What does operator ordering have to do with the density of paths?

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The density of paths enters all semiclassical path integral calculations and has a central role in a plethora of applications, from the Gutzwiller trace formula to quantum tunneling rates. Operator ordering also has taken the stage in path integration, particularly when it was mistakenly supposed (long ago) that ordering ambiguities could be resolved with this tool.

In the work I describe, these topics have a surprising relation, although the operator ordering issue that we deal with is at the $\hbar$ level, not $\hbar 2$, as in ordering ambiguities. We study what is known as the "operator Hamilton-Jacobi equation" which is an operator version of the classical equation and accomplishes the same thing, namely allows time dependence to be unraveled using a (quantum) canonical equation. It turns out that in the semiclassical approximation bringing this operator into "well-ordered" form (in which one canonical operator variable is always to the left of the other) is exactly compensated by the density of paths, nominally a classical quantity.