Physics Letters A 173 (1993) 317–325
North-Holland

PHYSICS LETTERS A

Field dependence of the collision integral and the Debye–Onsager relaxation effect in plasma systems

K. Morawetz
Max-Planck-Gesellschaft, AG "Theoretische Vielteilchensysteme", O-2500 Rostock, Germany

and

D. Kremp
Fachbereich Physik der Universität Rostock, O-2500 Rostock, Germany

Received 7 October 1992; revised manuscript received 28 November 1992; accepted for publication 30 November 1992
Communicated by L.J. Sham

An applied strong electric field leads to a field-dependent collision integral of the Boltzmann equation. The electric field contributes to the collision integral by the collisional broadening and the intra-collisional field effect and produces a nonlinear behaviour of the conductivity. Using a linear approximation of the collisional integral with respect to the electric field, there follows an additional term to the electrical conductivity of the plasma. This additional contribution can be considered as an analogue to the well-known Debye–Onsager relaxation effect of electrolyte theory. It is found that only the GKB ansatz of Lipavsky et al. reproduces the classical limit of this effect well known from Kadomtsev's result. The quantum effects result in a remarkable behaviour in the dependence of the mass ratio of scattering particles, i.e., in a maximum of the relaxation effect at special mass ratio numbers of interacting charged particles.

In this paper, we would like to consider a plasma system in a high applied electric field. The nonequilibrium properties of such a many particle system are to be described by a kinetic equation, e.g. by the well-known Boltzmann equation. In order to take into account a strong electric field, a generalization of the ordinary Boltzmann equation is necessary, especially the collision integral has to be modified. This problem of high field quantum transport was investigated in many papers [1–8]. It was found that the electric field produces two modifications of the collision integral:

(i) We have the collisional broadening effect, which means the energy conserving δ-function in the usually collisional integral has to be replaced by the more complicated cosine term and the distribution functions show a retardation. This effect is due to the finite collision duration time [9,6,10–12] and is well known in kinetic theory [13].

(ii) The applied electric field modifies the collision broadening on one side, and it gives rise to an additional retardation in the momentum of the distribution function on the other side. This effect is called intra-collisional field effect [5,4].

The aim of this paper is to discuss the generalization of the collision integral for high electric fields. Furthermore we consider this collision integral in linear approximation with respect to the external electric field as it was done in ref. [14]. Thus we obtain an additional term in the electric conductivity. The classical limit of this effect was first derived by Kadomtsev [15]. It is interesting to remark that this term is an analogue to the well-known Debye–Onsager–Falkenhagen relaxation contribution to the electric conductivity of electrolyte solutions. We can show that only the GKB ansatz for the $g^{>}$ and $g^{<}$ correlation function gives the correct Kadomtsev result.

In this paper we develop the quantum generalizations of the Kadomtsev result and obtain an interesting quantum behaviour, i.e., a maximum of this relaxation effect in dependence on the mass ratio of interacting particles.

We want to consider a system of charged particles of fermions under the influence of an applied constant electric field. Coupling the field in vector potential gauge, we have the Hamiltonian

$$H=\sum_i \int \mathrm{d}\boldsymbol{r}\, \Psi_i^*(\boldsymbol{r},t) H_i\left(\boldsymbol{p}-\frac{e}{c}A(t)\right)\Psi_i(\boldsymbol{r},t)+\tfrac{1}{2}\sum_{i,j}\int \mathrm{d}\bar{\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}\, \Psi_i^*(\boldsymbol{r},t)\,\Psi_j^*(\bar{\boldsymbol{r}},t)\,V_{ij}(\boldsymbol{r}-\bar{\boldsymbol{r}})\,\Psi_i(\boldsymbol{r},t)\,\Psi_j(\bar{\boldsymbol{r}},t)\,. \tag{1}$$

The Coulomb interaction V is

$$V_{ij}(\boldsymbol{r}-\bar{\boldsymbol{r}})=\frac{e^2 Z_i Z_j}{|\boldsymbol{r}-\bar{\boldsymbol{r}}|}\,.$$

The nonequilibrium properties of such a system are determined in a well-known manner by the correlation functions

$$g^{>}(1,2)=\frac{1}{\mathrm{i}}\langle \Psi(1)\Psi^*(2)\rangle, \qquad g^{<}(1,2)=-\frac{1}{\mathrm{i}}\langle \Psi^*(2)\Psi(1)\rangle\,. \tag{2}$$

It is useful to introduce different forms of nonequilibrium Green functions, e.g. the causal function is defined by

$$g(1,2)=\Theta(t_1-t_2)g^{>}(1,2)-\Theta(t_2-t_1)g^{<}(1,2)\,, \tag{3}$$

and the retarded and advanced functions are given by

$$g^{\mathrm{r/a}}(1,2)=\pm\Theta(\pm(t_1-t_2))[g^{>}(1,2)-g^{<}(1,2)]\,. \tag{4}$$

Here $\langle\ \rangle$ denotes the average value with respect to the unknown *nonequilibrium* statistical operator ρ.

The equations of motion for the nonequilibrium correlation functions may be derived in many different ways, using, e.g., Kadanoff and Baym's idea [16], Keldysh's nonequilibrium diagram technique [17] or the Martin–Schwinger hierarchy for the real-time Green function applying the condition of the weakening of initial correlations [18–21]. The result of the considerations are the following equations,

$$\left(\mathrm{i}\frac{\partial}{\partial t_1}+\frac{[(\hbar/\mathrm{i})\nabla_1-(e/c)A(t_1)]^2}{2m}\right)g_1^{\gtrless}(1,1')=\int \mathrm{d}\bar{r}_1\,\Sigma_{\mathrm{HF}}(r_1,\bar{r}_1,t_1)g_1^{\gtrless}(\bar{r}_1,r'_1,t_{1'})$$

$$+\int_{-\infty}^{t_1}\mathrm{d}\bar{1}\,[\Sigma^{>}(1,\bar{1})-\Sigma^{<}(1,\bar{1})]g_1^{\gtrless}(\bar{1},1')-\int_{-\infty}^{t'_1}\mathrm{d}\bar{1}\,\Sigma^{\gtrless}(1,\bar{1})[g_1^{>}(\bar{1},1')-g_1^{<}(\bar{1},1')]\,,$$

$$\left(-\mathrm{i}\frac{\partial}{\partial t'_1}+\frac{[-(\hbar/\mathrm{i})\nabla'_1-(e/c)A(t_{1'})]^2}{2m}\right)g_1^{\gtrless}(1,1')=\int \mathrm{d}\bar{r}_1\,g^{\gtrless}(r_1,\bar{r}_1,t_1)\Sigma_{\mathrm{HF}}(\bar{r}_1,r'_1,t_{1'})$$

$$+\int_{-\infty}^{t_1}\mathrm{d}\bar{1}\,[g_1^{>}(1,\bar{1})-g_1^{<}(1,\bar{1})]\Sigma^{\gtrless}(\bar{1},1')-\int_{-\infty}^{t'_1}\mathrm{d}\bar{1}\,g_1^{\gtrless}(1,\bar{1})[\Sigma^{>}(\bar{1},1')-\Sigma^{<}(\bar{1},1')]\,. \tag{5}$$

For further considerations it is convenient to introduce Wigner coordinates T, τ, R, r. Furthermore we subtract the second equation from the first one and take the times $t_1=t'_1$. In this way we obtain an equation for the time diagonal part of $g^{<}$. This is just the Wigner function $f_{\mathrm{W}}(p,T)$. It reads for the spatially homogeneous case [22]

$$\mathrm{i}\frac{\partial}{\partial T} f_{\mathrm{W}}(p,T)=\int_{-\infty}^{0}\mathrm{d}\tau\,[\{g^{>}(p),\Sigma^{<}(p)\}-\{g^{<}(p),\Sigma^{>}(p)\}]_{(T-\tau/2,\tau)(T-\tau/2,-\tau)}\,. \tag{6}$$

Here { , } is the anti-commutator of integrals of Wigner coordinates indicated as a subscript. This equation is exact in time.

In order to ensure the gauge invariance of the theory we have to introduce the kinematic momentum $\boldsymbol{k}=\boldsymbol{p}+e\boldsymbol{E}T$ in the case of constant electric fields [23]. This is the reason why gradient expansions fail with respect to the times [24].

To obtain an explicit form for the kinetic equation we have to determine the following quantities:

(i) The selfenergy $\Sigma^{\gtrless}$. For the selfenergy we use the screened Born approximation,

$$\Sigma^{\gtrless}(p,R,\tau,T)=\int\frac{\mathrm{d}\bar{\boldsymbol{p}}\,\mathrm{d}\bar{\boldsymbol{p}}'\,\mathrm{d}\boldsymbol{p}}{(2\pi)^9}(2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{p}'-\bar{\boldsymbol{p}}-\bar{\boldsymbol{p}}')[V_{\mathrm{s}}(\boldsymbol{p}-\bar{\boldsymbol{p}})]^2$$

$$\times g^{\lessgtr}(p',-\tau,R,T)g^{\gtrless}(\bar{p},\tau,R,T)g^{\gtrless}(\bar{p}',\tau,R,T)\,. \tag{7}$$

(ii) The correlation functions $g^{\gtrless}$ as a functional of the Wigner distribution. The first idea to solve these problems is the spectral representation for the nonequilibrium correlation functions $g^{\gtrless}$,

$$g^{<}=\mathrm{i}a(p,\omega,R,T)F(p,\omega,R,T),\qquad g^{>}=-\mathrm{i}a(p,\omega,R,T)[1-F(p,\omega,R,T)]\,. \tag{8}$$

Usually this ansatz is simplified to

$$g^{<}(p,\tau,R,T)=\mathrm{i}a(p,\omega,R,T)f_{\mathrm{W}}(p,R,T)\,, \tag{9}$$

with the Wigner distribution function f_{W}. This ansatz, the so-called Kadanoff–Baym ansatz, is only justified, if the quasiparticle picture is a valid approximation, that means if the spectral function of (8) reduces to a δ-distribution function at the quasiparticle energy.

It is known that the spectral function of free particles in an external electric field is given by [7]

$$A_0(k,\omega,R,T)=\frac{2\pi}{\lambda_E}\mathrm{Ai}[(1/\lambda_E)(\omega-k^2/2m)]\,, \tag{10}$$

where λ_E is defined by $\lambda_E^3=e^2E^2/8m$. Therefore, the Kadanoff–Baym ansatz (KB) cannot be applied in this case. A more systematic way of constructing an ansatz can be found in ref. [3]. Here a generalized Kadanoff–Baym ansatz (GKB) [3] is used, which is exact for Hartree–Fock approximations for the selfenergy. It takes the form for constant fields

$$g^{<}(k,\tau,R,T)=-\mathrm{i}\exp\left[-\frac{\mathrm{i}}{\hbar}\left(\epsilon_k\tau+\frac{e^2E^2}{24m}\tau^3\right)\right]f_{\mathrm{W}}(k-\tfrac{1}{2}eE|\tau|,R,T-\tfrac{1}{2}\tau)\,. \tag{11}$$

Introducing (11) in (7) for eq. (6) we finally obtain the following kinetic equation for the two particle scattering, where we have transformed to the gauge invariant variables $\boldsymbol{k}=e\boldsymbol{E}T+\boldsymbol{p}$ [22],

$$\frac{\partial}{\partial T}f_a+e\boldsymbol{E}\cdot\nabla_{\boldsymbol{k}_a}f_a=\sum_b I_{ab}\,,$$

$$I_{ab}=\frac{2}{\hbar^2}\int\frac{\mathrm{d}\bar{\boldsymbol{k}}'_a\,\mathrm{d}\bar{\boldsymbol{k}}_b\,\mathrm{d}\boldsymbol{k}'_b}{(2\pi\hbar)^9}\delta(\boldsymbol{k}_a+\boldsymbol{k}'_b-\bar{\boldsymbol{k}}_a-\bar{\boldsymbol{k}}'_b)V_{\mathrm{s}}^2(\boldsymbol{k}_a-\bar{\boldsymbol{k}}_a)$$

$$\times\int_0^\infty\mathrm{d}\tau\,2\cos\left\{\frac{1}{\hbar}\left[(\epsilon_a+\epsilon'_b-\bar{\epsilon}_a-\bar{\epsilon}'_b)\tau-\tfrac{1}{2}E\tau^2\cdot\left(\frac{e_a\boldsymbol{k}_a}{m_a}+\frac{e_b\boldsymbol{k}'_b}{m_b}-\frac{e_a\bar{\boldsymbol{k}}_a}{m_a}-\frac{e_b\bar{\boldsymbol{k}}'_b}{m_b}\right)\right]\right\}$$

$$\times\{f_a f'_b(1-\bar{f}_a)(1-\bar{f}'_b)-\bar{f}_a\bar{f}'_b(1-f_a)(1-f'_b)\}\,. \tag{12}$$

Here we have written, e.g., f_b for $f_b(k_b-eE\tau,\ T-\frac{1}{2}\tau)$ for simplicity. Furthermore it is assumed, that no charge or mass transfer will occur during the collision. Otherwise one would obtain an additional term in the cosine function proportional to τ^3.

Two modifications of the usual Boltzmann collision integral can be deduced from (12):

(i) We recognize a broadening of the δ-distribution function of the energy conservation and an additional retardation in the center-of-mass times of the distribution functions. This is known as collisional broadening (coll) and is a result of finite collision duration [6]. This effect can be observed even if no external field is applied. It is interesting to remark that this collisional broadening ensures the conservation of the total energy [13]. If this effect is neglected one gets the Boltzmann equation.

(ii) Secondly the electric field modifies the broadened δ-distribution function considerably by a term proportional to τ^2. At the same time the momentum of the distribution function becomes retarded by the difference time. This effect is named intra-collisional field effect (ICFE) and contains the retardation of distribution functions with respect to the momentum, which we will call ICF, and the part changing the cosine term, which will be named ICB.

We note that the part ICB of the intra-collisional field effect vanishes in the case of identical particles as may be verified by the momentum-conserving δ-distribution in (12).

In order to simplify the collision term (12) we introduce a new variable,

$$\boldsymbol{k}'=\boldsymbol{Q}+\tfrac{1}{2}\boldsymbol{q}, \qquad \bar{\boldsymbol{k}}'=\boldsymbol{Q}-\tfrac{1}{2}\boldsymbol{q},$$

and taking into account the conservation of momentum, we obtain

$$\frac{2}{\hbar^2}\int\frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{2\pi^6}V_s^2(\boldsymbol{q})\int_0^\infty \mathrm{d}\tau\,2\cos\left\{\frac{1}{\hbar}\left[-\left(\frac{q^2}{2m_a}+\frac{\boldsymbol{k}\cdot\boldsymbol{q}}{m_a}-\frac{\boldsymbol{q}\cdot\boldsymbol{Q}}{m_b}\right)\tau+\tfrac{1}{2}\boldsymbol{E}\cdot\boldsymbol{q}\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)\tau^2\right]\right\}$$
$$\times[f_a(\boldsymbol{k}^*)f_b(\boldsymbol{Q}^*+\tfrac{1}{2}\boldsymbol{q})-f_a(\boldsymbol{k}^*+\boldsymbol{q})f_b(\boldsymbol{Q}^*-\tfrac{1}{2}\boldsymbol{q})]\,. \qquad (13)$$

For k^* and Q^* one obtains different expressions using the KB or the GKB ansatz, respectively.

For the KB ansatz one gets

$$\boldsymbol{k}^*=\boldsymbol{k}-\tfrac{1}{2}e\boldsymbol{E}\tau, \qquad \boldsymbol{Q}^*=\boldsymbol{Q}-\tfrac{1}{2}e\boldsymbol{E}\tau,$$

whereas for the GKB ansatz (11) one would obtain

$$\boldsymbol{k}^*=\boldsymbol{k}-e\boldsymbol{E}\tau, \qquad \boldsymbol{Q}^*=\boldsymbol{Q}-e\boldsymbol{E}\tau.$$

Therefore the comparison of the two different types of ansatz shows, that the GKB will lead to a double ICF, which agrees with the one found from density operator techniques [25]. We will indicate it by the factor $L=2$. Further we are only interested in the modification of the transport theory due to the electric field. That means we can neglect the time retardation in the distribution function, but include the momentum retardation of the distribution function due to the field. Therefore we obtain the usual Boltzmann equation for the case of vanishing electric fields.

To study the influence of the intra-collisional field effects ICF and ICB in more detail, we linearize eq. (13) with respect to the electric field. In that way we obtain two additional terms in the Boltzmann collision integral,

$$I^a=\sum_b I_{\mathrm{B}}^{ab}+I_{\mathrm{ICF}}^{ab}+I_{\mathrm{ICB}}^{ab}, \qquad (14)$$

$$I^{ab}_{\rm ICF}=-\frac{2}{\hbar^2}L\int\frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{2\pi^6}V_s^2\int_0^\infty \mathrm{d}\tau\Big(\frac{\tau}{2}\cos(a\tau)e\boldsymbol{E}\cdot\nabla_k[f_a(\boldsymbol{k}+\boldsymbol{q})f_b(\boldsymbol{Q}-\tfrac{1}{2}\boldsymbol{q})-f_a(\boldsymbol{k})f_b(\boldsymbol{Q}+\tfrac{1}{2}\boldsymbol{q})]$$

$$+\frac{e_b m_a}{e_a m_b}\frac{\tau}{2}[e\boldsymbol{E}\cdot\nabla_k\cos(a\tau)][f_a(\boldsymbol{k}+\boldsymbol{q})f_b(\boldsymbol{Q}-\tfrac{1}{2}\boldsymbol{q})-f_a(\boldsymbol{k})f_b(\boldsymbol{Q}+\tfrac{1}{2}\boldsymbol{q})]\Big),$$

$$I^{ab}_{\rm ICB}=-\frac{2}{\hbar^2}\Big(1-\frac{e_b m_a}{e_a m_b}\Big)\int\frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{2\pi^6}V_s^2\int_0^\infty \mathrm{d}\tau\frac{\tau}{2}[e\boldsymbol{E}\cdot\nabla_k\cos(a\tau)][f_a(\boldsymbol{k}+\boldsymbol{q})f_b(\boldsymbol{Q}-\tfrac{1}{2}\boldsymbol{q})-f_a(\boldsymbol{k})f_b(\boldsymbol{Q}+\tfrac{1}{2}\boldsymbol{q})], \tag{15}$$

with the abbreviations

$$a=-\frac{1}{\hbar}\Big(\frac{q^2}{2m_a}+\frac{\boldsymbol{k}\cdot\boldsymbol{q}}{m_a}-\frac{\boldsymbol{q}\cdot\boldsymbol{Q}}{m_b}\Big).$$

The ICF of the retardation of f_b could be rewritten by changing $\boldsymbol{Q}-\frac{1}{2}e\boldsymbol{E}\tau$ to $\boldsymbol{Q}$. Neglecting back scattering the first term of eq. (14) is sometimes written at the "drift side" of the kinetic equation (12) as a renormalization of the driven electric field [26,15]. The second term turned out to be necessary in order to get the correct classical limit, as we will see below.

It was convenient to rearrange the additional collisional integral (14), which was possible by writing

$$\frac{\tau^2}{2}\cos(a\tau)e\boldsymbol{E}\cdot\boldsymbol{q}=-\frac{\tau}{2}e\boldsymbol{E}\cdot\nabla_k\cos(a\tau).$$

Further on we use the equivalence

$$\int_0^\infty \mathrm{d}\tau\,\tau\cos(a\tau)=-\mathrm{P}(1/a^2),$$

where P denotes the principal value. After some rearrangement we arrive at

$$\Delta I^{ab}=I^{ab}_{\rm ICF}+I^{ab}_{\rm ICB}=(L-1)\Big(1-\frac{e_b m_a}{e_a m_b}\Big)m_2^2\int\frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{(2\pi\hbar)^6}V(q)^2$$

$$\times e_a\boldsymbol{E}\cdot\nabla_p(f^a_{p+q}F^b_{Q-q/2}-f^a_pF^b_{Q+q/2})\Big(\boldsymbol{Q}\cdot\boldsymbol{q}-\frac{m_b}{m_a}\frac{q^2}{2}-\frac{m_b}{m_a}\boldsymbol{k}\cdot\boldsymbol{q}\Big)^{-2}$$

$$+\Big(1+(L-1)\frac{e_b m_a}{e_a m_b}\Big)e_a\boldsymbol{E}\cdot\nabla_p m_2^2\int\frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{(2\pi\hbar)^6}V(q)^2(f^a_{p+q}F^b_{Q-q/2}-f^a_pF^b_{Q+q/2})\Big(\boldsymbol{Q}\cdot\boldsymbol{q}-\frac{m_b}{m_a}\frac{q^2}{2}-\frac{m_b}{m_a}\boldsymbol{k}\cdot\boldsymbol{q}\Big)^{-2}. \tag{16}$$

In the following steps we are interested only in corrections to the flux, and therefore we use the balance equation for momentum

$$\frac{\partial}{\partial t}\langle\boldsymbol{p}\rangle-ne_a\boldsymbol{E}(1+\delta E^{\rm rel}/E)=\sum_b\langle\boldsymbol{p}I^{ab}_{\rm B}\rangle, \tag{17}$$

where $\delta E^{\rm rel}=\langle p\Delta I^{ab}\rangle$ is a renormalization of the external field similar to the Debye–Onsager relaxation field in the theory of electrolyte transport [27–29]. This effect can be shown to be a result of the deformation of the two-particle correlation function by an applied electric field.

With the help of the additional integration over $\boldsymbol{p}$ it is possible to simplify the integral considerably. To achieve this, we first change the integration variables to p and Q, in f and F, respectively. Then it is easy to see that most terms cancel each other. The term which remains and determines the additional flux due to the field has the following form,

$$ne_a \delta E = \sum_b \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3} \Delta I_{ab} = (L-1)\left(1 - \frac{e_b m_a}{e_a m_b}\right) \frac{m_b^2}{m_a kT_a}$$
$$\times \int \frac{\mathrm{d}\boldsymbol{p}\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{Q}}{(2\pi\hbar)^9} V(q)^2 e_a \boldsymbol{E}\cdot\boldsymbol{p} f_p^a F_Q^b \boldsymbol{q} \left[\boldsymbol{Q}\cdot\boldsymbol{q} + \left(1 + \frac{m_b}{m_a}\right)\frac{q^2}{2} - \frac{m_b}{m_a}\boldsymbol{p}\cdot\boldsymbol{q}\right]^{-2} . \tag{18}$$

As can be seen immediately from (18), we get no effect if we use the KB ansatz. Therefore this case will not be considered further.

The integrals can easily be handled starting with the q integration. If we consider a test system of electrons and negatively charged ions, we can assume the statically screened potential V_s to be a Debye one with the screening parameter κ_D,

$$\kappa_D^2 = \sum_b \frac{4\pi e_b^2}{\epsilon_0} \frac{n_b}{kT_b} .$$

The resulting additional flux, which was expressed as a renormalizing field, finally becomes

$$\delta E = -\frac{\kappa_D e_a^2}{3\epsilon_0} \left(\sum_b \frac{n_b e_b^2 (m_a + m_b) m_b}{(m_a kT_b + m_b kT_a)^2} \{1 + x^2 - (1.5 + x^2)\sqrt{\pi}\exp(x^2)[1 - \Phi(x)]\} \right.$$
$$\left. \times (L-1)(1 - e_b m_a / e_a m_b) \right) \left(\sum_b \frac{e_b^2 n_b}{kT_b} \right)^{-1} \equiv -\frac{\kappa_D e_a^2}{6\epsilon_0 kT_a} \zeta . \tag{19}$$

Here we assume a Maxwellian corresponding to the field linearization. The parameter x is the product $(\lambda_{ab}\kappa_D)$ of the screening parameter and the thermal de Broglie wavelength [30] of *two particles*,

$$\lambda_{ab} = \hbar \left(\frac{1}{m_a} + \frac{1}{m_b}\right)\left(\frac{2kT_a}{m_a} + \frac{2kT_b}{m_b}\right)^{-1/2}$$

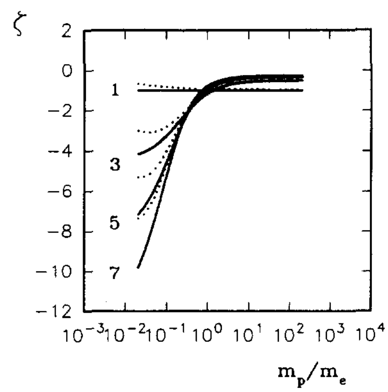

Fig. 1. The ICB effect only for four different electron temperatures (1, 3, 5, 7 times the ion temperature) versus mass ratio together with the quantum result (dotted line). The temperature of ions is 10 000 K and the electron density 10^{18} cm^{-3}.

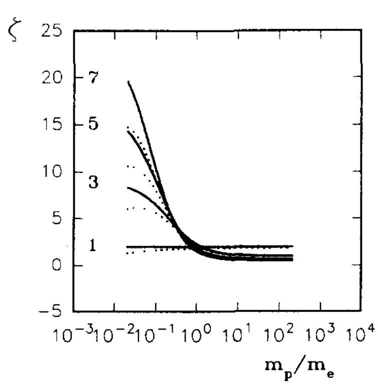

Fig. 2. The ICF effect only for four different electron temperatures (1, 3, 5, 7 times the ion temperature) versus mass ratio together with the quantum result (dotted line). Temperature and density as in fig. 1.

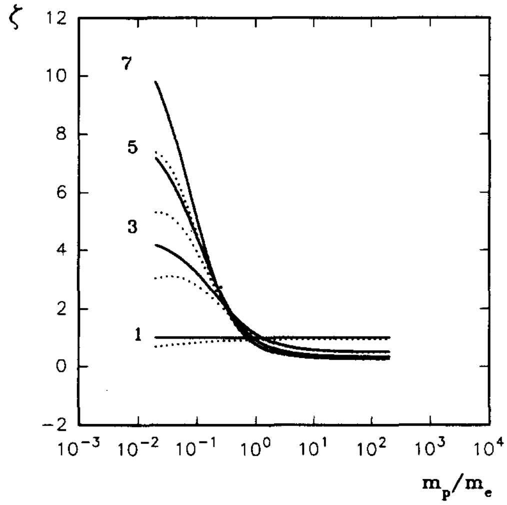

Fig. 3. The net relaxation effect ICFE for four different electron temperatures (1, 3, 5, 7 times the ion temperature) versus mass ratio together with the quantum result (dotted line). Temperature and density as in fig. 1.

and thus describes the quantum *interference* effects. Equation (19) shows that the ICFE turned out to have a classical counterpart even in linear response.

Various special cases can be discussed. In figs. 1–3 we plotted the behaviour for the different parts of the ICFE versus the mass ratio of scattered particles in a two-component plasma. The behaviour of ICB only and ICF as well are to be seen in figs. 1 and 2. This means that the modification of the cosine term, signed as ICB, leads to an enhancement of the applied electric field. In contrast, the momentum retardation of the distribution function in (13), which was named ICF, lowers the applied field. The net effect of ICFE consists in a decrease of the flux of particles or a suppression of the applied electric field.

The electron density of the hydrogen plasma is assumed to be 10^{18} cm^{-3} and the temperature of ions is 10000 K. It can be observed that, with increasing electron temperature and diminishing of the electron–ion mass ratio, the effect is enhanced. The quantum corrections result in a slight decrease of this behaviour. This becomes more important at higher densities and lower temperatures, i.e. with a larger screening parameter κ_D and/or a larger thermal wavelength. In the classical case, we obtain for large mass ratios and equal temperatures

$$\delta E_{\mathrm{class}} = -\frac{\kappa_D e_a^2}{3\epsilon_0 kT}\frac{1}{1+e_a^2 n_a/e_b^2 n_b} = -\frac{\kappa_D e_a^2}{6\epsilon_0 kT_a}\zeta\,. \tag{20}$$

This result is in full agreement with the one obtained in ref. [30]. So we can see that for equal but oppositely charged particles the factor in front of (20) is just $\frac{1}{6}$. This is the lowering of the conductivity known as Debye–Onsager relaxation effect [15,26,30–32]. It is well understood from the theory of electrolytes [27], where the conductivity becomes smaller because the electric field, acting on a charged particle within a plasma, is partially compensated by the nonequilibrium screening cloud.

For twofold ionized ions and electrons we obtain a factor of $\frac{2}{9}$. The comparison between a plasma consisting of electrons and twofold ionized ions and another with singly ionized ions is shown in figs. 4–6.

Towards lower mass ratios the quantum corrections yield an interesting maximum of the relaxation effect for mass ratios of electrons/ions between 5 and 6. In other words, particles with double charges and masses 5 or 6 times larger than those of the surrounding particles will experience a maximum of retardation. The qualitative behaviour can be understood if we imagine the collective motion of charged particles and their screening cloud due to an applied electric field. The smaller the mass of the screening particles is, the larger is the acceleration of them against the field direction and the resulting enhancement of retardation. The quantum result, on the other hand, means that for the motion of heavier ions and their screening, a nonlinear behaviour occurs,

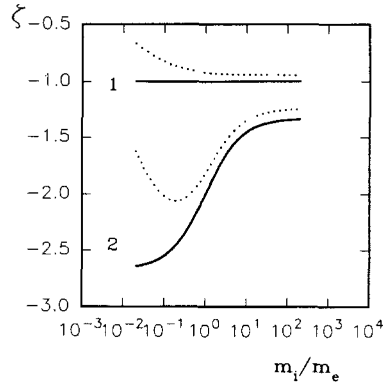

Fig. 4. The ICB effect for two different charge ratios (1, 2 times the electron charges) of the interacting particles versus mass ratio in comparison with the quantum result (dotted line). Temperature and density as in fig. 1.

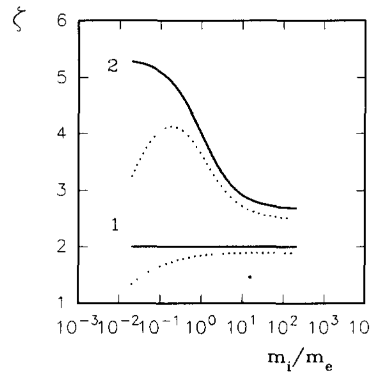

Fig. 5. The ICF effect for two different charge ratios (1, 2 times the electron charges) of the interacting particles versus mass ratio in comparison with the quantum result (dotted line). Temperature and density as in fig. 1.

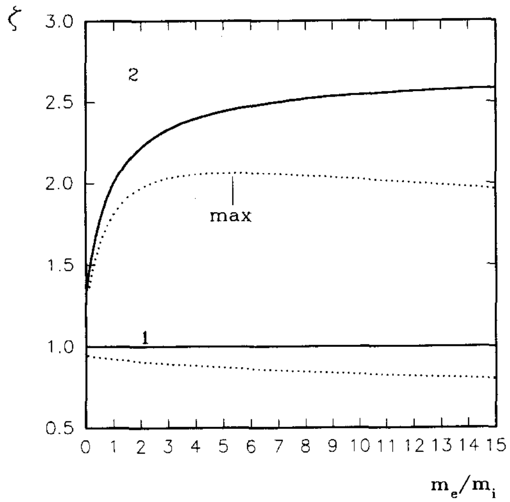

Fig. 6. The net relaxation effect ICFE for two different charge ratios (1, 2 times the electron charges) of the interacting particles versus mass ratio in comparison with the quantum result (dotted line). Temperature and density as in fig. 1.

which cannot be expected in simple classical ideas. The latter is essentially dependent on the thermal de Broglie wavelength of the interacting *two* particles and has therefore to be considered as an effect of quantum interference.

Finally let us repeat that we will get no relaxation effect at all, if we use the KB ansatz. Only the GKB ansatz reproduces the well-known classical relaxation effect. Therefore we consider this observation as a further justification of the GKB ansatz.

Forthcoming calculations should include the neglected collisional broadening. This concerns the time retardation of the distribution functions in the generalized Boltzmann equation, but not the considered one of momentum relaxation due to the field. The linearization of the electric field presented here should also be overcome by some analytical calculations in order to understand the effects obtained numerically.

We like to thank Professor Kraeft for stimulating discussions. One of the authors (K.M.) wishes to thank Professor Jauho in Copenhagen for the benefit of several stays at NBI.

References

[1] I.B. Levinson, Fiz. Tverd. Tela 6 (1965) 2113.
[2] N.N. Bytchkova, C.W. Peletminsky and A.A. Jatsenko, Teor. Mat. Fiz. 45 (1980) 93.
[3] P. Lipavsky, V. Spicka and B. Velicky, Phys. Rev. B 34 (1986) 6933.
[4] A.P. Jauho, in: Quantum transport in semiconductors, eds. D.K. Ferry and C. Jacoboni (Plenum, New York, 1991), ch. 7.
[5] P. Lipavsky, F.S. Khan, F. Abdolsalami and J.W. Wilkins, Phys. Rev. B 43 (1991) 4885.
[6] S.K. Sarker, J.H. Davies, F.S. Khan and J.W. Wilkins, Phys. Rev. B 33 (1986) 7263.
[7] F.S. Khan, J.H. Davies and J.W. Wilkins, Phys. Rev. B 36 (1987) 2578.
[8] H. Haug and S.W. Koch, Quantum theory of the optical and electronic properties of semiconductors (World Scientific, Singapore, 1990).
[9] Y.C. Chang, D.Z.Y. Ting, J.Y. Tang and K. Hess, Appl. Phys. Lett. 42 (1983) 76.
[10] M. Schlanges, Ph.D. thesis, Rostock University (1985).
[11] K. Kim, B.A. Mason and K. Hess, Phys. Rev. B 36 (1987) 6567.
[12] P. Lipavsky, F.S. Khan, A. Kalvova and J.W. Wilkins, Phys. Rev. B 43 (1991) 6650.
[13] Yu.L. Klimontovich, Kinetic theory of nonideal gases and nonideal plasmas (Academic Press, New York, 1975).
[14] R.Kh. Amirov and O.G. Zudeev, J. Phys. Condens. Matter 2 (1990) 6771.
[15] B.B. Kadomtsev, Zh. Eksp. Teor. Fiz. 33 (1958) 151 [Sov. Phys. JETP 33 (1958) 117].
[16] L.P. Kadanoff and G. Baym, Quantum statistical mechanics (Benjamin, New York, 1964).
[17] L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515.
[18] P.C. Martin and J. Schwinger, Phys. Rev. 115 (1959) 1342.
[19] D. Kremp, M. Schlanges and Th. Bornath, in: Kinetic equations and nonlinear optics in semiconductors, ed. H. Stolz (Berlin, 1985) p. 33.
[20] P. Danielewicz, Ann. Phys. (NY) 197 (1990) 154.
[21] W. Botermans and R. Malfliet, Phys. Rep. 198 (1990) 115.
[22] A.P. Jauho and J.W. Wilkins, Phys. Rev. B 29 (1984) 1919.
[23] R. Bertoncini and A.P. Jauho, Phys. Rev. B 44 (1991) 3655.
[24] G.D. Mahan, Phys. Rep. 145 (1987) 251.
[25] V.P. Seminozhenko, Phys. Rep. 91 (1982) 103.
[26] W. Ebeling, Ann. Phys. (Leipzig) 33 (1976) 5.
[27] H. Falkenhagen, Elektrolyte (Hirzel, Stuttgart, 1953).
[28] W.D. Kraeft, H. Falkenhagen and W. Ebeling, in: Ionic interaction, ed. S. Petrucci (Academic Press, New York, 1971) ch. 1, p. 1.
[29] D. Kremp, D. Kraeft and W. Ebeling, Ann. Phys. (Leipzig) 18 (1966) 246.
[30] Y.L. Klimontovich and W. Ebeling, Zh. Eksp. Teor. Phys. 63 (1972) 904.
[31] W. Ebeling and G. Röpke, Ann. Phys. (Leipzig) 36 (1979) 429.
[32] G. Röpke, Phys. Rev. A 38 (1988) 3001.