COMPUTER SIMULATION of the Quantum Universe

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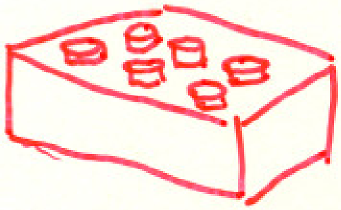
$$Z(G,\Lambda) = \int \mathcal{D}[g_{\mu\nu}]\, e^{-S([g_{\mu\nu}],\Lambda,G)}$$

$$[g_{\mu\nu}] \equiv \text{Geometry} \sim g_{\mu\nu}$$

Need regularization (cut off a)

Use building blocks (BB), also called Dynamical Triangulation (DT)

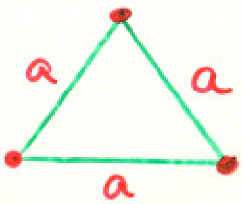
Use building blocks!

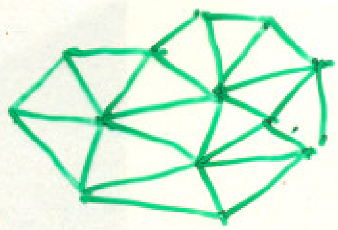


Danish high tech

one dimension

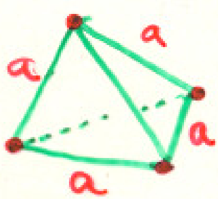
two dimensions

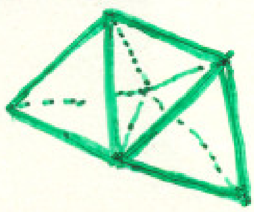




triangles

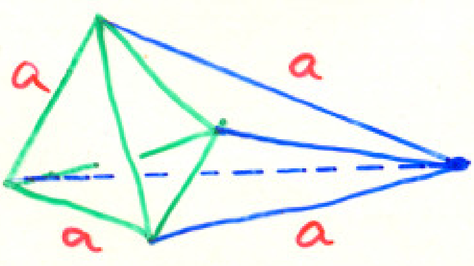
three dimensions





tetrahedra

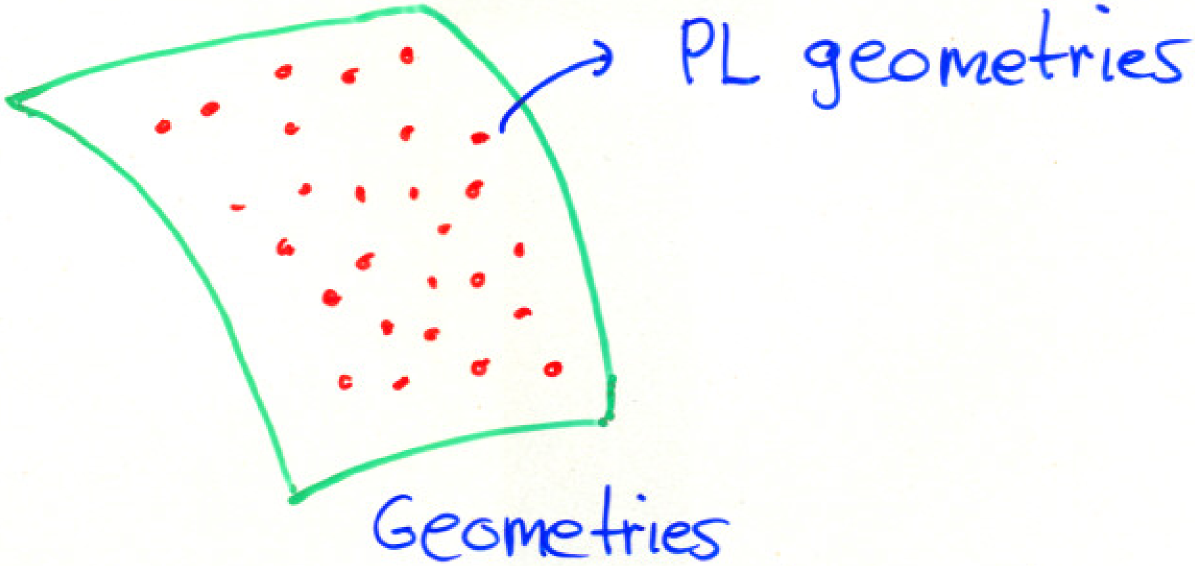
four dimensions



Four-simplices

Gluing together BBs create piecewise linear geometries:

Manifestly coordinate independent



Assumption: Class of PL-geometries made from DT becomes dense in space of geometries for $a \to 0$

TOY MODEL: 2d Euclidean Q.G.

$$Z(\Lambda) = \int \mathcal{D}[g_{\mu\nu}]\, e^{-\Lambda V_2(g)}$$

$$V_2(g) = \int d^2\xi \sqrt{g(\xi)}$$

$$\int d^2\xi \sqrt{g(\xi)}\, R(\xi) = 2\pi \chi$$

Conformal gauge: $g_{\mu\nu} = e^{\phi}\, \delta_{\mu\nu}$

$$Z(\Lambda) = \int \mathcal{D}\phi\; e^{-\frac{26}{48\pi^2}\int d^2\xi \left((\partial\phi)^2 + \Lambda e^{\phi} \right)}$$

Quantum Liouville theory.

cut-off issue subtle:

Diffeomorphism invariant cut-off a

From point of view of geometries the model is trivial and entirely <u>entropic</u>

$$Z(\Lambda) = \int_0^\infty dV\, e^{-\Lambda V} \int \mathcal{D}[g_{\mu\nu}]\, \delta(V_2(g) - V)$$

$$= \int_0^\infty dV\, e^{-\Lambda V}\, \mathcal{N}(V)$$

Can we count $\mathcal{N}(V)$?

Yes, using BB's

$V_2 = N_2 a^2$, $\mathcal{N}(N) \sim N^{\gamma-3} e^{\mu_c N}$

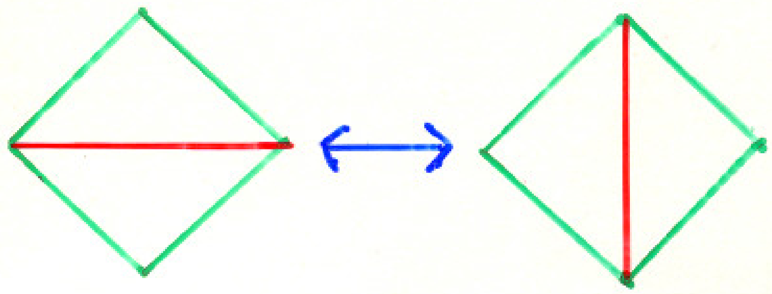
$\Lambda = \frac{\mu_c}{a^2} + \Lambda_R$ (additive renormalization)

Everything agrees with continuum quantum Liouville calculations.

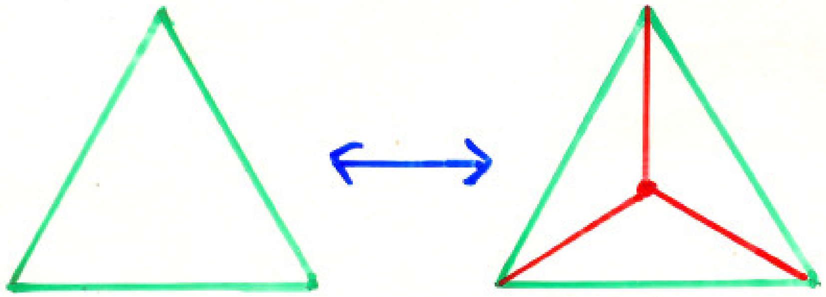
Conclusion 1: DT provides a good regularization of $\int \mathcal{D}[g_{\mu\nu}]$ in 2d

Conclusion 2: One can use MC-simulations to calculate critical exponents in 2d EQG.

<u>updating</u> in 2d for fixed N:



Ergodic for fixed N and topology



Additional <u>move</u> if N varies

4D: We do not know if there exists a QFT of QG ?
(non-renormalizable)

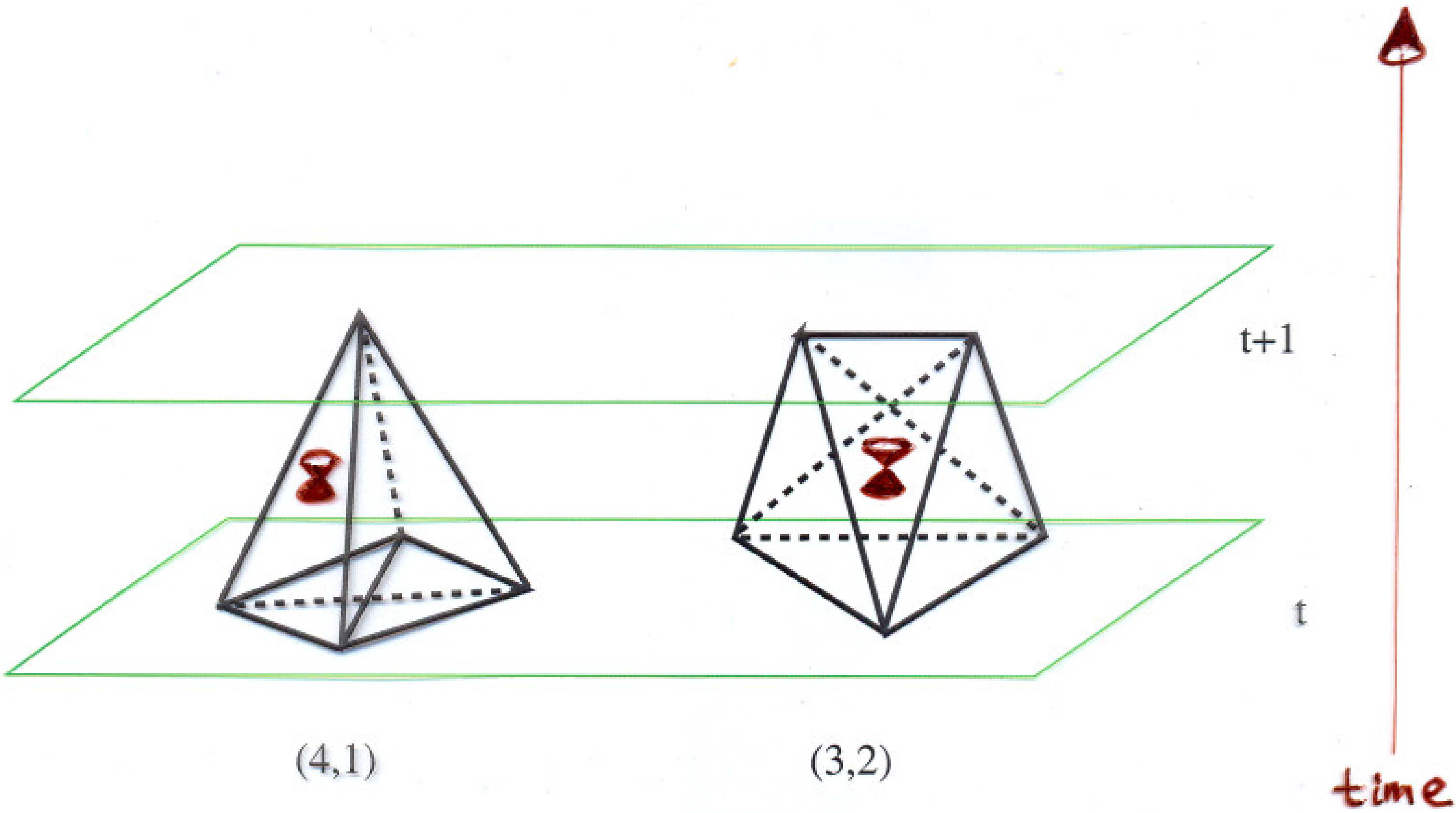
DT with lattice cut-off a provides a non-perturbative def. of the path integral

However, to obtain a (seemingly) interesting continuum limit ($a \to 0$) one has to impose two add. conditions:

(1) Existence of a time foliation

(2) Only causal geometries are included in the path integral (Lorentzian!)

(1) + (2) : **Causal DT (CDT)**



elementary simplicial building blocks,
cut out of 4d Minkowski space

(a): Each Lorentzian CDT has an analytic continuation to the Euclidean sector.

(b): In 2d the relation between CDT and EQG has been worked out in detail.

In 4d we have presently no analytic tools, but MC simulations are available in the Euclidean sector of CDT: 5 moves

Action: The Regge action for PL-geometries (Geometric)

Identical BBs makes it trivial:

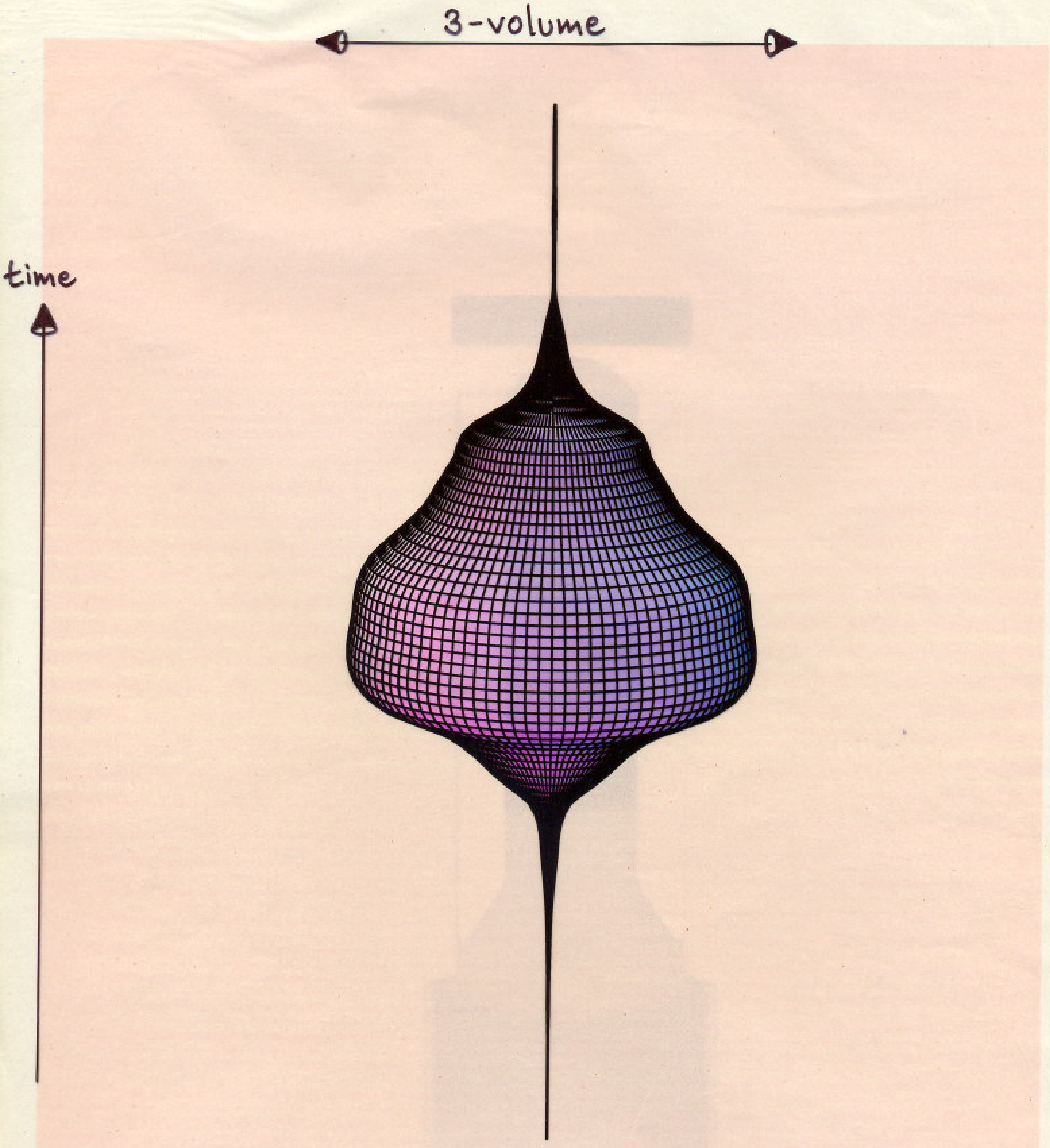
$$S_{Regge}(T) = -\frac{1}{G_0} N_2(T) + \Lambda_0 N_4(T)$$

$$Z(G,\Lambda) = \sum_{T} e^{\frac{1}{G_0} N_2(T) - \Lambda_0 N_4(T)}$$

$$= \sum_{N_2, N_4} e^{\frac{1}{G_0} N_2 - \Lambda_0 N_4} \mathcal{N}(N_2, N_4)$$

Again purely entropic, like in 2d!

What do we see in computer simulations?



Dynamically generated four-dimensional quantum universe in CDT
(typical path integral configuration at fixed four-volume N = 91.000)

what to measure ?

$$C(\Delta) = \frac{1}{T} \sum_{t=1}^{T} \langle V_3(t)\, V_3(t+\Delta) \rangle$$

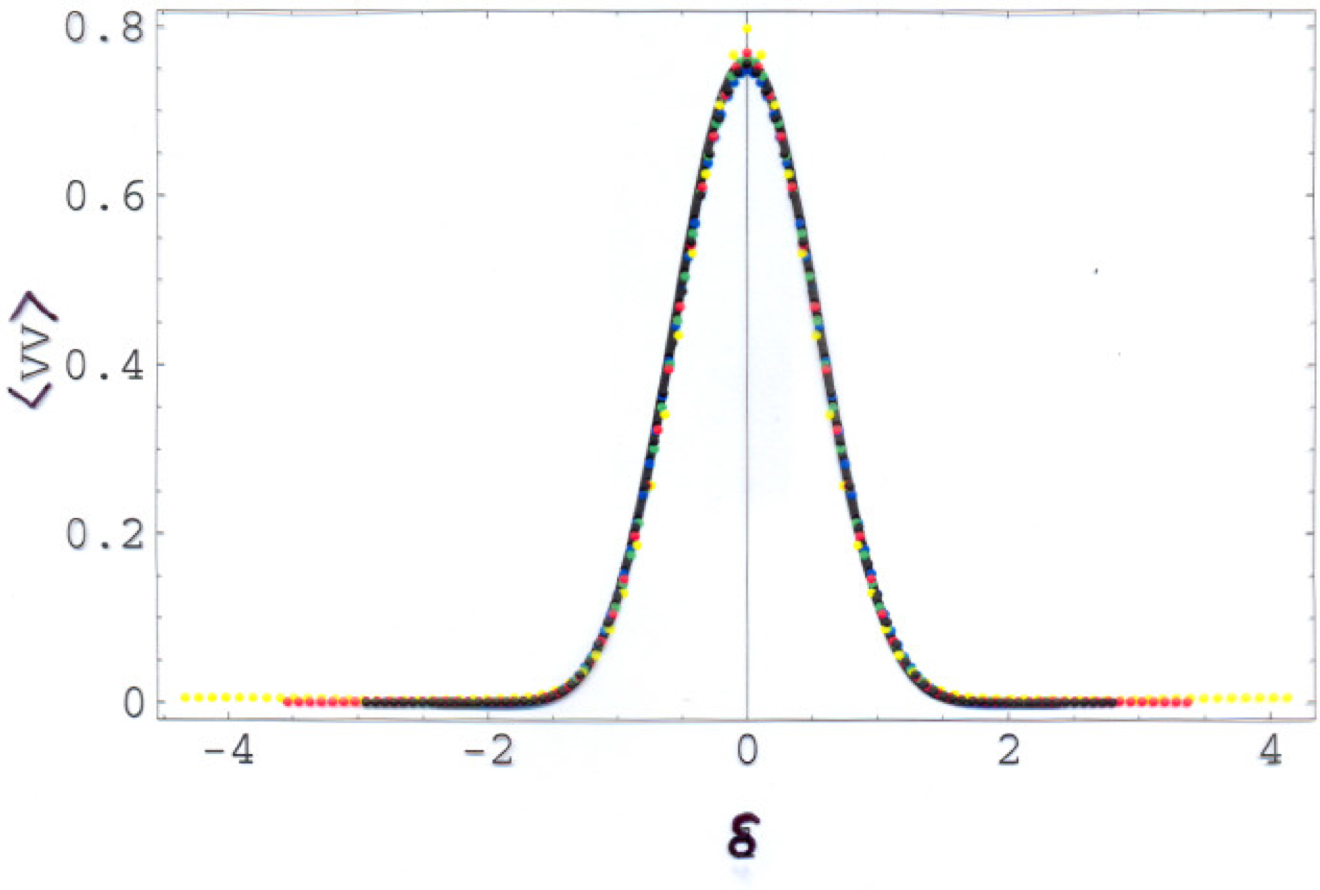
It measures extension of "blob"
as a function of Δ.

$$\boxed{\Delta_{max} \sim N_4^{1/4}}$$

Deduced from standard finite size
scaling analysis : $\Delta \sim N_4^{\alpha}$

Extension of universe scales as
canonical time

$N_y = 22250, 45500,$
$181000, 362000$



But we can say much more:

$C(\Delta)$-curve perfectly described by:

$$S = \frac{1}{G} \int_0^T dt \left(a \dot{a}^2 + a - \hat{\lambda} a^3 \right)$$

$V_3(t) = a^3(t)$, $V_4 = \int dt\, a^3(t)$ fixed

($\hat{\lambda}$ Lagrange multiplier)

How does S arise: Assume spatial isotropy and homogeneity:

$$ds^2 = dt^2 + a^2(t)\, d\Omega_3$$

Minisuperspace action $\sim a(t)$, but here found by <u>integrating out</u> all other degrees of freedom except $a(t)$

Solution: $a_{cl}(t) = R \cos t/R$

$\left(\frac{1}{R^2} \sim \hat{\lambda} \sim 1/N_4^{1/2} \right)$

$a_{cl}(t) \sim S^4$, 4d - Euclidean deSitter

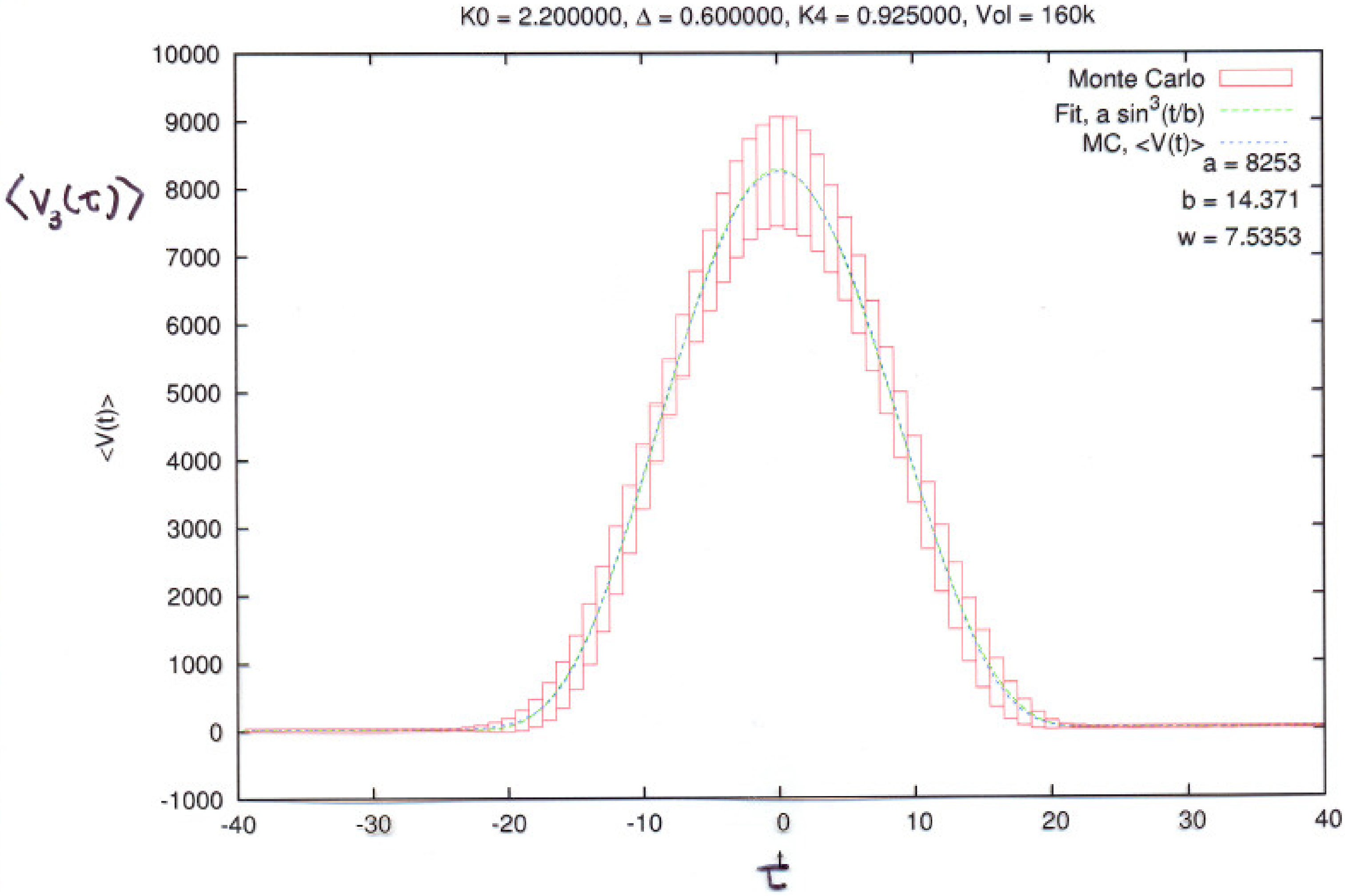
1. $a_{cl}(t)$ reproduces very well the observed $C(\Delta)$

2. Like in 2d a trivial entropic action has resulted in a non-trivial continuum action (universality ?)

3. Quantum fluctuations ? (test G)

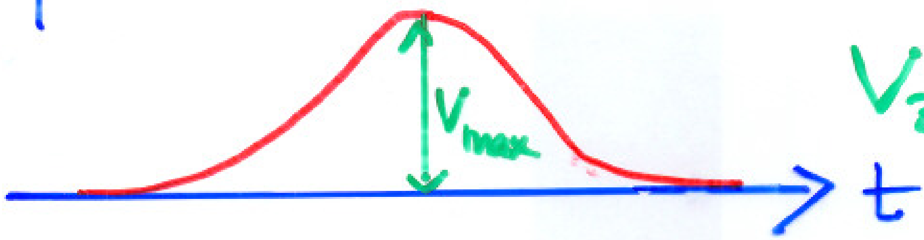
$$\langle (V_3(t) - V_3^{cl}(t))(V_3(t+\Delta) - V_3^{cl}(t+\Delta)) \rangle$$

First look at $\langle V_3(t) - V_3^{cl}(t) \rangle$



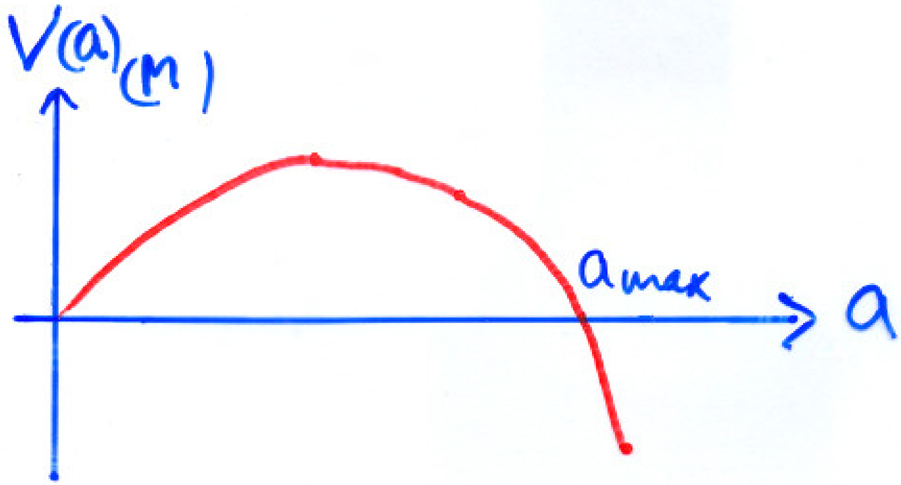
shape $\langle V_3(\tau)\rangle$ of the universe, fitted to $\propto \sin^3\left(\frac{\tau}{\beta}\right)$, with typical quantum fluctuations

(c.f. $ds^2 = d\tau^2 + a^2(\tau)\, d\Omega^2_{(3)}$)



$V_3(t) = a^3(t)$

We observe a bounce !



Back to Lorentzian signature:

Bounce ⟶ tunneling

$$e^{-\frac{1}{\hbar} S^{E}_{cl}(\text{Bounce})} \sim \Psi_0(a_{max})$$

Do we have a theory of gravity?

Include matter and check that it attracts correctly

See some "transverse" gravitons

Work in progress

1. Size of universe $R \sim 10$

2. How to define distance (We are integrating over geometries)

3. We are confined in MC to Euclidean sector