Vortex-Line Percolation in a Three-Dimensional Complex $|\psi|^4$ Theory

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The functional measure of the three-dimensional complex $|\psi|^4$ theory allows for line-like topological excitations which can be related to vortex lines in superfluid helium by universality arguments. Upon approaching the λ point, these lines proliferate and destroy the superfluidity. To study the phase transition from this geometrical point of view, we investigated the statistical properties of the emerging vortex-loop network in the vicinity of the critical point by means of high-precision Monte Carlo simulations. For comparison the standard magnetic properties of the system were considered as well. Using sophisticated embedded cluster update techniques we examined if both of them exhibit the same critical behaviour leading to the same critical exponents and therefore to a consistent description of the phase transition. Different percolation observables are taken into account and compared with each other. We find that different definitions for constructing the vortex-loop network lead to slightly (but statistically significantly) different results in the thermodynamic limit, and that the percolation thresholds are close to but do not really coincide with the thermodynamic phase transition point.