Atomic Bose-Einstein condensates and Rydberg gases

Sebastian Wüster

Summer-term 2013

mpipks
Finite Systems

MAX-PLANCK-GESELLSCHAFT

International Max Planck Research School
"Dynamical Processes in Atoms, Molecules and Solids"

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Information

- contact details: *Sebastian Wüster,*
  Max Planck Institute for the
  Physics of Complex Systems,
  Noethnitzer Str. 38, Dresden
  Office: 2B8

  email://sew654@pks.mpg.de

  Phone: +49 351 871 2208
  web: http://www.mpipks-dresden.mpg.de/~sew654/teaching.html

- Lecture: *Wednesdays: 16:40 - 18:10*
  Room BZW/ A120/ P
  13 lectures (10.4. - 17.7., not 22.5, 5.6.)
  beamer introduction, then chalk and blackboard + movies and images

Literature

(1) C.J. Pethick and H. Smith, *“Bose-Einstein Condensation in Dilute Gases”* , Cambridge University Press (2002)

(2) L.P. Pitaevskii and S. Stringari *“Bose-Einstein Condensation”,* Oxford University Press (2003)

(3) F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari “*Theory of Bose-Einstein condensation”,* Rev. Mod. Phys. **71**, 463 (1999)

(4) T. F. Gallagher, *“Rydberg atoms”,* Cambridge University Press (1994)

Outline

1. Introduction
    1.1. Motivation
    1.2. Revision

2. Ultra-cold atomic gases
    2.1. Quantum statistical physics
    2.2. Trapping and cooling of atoms
    2.3. Interactions between atoms

3. Bose-Einstein condensates
    3.1. Field operator
    3.2. Gross-Pitaevskii theory
    3.3. Excitations of the condensate

4. Rydberg atoms
    4.1. Quantum defects
    4.2. Rydberg interactions and external fields
    4.3. Rydberg excitation and decay
    4.4. Dipole blockade
    4.5. Field ionization
    4.6. Applications in quantum information

1. Introduction

- Purpose of 1.1. motivation:
  - get you excited about the field of ultra-cold atoms (quantum-atom-optics)
  - show for what the material presented later is important
  - personal view: highlight use of BEC for inter-disciplinary physics (quantum simulation)
- Purpose of 1.2. revision:
  - remind you of required material from quantum mechanics and atomic physics
  - make sure everyone has similar starting point
  - will **not** replace prior study of these topics, if you notice that you are not familiar with some of the physics, please recap.

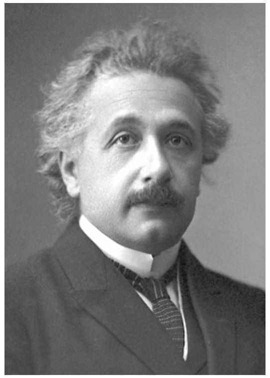
Survey:

Please indicate for the following physics topics how familiar you feel that you are with them:

	never heard	*not had in lecture*	*remember what it is, but no details*	*heard and can revise the details*	*fully familiar*
quantum oscillator:	☐	☐	☐	☐	☐
second quantisation:	☐	☐	☐	☐	☐
quantum field theory:	☐	☐	☐	☐	☐
quantum statistical physics:	☐	☐	☐	☐	☐
Bose-Einstein statistics:	☐	☐	☐	☐	☐
Zeeman effect:	☐	☐	☐	☐	☐
Stark effect:	☐	☐	☐	☐	☐
two-level atom interacting with light field:	☐	☐	☐	☐	☐
hyperfine splitting:	☐	☐	☐	☐	☐
quantum scattering-theory/ partial wave expansion:	☐	☐	☐	☐	☐

1.1. Motivation

(part1) Bose-Einstein Condensation



Albert Einstein

Bose 1924: Identical quantum particles (e.g. photons) have to be treated as indistinguishable. This affects the counting of the possible ways to realize a given state.

S. N. Bose, Z. Phys. **26** (1924) 178.

Satyendra Nath Bose

Einstein 1925: This results at T=0 in a case where all **bosons** occupy the same quantum state.

A.Einstein, Sitzungsber. Kgl. Preuss. Akad. Wiss. page 261 (1924).
A.Einstein, Sitzungsber. Kgl. Preuss. Akad. Wiss. page 3 (1925).

=> Bose-Einstein Statistics: Occupation of state with energy E: $n = \frac{1}{e^{(E-\mu)/k_B T} - 1}$

=> Bose-Einstein Condensation: At T=0, all Bosons will occupy **the same** lowest energy quantum state

What is Bose-Einstein condensation (BEC)?

High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$:

Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0:

Pure Bose condensate

"Giant matter wave"

Experimental Condensation in dilute alkali gases

theoretical prediction of BEC 1925
experimental realization 1995

Nobel Prize in Physics (2001),
W. Ketterle ,E.A. Cornell and C.E.Wieman

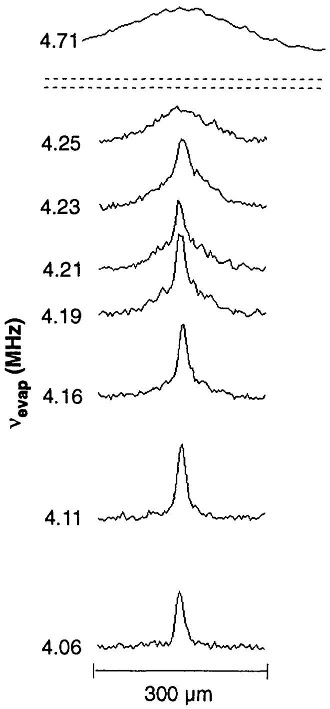
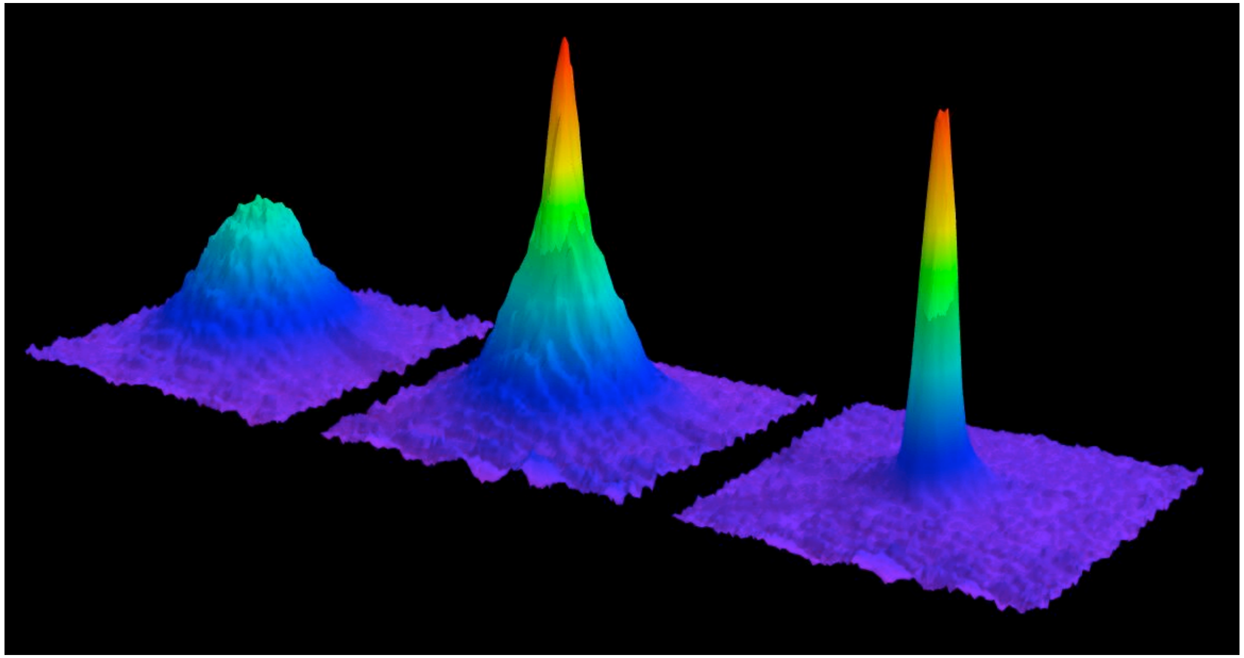


Fig. 4. Horizontal sections taken through the velocity distribution at progressively lower values of ν_{evap} show the appearance of the condensate fraction.

SCIENCE • VOL. 269 • 14 JULY 1995



K.B. Davis *et al.*, Phys. Rev. Lett. **75** (1995) 3969.
M. H. Anderson *et al.* , Science **269** (1995) 198.
C.C. Bradley *et al.*, Phys. Rev. Lett. **75** (1995) 1687.

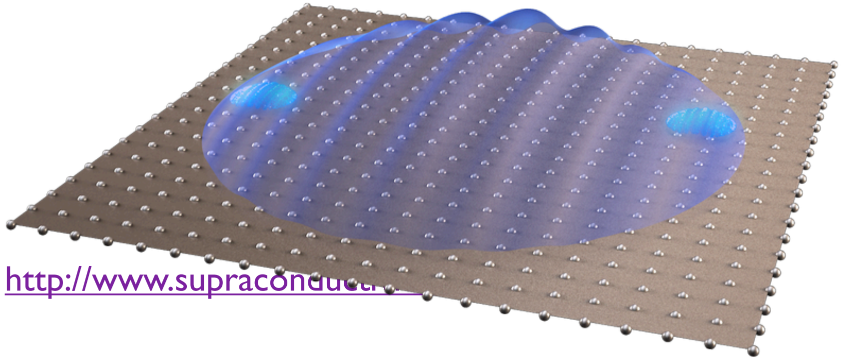
T_{crit} = *170 nK*

crucial techniques: laser-cooling, magneto-optical trap, evaporative cooling => chapter 2

BEC in other physical systems

(some examples)

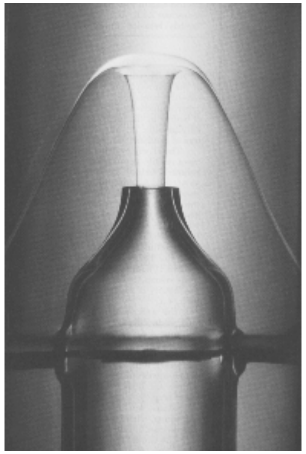
Cooper-pair condensates in superconductors



http://www.supracondu

- more complicated than direct BEC of alkali-gases, since we first have to bind two Fermions to make a Boson.
- Can see similar effect in degenerate Fermi gases.

liquid helium, superfluidity



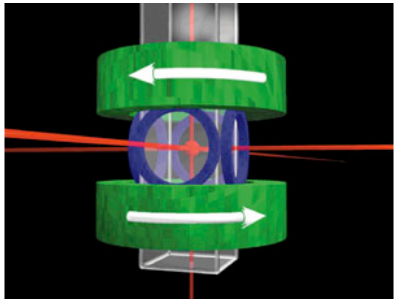
- Helium is strongly interacting, only shows very small condensed fractions

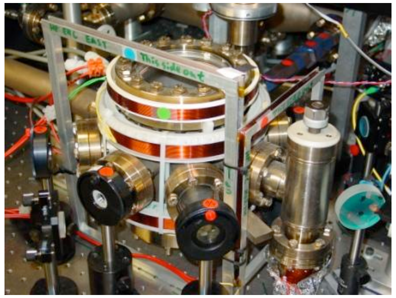
exciton-polaritons in semi-conductor micro cavities

Various atom traps

- Experimental techniques to cool down to the nK regime are anyway quite sophisticated => They then also allow a tremendous degree of control over the BEC/ ultra-cold gas, once it has been created.

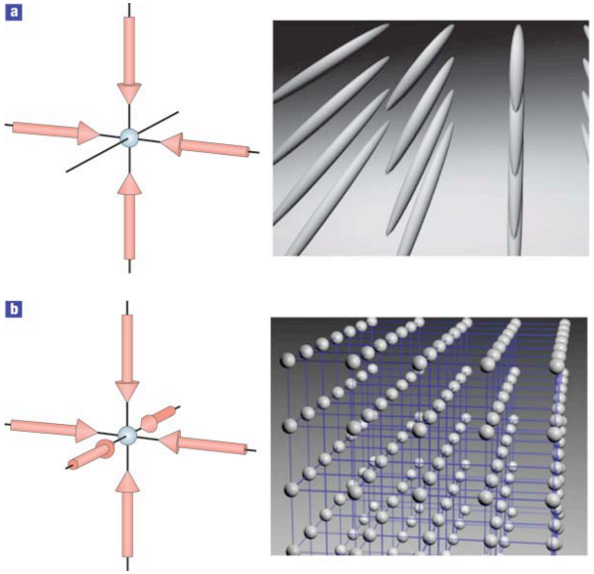
magneto optical trap





http://www.physics.otago.ac.nz/research/jackdodd/resources/ResourceFiles/galleryimages/

optical lattice



I. Bloch, Nature Phys. **1** (2001) 23.

ring trap

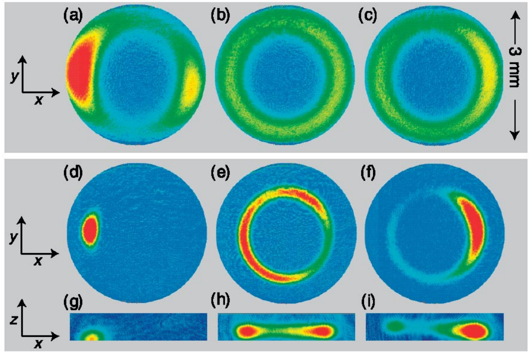
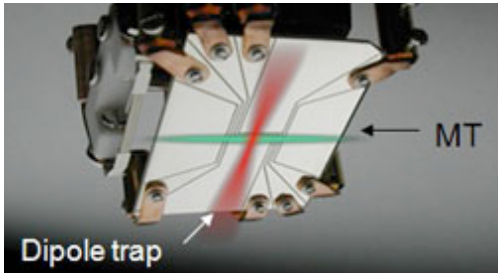


FIG. 2 (color). Atoms in a ring-shaped magnetic trap. Shown

S. Gupta *et al.*, Phys. Rev. Lett. **95** (2005) 143201.

atom chip

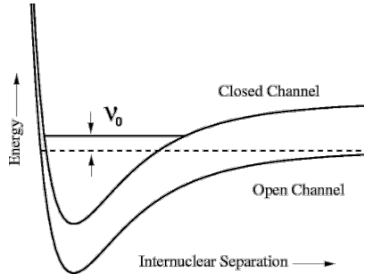


http://www.physics.uq.edu.au/BEC/images/experimental/research_combinedtrap.jpg

- Can confine atoms using magnetic field, electrical fields (from lasers) or both.

Experimental control

Feshbach resonance (interaction control)



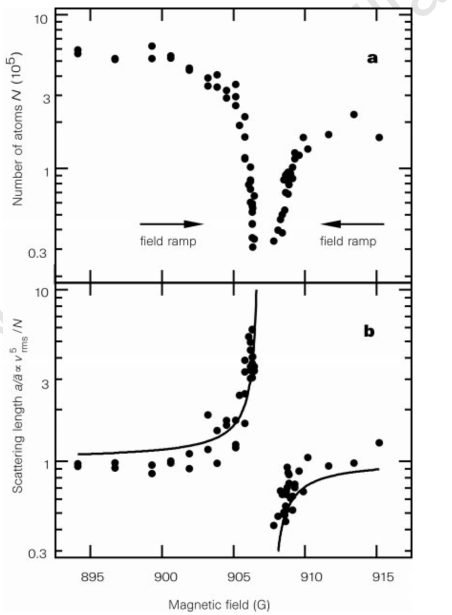


Figure 2 Observation of the Feshbach resonance at 907 G using time-of-flight absorption imaging. **a**, Number of atoms in the condensate versus magnetic field.

S. Inouye *et al.*, Nature **151** (1998) 392.

Phase imprinting (motion control)

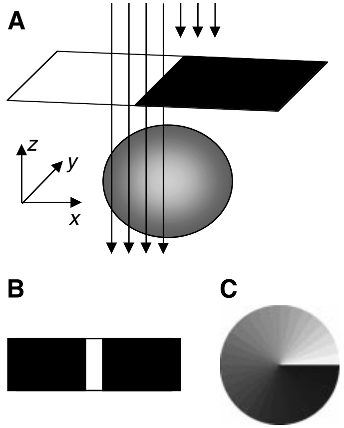


Fig. 1. (**A**) Writing a phase step onto the condensate. A far-detuned uniform light pulse

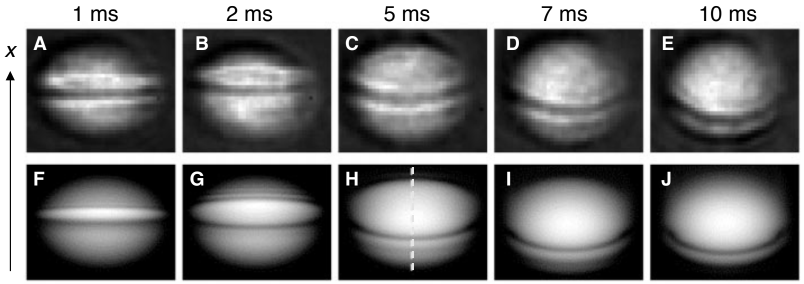
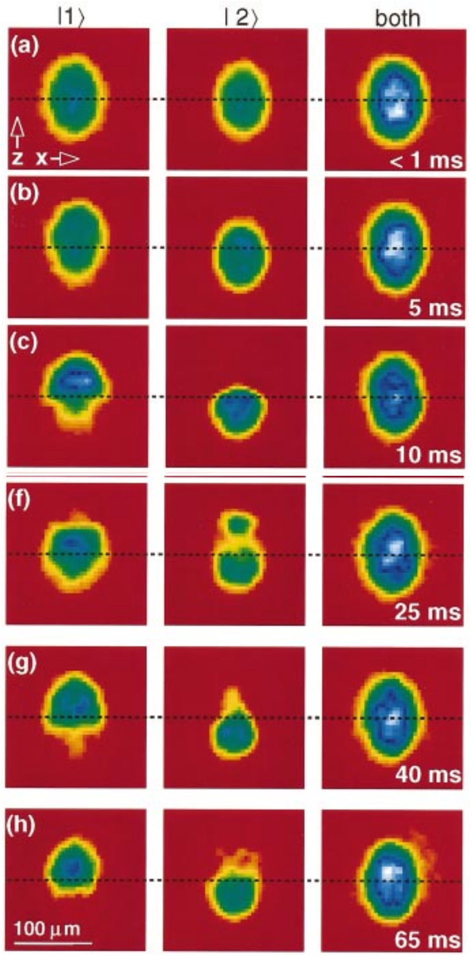


Fig. 3. Experimental (**A** to **E**) and theoretical (**F** to **J**) images of the integrated BEC density for various times after we imprinted a phase step of ~1.5π on the top half of the condensate with a

J. Denschlag *et al.*, Science **287** (2000) 97.

transitions (internal state control)

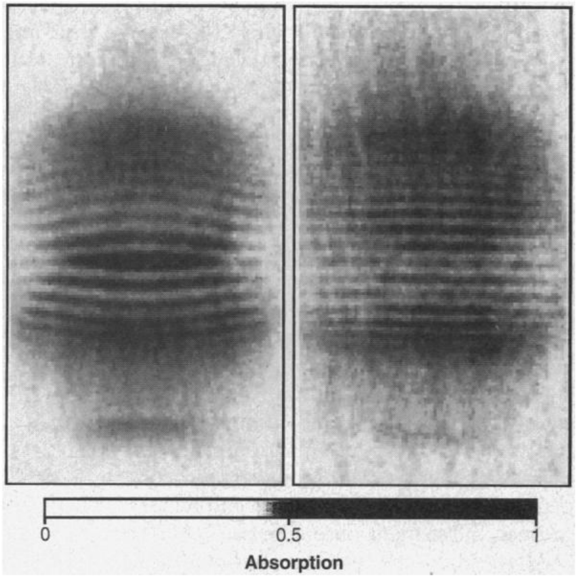


D. S. Hall *et al.*, Phys. Rev. Lett. **81** (1998) 1539.

Macroscopic matter waves

- *single quantum wave-function occupied by* ***many atoms*** *can easily be imaged*
- *the system is analogous to many photons in a laser cavity => atom optics*

two condensates interfering



M.R. Andrews *et al.*, Science **275** (1997) 637.

scattering partial waves

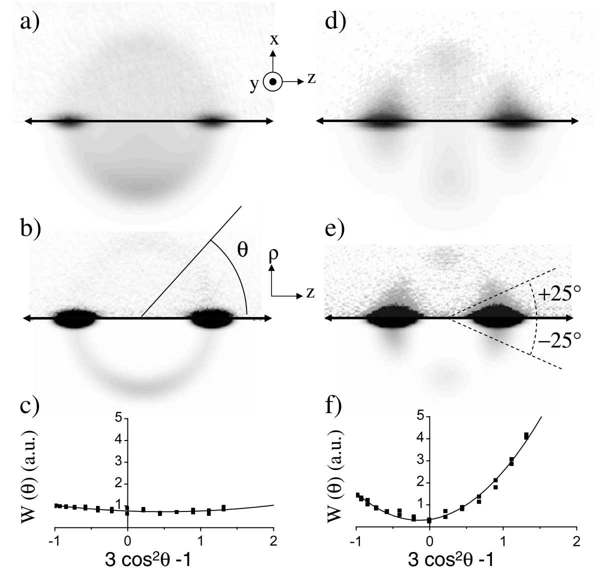


FIG. 1. (a) Optical density of the scattering halo of two ^{87}Rb condensates for collision energy $E/k_B = 138(4)$ μK, measured

Ch. Buggle *et al.*, Phys. Rev. Lett. **93** (2004) 173202.

atom laser

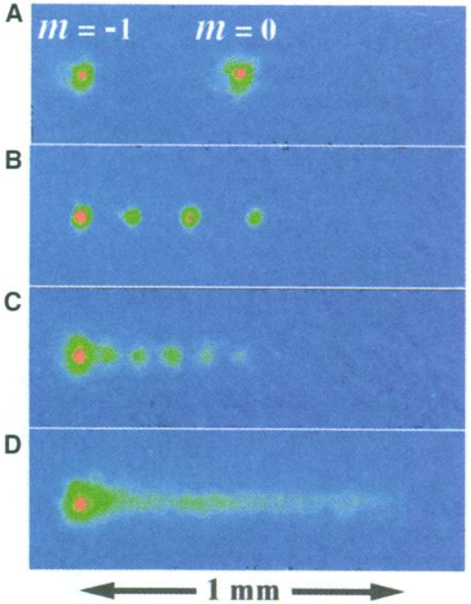


Fig. 4. (**A** to **C**) One, three, and six 6-µs Raman pulses, respectively, were applied to the condensate. (**D**) Firing 1-µs Raman pulses at the

E.W. Hagley *et al.*, Science **283** (1999) 1706.

Vortices

Rotating condensates: Circulation is quantized, condensate must rotate via vortices.

$$\oint_C \mathbf{v} \cdot \mathbf{dl} = \frac{h}{m} q$$

Vortices in stirred BEC (q=1)

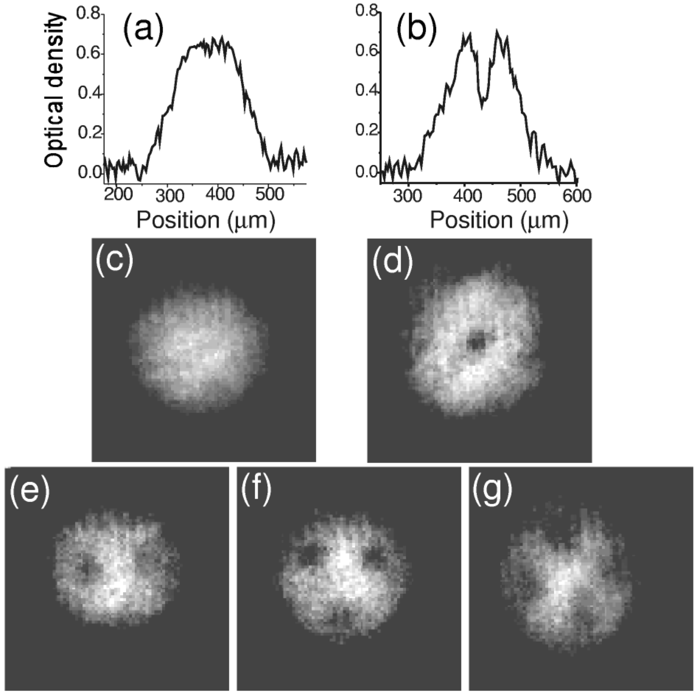
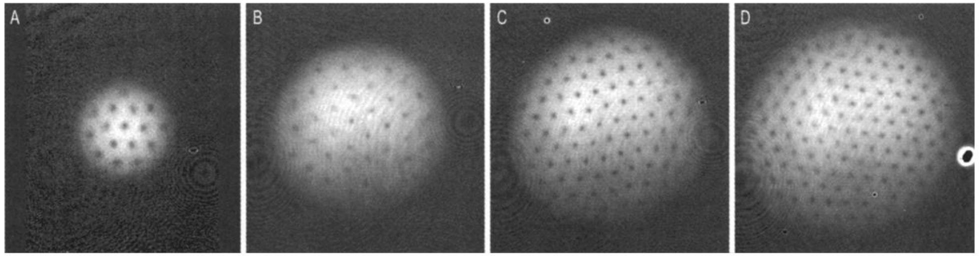


FIG. 1. Transverse absorption images of a Bose-Einstein condensate stirred with a laser beam (after a 27 ms time of flight).

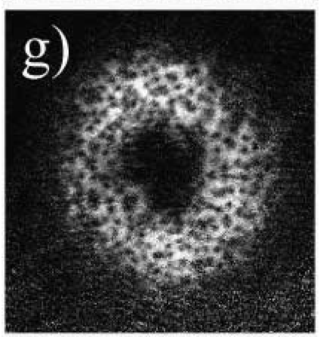
K.W. Madison *et al.*, Phys. Rev. Lett. **84** (2000) 806.

Abrikosov Lattice



J.R. Abo-Shaeer *et al.*, Science **292** (2001) 476.

Giant vortices (q>>1)



P. Engels *et al.*, Phys. Rev. Lett. **90** (2003) 170405.

Quantum simulation

quantum simulation: *new ways to understand complicated (many-body) quantum systems that cannot be fully simulated on a classical computer.*

R. Feynman, *Int. J. Theor. Phys.* **21** (1982) 467.

Size of Hilbert space of N particle, M mode system: M^N

analog quantum simulator: find a quantum system with the same Hamiltonian but easier access to parameters and measurements.

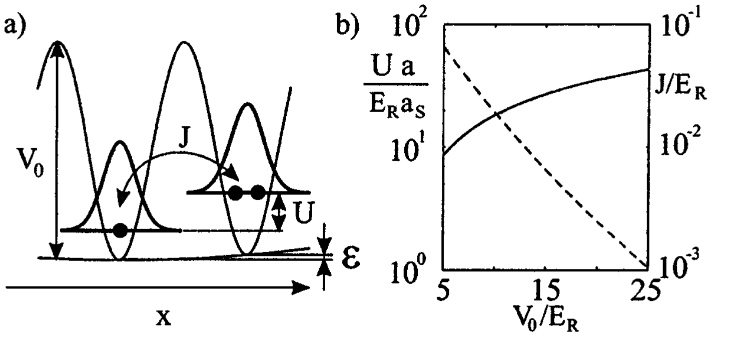
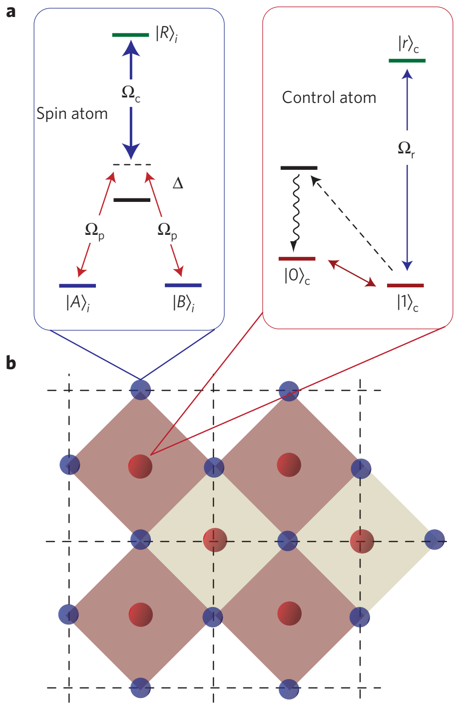


FIG. 1. (a) Realization of the BHM in an optical lattice (see text). The offset of the bottoms of the wells indicates a trapping

D. Jaksch *et al.*, Phys. Rev. Lett. **81** (1998) 3108.

digital quantum simulator: find a system that flexibly can evolve according to a Trotter decomposition of the time-evolution operator.

$$e^{-iHt/\hbar} = \lim_{n\rightarrow\infty}\left(\prod_k e^{-iH_k t/n\hbar}\right)^n$$



H. Weimer *et al.*, Nature Phys. **6** (2010) 382.

Condensed matter analogues

(more on quantum simulation)

Mott-Insulator / Superfluid quantum phase transition

$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger \hat{a}_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Lattice strength decides type of ground-state

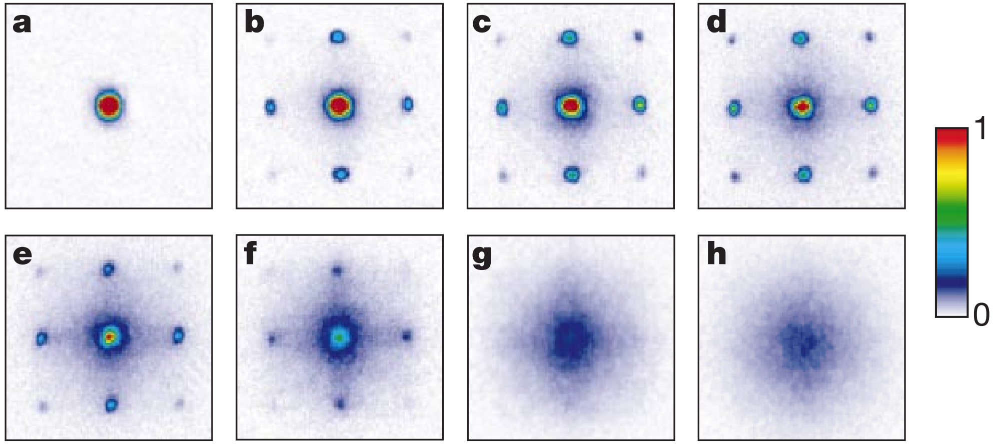
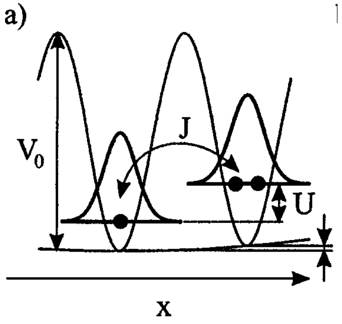
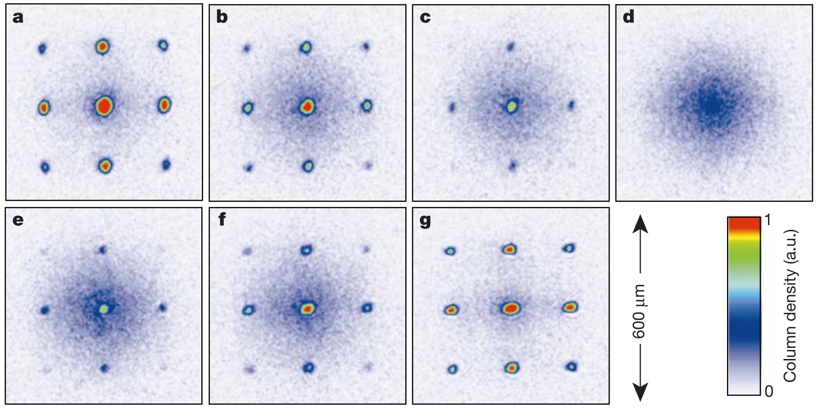


Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.

M. Greiner *et al.* Nature **415** (2002) 39.



collapse and revival



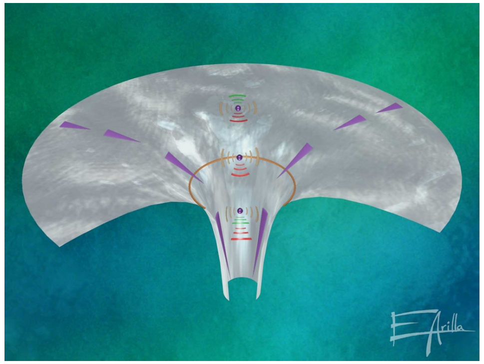
M. Greiner *et al.* Nature **419** (2002) 51.

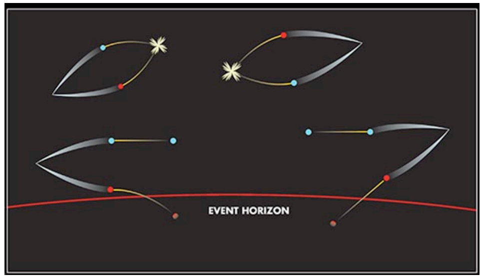
General relativity analogues

(more on quantum simulation)

Analog Gravity: *excitations (sound-waves) in a Bose-Einstein condensate behave in some regime like particles propagating in general relativistic space-times.* C. Barcelo *et al.*, Living Rev. Relativity **14**, (2011), 3

Hawking radiation

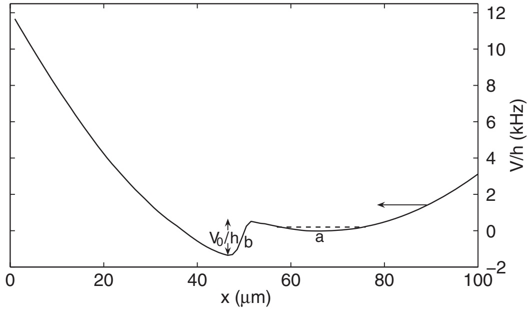


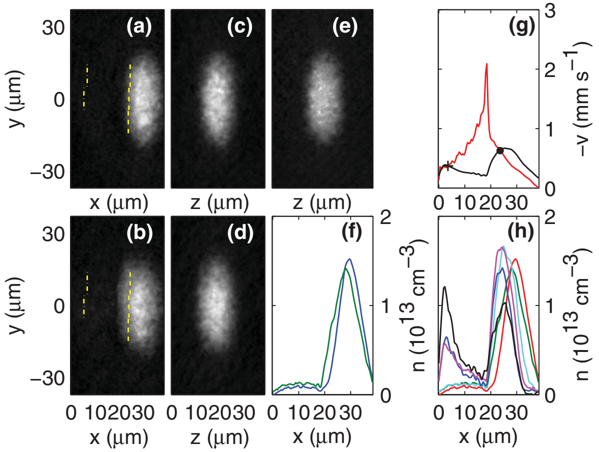


S. Hawking, Nature **248** (1974) 30.

analogue Hawking radiation in (fluids) Bose Einstein condensates

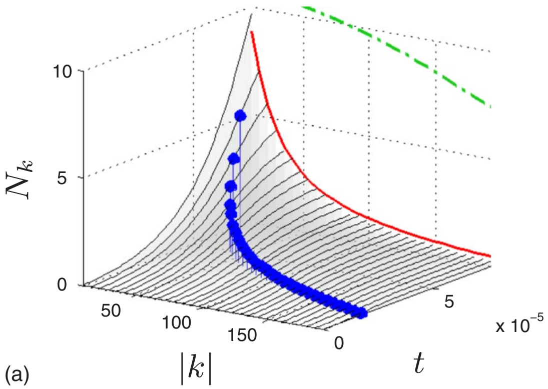
W. G. Unruh, Phys. Rev. Lett. **46** (1981) 1351.





O. Lahav, *et al.* Phys. Rev. Lett. **105** (2010) 240401.

analogue cosmological particle creation radiation



(part 2) Rydberg atoms

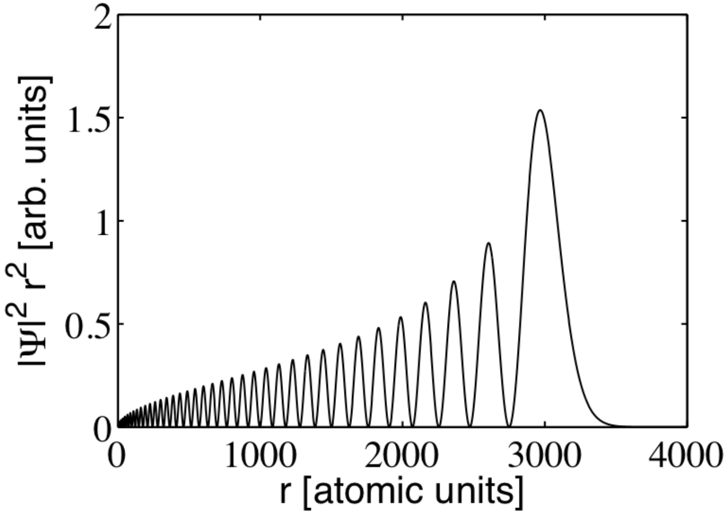
Johannes Rydberg

Rydberg 1880: Wavelengths of alkali spectral lines behave according to:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

- atoms in states with large principal quantum number n ~ 40-100.
- Large size ~ n^2 (85nm for n=40)
- Large polarizability ~ n^7
- Long life times ~ n^3 (40µs for n=40)
- long range interactions
  $V \sim n^4/d^3$ dipole-dipole
  $V \sim n^{11}/d^6$ Van-der-Waals

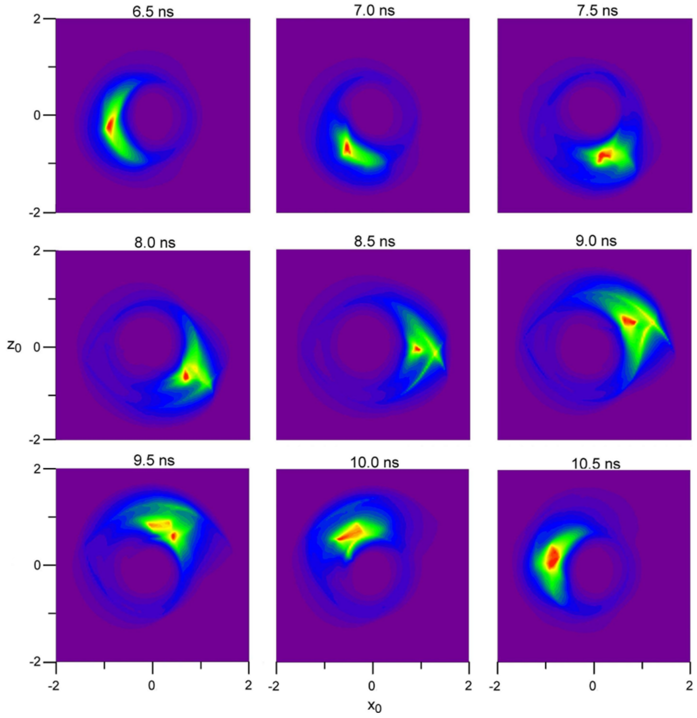
Li, n=40, radial density



Almost classical orbits

semi-classical description: *de-Broglie wavelength compared to orbital radius becomes smaller and smaller => Understanding that can be gained from classical pictures increases*

circular wave packet, Bohr-atom



J. J. Mestayer *et al.*, Phys. Rev. Lett. **100** (2010) 243004.

classical and quantum dynamics

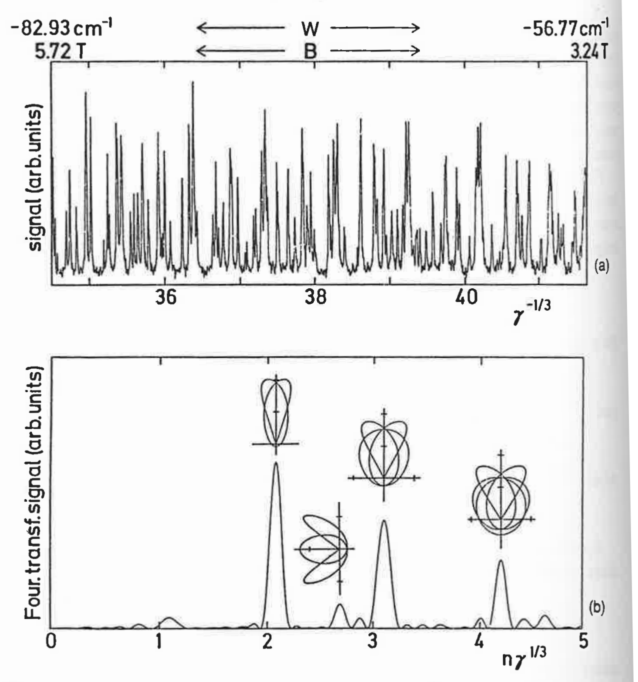


Fig. 9.10 (a) Scaled energy spectrum at $\tilde{W} = -0.45$ as a function of $\gamma^{-1/3}$. Range of excitation energy $-77.7\ \mathrm{cm}^{-1} \leq W \leq -54.3\ \mathrm{cm}^{-1}$ and field strength $5.19\ \mathrm{T} \geq B \geq 3.03\ \mathrm{T}$. (b) Fourier transformed action spectrum of (a); closed orbits correlated to respective resonances in ρ, z projection; z coordinate vertical (from ref. 27).

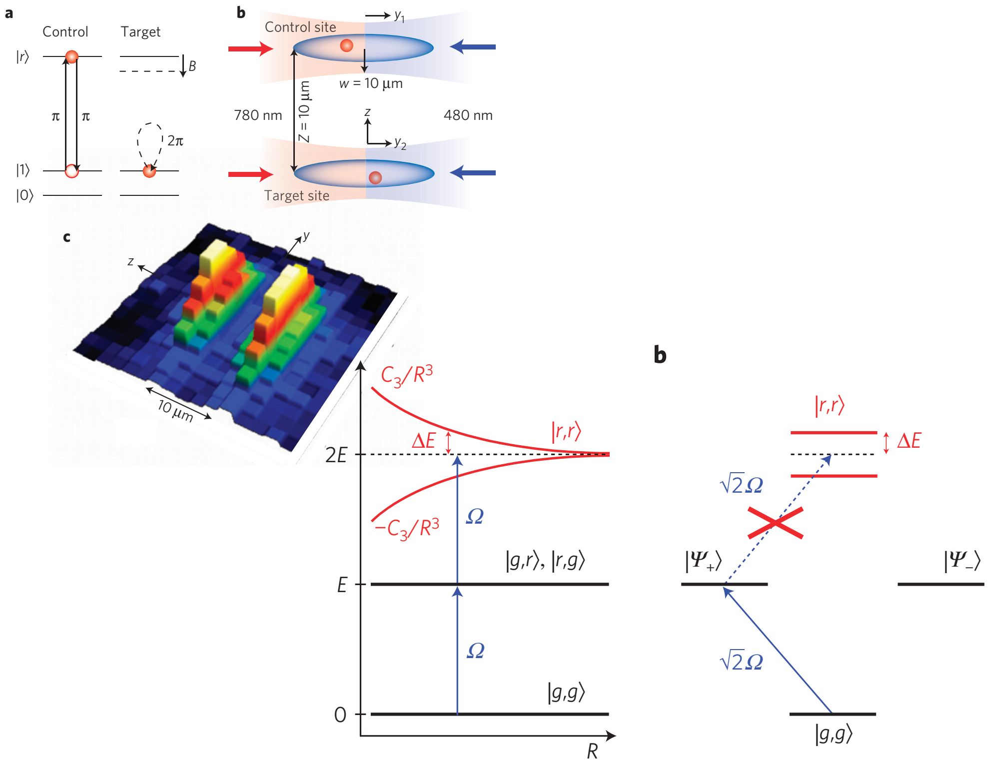
W. Wintgen *et al.*, Phys. Rev. A. **36** (1987) 131.

also can study quantum chaos

Rydberg-dipole blockade

Interactions between Rydberg atoms at micrometer distances is 12 OOM larger than between ground-state atoms. Micrometer distances allow individual laser addressing. => interesting for quantum computation

The blockade effect



E. Urban *et al.*, Nature Phys. **5** (2009) 110.

A. Gaëtan, *et al.*, Nature Phys. **5** (2009) 115.

Application to quantum gates

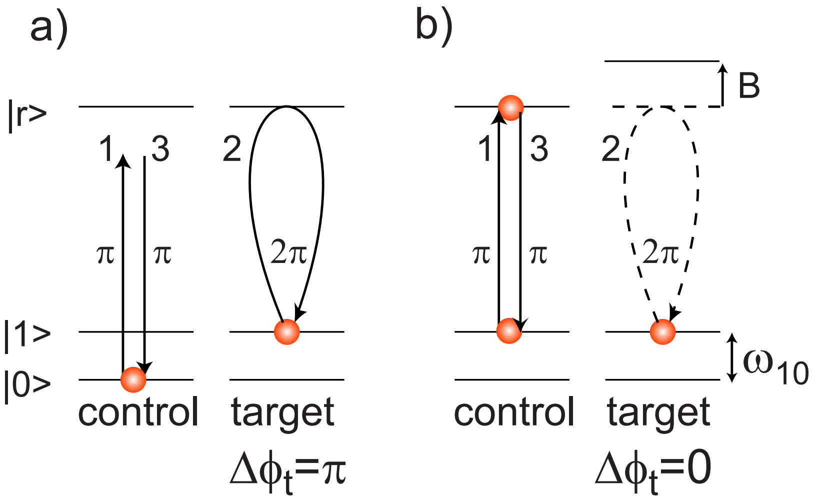


FIG. 2. (Color online) Rydberg blockade controlled phase gate

D. Jaksch, *et al.* Phys. Rev. Lett. **85** (2000) 2208.

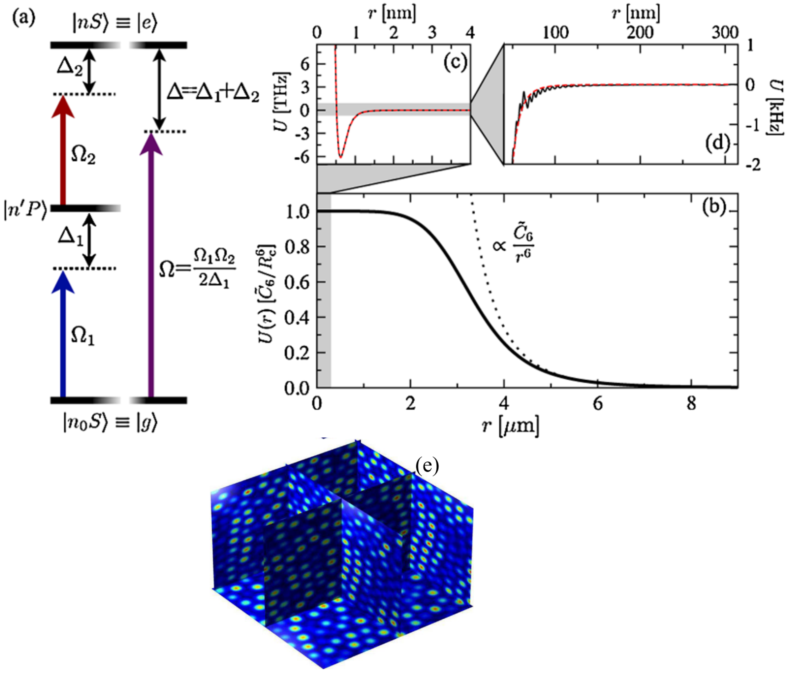
M. D. Lukin, *et al.* Phys. Rev. Lett. **87** (2001) 037901.

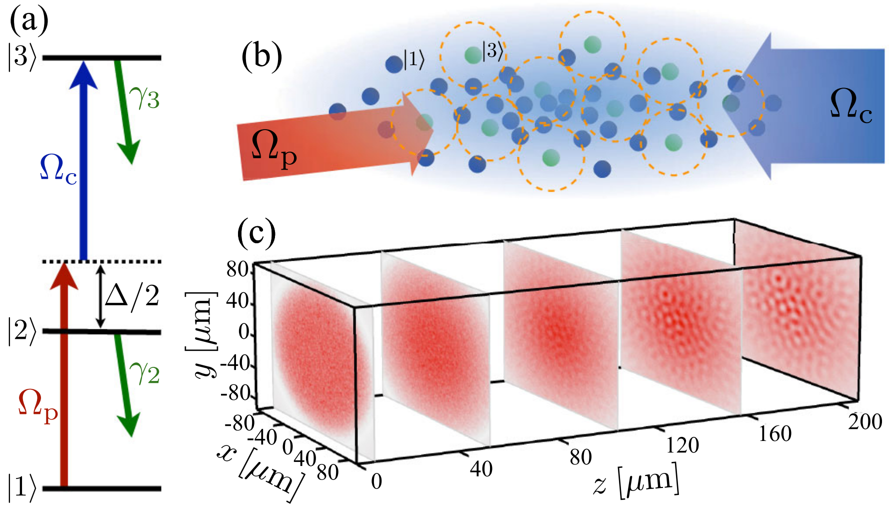
M. Saffman, *et al.* Rev. Mod. Phys. **82** (2010) 2312.

Rydberg dressing and EIT

Dressing: *Off resonant coupling between ground-state and Rydberg state: Very small Rydberg admixture makes ground-state inherit some of the extreme Rydberg properties.*

EIT: *Electromagnetically induced transparency. Quantum interference effects leads to zero population in middle level. Extremely sensitive to interactions acting on |3>.*





L. Santos, *et al.* , Phys. Rev. Lett. **85**, 1791 (2000).

N. Henkel, *et al.* , Phys. Rev. Lett. **104**, 195302 (2010).

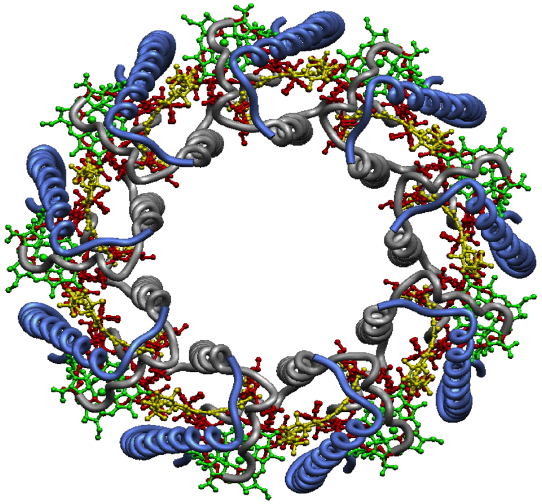
S. Sevincli, *et al.* , Phys. Rev. Lett. **107**, 153001 (2011).

Excitation transport

(more on quantum simulation)

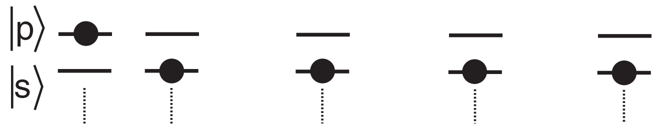
resonant dipole-dipole interactions:
Fluctuations between dipoles of different atoms (molecules) can lead to excitation transport.

Photosynthetic light-harvesting complex

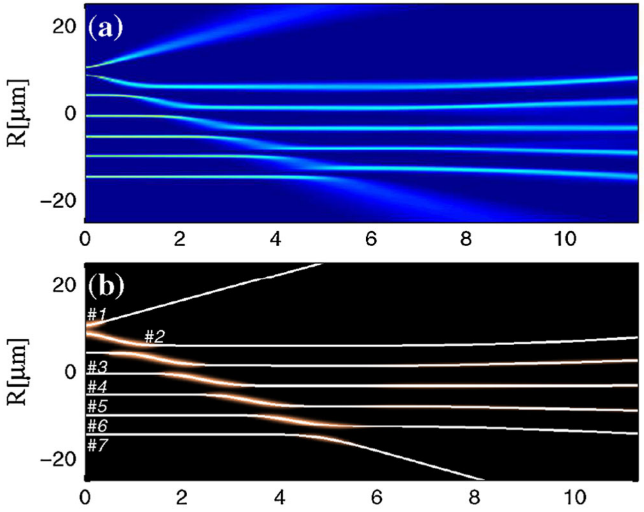


G. McDermott *et al.*
Nature **374** (1995) 517.

Chain of Rydberg atoms



Adiabatic excitation transport



S. Wüster *et al.* , Phys. Rev. Lett. **105**, 053004 (2010).

1.2. Revision

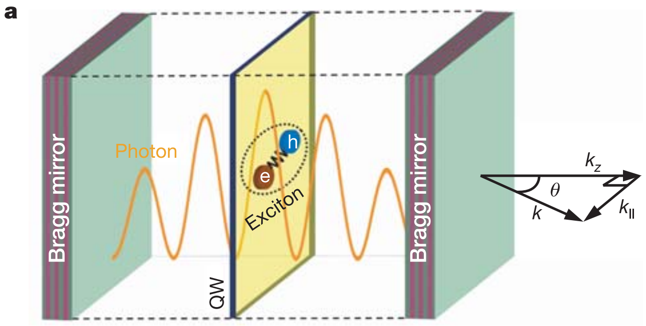
Let's move to blackboard

Appendix

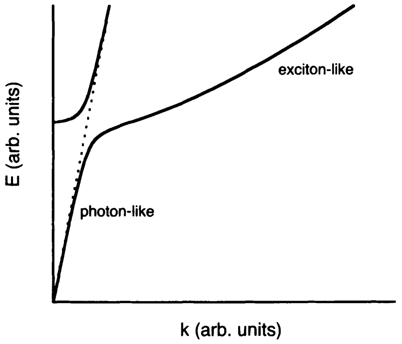
BEC in other physical systems

(some examples)

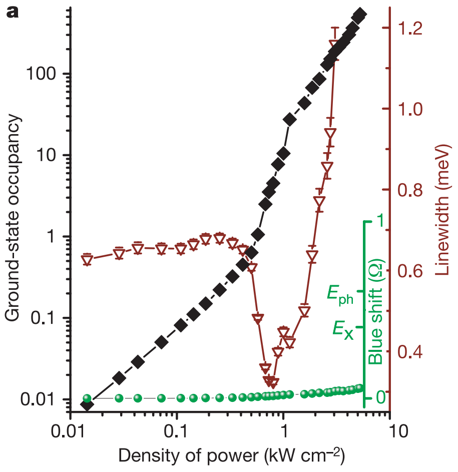
exciton polaritons
in semiconductor
microcavity



quantum well inside resonator



dispersion relation



condensation

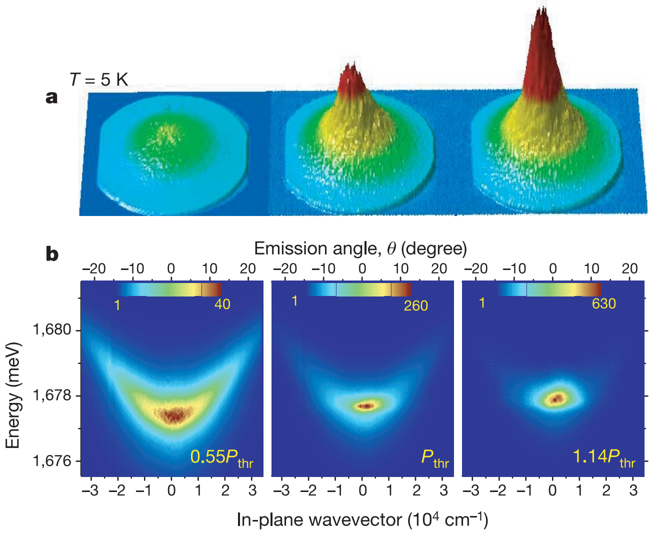
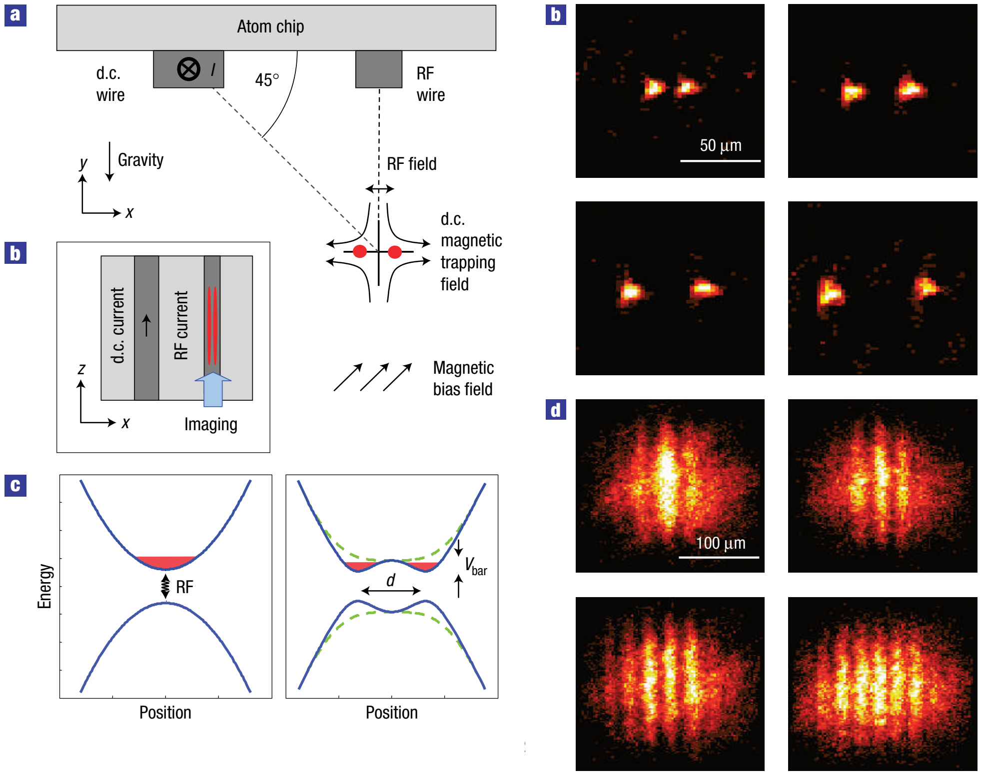


Figure 2 | Far-field emission measured at 5 K for three excitation intensities. Left panels, 0.55 P_{thr}; centre panels, P_{thr}; and right panels,

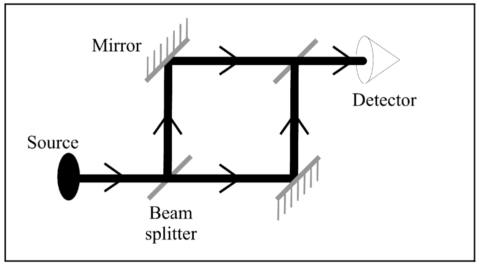
Atom interferometry

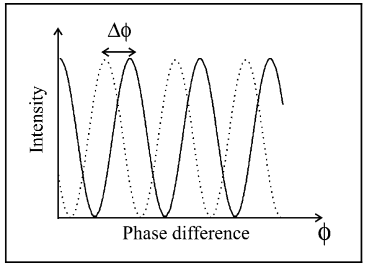
interferometer on an atom-chip



T. Schumm *et al.*, Nature Phys. **1** (2005) 57.

atom interferometer vs. optical interferometer





$$\Delta\Phi_{\text{light}} = \frac{4\pi}{\lambda c}\Omega\cdot A$$

(Sagnac)

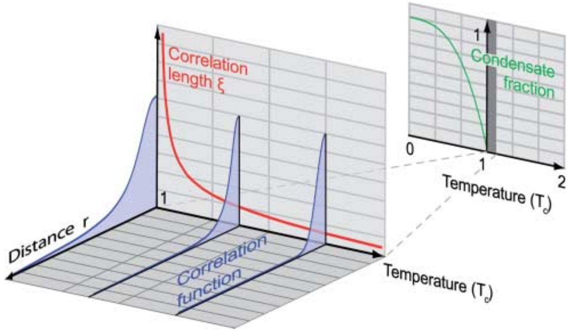
$$\begin{aligned}\Delta\Phi_{\text{atom}} &= \frac{4\pi}{\lambda_{\text{dB}}v}\Omega\cdot A\\ &= \frac{\lambda c}{\lambda_{\text{dB}}v}\Delta\Phi_{\text{light}}\\ &= \frac{mc^2}{\hbar\omega}\Delta\Phi_{\text{light}},\end{aligned}$$

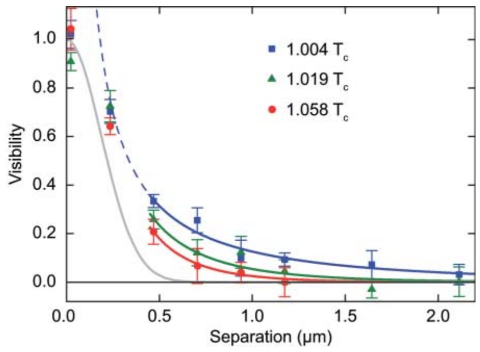
atoms are 11 orders of magnitude more sensitive than photons

e.g. R.M. Godun *et al.*, Contemp. Phys. **42** (2001) 77.

Ultra-cold phase transition

statistical mechanics and phase transition of interacting system





T. Donner *et al.*, Science **315** (2007) 1556.

$$\frac{\Delta T_c}{T_c} = C_0 a n^{1/3}$$

C0 = 1.3

V. A. Kashurnikov *et al.*, Phys. Rev. Lett. **87** (2001) 120402.

thermometry

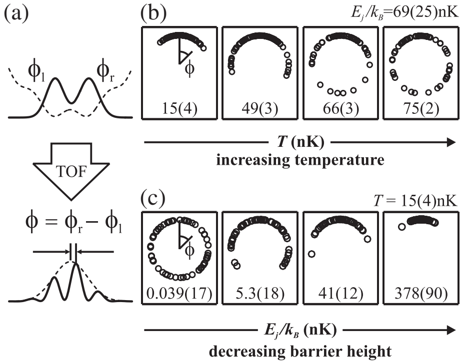


FIG. 1. Observation of thermal phase fluctuations. The experi-

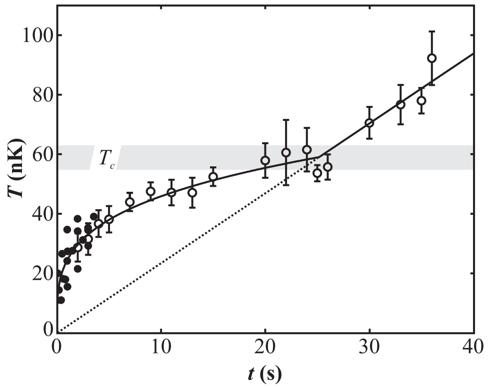


FIG. 4. Heating up of a Bose gas. The filled circles correspond

R. Gati *et al.*, Phys. Rev. Lett. **96** (2006) 130404..