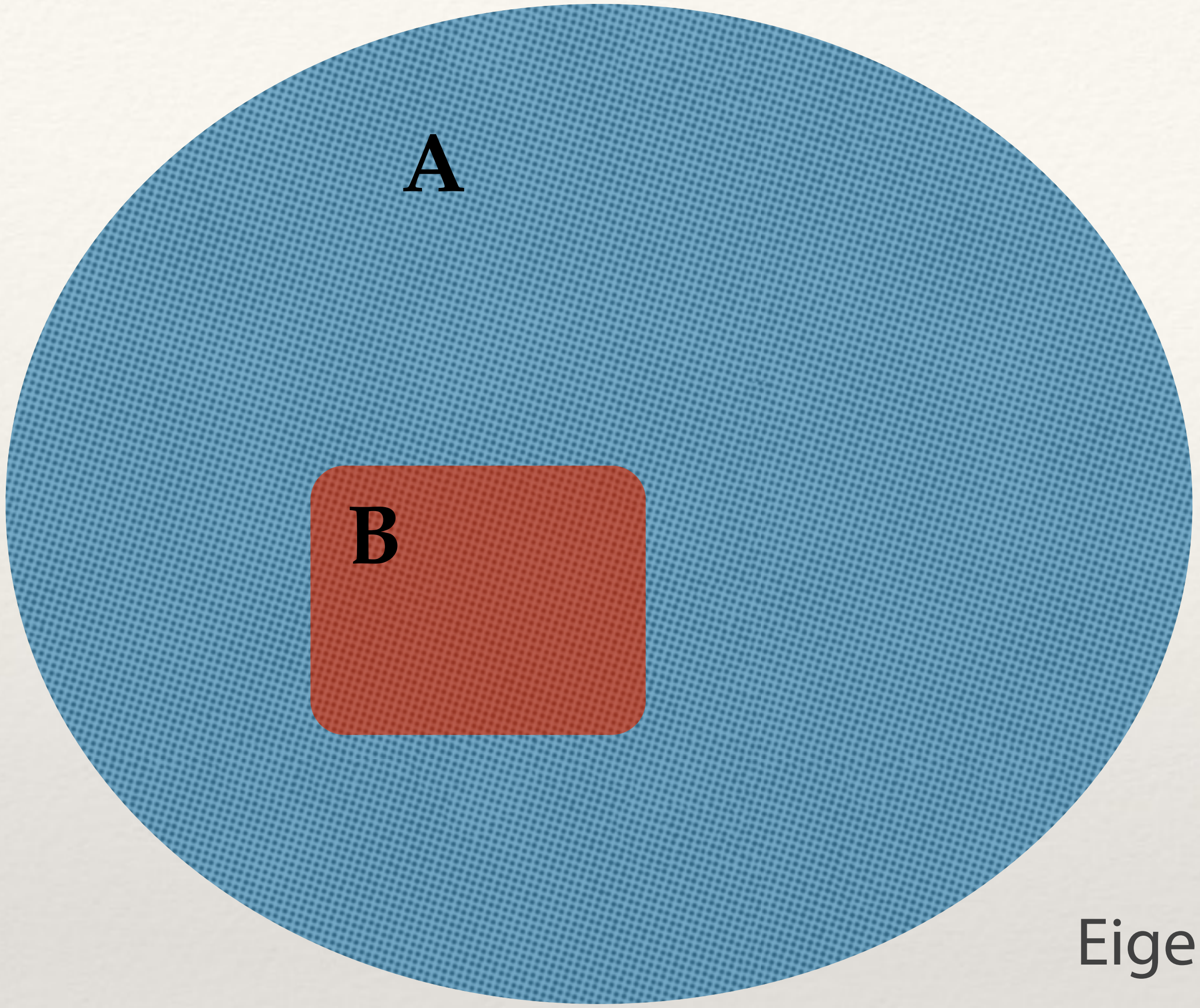
Bera, Schomerus, Heidrich-Meisner, JHB PRL 2015

Many-body localization as eigenstate quantum phase transition



Basko, Aleiner and Altshuler Ann. Phys. 2006; Oganesyan and Huse PRB 2007; …

Entanglement entropy



$$|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\rho_B = \mathrm{Tr}_A\, |\psi\rangle\langle\psi|$$

$$S = -\mathrm{Tr}\,\rho_B \log \rho_B$$

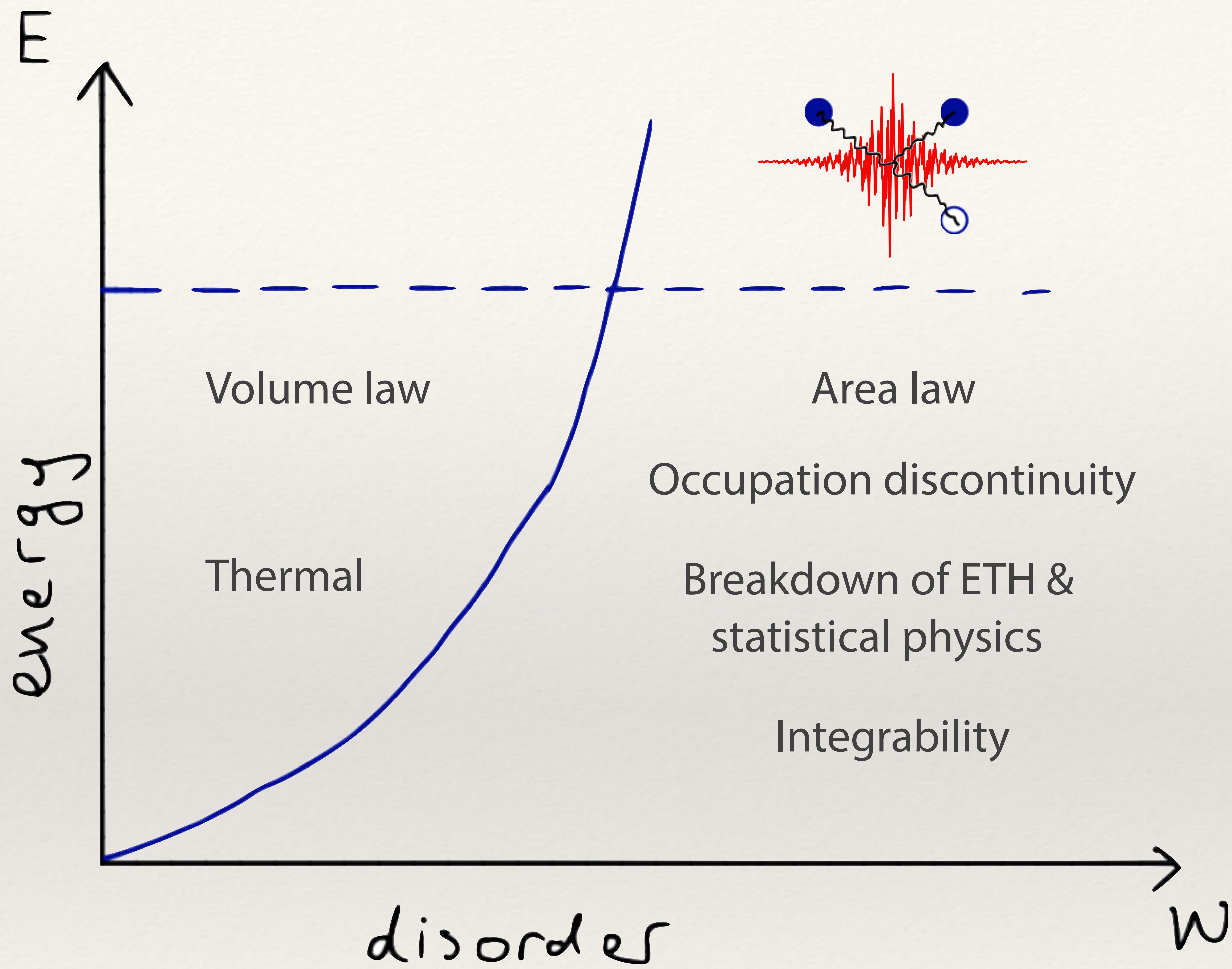
Eigenstate thermalization hypothesis

$$H|\psi\rangle = E|\psi\rangle$$

$$\rho_B \sim e^{-\beta H_B}$$

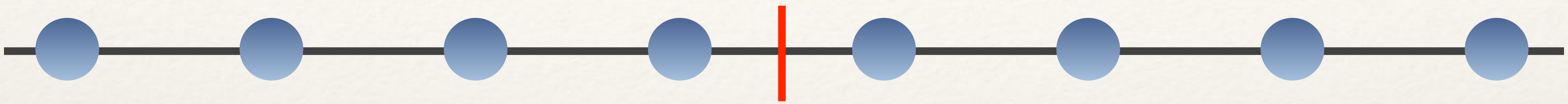
Deutsch PRA 1991, Srednicki PRE 1994
Popescu et al. Nature 2006, Rigol et al. Nature 2008

Many-body localization as eigenstate quantum phase transition



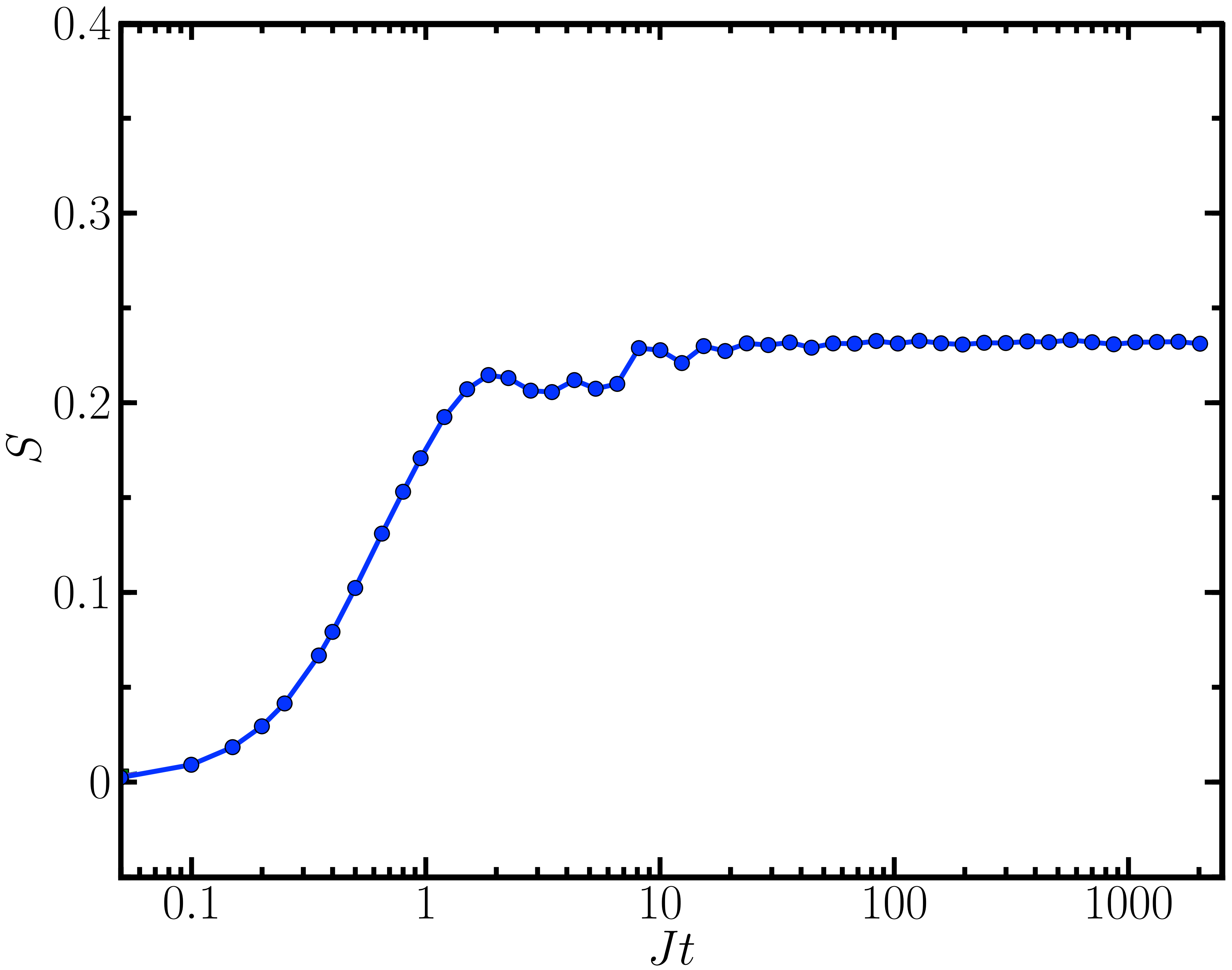
Basko, Aleiner and Altshuler Ann. Phys. 2006; Oganesyan and Huse PRB 2007; …

Global quench from a charge density wave — Anderson insulator



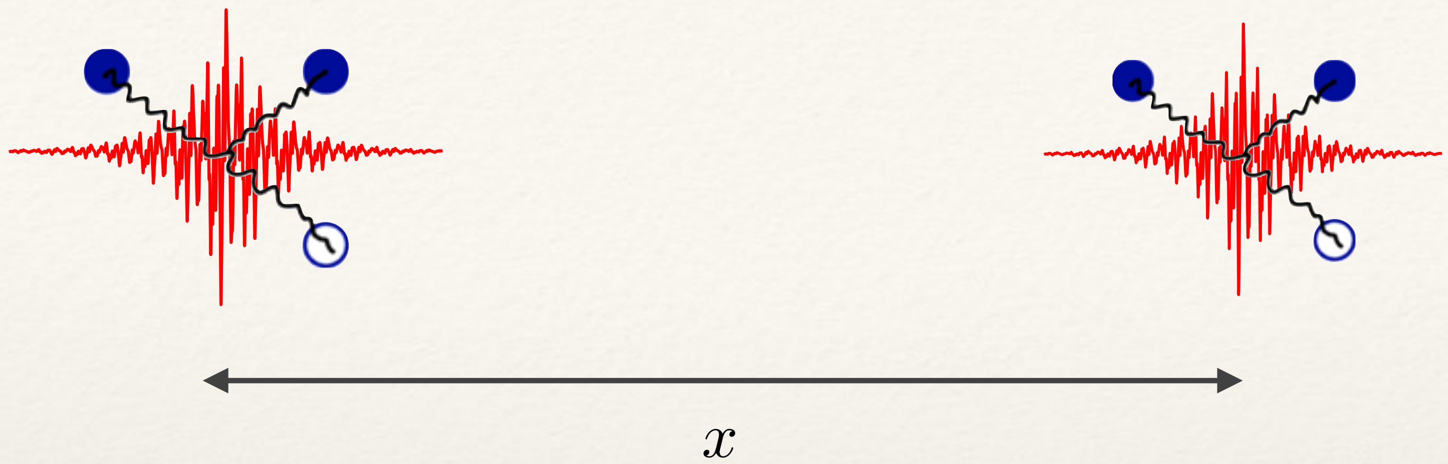
$$|\psi(t)\rangle = \exp(-iHt) \prod_{i=1}^{L/2} c_{2i}^{\dagger} |0\rangle$$

Entanglement vs. time — starting from a product state



JHB, Pollmann and Moore PRL 2012

Quasiparticle interaction leads to dephasing in dynamics



$$J_{ij} \sim J_0 e^{-|x_i - x_j|/\xi^*}$$

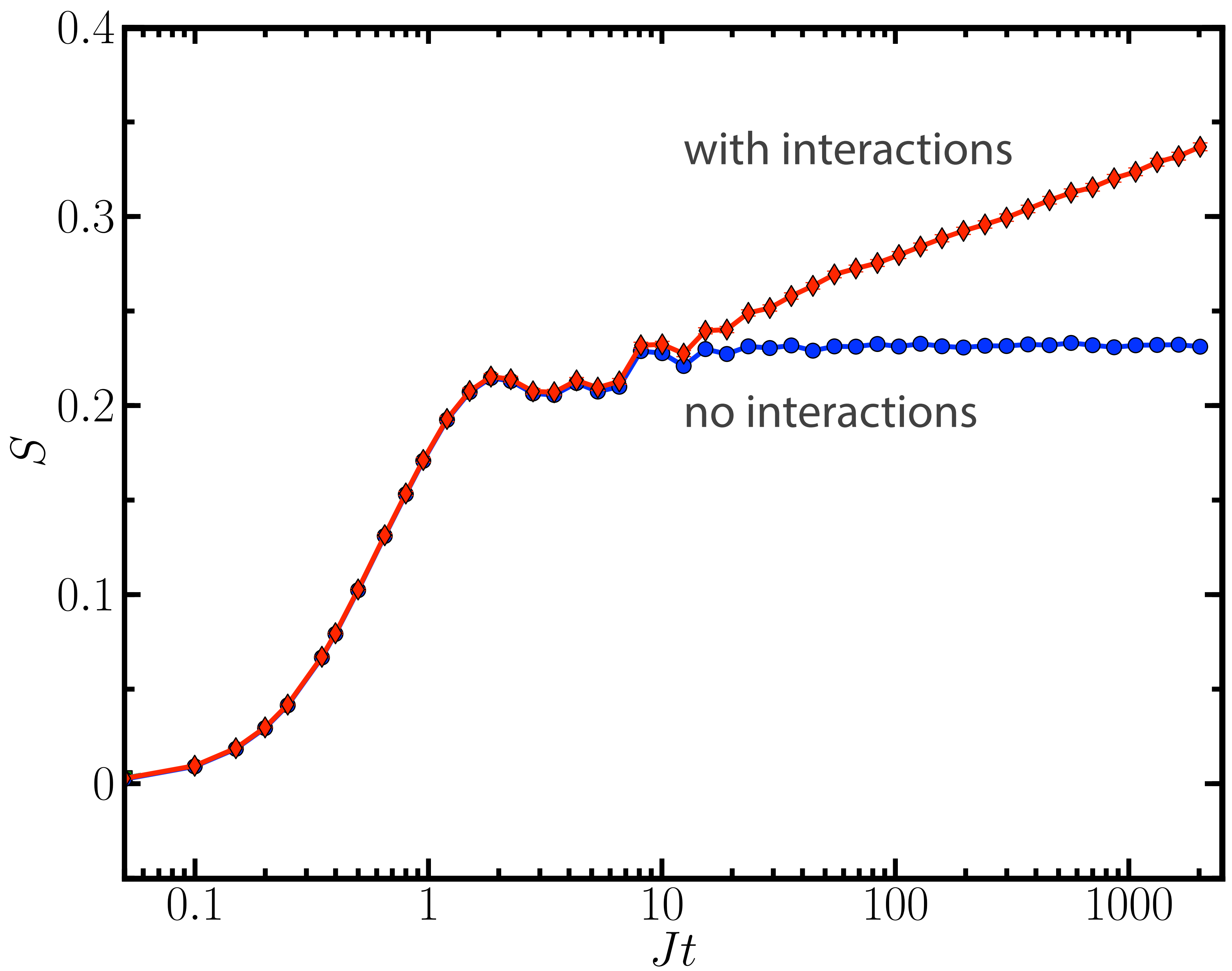
$$t_d \sim J_0^{-1} e^{x/\xi^*}$$

$$x_d(t) \sim \xi^* \log(J_0 t)$$

$$S(t) \sim \xi^* \log(J_0 t)$$

Serbyn, Papic, Abanin PRL 2013

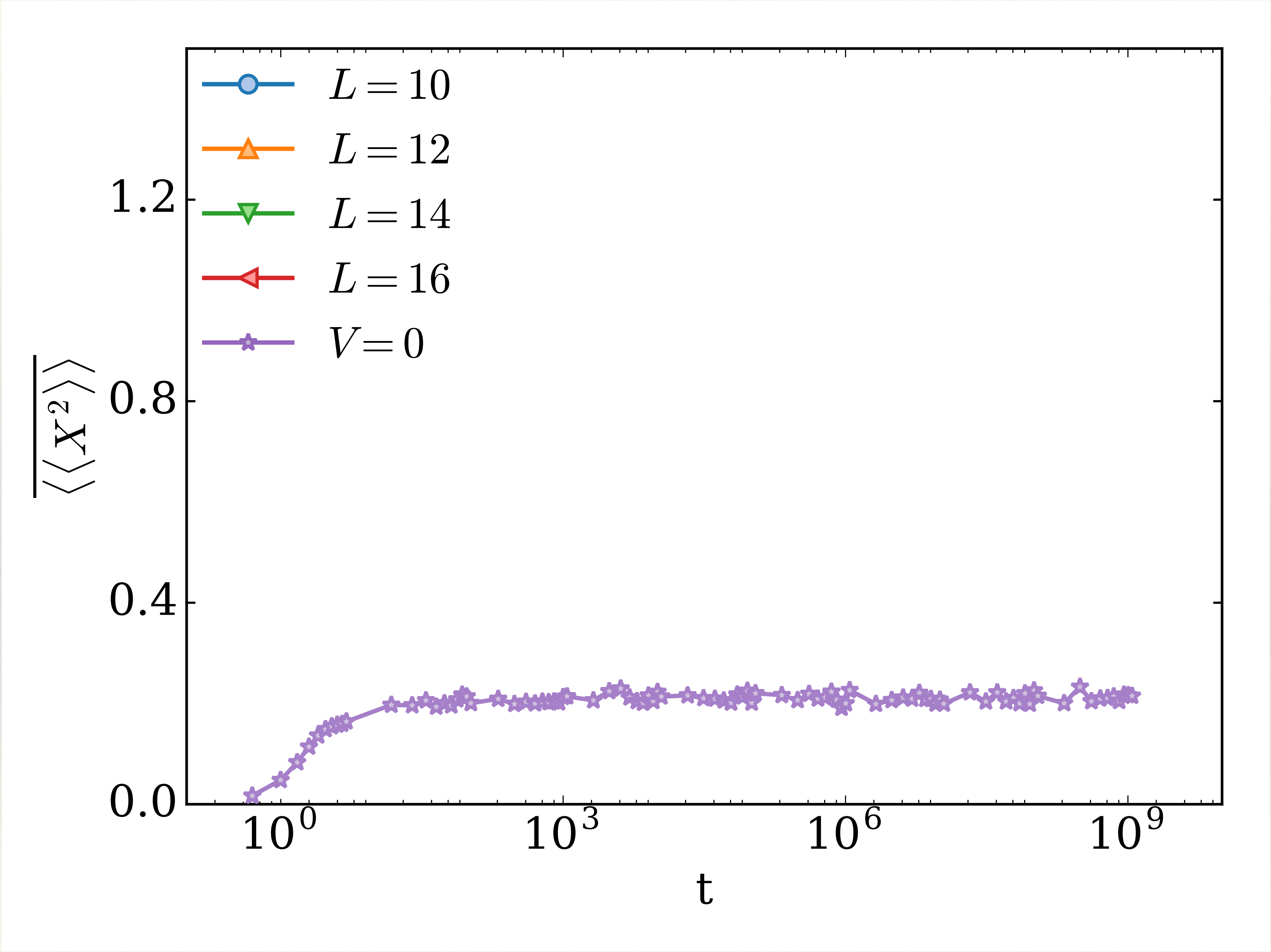
Entanglement vs. time — starting from a product state



JHB, Pollmann and Moore PRL 2012

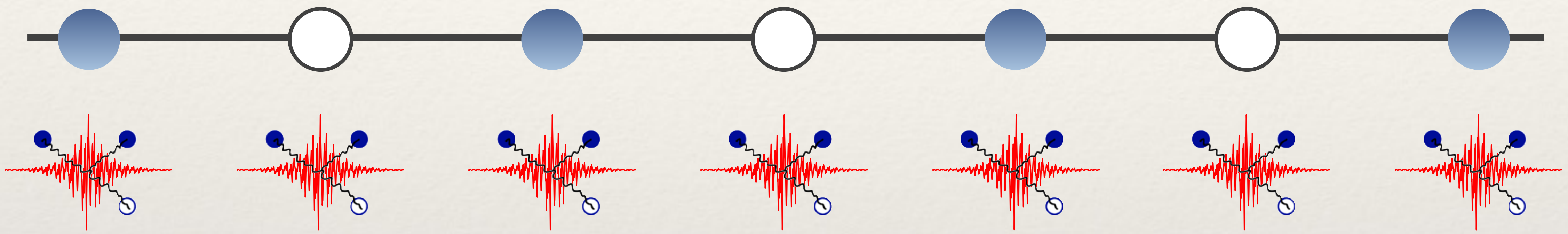
Mutual information after quench from product state

$$\langle\langle X^2\rangle\rangle = \sum_j j^2 \mathcal{I}_j(t) - \left(\sum_j j\mathcal{I}_j\right)^2 \qquad \mathcal{I}_j = \mathcal{I}(A=[1], B=[j])$$



De Tomasi, Bera, JHB, Pollmann PRL 2017

Charge density wave provides partial occupation of quasiparticles

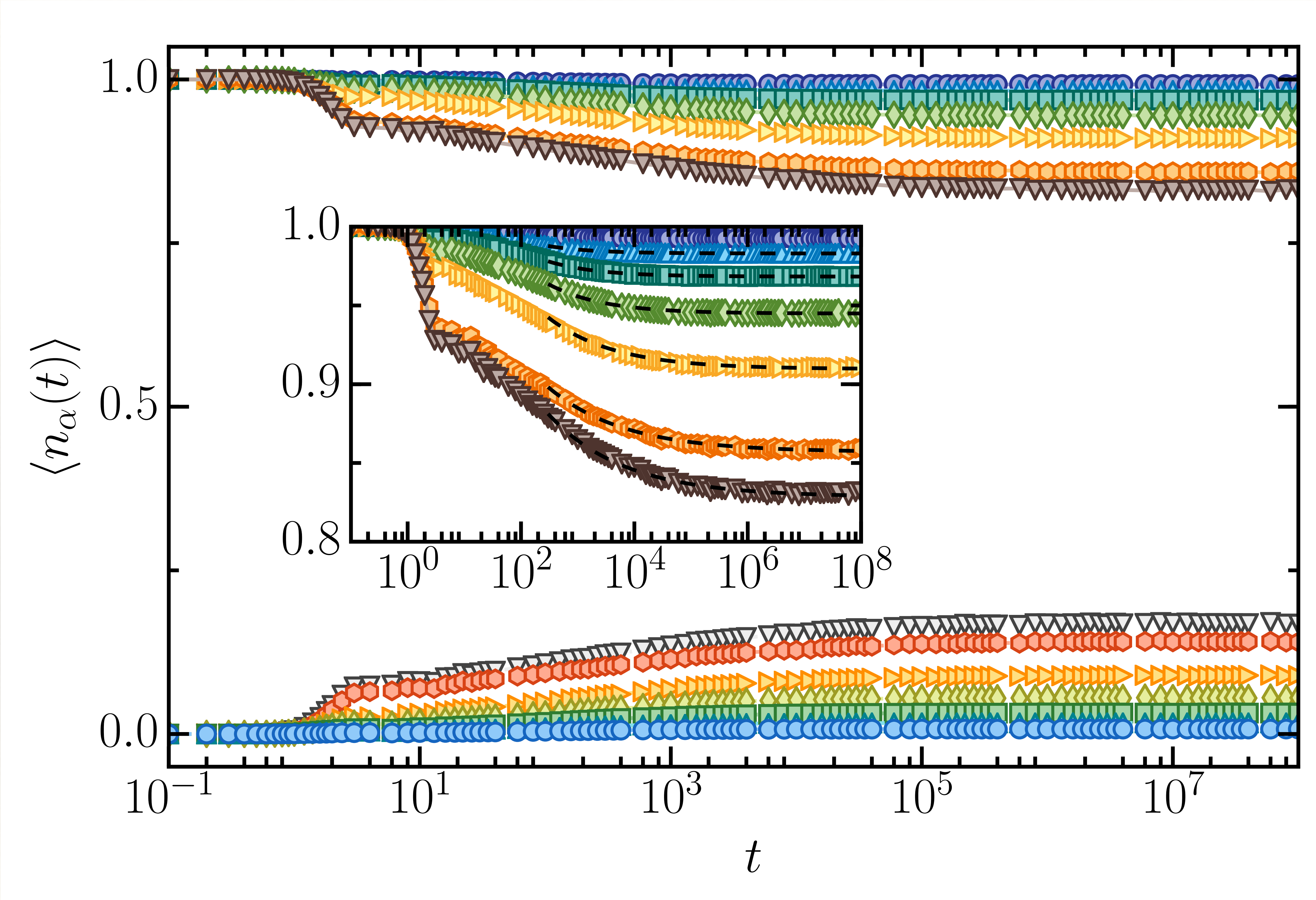


$$|\mathrm{CDW}\rangle \sim |1010101\rangle + \ldots$$

Lezama, Bera, Schomerus, Heidrich-Meisner, JHB arXiv:1703.04398

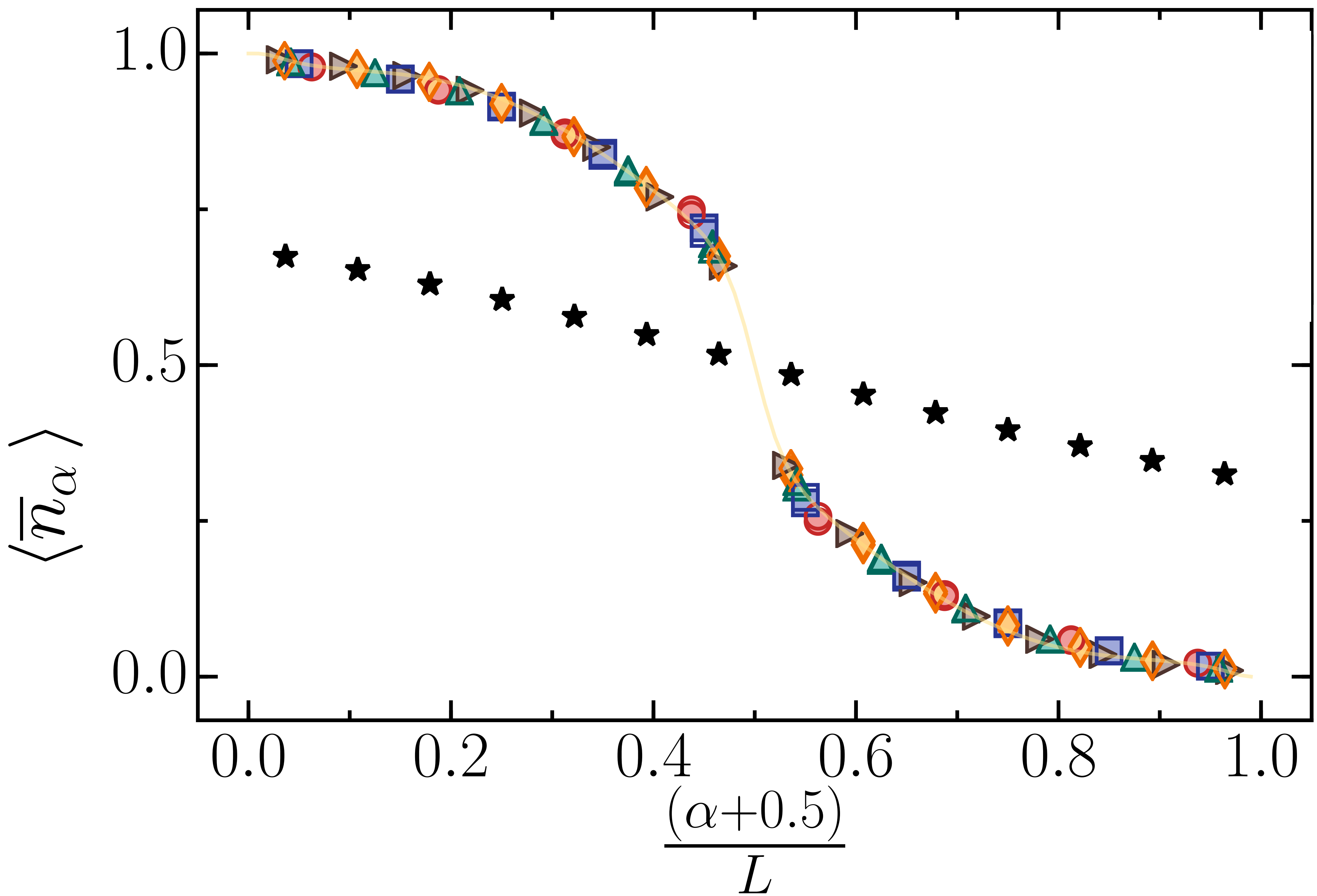
Initial correlations in CDW state destroyed by dephasing

$$\rho_{ij}(t) = \langle \psi(t)| c_i^\dagger c_j |\psi(t)\rangle \qquad \rho(t)\phi_\alpha(t) = n_\alpha(t)\phi_\alpha(t)$$



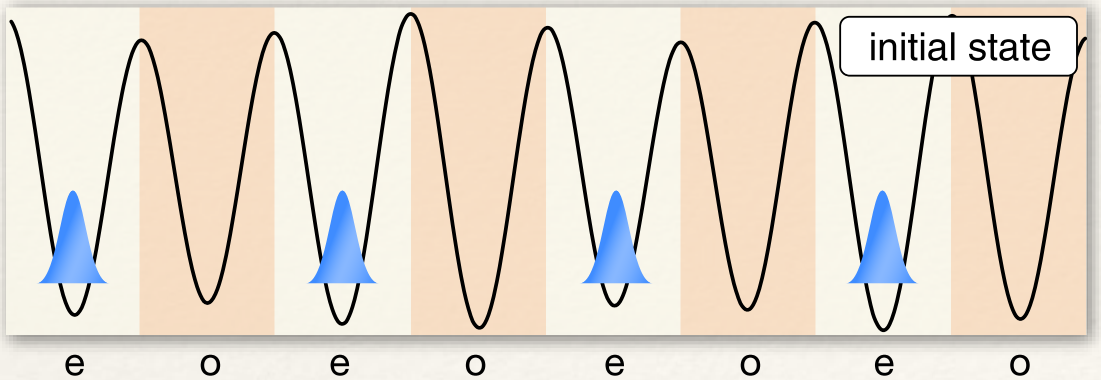
Lezama, Bera, Schomerus, Heidrich-Meisner, JHB arXiv:1703.04398

Long time occupations nonthermal with smeared discontinuity



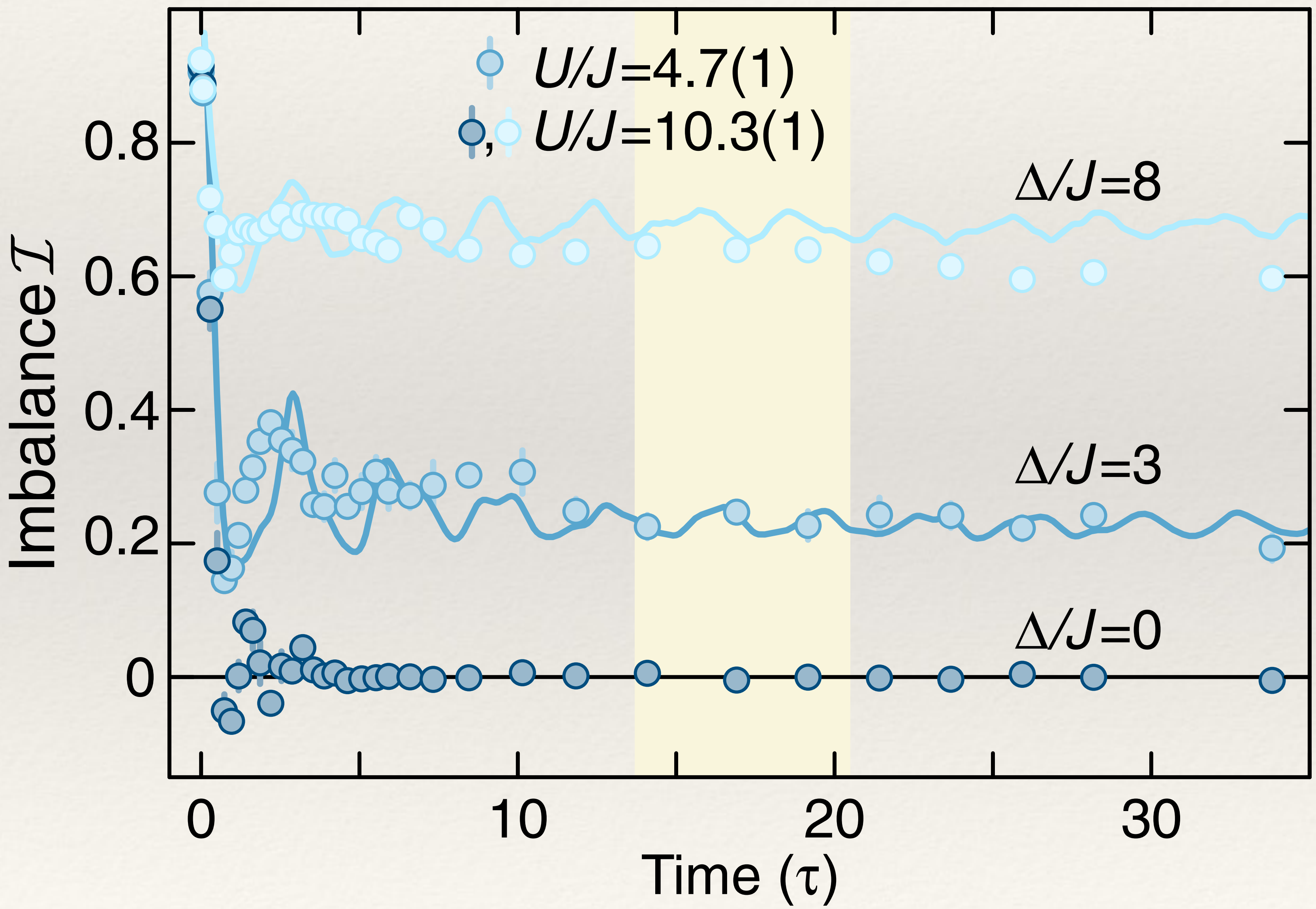
Lezama, Bera, Schomerus, Heidrich-Meisner, JHB arXiv:1703.04398

Many-body localization may be experimentally realized in cold atoms in a quasiperiodic lattice



Aubry-André model

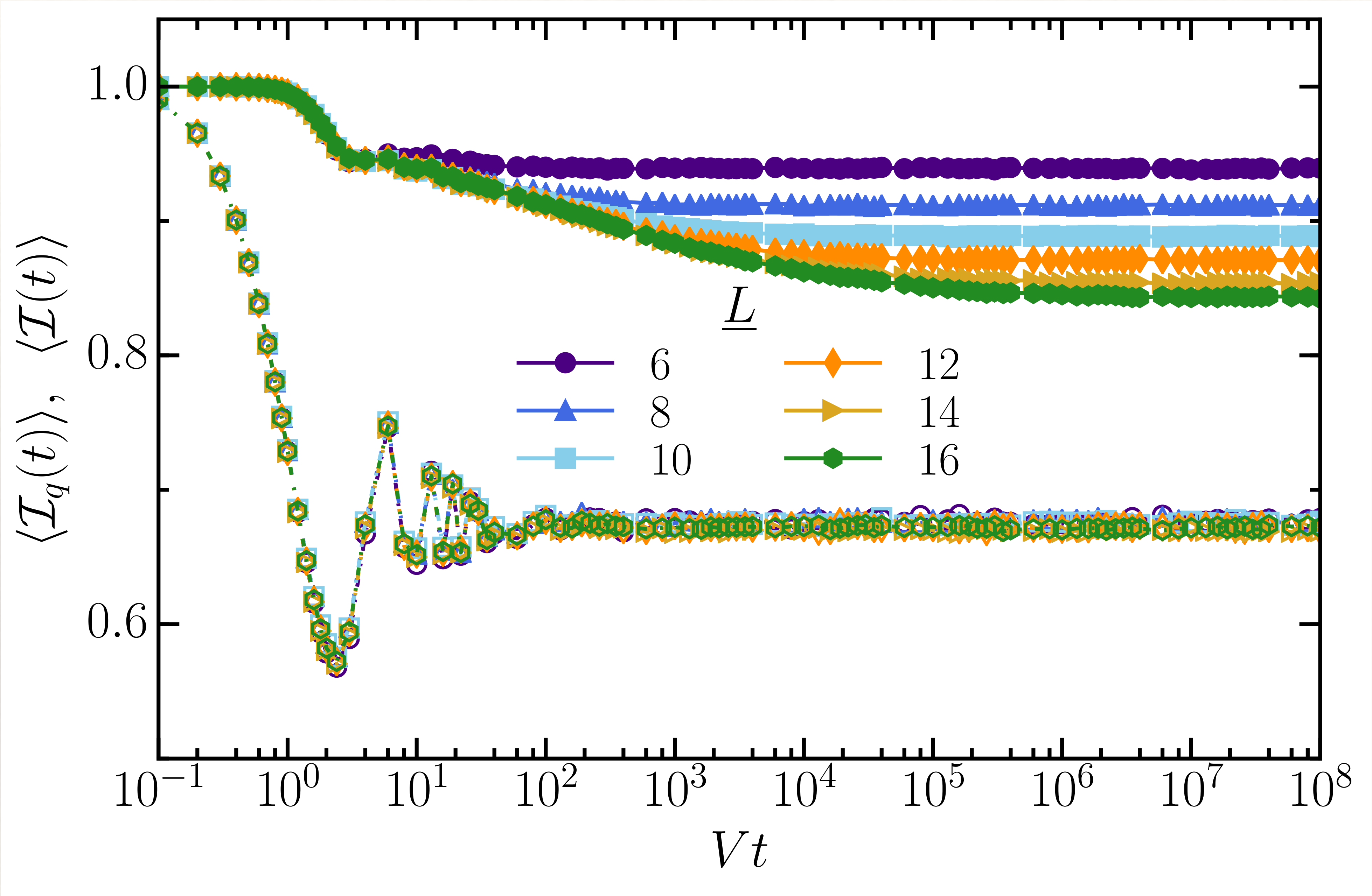
$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}.$$



Schreiber et al, Science 2015; see also Choi et al Science 2016 for 2D

Occupation imbalance similar to density imbalance, but with slower relaxation towards steady state

$$\mathcal{I} = \frac{N_+ - N_-}{N}$$



Lezama, Bera, Schomerus, Heidrich-Meisner, JHB arXiv:1703.04398

Collaborators

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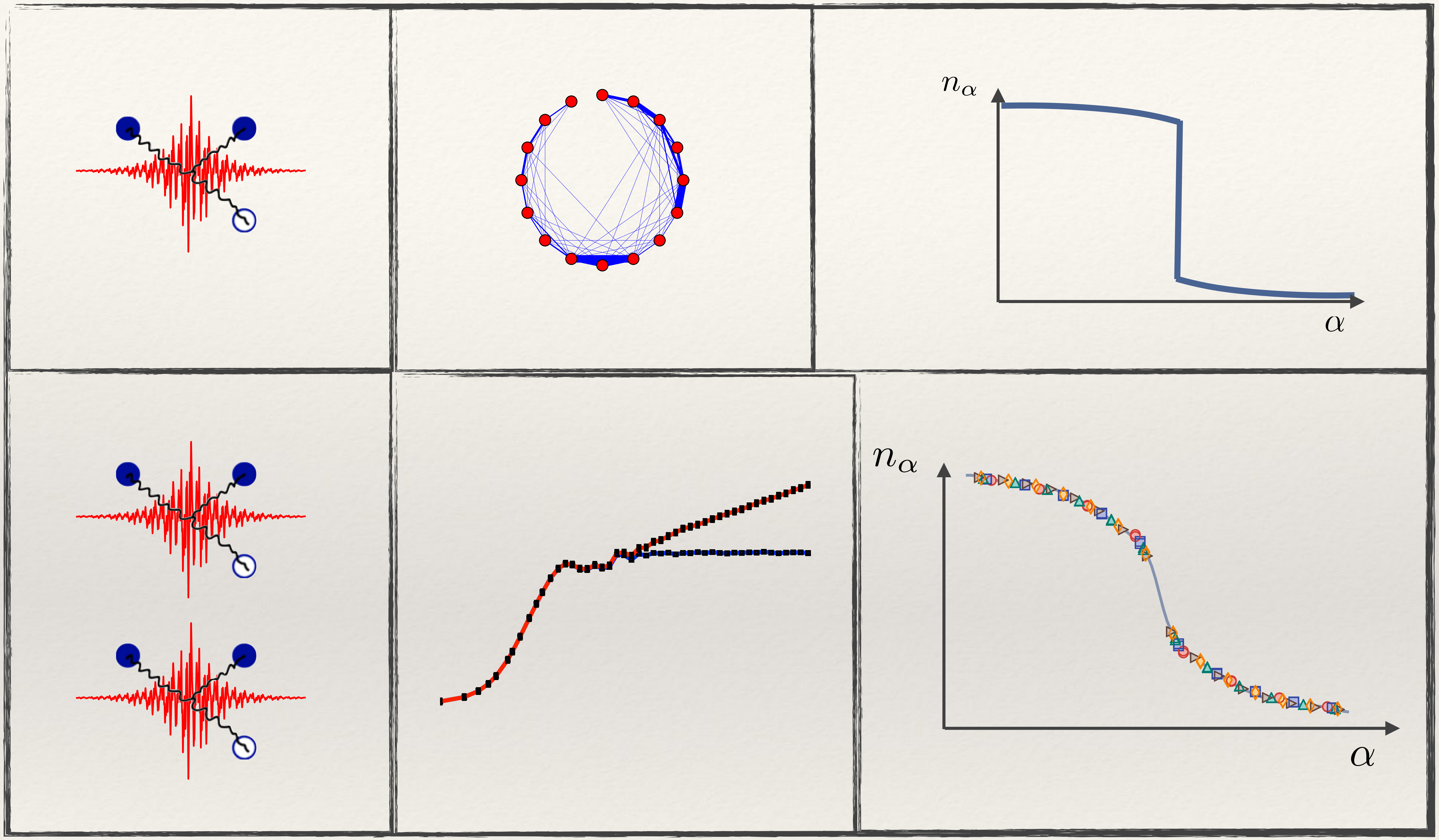
Summary



JHB, Pollmann, Moore PRL 2012; Kjäll, JHB, Pollmann PRL 2014
Bera, Schomerus, Heidrich- Meisner, JHB PRL 2015; De Tomasi, Bera, JHB, Pollmann PRL 2017
Bera, Martynec, Schomerus, Heidrich-Meisner, JHB Ann. Phys. 2017; Lezama, Bera, Schomerus, Heidrich-Meisner, JHB arXiv:1703.04398