Transmutation of momentum into position in magnetic vortices

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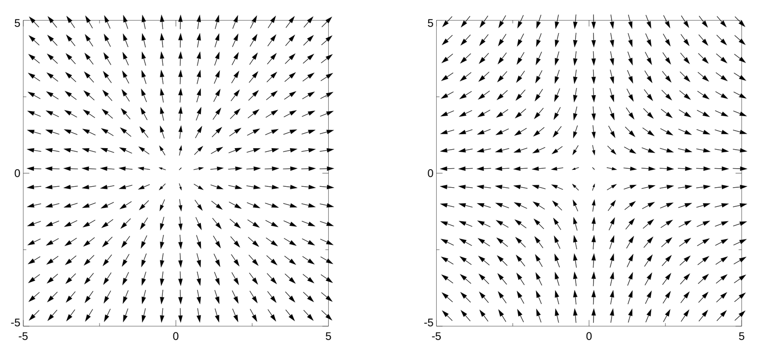
Abstract

We show that transmutation of linear momentum into position may occur in a system of three magnetic vortices thanks to a direct link between topology and dynamics in a ferromagnet. This happens via exchange between the linear momentum of a vortex-antivortex pair and the position of a single vortex during a semi-elastic scattering process. Vortex polarity switching occurs in the case of inelastic collisions. [1]

1 Magnetic vortices

Vortex ($S=1$), Antivortex ($S=-1$)

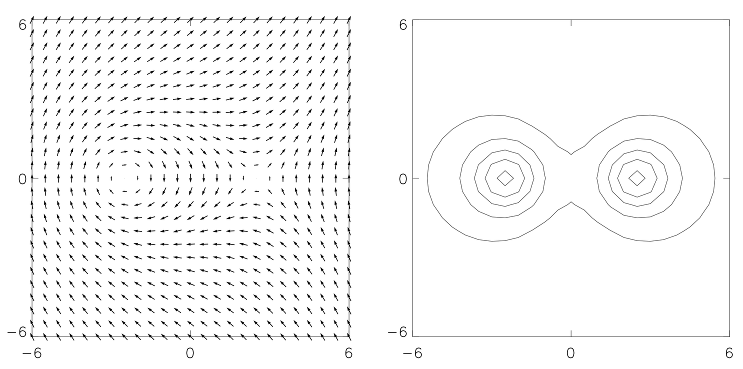


A vortex is a magnetization [$\boldsymbol{m}=(m_x,m_y,m_z)$] configuration of the form:

$$m_z = \lambda \cos\Theta(\rho),$$
$$m_x + i\,m_y = \sin\Theta(\rho)\,e^{iS(\phi-\phi_0)}.$$

(ρ,ϕ): polar coordinates
$S=\pm1,\pm2,\dots$ is the winding number
$\lambda=\pm1$ is the vortex polarity
ϕ_0: constant angle.

Vortex-antivortex (VA) pairs [2]



Same vortex polarities: the pair undergoes Kelvin motion in a direction perpendicular to the line connecting the vortex and the antivortex with velocity (v) that is inversely proportional to the distance (d) between the two vortices [3].

Opposite vortex polarities: the pair undergoes rotational motion around a fixed guiding center [4].

Topological charge: Skyrmion number

$$\mathcal{N} \equiv \frac{1}{4\pi}\int n\,dxdy, \quad n \equiv \frac{1}{2}\epsilon_{\mu\nu}\,(\partial_\nu \boldsymbol{m}\times\partial_\mu \boldsymbol{m})\cdot\boldsymbol{m}.$$

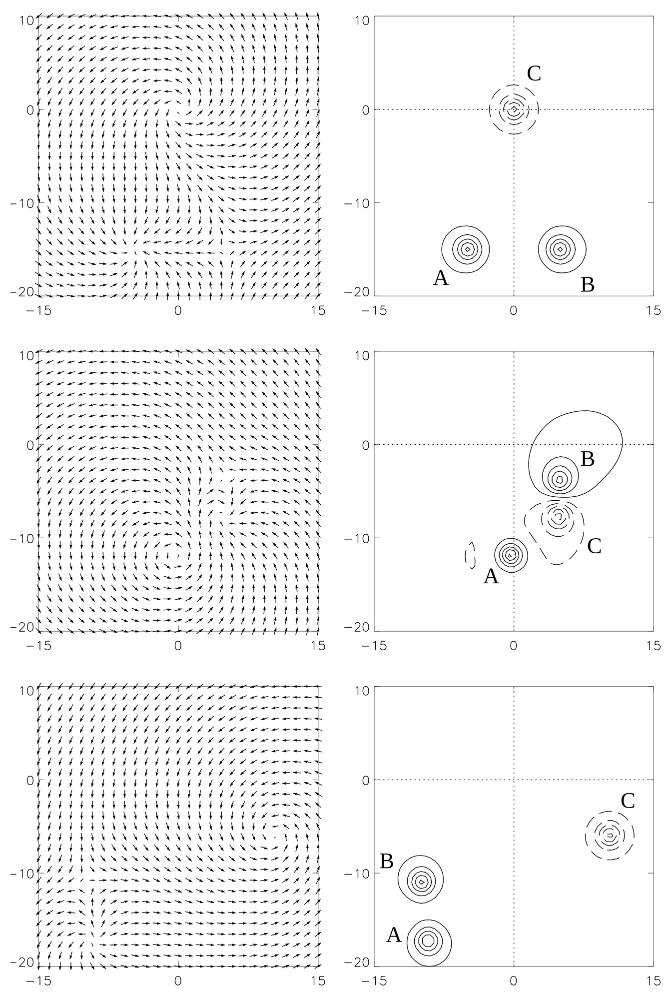
For vortices: $\mathcal{N}=-\frac{1}{2}\,S\,\lambda$.

Linear momentum

$$P_x = -\int y\,n\,dxdy,$$
$$P_y = \int x\,n\,dxdy. \qquad (1)$$

2 Vortex pair scattering

VA pair velocity $v=0.1$



Vectors: (M_x,M_y), Contours: M_z

Solid line contours: positive polarity ($M_z>0$)
Dashed line contours: negative polarity ($M_z<0$)

Process: A single vortex C (negative polarity) is initially at rest at position $R_C=(0,0)\,[\Rightarrow P_C=(0,0)\,]$
A Kelvin pair (AB) (positive polarity) moves along y axis [velocity $v=0.1$, momentum $P_{AB}=(0,62)$] and collides with C.

Result: Vortex C is shifted to a new position $R_C=(10,-6)\,[\Rightarrow P_C=(2\pi\times10, 2\pi\times-6)\,]$.
Pair AB has changed its momentum to $P_{AB}=(-36,-17)$

Violation of momentum conservation? No!

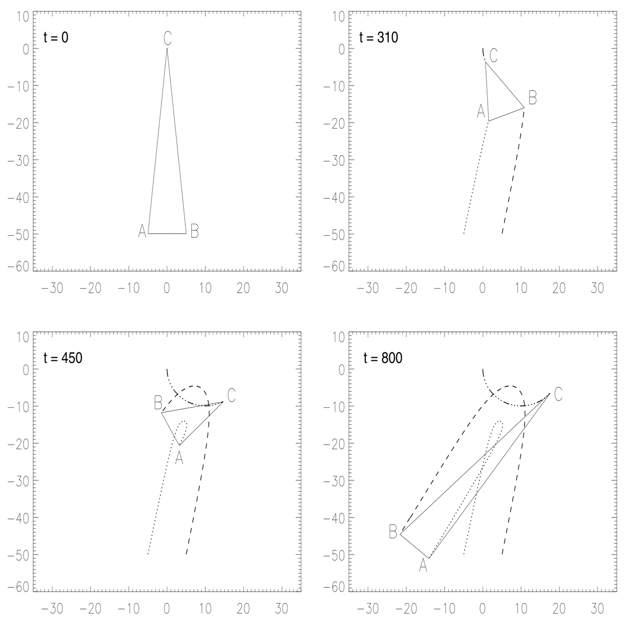
Conservation of momentum (1) means:

$$\Delta P_{AB} + \Delta P_C = 0$$

or

$$\Delta P_{AB} + 2\pi\times\Delta R_C = 0$$

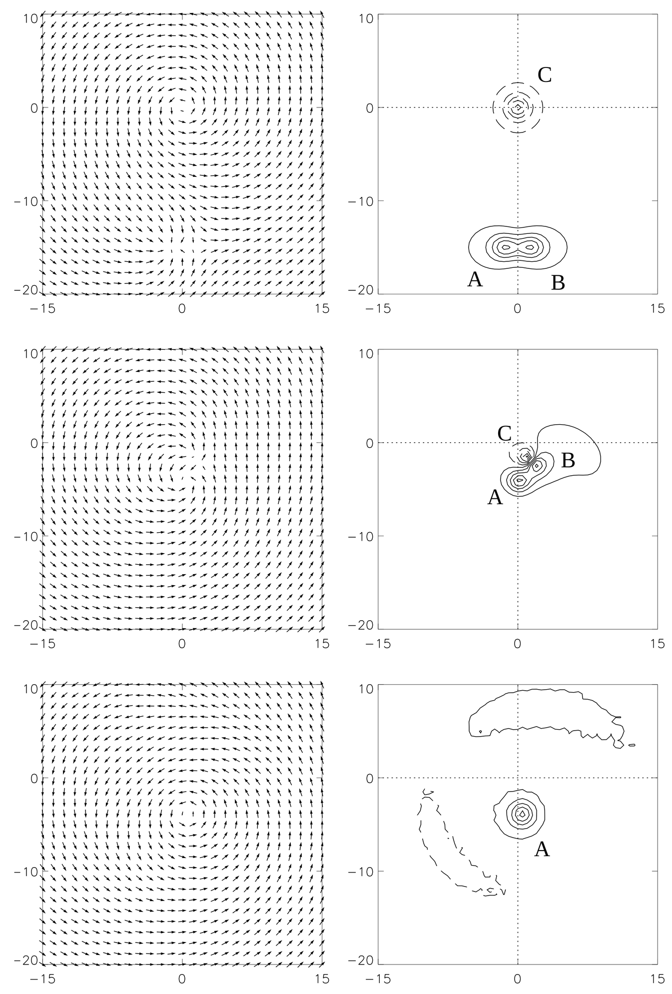
i.e., we have transmutation of VA pair momentum into vortex position!



The vortex dynamics has also been simulated using collective coordinates.

3 Switching of vortex polarity

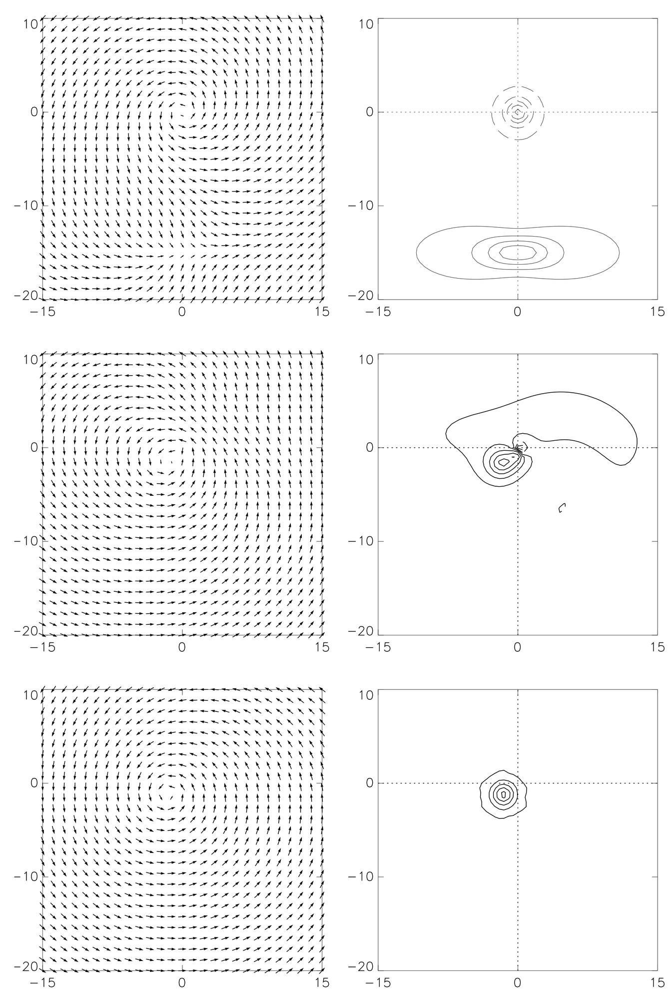
VA pair velocity $v=0.5$



Antivortex B annihilates with vortex C (negative polarity). Vortex A (positive polarity) remains.

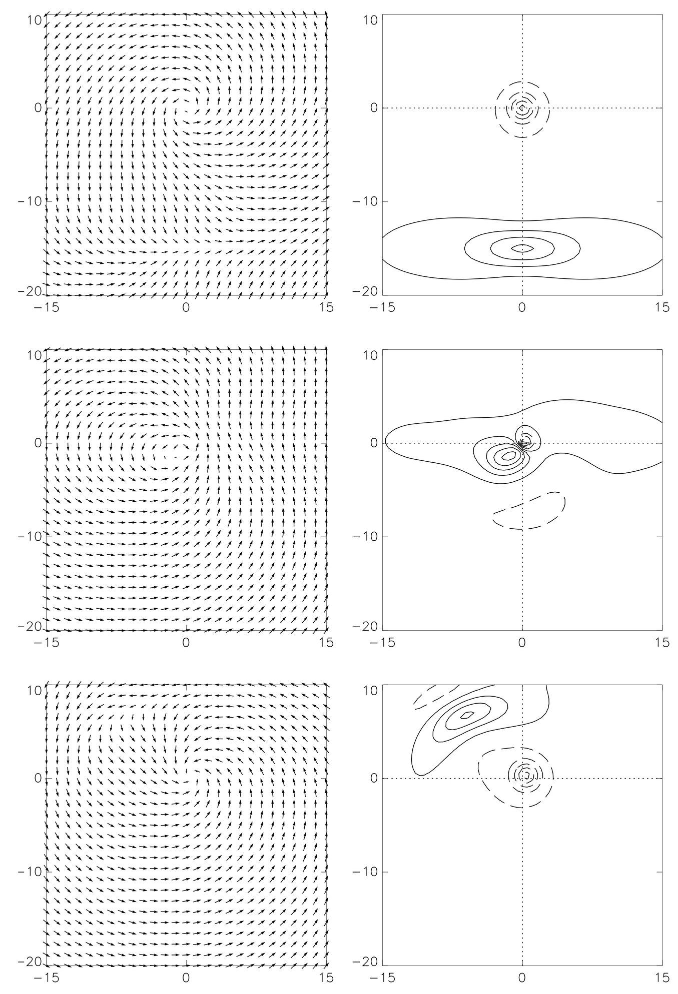
We thus obtain vortex polarity switching [5–7].

Solitary wave velocity $v=0.9$



Magnetic solitary wave (velocity $v=0.9$) collision onto a vortex C (negative polarity). A generated antivortex annihilates with vortex C. A vortex with positive polarity remains.

Solitary wave velocity $v=0.95$



Magnetic solitary wave (velocity $v=0.95$) scattering onto a vortex C.

4 Summary

In a collision of a VA pair (positive polarity) against a single vortex (negative polarity) we have the following (v: velocity of VA pair):

- $v<0.3$: Scattering of VA pair ⇒ transmutation of its momentum into vortex position.
- $0.3<v<0.9$: Annihilation of vortex and antivortex ⇒ Switching of vortex polarity.
- $v>0.9$: Scattering of magnetic soliton onto vortex.

We see that: the unusual definition of the linear momentum (1) is physically relevant.

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