Path integral derivation of Lifshitz tails

Virulh Sa-yakanit

Chulalongkorn-University, Bangkok, Thailand

The first attempt to derive the Lifshitz conjecture is given by Friedberg and Luttinger based on the reformulation of the problem in terms of the Brownian motion. They shown that the solution of this problem is equivalent to the knowing the D(E)for small energy. A variational method is used for solving this problem and the solution is reduced to solving the nonlinear differential equation. This equation is solved in the long time limit and obtained the Lifshitz conjecture.

It was Lifshitz who was first presented the intuitive arguments and shown that the DOS should be behave as $\sim exp(-E^{-(3/2)})$. The Lifshitz's ideas are as follow. The probability of finding a large region of volume V,being free of impurities is proportional to $exp(-pV.)$.Then the main contribution to the probability of finding a low-level E for a system will be proportional to the probability of finding the region V whose energy level is E. The lowest-level E in the empty volume of radius R_0 with the boundary conditions that the wave function vanish on its surface is given by $E = ((\pi^2)/(2R_02))$;or $R_0 = \sqrt{(}(\pi^2/(2E)))$.

In this paper we derive the Lifshitz conjecture using the Feynman path integral formulation developed in our previous paper for handling the heavily doped semiconductors. With out the Gaussian approximation we derive the average propagator for general statistics or Poisson distribution. After performing the random average the system becomes translation invariant and therefore it is reasonable to model the system with nonlocal harmonics oscillator trial action. A variational principle is used to obtain the Lifshitz conjecture for large time limit.A comparison with the method developed by Friedberg and Luttinger based on the reformulation of the problem in terms of the Brownian motion is given.