

Dynamical structure factor of correlated hot nuclear matter

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Starting from a quantum statistical approach to the density–density response function, its relation to the compressibility and to the equation of state of nuclear matter at finite temperature is investigated. In particular, the contribution of two-particle correlations to the dynamical structure factor is taken into account, which becomes important at low temperatures in the low-density limit. It is shown that this treatment for the structure factor is consistent with the equation of state up to the second virial coefficient.

Recently, there were some efforts to describe the disintegration of hot nuclear matter by considering the time evolution of the single particle distribution. The use of a kinetic equation such as the Boltzmann–Uehling–Uhlenbeck equation in a self-consistent mean field [1] or its extension by including a fluctuating part of the collision integral [2] is appropriate to describe the single-particle distribution in the low-density region. The inclusion of cluster formation leads to a coupled system of kinetic equations for the single particle distribution and the cluster distribution functions [3,4].

The relevant quantity to describe the disintegration process in dense nuclear matter, namely the growth of density fluctuations, can be treated employing a generalization of Landau’s kinetic theory to finite temperatures and finite wave numbers [5–7]. Instabilities are found which are used to analyze the process of fragmentation. The influence of density fluctuations on the scattering process has been considered in ref. [8] leading to an enhanced equilibration rate

An other approach to nonequilibrium, especially to describe fluctuations, is given by the theory of linear response [9,10]. This approach based on the fluctuation–dissipation theorem is an alternative of kinetic theory. In its simplest version, first order deviations from the equilibrium distribution due to small external perturbations are evaluated. The evaluation of the response to the perturbation can be carried out within a systematic quantum statis-

tical approach using the method of thermodynamic Green’s functions, which accounts for degeneration and strong correlations in the medium. However, up to now most of the calculations have been performed on the quasiparticle level. The systematic inclusion of correlations, in particular bound states, is an open problem that will be addressed within this letter.

Within linear response theory the response function $\chi(\mathbf{q}, \omega)$ [11] describes the density response to an external potential $U(\mathbf{q}, \omega)$,

$$\langle \varrho_{\mathbf{q}}(\omega) \rangle = \chi(\mathbf{q}, \omega) U(\mathbf{q}, \omega) \quad (1)$$

The density response $\langle \varrho_{\mathbf{q}}(\omega) \rangle$ is given by the Fourier transform of the time-dependent mean value of $\varrho_{\mathbf{q}} = \sum_p a_{p-q/2}^+ a_{p+q/2}$, where the summation index includes momentum, spin and isospin variables.

The dynamical structure factor (form factor, van Hove function) is defined in terms of the density fluctuations in the thermodynamic equilibrium

$$S(\mathbf{q}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \langle \varrho_{\mathbf{q}}^+(t) \varrho_{\mathbf{q}} \rangle e^{i\omega t} \quad (2)$$

It is related to the response function $\chi(\mathbf{q}, \omega)$ in eq. (1) according to the fluctuation–dissipation theorem [11].

The dynamical structure factor (2) is of interest not only in connection with scattering experiments and

with the mode spectrum, but it is also related to the isothermal compressibility $\kappa(\beta, \mu)$, see e.g. ref. [9],

$$\kappa = \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial \mu} \right)_T = \frac{\beta}{\Omega_0 \rho^2} \lim_{q \rightarrow 0} \int_{-\infty}^{\infty} d\omega S(q, \omega), \quad (3)$$

where

$$\rho = \frac{1}{\Omega_0} \sum_p \langle a_p^\dagger a_p \rangle$$

denotes the density of nucleons, Ω_0 the normalization volume and $\beta = (k_B T)^{-1}$ the inverse temperature. This gives the possibility to obtain an equation of state of the form $\rho = \rho(\beta, \mu)$ from the dynamical structure factor by integrating eq. (3). The equation of state for the thermodynamic potential p is obtained using the thermodynamic relation $p(\beta, \mu) = \int_{-\infty}^{\mu} \rho(\beta, \mu') d\mu'$.

For the evaluation of the response function $\chi(\mathbf{q}, \omega)$ or the correlation function $S(\mathbf{q}, \omega)$, respectively, we use the technique of thermodynamic Green functions. We consider the density-density correlation function $L(\mathbf{q}, z)$ as the Fourier transform of

$$\begin{aligned} L(121^+2^+) &= \frac{1}{(i\hbar)^2} \langle T \{ \psi^+(1^+) \psi(1) \psi^+(2^+) \psi(2) \} \rangle \\ &- \frac{1}{(i\hbar)^2} \langle \psi^+(1^+) \psi(1) \rangle \langle \psi^+(2^+) \psi(2) \rangle, \end{aligned} \quad (4)$$

where 1 denotes $(x_1, \sigma_1, \tau_1, t_1)$. The quantity $L(\mathbf{q}, z)$ is related to $\chi(\mathbf{q}, \omega)$ and $S(\mathbf{q}, \omega)$ according to the fluctuation-dissipation theorem

$$\text{Im } \chi(\mathbf{q}, \omega) = -\text{Im } L(\mathbf{q}, \omega - i0), \quad (5)$$

$$S(\mathbf{q}, \omega) = \frac{1}{\pi} \frac{1}{e^{\beta\omega} - 1} \text{Im } L(\mathbf{q}, \omega - i0). \quad (6)$$

Thus one finds for the compressibility (3)

$$\begin{aligned} \kappa(\beta, \mu) &= \frac{\beta}{\Omega_0 \rho^2} \lim_{q \rightarrow 0} \int \frac{d\omega}{\pi} (e^{\beta\omega} - 1)^{-1} \\ &\times \text{Im } L(\mathbf{q}, \omega - i0). \end{aligned} \quad (7)$$

This expression for the compressibility $\kappa(\beta, \mu)$ can be confronted with the standard approach to the ther-

modynamic properties using the normalization condition for the single-particle Green function

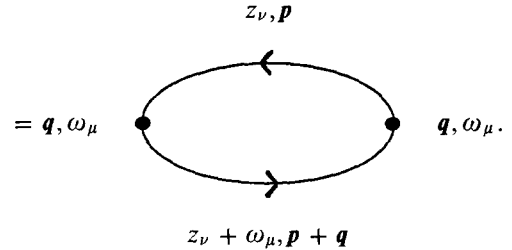
$$\begin{aligned} \rho(\beta, \mu) &= \frac{1}{\Omega_0} \sum_p \int \frac{d\omega}{\pi} \frac{1}{e^{\beta(\omega - \mu)} + 1} \\ &\times \text{Im } G_1(p, \omega - i0). \end{aligned} \quad (8)$$

This is used to control approximations in evaluating $L(\mathbf{q}, z)$ on the level of the equation of state. We will demonstrate this within a perturbative approach to $L(\mathbf{q}, z)$, which is related to the polarization function $\Pi(\mathbf{q}, z)$ by the well-known relation

$$L(\mathbf{q}, z) = \frac{\Pi(\mathbf{q}, z)}{1 - V(\mathbf{q})\Pi(\mathbf{q}, z)}. \quad (9)$$

Using a diagram representation of $L(\mathbf{q}, z)$, the lowest order approximation with respect to the interaction is given by the closed loop

$$L(\mathbf{q}, \omega_\mu) \approx \Pi_1^0(\mathbf{q}, \omega_\mu)$$



Performing the summation over the fermionic Matsubara frequencies z_ν and continuing analytically from the bosonic Matsubara frequencies ω_μ , we obtain the well-known result for the polarization function in one-loop approximation

$$\Pi_1^0(\mathbf{q}, z) = \sum_p \frac{f_{p-q/2} - f_{p+q/2}}{z + E_{p-q/2} - E_{p+q/2}}. \quad (10)$$

Here f_p denotes the Fermi distribution function. The corresponding compressibility

$$\kappa_0(\beta, \mu) = \frac{\beta}{\Omega_0} \frac{1}{\rho^2} \sum_p f_p (1 - f_p) \quad (11)$$

coincides with that derived from the normalization condition (8) for an ideal fermion gas.

For interacting fermions with an interaction potential $V(q)$, however, this approximation is not sufficient. In low-density nuclear matter two-particle correlations and possible bound states become important. Within the Green function approach, higher orders in the interaction can be obtained from additional diagrams for the density-density correlation function $L(q, z)$ or the polarization function $\Pi(q, z)$, respectively. The summation of ring diagrams [9] and the inclusion of self-energy and vertex corrections [12] are mentioned here. However these are improvements on the single-particle level that are not capable to describe the formation of two-particle correlations.

Therefore we want to focus on the formation of two-particle correlations and consider the equation of state $\varrho(\beta, \mu)$ which is related to $L(q, z)$ via the compressibility κ by eq. (7). An exact expression is given up to the second virial coefficient by the Beth-Uhlenbeck formula, see ref. [13]:

$$\varrho(\beta, \mu) = \frac{1}{\Omega_0} \sum_p f_p + \frac{2}{\Omega_0} \sum_{P\alpha} \int_{-\infty}^{\infty} \frac{dE}{\pi} g\left(E + \frac{P^2}{4m}\right) D_{\alpha P}(E), \quad (12)$$

with

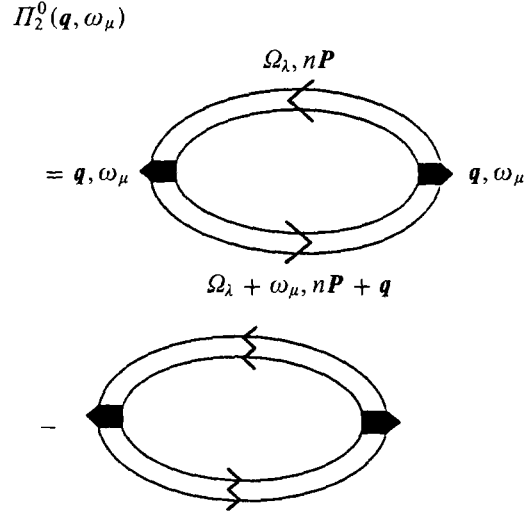
$$D_{\alpha P}(E) = c_{\alpha} \left(\sum_m \pi \delta(E - E_{\alpha m P}) + \frac{\partial}{\partial E} \delta_{\alpha P}(E) \right). \quad (13)$$

It contains the contribution of bound states ($E_{\alpha m P}$) and of scattering states with the scattering phase shift $\delta_{\alpha P}(E)$. Here α denotes the channel of two-particle states, P its total momentum and m the internal quantum number for the bound states. $g(E)$ is the two-particle (Bose) distribution function including the chemical potentials of both particles, and c_{α} represents the degeneration factor of the channel α .

The interesting point considered here is the question, which approximation has to be performed for $L(q, z)$ in order to arrive at this correct expressions for the second virial coefficient (12) using eq (3). Such an approximation should be of second order in the density, but of arbitrary order in the interaction potential. This problem is solved by a cluster decomposition for $\Pi(q, z)$:

$$L(q, z) \approx \Pi(q, z) = \Pi_1^0(q, z) + \Pi_2^0(q, z) + \dots, \quad (14)$$

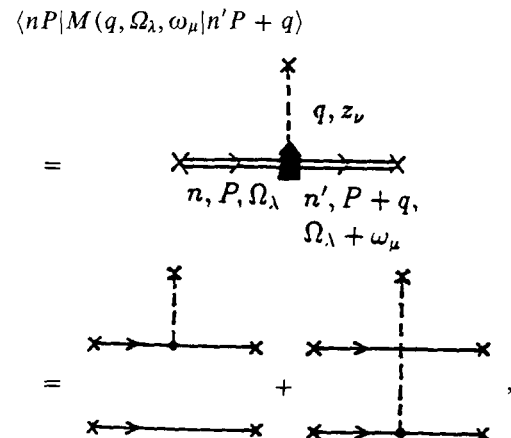
where $\Pi_1^0(q, z)$ is the one-particle contribution to the polarization function (10) and $\Pi_2^0(q, z)$ is the two-particle contribution to the polarization function [14]:



The evaluation of this diagram is simplified by using the two-particle representation of G_2 and of the two-particle vertex M . In this representation the two-particle Green's function has the following form with internal quantum number $n = (\alpha, m)$:

$$\langle nP | G_2(z) | n'P' \rangle = \frac{1}{z - E_{n,P}} \delta_{nn'} \delta_{PP'}. \quad (15)$$

In the low-density limit considered here the solution of the two-particle problem can be used. For the two-particle vertex M we have (crosses denoting amputation)



$$\begin{aligned}
M_{nn'}(q) &= \langle nP | M(\Omega_\lambda, q, \omega_\mu) | n'P + q \rangle \\
&= \frac{1}{\beta} \sum_{z_\nu, 121'2'} \psi_{nP}^*(p_1 p_2) (\Omega_\lambda - E_1 - E_2) \\
&\times \left(\frac{1}{\Omega_\lambda - z_\nu - E_2} \frac{\delta_{1',1+q}}{z_\nu - E_1} \frac{\delta_{2',2}}{z_\nu + \omega_\mu - E_{1+q}} \right. \\
&+ \left. \frac{1}{\Omega_\lambda - z_\nu - E_1} \frac{\delta_{2',2+q}}{z_\nu - E_2} \frac{\delta_{1',1}}{z_\nu + \omega_\mu - E_{2+q}} \right) \\
&\times (\Omega_\lambda + \omega_\mu - E_{1'} - E_{2'}) \psi_{n'P+q}(p_1' p_2') \\
&= \sum_{p_1 p_2} \psi_{nP}^*(p_1 p_2) [\psi_{n'P+q}(p_1 + q, p_2) \\
&+ \psi_{n'P+q}(p_1, p_2 + q)]. \quad (16)
\end{aligned}$$

Here $\psi_{nP}(p_1 p_2) = \langle p_1 p_2 | nP \rangle$ are the solutions of the two-particle Schrödinger equation. Using eqs. (15) and (16) we end up with the following expression for the two-particle contribution to the polarization function:

$$\begin{aligned}
\Pi_2^0(q, z) &= \sum_{nn'P} \frac{g(E_{nP}) - g(E_{n'P+q})}{z + E_{nP} - E_{n'P+q}} |M_{nn'}(q)|^2 \\
&- \sum_{p_1 p_2} \frac{g(E_{p_1} + E_{p_2}) - g(E_{p_1+q} + E_{p_2})}{z + E_{p_1} - E_{p_1+q}}. \quad (17)
\end{aligned}$$

This result for the two-particle polarization function is formally similar to the single-particle polarization function (10). The single-particle (Fermi) distribution $f(E_p)$ has been replaced by the two-particle (Bose) distribution $g(E_{nP})$. The summations over n, n' include bound states as well as scattering states.

Here we concentrate on the thermodynamic properties obtained from the corresponding approximation for the dynamical structure function (2) in the long wave limit. The contribution of the bound states and of the scattering states in (17) is treated separately. Considering the vertex M for bound-bound or bound-scattering transitions, we find from (16) in the limit $q \rightarrow 0$ using the orthogonality relation

$$\lim_{q \rightarrow 0} M_{nn'}(q) = 2\delta_{nn'}. \quad (18)$$

The orthogonality condition holds for scattering states as well, however for interacting particles the normalization condition is given in terms of phase shifts $\delta(E_p)$ (see ref. [15])

$$\lim_{q \rightarrow 0} M_{pp'}(q) = \delta(p - p') \left(1 + \frac{\partial}{\partial E_p} \delta(E_p) \right), \quad (19)$$

where the internal quantum numbers n, n' are replaced by the relative momenta p, p' . The first term $\delta(p - p')$ of eq. (19) when introduced in eq. (17) is compensated by the second term in eq. (17).

Performing this way, the following expression for the compressibility is obtained:

$$\begin{aligned}
\kappa(\beta, \mu) &= \frac{1}{\rho^2} \frac{\beta}{\Omega_0} \left\{ \sum_p f_p (1 - f_p) \right. \\
&+ 4 \sum_{P\alpha} \int_{-\infty}^{\infty} \frac{dE}{\pi} g \left(E + \frac{P^2}{4m} \right) \\
&\times \left[1 + g \left(E + \frac{P^2}{4m} \right) \right] D_{\alpha P}(E) \left. \right\}, \quad (20)
\end{aligned}$$

with $D_{\alpha P}$ given by (13). This expression coincides with the compressibility one finds from the Beth-Uhlenbeck formula (12).

To demonstrate the significance of two-particle correlations to the density-density correlation function L in the low density limit, the ratio κ/κ_0 of the compressibility κ including two-particle contributions (20) to the single-particle approximation κ_0 (11) is shown for symmetric nuclear matter in fig. 1. It is plotted as a function of the total density ρ for three different temperatures $T = 10$ MeV, $T = 20$ MeV and $T = 30$ MeV. The binding energies $E_{\alpha nP}$ and the scattering phase shifts $\delta_\alpha(E)$ in eq. (20) are derived from the separable Yamaguchi nucleon-nucleon interaction. With increasing density the compressibility ratio (fig. 1) is reduced at a given temperature. The influence of two-particle correlations on the compressibility is especially pronounced at low temperature. This behaviour is due to the larger ratio of two-particle correlations (bound and scattering states) at high density and low temperature, respectively. The reduction of the compressibility is in accordance with the principle of Brown-Le Chatelier that at higher densities the system is forced to form bound states so that the pressure is reduced.

The expression for the compressibility (20) is correct up to the second order with respect to the fugacity, i.e. in the low density region ($\rho < 0.1\rho_0$). Besides by the neglect of the already mentioned ring summation (9), it is limited by self-energy and vertex

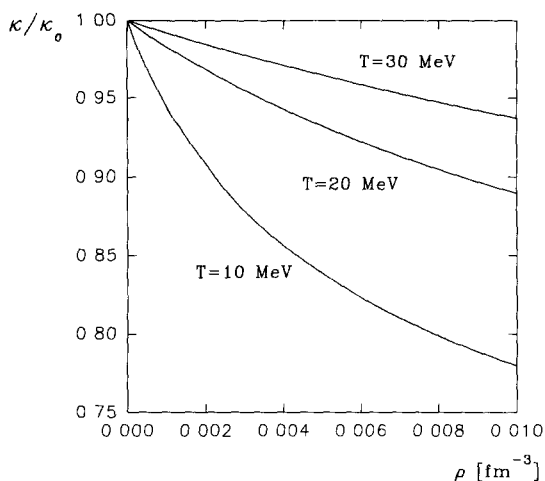


Fig. 1. The ratio κ/κ_0 of the compressibility κ from eq. (20) of nuclear matter including two-particle correlations to the single-particle approximation to the compressibility κ_0 as a function of the total nucleon density ρ and temperatures T of 10, 20 and 30 MeV. The scattering phase shifts and bound state (deuteron) energy are derived from the Yamaguchi separable interaction. The compressibility is significantly reduced due to correlations.

corrections [12]. Thus, for nuclear matter, the Pauli blocking [13,16] has an essential effect on the two-particle states leading to a suppression of the two-particle contribution with increasing density. In particular, the bound states disappear for densities higher than the Mott density.

Within the thermodynamic approach it can be shown that the instability region for the nuclear matter phase transition, that was not considered in this paper, is considerably reduced if correlations and bound state formation are included. With respect to a larger region of the nucleon density, the relevance of two-particle correlations for the equation of state (8) has been discussed in refs. [13,17]. It has also been discussed that the instability region of the nuclear-matter phase transition is decreased if particle-particle hole-hole diagrams are included in evaluating the potential energy, see ref. [18]. We will discuss the corresponding higher order effects in the density-density correlation function and their relation to the generalized Beth-Uhlenbeck formula [13] in a forthcoming paper.

We have shown that consistent results are obtained from the density response function and the dynamical

structure function, if the contribution of two-particle clusters to the polarization function is taken into account. The correct virial expansion of the equation of state (Beth-Uhlenbeck formula) has been reproduced which includes the contribution of bound states as well as scattering states.

The approach to the thermodynamic properties starting from the treatment of the dynamical structure factor (3) seems to be more involved than the normalization condition (8). However, the thermodynamic limit may serve as a consistency check to get the relevant approximations for the density response function and the dynamical structure factor, which contain important information about the dynamical properties of the dense interacting system.

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