Spin-1/2 fermions:
Crossover from weak to strong attractive interaction

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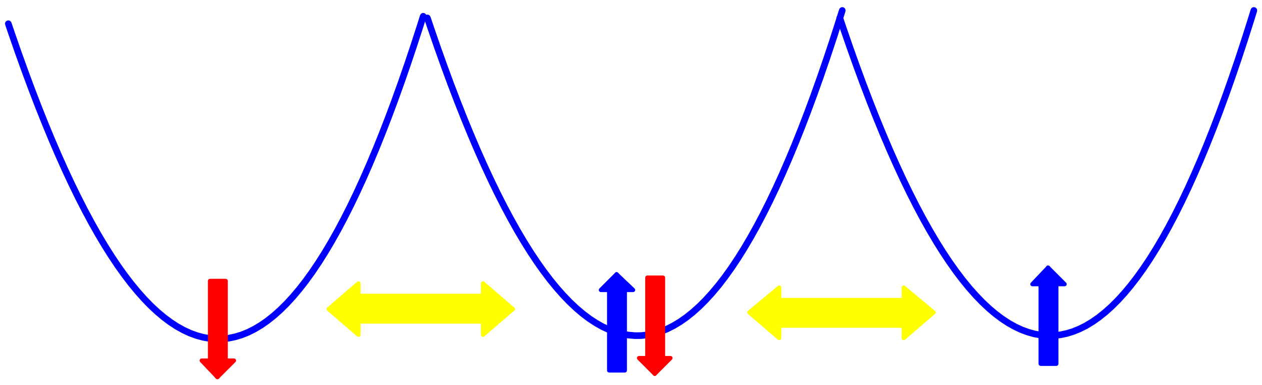
Spin-1/2 Fermions in an Optical Lattice

- optical lattice stabilizes molecules

- BCS state appears in the lattice
- Mott insulator appears in the lattice

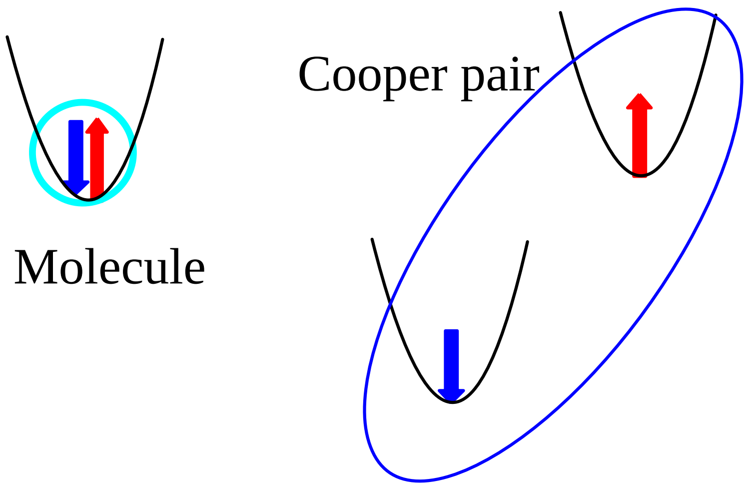


Attractive Interaction: Molecules vs Cooper Pairs

Cooper pairs are global objects with $U(1)$–symmetry breaking

Molecules are local objects with broken translational symmetry

Molecules can BE condense $\longrightarrow U(1)$–symmmetry breaking



Hamiltonian for short-range interaction:

$$H = -\bar{t} \sum_{<r,r'>} \sum_{\sigma=\uparrow,\downarrow} c^{\dagger}_{r\sigma} c_{r'\sigma} - J \sum_{<r,r'>} c^{\dagger}_{r\uparrow} c_{r'\uparrow} c^{\dagger}_{r\downarrow} c_{r'\downarrow}$$

$$-\mu \sum_{r} \sum_{\sigma=\uparrow,\downarrow} c^{\dagger}_{r\sigma} c_{r\sigma}$$

Variational States

BCS state:

$$|\Phi_{BCS}\rangle = \prod_{k} (u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})|0\rangle$$

molecular state:

$$|\Phi_m\rangle = \prod_{r} (u_r + v_r c^{\dagger}_{r\uparrow} c^{\dagger}_{r\downarrow})|0\rangle$$

$$|u_k|^2 + |v_k|^2 = |u_r|^2 + |v_r|^2 = 1$$

Mean-Field Theory

$$\langle \Phi_{BCS} | H | \Phi_{BCS} \rangle = - \int_k (\epsilon_k + \mu) |v_k|^2 - J \Big| \int_k u_k v_k^* \Big|^2$$

minimum:

$$\Delta_k = J \int_p \frac{\Delta_p}{\epsilon_p^2 + \Delta_p^2}, \qquad v_k^2 = \frac{\Delta_k}{\epsilon_k^2 + \Delta_k^2}$$

$$\langle \Phi_m | H | \Phi_m \rangle = -\mu \sum_r |v_r|^2 - 2J \sum_{<r,r'>} (u_r v_r^*)(v_{r'} u_{r'}^*)$$

minimum: $v_r = v$

$$v^2 = \frac{1}{2}(1 + \frac{\mu}{2J})$$

constant v_r, v_k

$$\langle \Phi_{\ldots} | H | \Phi_{\ldots} \rangle = J v^4 - a v^2$$

Molecular Gas

$$\bar{t} = 0$$

Hamiltonian:

$$H = -J \sum_{<r,r'>} c^{\dagger}_{r\uparrow} c_{r'\uparrow} c^{\dagger}_{r\downarrow} c_{r'\downarrow} - \mu \sum_{r} \sum_{\sigma=\uparrow,\downarrow} c^{\dagger}_{r\sigma} c_{r\sigma}$$

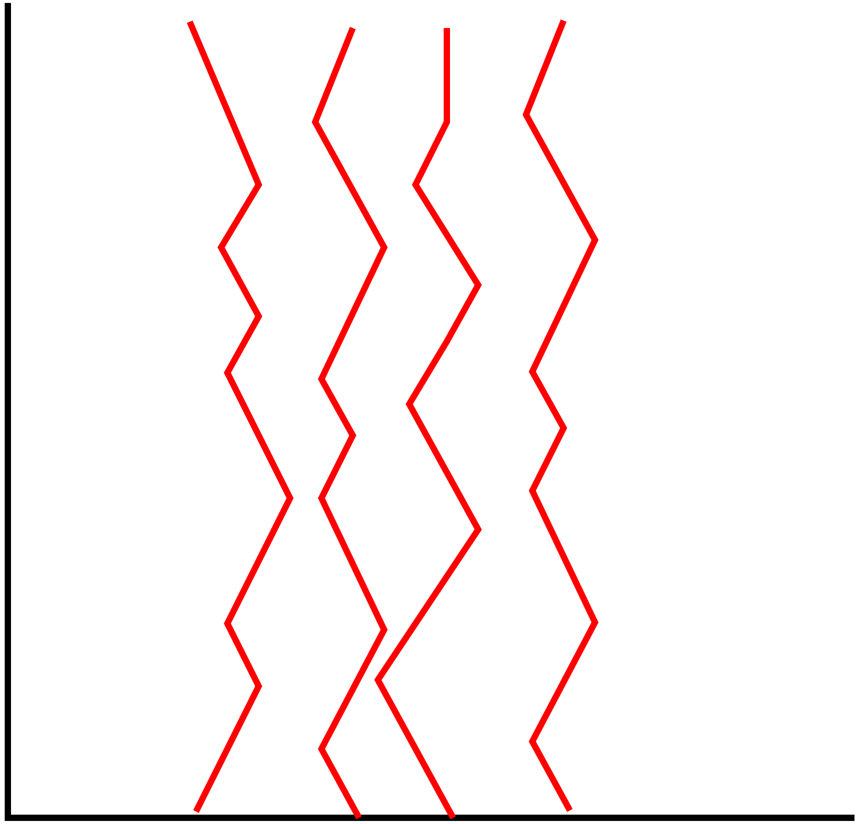
Exactly solvable models:

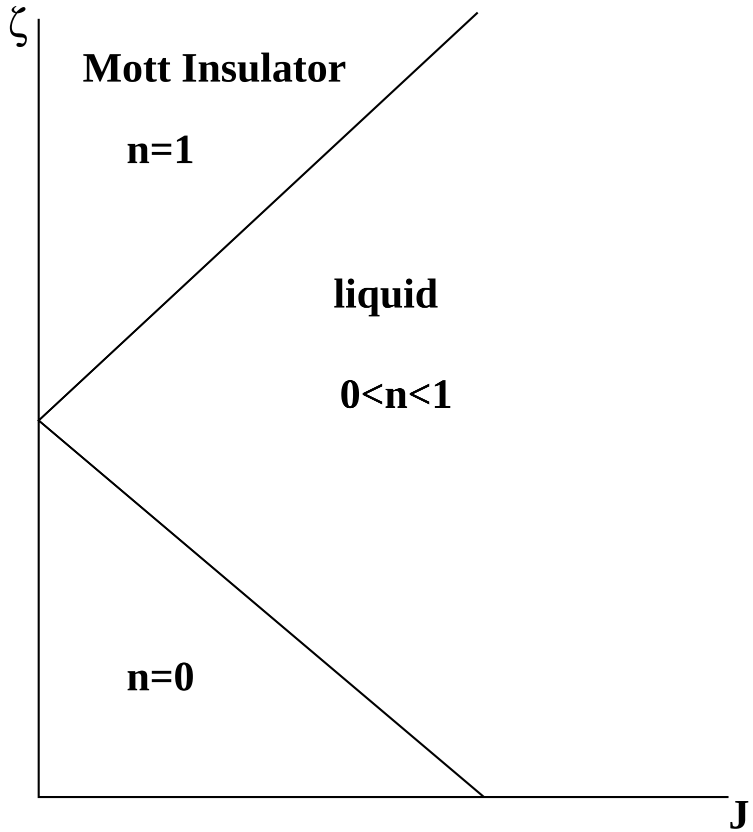
1D hard-core Bose gas

N-component Bose gas: $N \to \infty$

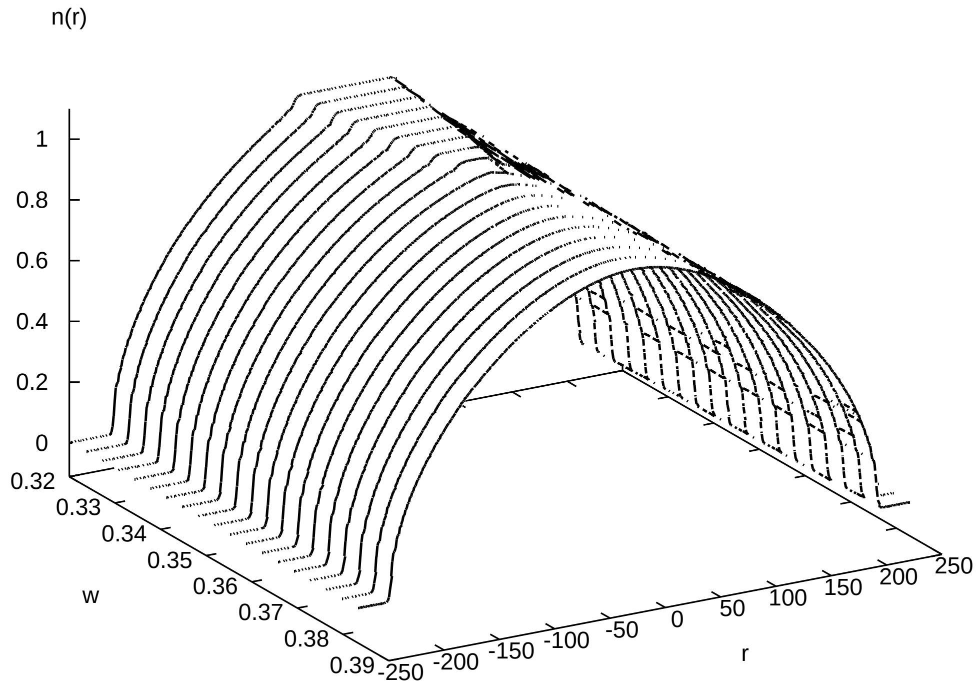
Theory of 1D Hard-Core Bose Gas:

Hard-core Bosons $\longleftrightarrow$ Free Fermions

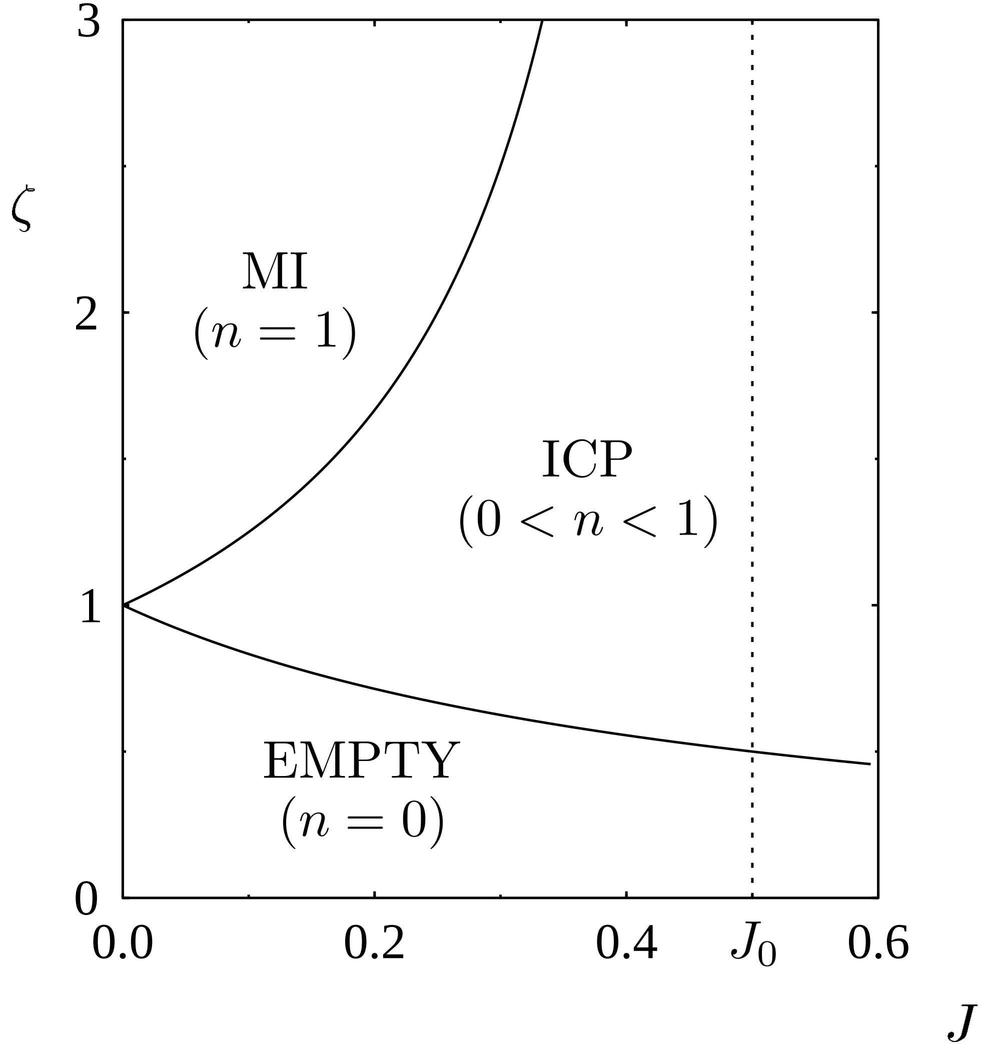




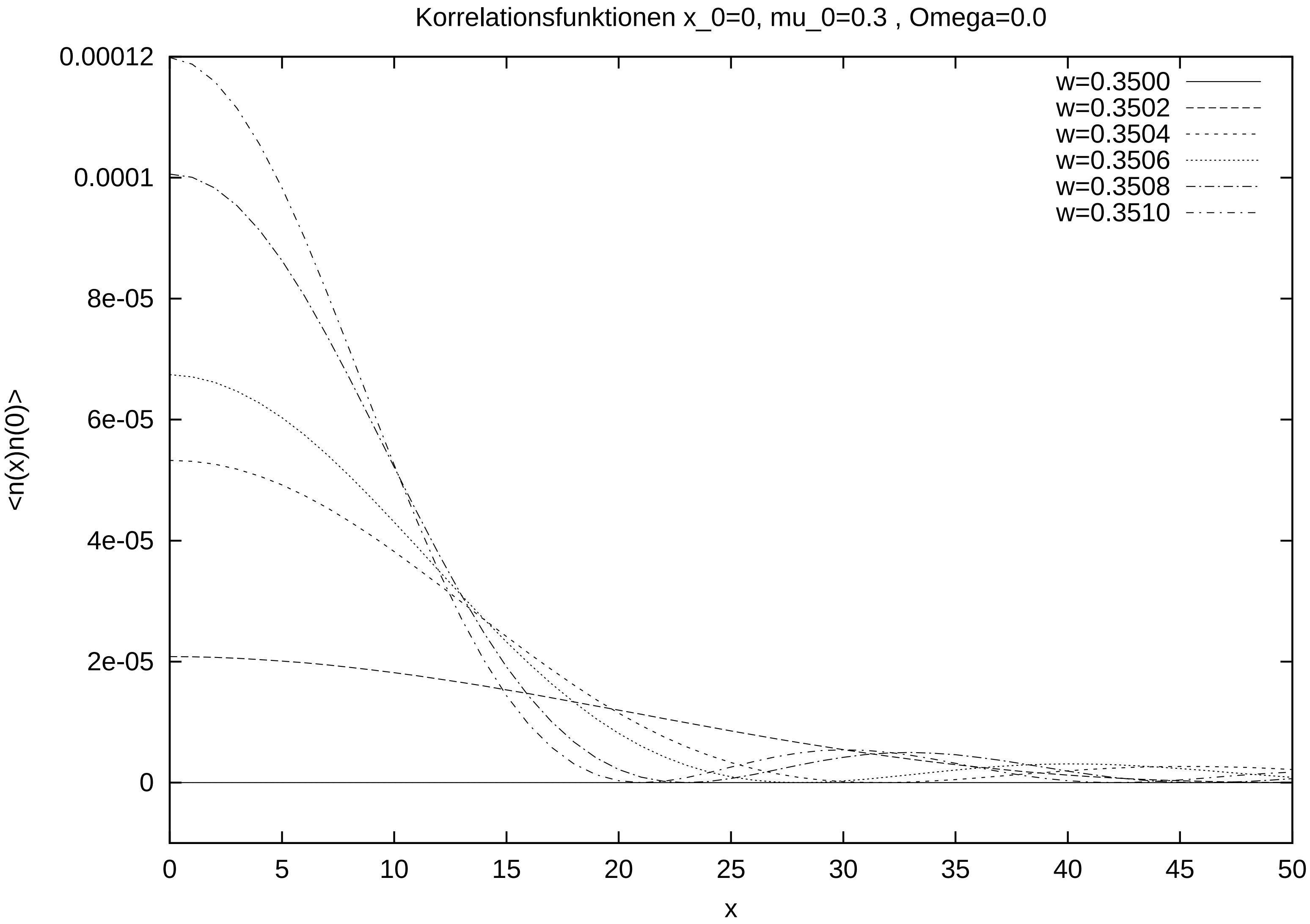
Density of a 1D Bose Gas in an optical lattice with harmonic potential



$T = 0$ Phase Diagram of 1D Bose Gas

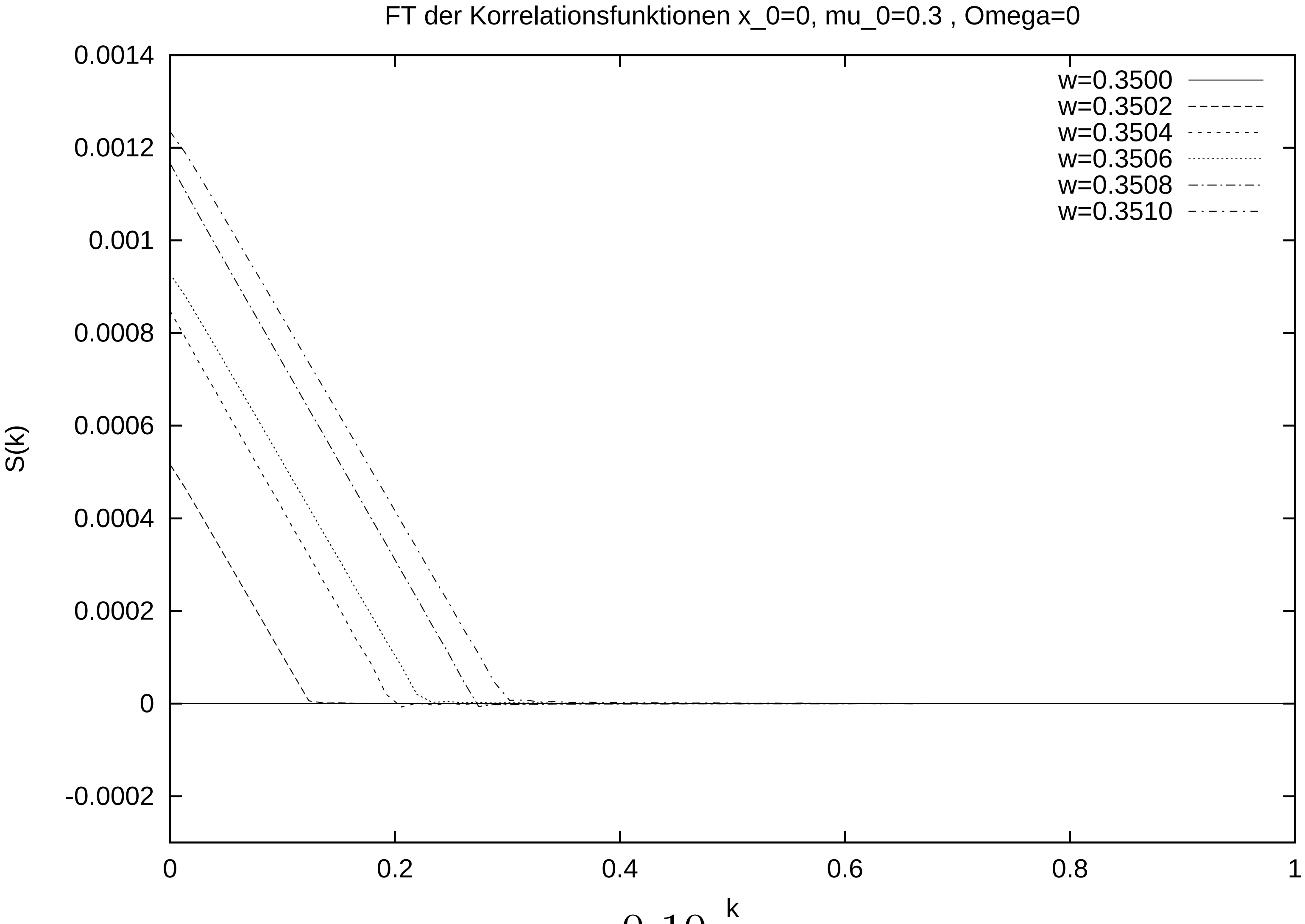


Density-Density Fluctuations



Static Structure Factor

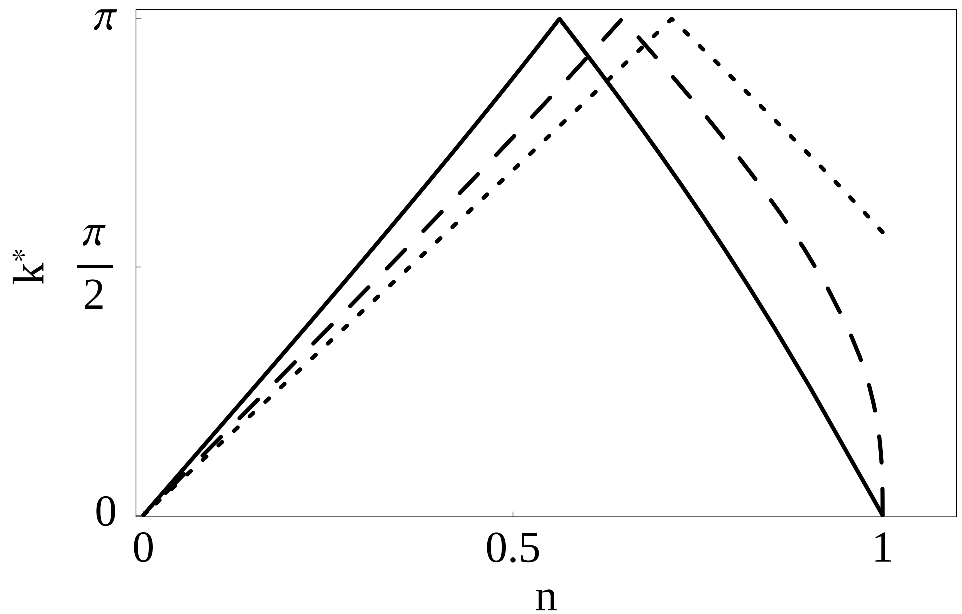
$$\xi_{ol} = 1/k^{*} \sim |\zeta - \zeta_c|^{-1/2}$$



0-10

Characteristic wavevector k^*

for $J = 0.2$ (solid), $J = 0.5$ (dashed), $J = 0.8$ (dotted)



Tonks-Girardeau gas: $k^* = k_F = 2\pi n$

N-component Bose Gas

$$H = -\sum_{<r,r'>} \sum_{\sigma,\sigma'=1}^{N} J_{\sigma\sigma'} c^{\dagger}_{r\sigma} c_{r'\sigma} c^{\dagger}_{r\sigma'} c_{r'\sigma'}$$

special cases:

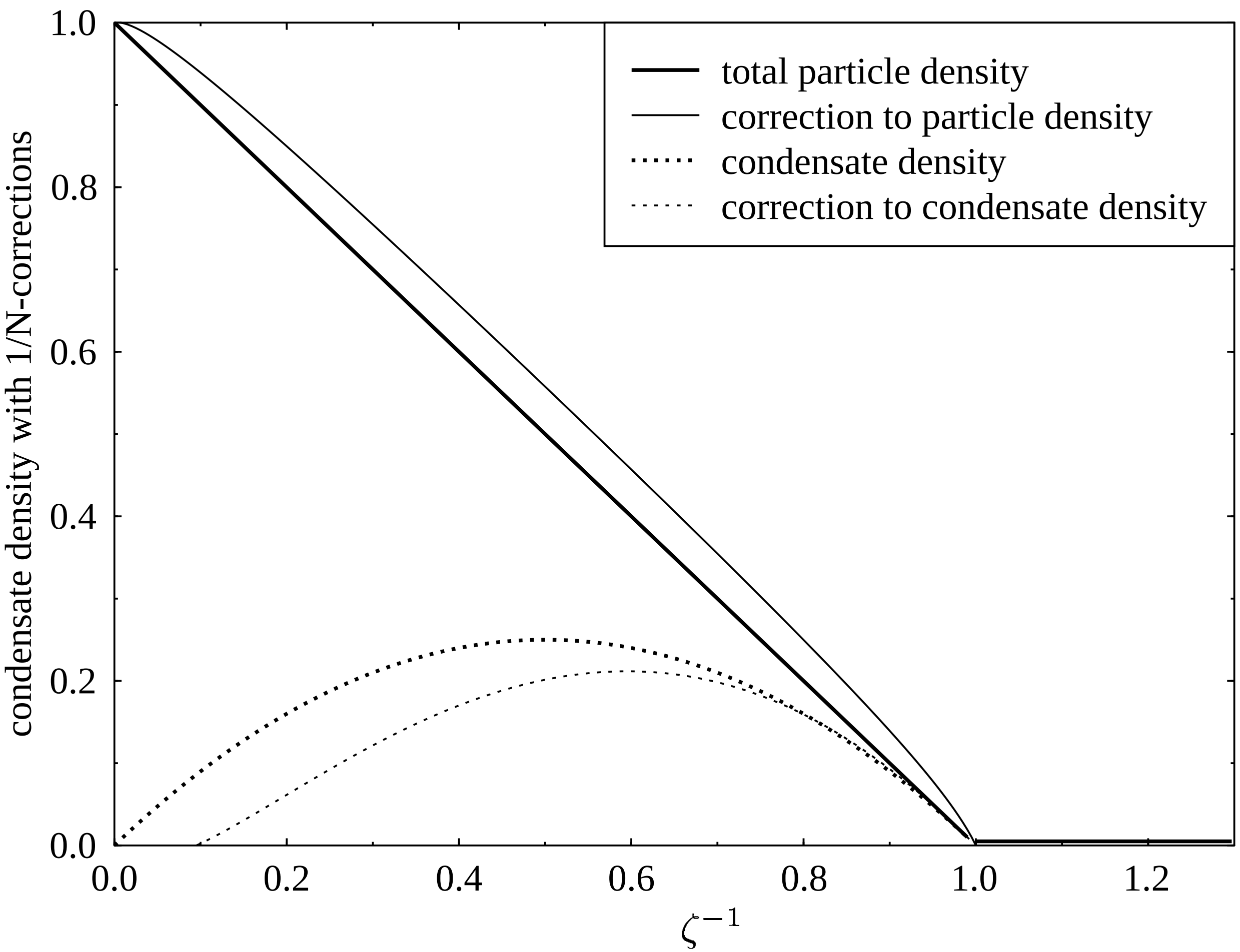
$$J_{\sigma\sigma'} = J/N$$

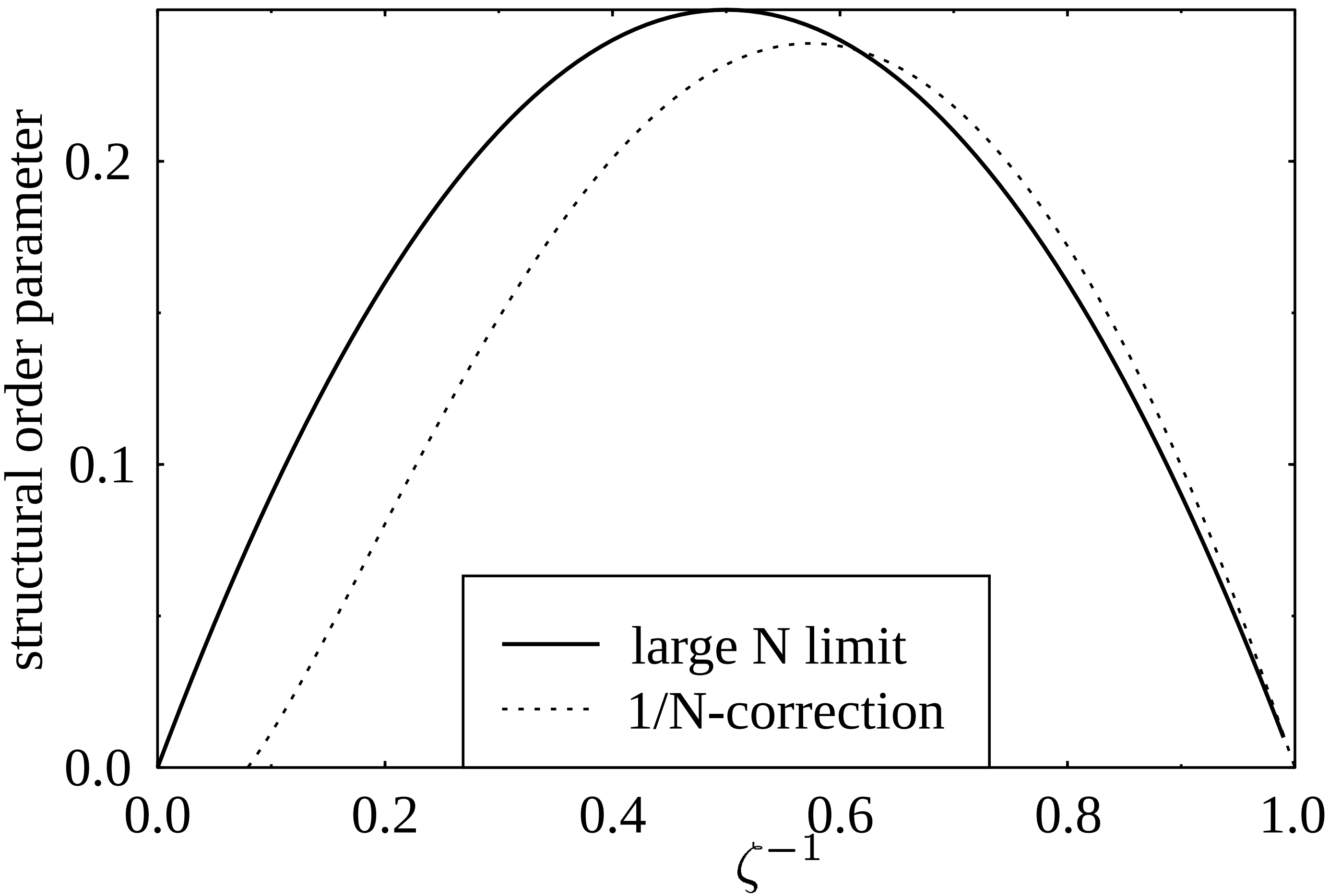
$$J_{\sigma\sigma'} = J\delta_{\sigma\sigma'}$$

– $N \to \infty$ can be solved exactly

– $1/N$-expansion is available

Total density & condensate density in the N-component Bose Gas




structural order parameter
0.0
0.1
0.2
0.0
0.2
0.4
0.6
0.8
1.0
ζ^{-1}
large N limit
1/N-correction

Atom-Molecule Mixture: Functional Integral

partition function:

$$Z = Tre^{-\beta H} = \int e^{-S}\mathcal{D}[\psi]$$

action:

$$S = \sum_x (\psi_{x,\uparrow}\bar{\psi}_{x,\uparrow} + \psi_{x,\downarrow}\bar{\psi}_{x,\downarrow}) - \sum_{r,r',t} (\zeta\delta_{r,r'} + \bar{t}w_{r,r'})(\psi_{r,t,\uparrow}\bar{\psi}_{r',t+1,\uparrow} + \psi_{r,t,\downarrow}\bar{\psi}_{r',t+1,\downarrow})$$

$$-J\sum_{r,r',t} \psi_{r,t,\uparrow}\bar{\psi}_{r',t+1,\uparrow}\psi_{r,t,\downarrow}\bar{\psi}_{r',t+1,\downarrow}$$

$\zeta = e^{\mu}$

Densities

averaging:

$$\langle ... \rangle = \frac{1}{Z} \int ... e^{-S} \mathcal{D}[\psi]$$

fermionic density

$$n_{f,x} = -\langle \psi_{x,\uparrow} \bar{\psi}_{x,\uparrow} + \psi_{x,\downarrow} \bar{\psi}_{x,\downarrow} + 2\psi_{x,\uparrow} \bar{\psi}_{x,\uparrow} \psi_{x,\downarrow} \bar{\psi}_{x,\downarrow} \rangle$$

molecular density

$$n_{m,x} = \langle 1 + \psi_{x,\uparrow} \bar{\psi}_{x,\uparrow} + \psi_{x,\downarrow} \bar{\psi}_{x,\downarrow} + \psi_{x,\uparrow} \bar{\psi}_{x,\uparrow} \psi_{x,\downarrow} \bar{\psi}_{x,\downarrow} \rangle$$

Conventional Form of the action

time shift

$$\bar{\psi}_{r,t,j} \to \bar{\psi}_{r,t-1,j} \quad (j = \uparrow, \downarrow)$$

action

$$S = \sum_x (\psi_{x,\uparrow} \partial_t^T \bar{\psi}_{x,\uparrow} + \psi_{x,\downarrow} \partial_t^T \bar{\psi}_{x,\downarrow})$$

$$- \sum_{r,r',t} (w_\parallel \delta_{r,r'} + w_{r,r'})(\psi_{r,t,\uparrow} \bar{\psi}_{r',t,\uparrow} + \psi_{r,t,\downarrow} \bar{\psi}_{r',t,\downarrow})$$

$$-w_\perp \sum_{r,r',t} \psi_{r,t,\uparrow} \bar{\psi}_{r',t,\uparrow} \psi_{r,t,\downarrow} \bar{\psi}_{r',t,\downarrow}.$$

decoupling by a Hubbard-Stratonovich field:

$$(i\phi_x + \chi_x)\psi_{x,\uparrow}\psi_{x,\downarrow} + h.c.$$

ϕ, χ are complex fields

Effective Bosonic Action

$$S_{eff} = (\phi, v^{-1}\phi) + \frac{1}{2J}(\chi, \chi) - \frac{1}{2}\log(\det A)$$

antisymmetric space-time matrix:

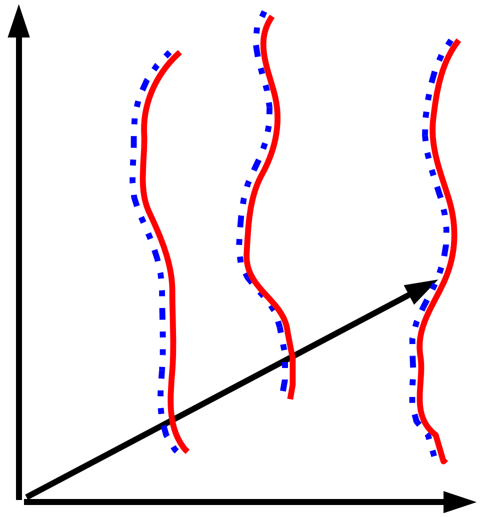
$$A = \begin{pmatrix} 0 & i\phi + \chi & \zeta + \bar{t}w - \partial_t^T & 0 \\ -i\phi - \chi & 0 & 0 & \zeta + \bar{t}w - \partial_t^T \\ -\zeta - \bar{t}w + \partial_t & 0 & 0 & i\bar{\phi} + \bar{\chi} \\ 0 & -\zeta - \bar{t}w + \partial_t & -i\bar{\phi} - \bar{\chi} & 0 \end{pmatrix}$$

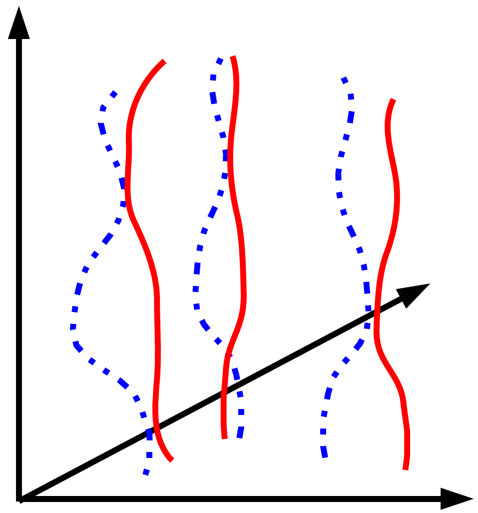
$$v^{-1} = (w + 2\mathbf{1})^{-1}/J.$$

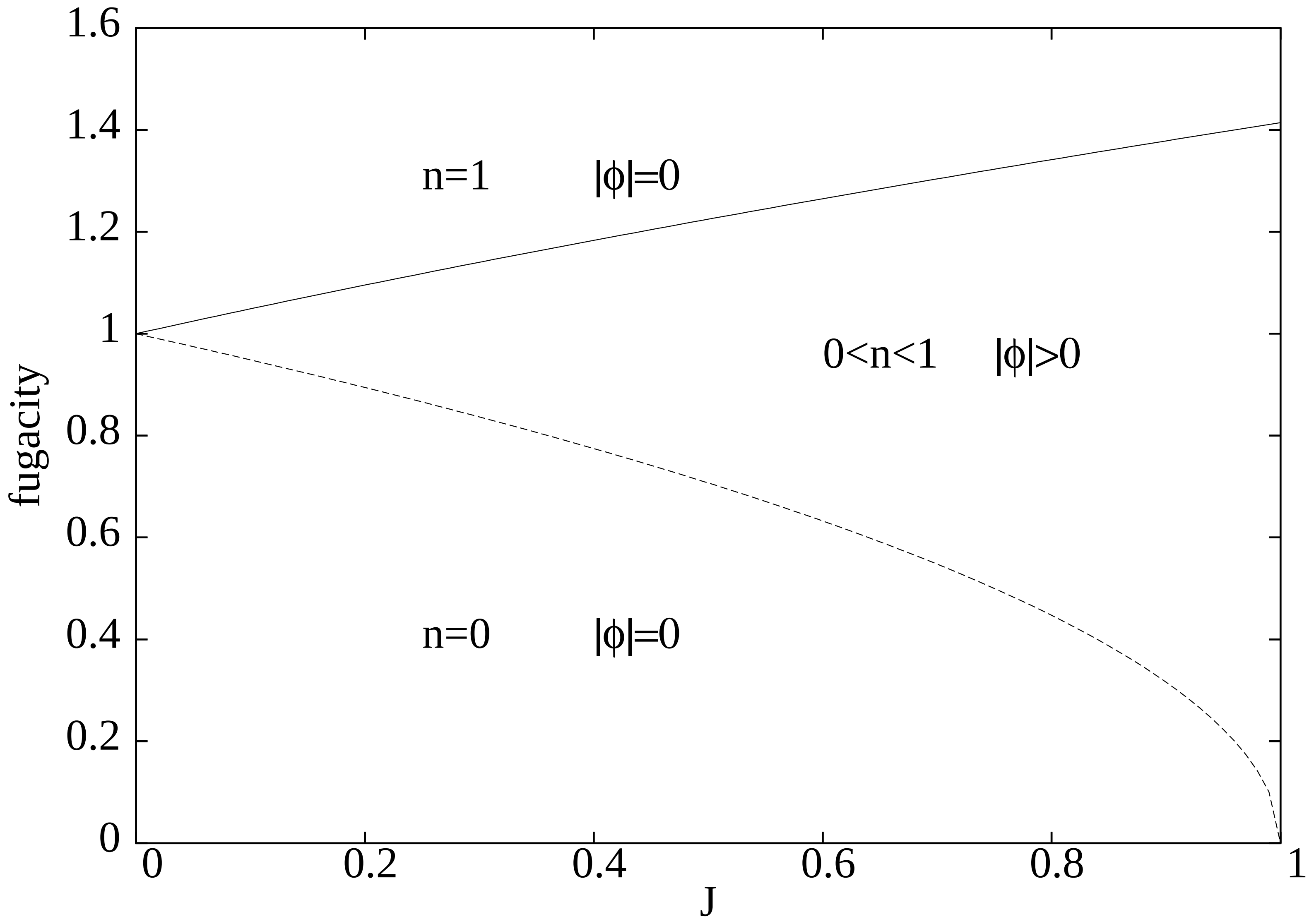
Partition Function:

$$Z = \int e^{-S_{eff}} \mathcal{D}[\phi]$$

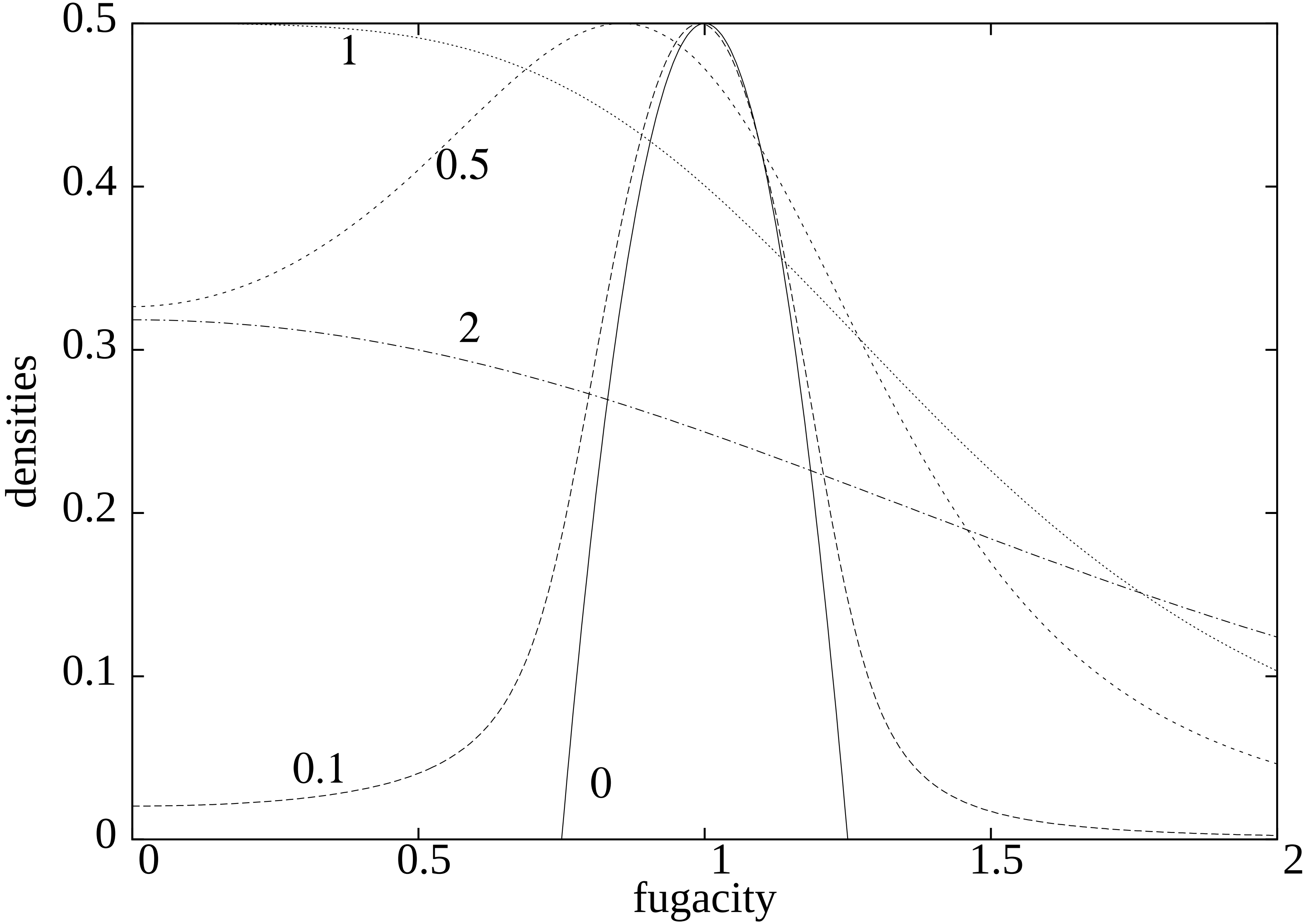
Dissociation of Atoms



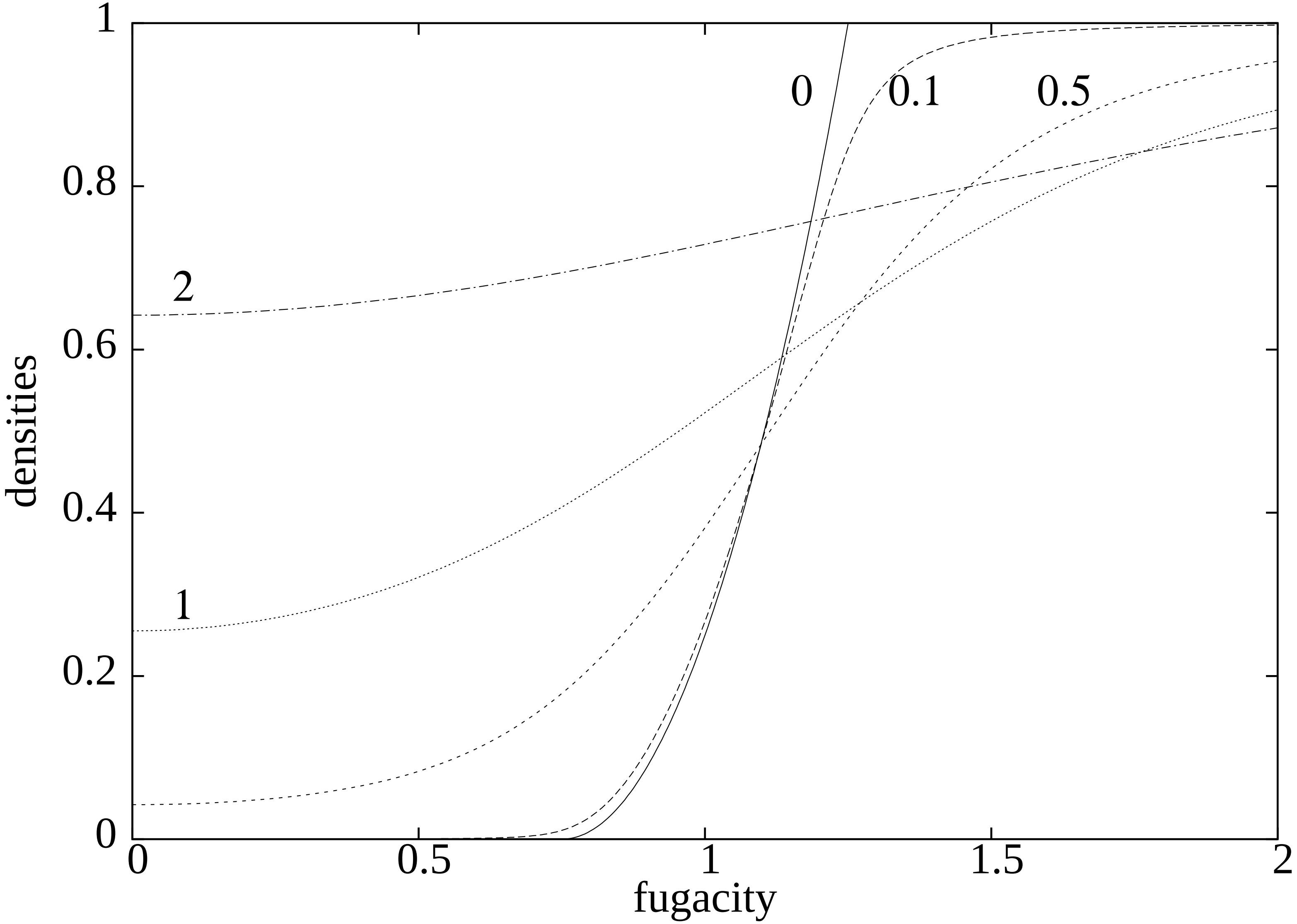




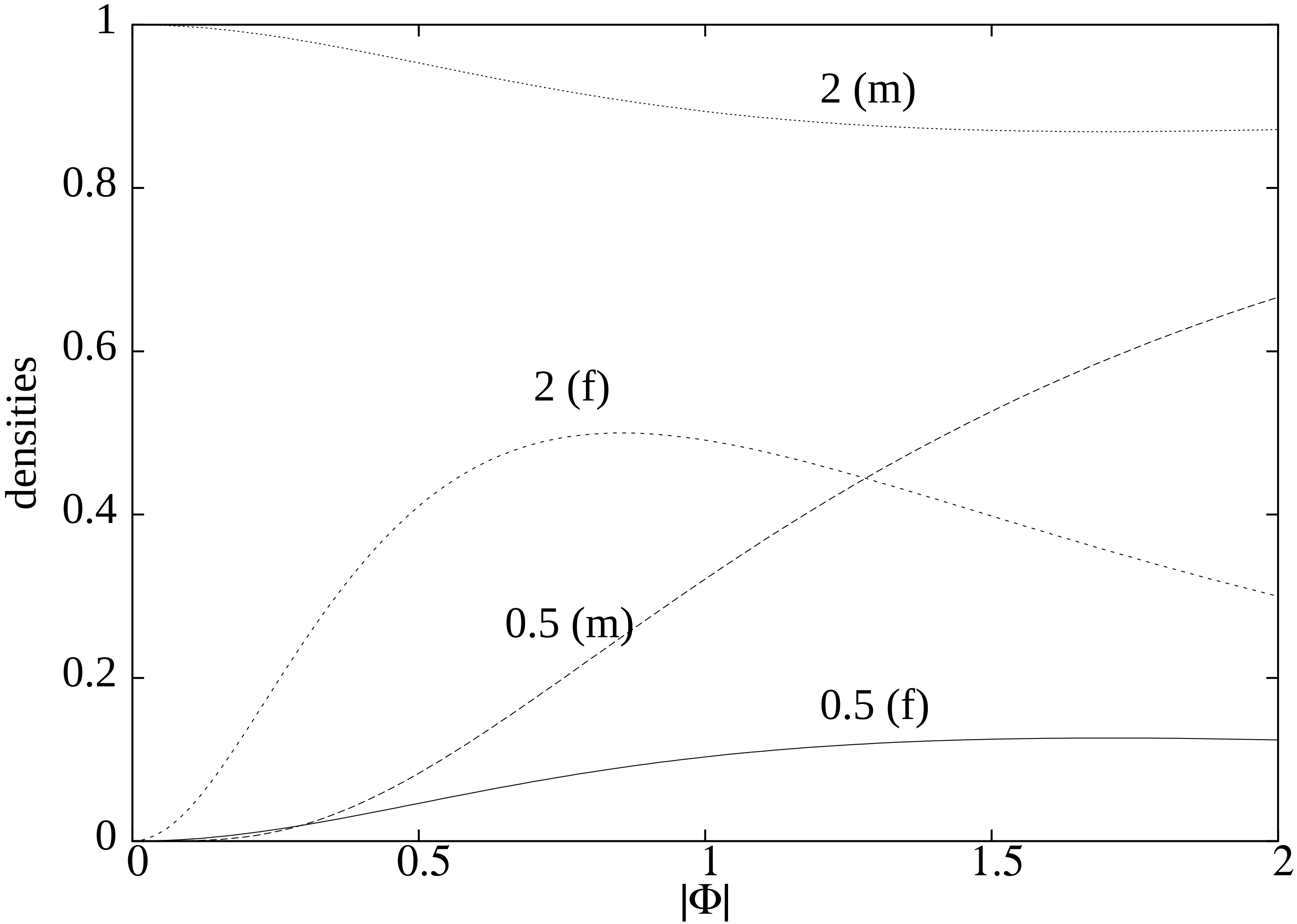
$T = 0$ phase diagram for molecules ($\bar{t} = 0$)



density of individual fermions (dissociated atoms)



density of molecules



densities vs. interaction

Mixture of two Fermion Gases

Two types of fermionic atoms with different mass: e.g. ^{6}Li and ^{40}K
Hamiltonian

$$H = -\bar{t}_c \sum_{\langle r,r'\rangle} c_r^\dagger c_{r'} - \bar{t}_f \sum_{\langle r,r'\rangle} f_r^\dagger f_{r'} + \sum_r \Big[-\mu(c_r^\dagger c_r + f_r^\dagger f_r) + U f_r^\dagger f_r c_r^\dagger c_r \Big]$$

f atoms are heavy: $\bar{t}_f \approx 0$

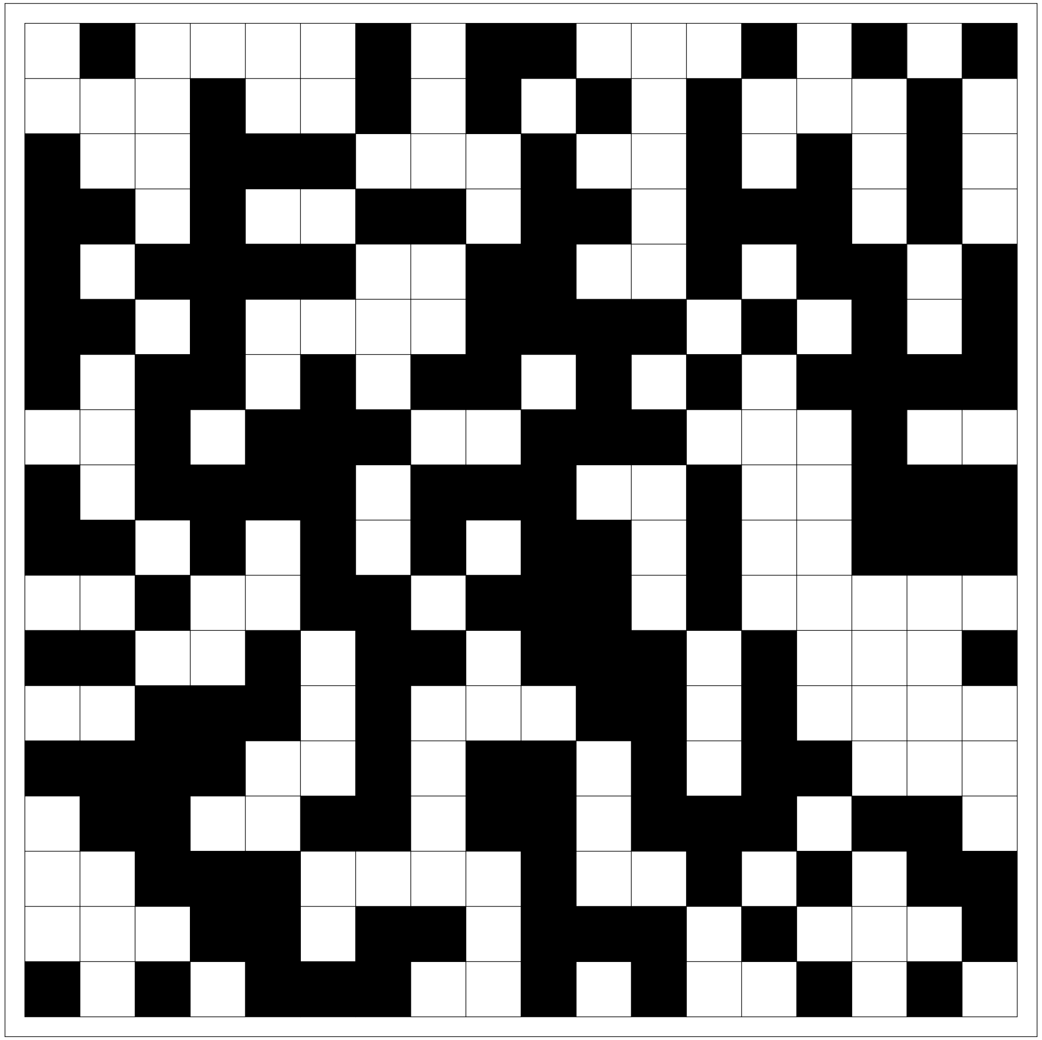
Realization as Correlated Disorder: Green's function (for $\Delta \to 0$)

$$G = \langle [-\partial_t + 1 - \Delta(\mu - \hat{t} + U\mathbf{I})]^{-1} \rangle_{\text{Ising}}$$

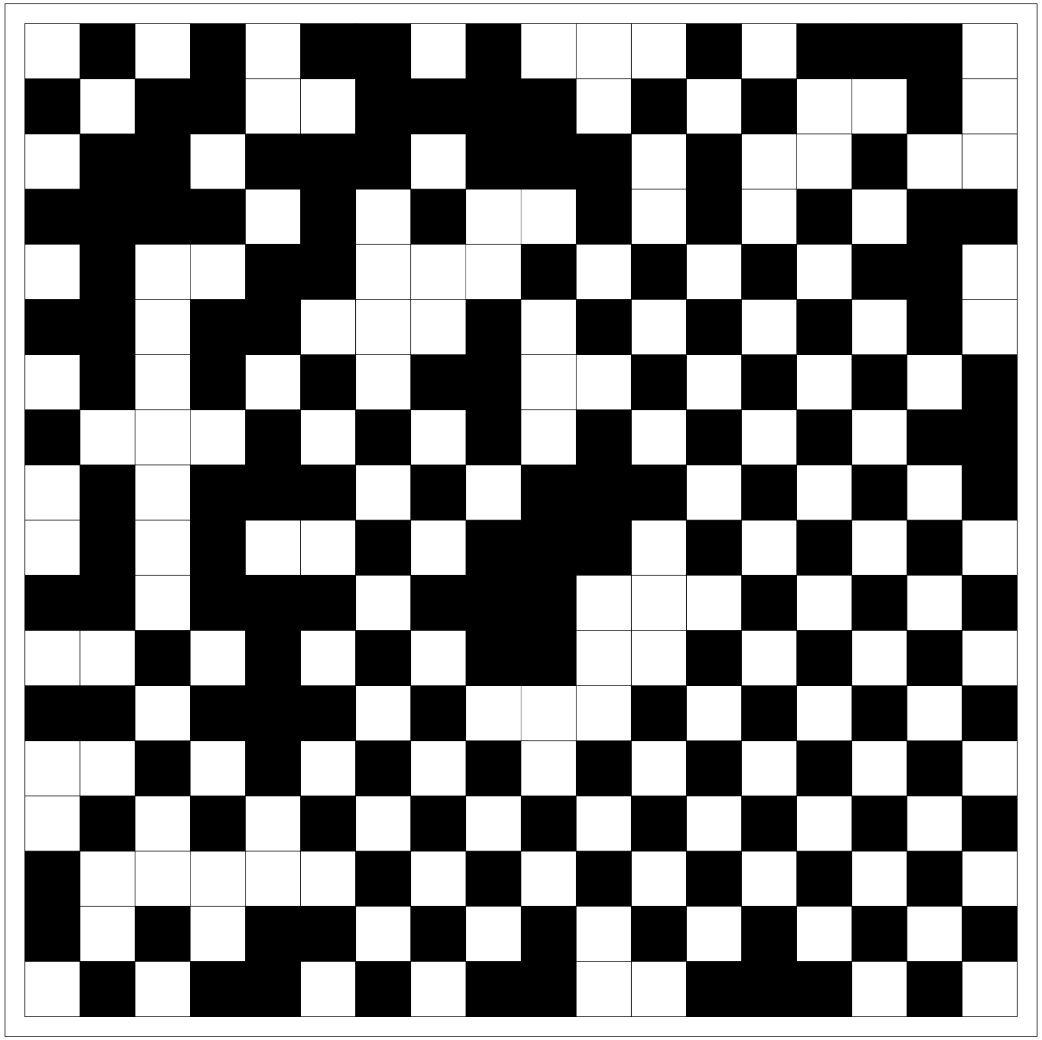
average $\langle ... \rangle_{\text{Ising}}$ with respect to the distribution ($\mathbf{I} = (1-S)/2$)

$$P(\{S(r)\}) = \frac{e^{\mu\beta \sum_r \mathbf{I}(r)} \det[\mathbf{1} + \{1 - \Delta(\mu - \hat{t} + \mathbf{I})\}^{\beta/\Delta}]}{\sum_{\{S(r)=\pm 1\}} e^{\mu\beta \sum_r \mathbf{I}(r)} \det[\mathbf{1} + \{1 - \Delta(\mu - \hat{t} + U\mathbf{I})\}^{\beta/\Delta}]}$$

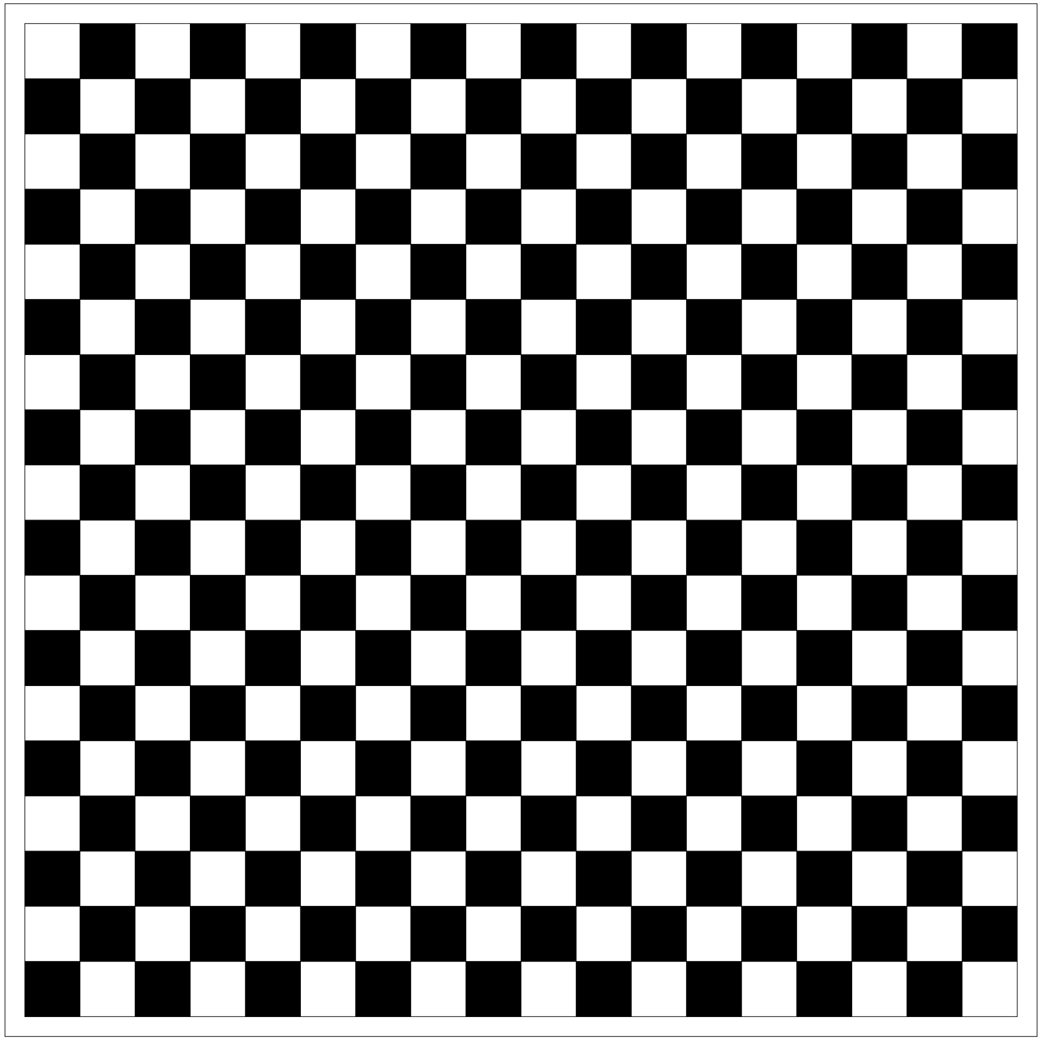
Typical Configurations of Heavy Atoms



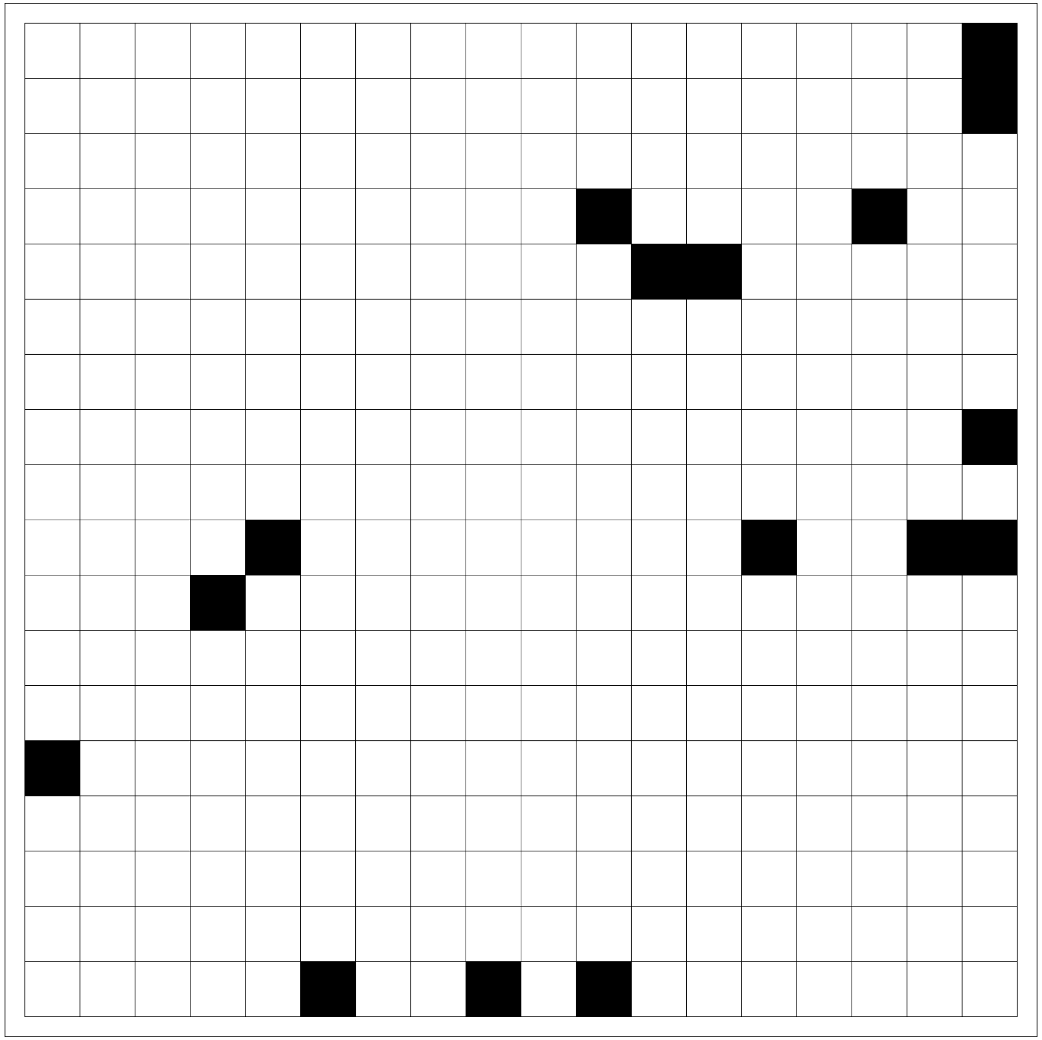
Paramagnetic phase: $t = 1$, $U = 3\,,\ \mu = U/2$, $\beta = 3$



$t = 1,\ U = 3\,,\ \mu = U/2,\ \beta = 7$

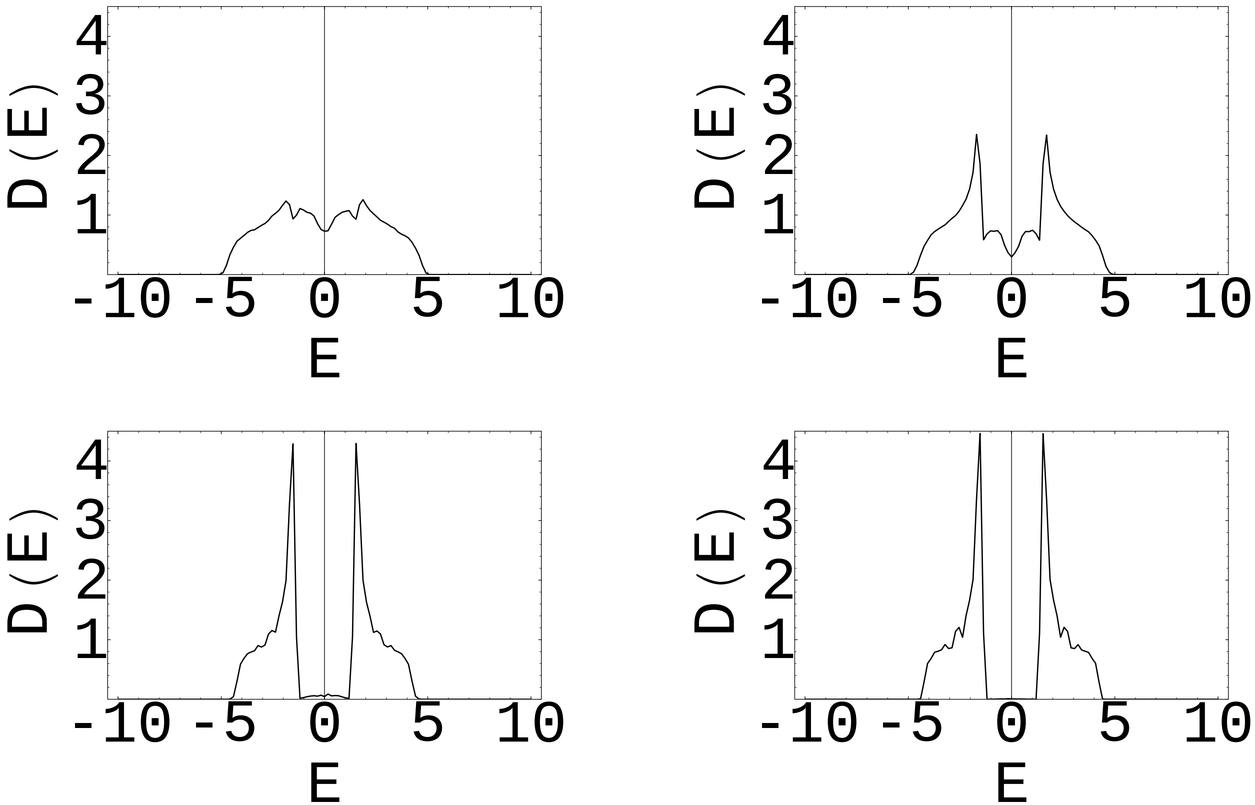


$t = 1,\, U = 3\,,\ \mu = U/2,\, \beta = 14$

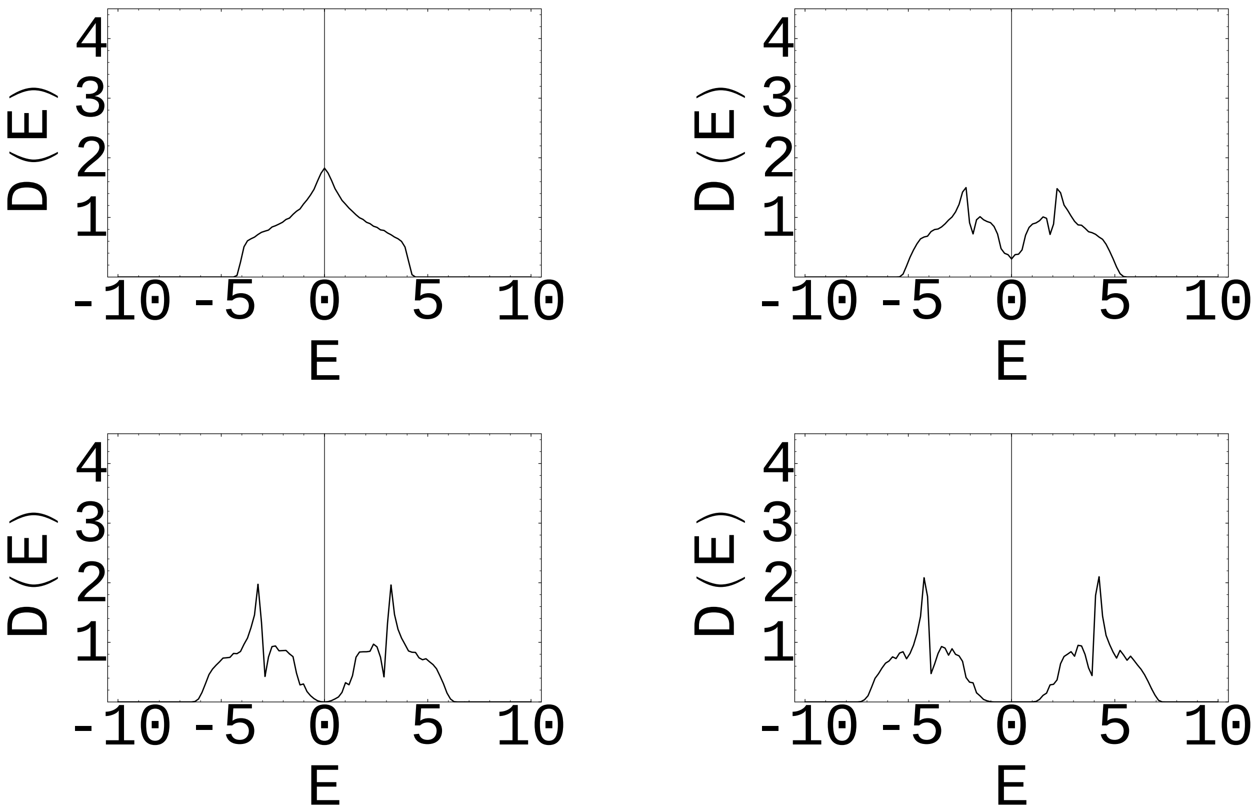


$t = 1,\ U = 8\ ,\ \mu = 0.8U,\ \beta = 14$

Density of States for Light Atoms



DOS for $U = 3\,,\ \mu = U/2$ (half filling) and decreasing temperatures.
$\beta = 3, 7, 10, 14$



DOS at $\beta = 3$ for increasing interaction U and half-filling ($\mu = U/2$).

$U = 1, 4, 6, 8$

Conclusions

BCS vs. molecular states:

Mean-field approach does not provide unique solution

Model for tunneling of fermions and molecules:

Molecules and fermions compete in an optical lattice

Molecular Gas:

1D and $N \to \infty$ can be treated exactly for BEC and Mott insulator

Mixture of fermionic atoms and molecules:

Interaction favors formation of molecules at low density and dissociation at high densities

Mixture of light and heavy atoms:

Heavy atoms represent correlated disorder for light atoms