Dynamically Induced Entanglement and Decoherence

The Quantum to Classical Crossover

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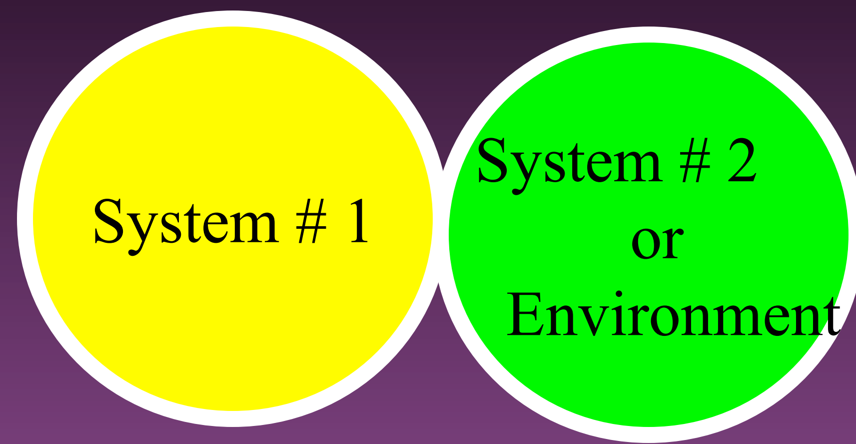
Coupled Quantum Mechanical Systems



- Initial pure state

$$\rho_0 = |\Psi \otimes \Phi\rangle\langle\Psi \otimes \Phi|$$

- Reduced density matrix

$$\rho_r(t) = \mathrm{Tr}_\Phi\{\exp[-i\mathcal{H}t]\ \rho_0\ \exp[i\mathcal{H}t]\}$$

$$\begin{cases} \Phi = \Psi' & \Rightarrow \text{entanglement} \\ \Phi = \Phi_{env} & \Rightarrow \text{decoherence} \end{cases}$$

- Purity

$$\mathcal{P}(t) = \mathrm{Tr}_\Psi\{\rho_r^{\,2}(t)\}$$

The Purity as a characterizing tool : $0 \leq \mathcal{P}(t) \leq 1$

Entanglement in bipartite chaotic systems

Semiclassical vanishing coupling

- Two-Body Hamiltonian

$$\mathcal{H} = H_1 \otimes I_2 + I_1 \otimes H_2 + \hbar\mathcal{U}$$

- Initial product two-particle state / localised Gaussian wavepackets

$$|\Psi_1\rangle \otimes |\Psi_2\rangle \equiv |\Psi_1,\Psi_2\rangle \quad , \quad \rho_0 = |\Psi_1,\Psi_2\rangle\langle \Psi_1,\Psi_2|$$

$$\langle x_i |\Psi_i\rangle \propto \exp[i p_{0i}(x_i - r_{0i})] \exp[-(x_i - r_{0i})^2 / 2\sigma^2]$$

- Calculate the reduced density matrix and its purity

$$\rho_1(t) = \mathrm{Tr}_2\{\exp[-i\mathcal{H}t]\, \rho_0 \exp[i\mathcal{H}t]\} \ , \ \mathcal{P}(t) = \mathrm{Tr}_2\{ \rho_1(t)^2\}$$

- Semiclassical two-particle propagators

Fast one-particle actions

$$\langle x_1,x_2 \,|\, \exp[-i\mathcal{H}t] \,|y_1,y_2\rangle = (-i)^d \sum_{s,s'} C_{s,s'}^{1/2} \exp[\, i\{S_s + S_{s'} + \mathcal{S}_{s,s'}\}]$$

Classical paths determined by H_1 and H_2

Slow two-particle action

Semiclassical Purity

Fast one-particle phases

Slow two-particle phase

$$\mathcal{P}(t) \propto \int d\mathbf{x}_1 d\mathbf{y}_1 \sum_{s,l,k,m} A_s A_l^\dagger A_k A_m^\dagger \exp[i\Phi_1]$$

$$\times \int d\mathbf{x}_2 d\mathbf{y}_2 \sum_{s',l',k',m'} A_{s'} A_{l'}^\dagger A_{k'} A_{m'}^\dagger \{\exp[i\Phi_2] \times \exp[i\Phi_{12}]\}$$

Stationary Phase Approximation over one-particle phase :

$$\Phi_1 = S_s(r_1,x_1;t) - S_m(r_1,x_1;t) + S_k(r_1,y_1;t) - S_l(r_1,y_1;t)$$

SPA ➡ ($s = m$, $k = l$) and ($s' = m'$, $k' = l'$)

Slow two-particle phase reduced to :

$$\Phi_{12} = S_{s,s'} - S_{k,s'} + S_{k,k'} - S_{s,k'}$$

Semiclassical Purity

Purity as sum over four classical trajectories : s,k,s’,k’

$$\mathcal{P}(t) \propto \int d\mathbf{x}_1 d\mathbf{y}_1 \int d\mathbf{x}_2 d\mathbf{y}_2 \sum_{s,k} \sum_{s',k'} |\mathcal{A}_s|^2 \, |\mathcal{A}_k|^2 \, |\mathcal{A}_{s'}|^2 \, |\mathcal{A}_{k'}|^2 \, \exp[i\, \Phi_{12}]$$

- Diagonal contribution; s = k , s’ = k’ ; decays with amplitudes
- Nondiagonal contribution; decays with phase

Semiclassical Purity

$$\mathcal{P}(t) = \exp[-\lambda_1 t] + \exp[-\lambda_2 t] + \exp[-2\Gamma t]$$

Diagonal contribution of system # 1
Classical Term

Diagonal contribution of system # 2
Classical Term

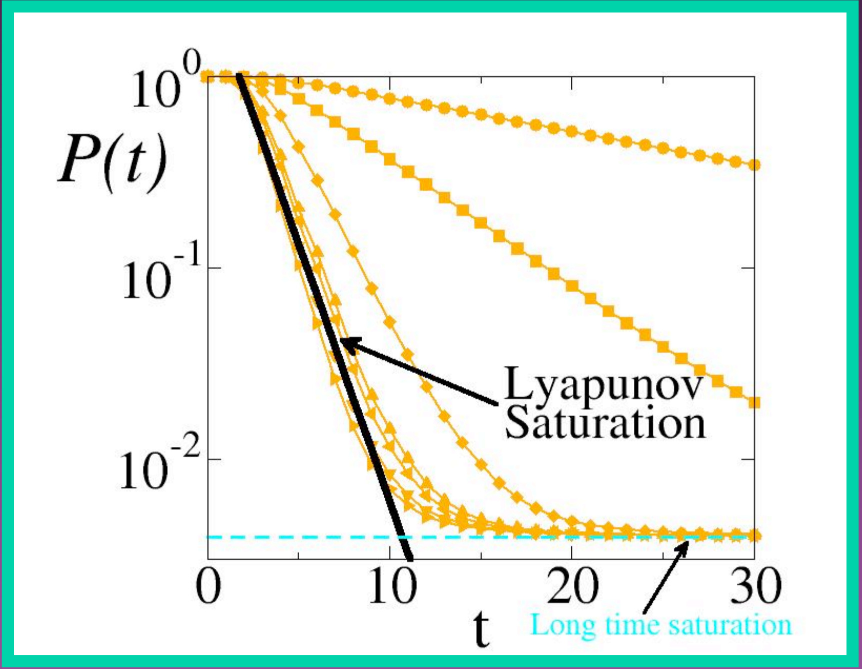
Non diagonal contribution
(Interaction dependent)
Quantum Term

$\Gamma \propto \langle \mathcal{U}_{ss'}, \mathcal{U}_{ss'} \rangle$

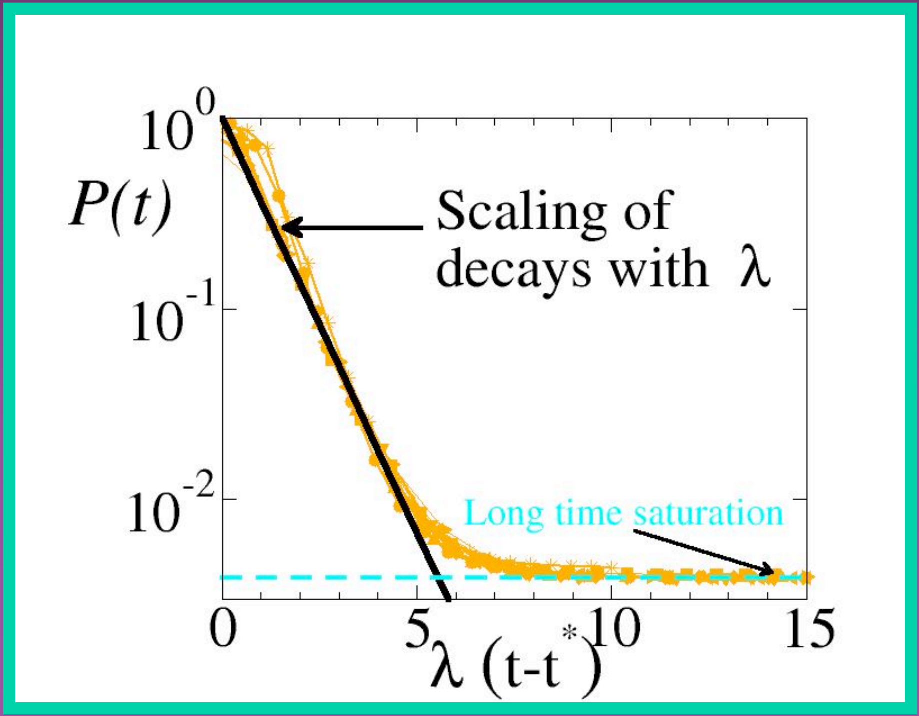
Miller & Sarkar, PRE 99
Zurek ,Rev Mod Phys 03

Tanaka & al., PRE 02
Znidaric & Prosen, J. Phys A 03

Lyapunov Regime



Chaotic system : kick rotator map



From Entanglement to Decoherence

System # 2 $\Rightarrow$ Environment

- Much shorter time scales : Faster Hamiltonian flow,

  Increased chaoticity $\lambda_2 >> \lambda_1$

- Bigger Hilbert space : $N_2 >> N_1$

- Unprepared initial state $\Rightarrow$ superposition of many Gaussian

  $M_2 >> 1$

$$\langle x_2 | \Psi_2 \rangle \propto M_2^{-1} \Sigma_\alpha \{ \exp[i\, \mathbf{p}_{0\alpha}(\mathbf{x}_2 - \mathbf{r}_\alpha)\,] \exp[-\,(\mathbf{x}_2 - \mathbf{r}_\alpha)^2 / 2\sigma^2 \}$$

Semiclassical Purity (Decoherence case)

$$\mathcal{P}(t) = \exp[-\lambda_1 t] + M_2^{-1}\exp[-\lambda_2 t] + \exp[-2\Gamma t]$$

Diagonal contribution of System Classical Term

Diagonal contribution of Environment Classical Term

Non diagonal contribution (Interaction dependent) Quantum Term

$$\Gamma \propto \langle \mathcal{U}_{ss^{\alpha}} \mathcal{U}_{ss^{\delta}} \rangle$$

Quantum-to-Classical Crossover

Wigner function as a phase space representation

Classical dynamics generated by Poisson bracket

Quantum contribution dominates after the Ehrenfest time

$$\partial_t W = \{W, \mathcal{H}\}_{PB} + \sum \frac{\hbar^{2n}(-1)^n}{2^{2n}(2n+1)!} \partial_x^{2n+1} V \, \partial_p^{2n+1} W$$

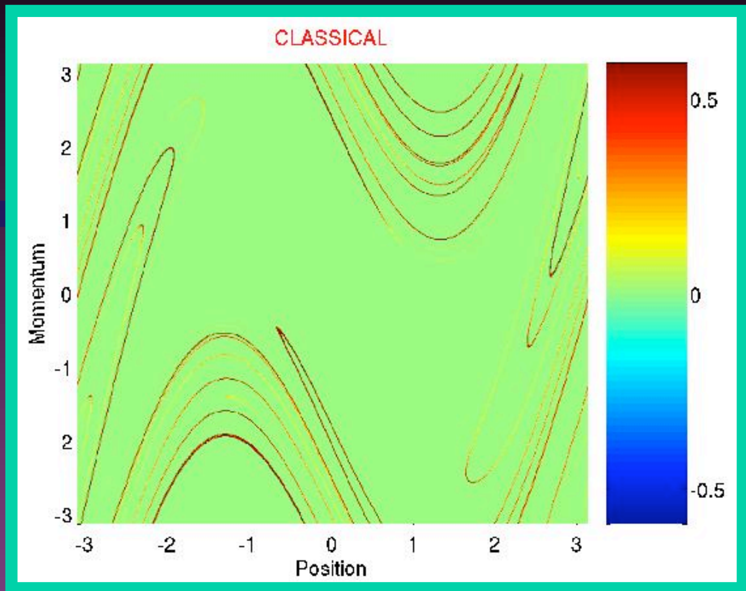
How to kill the quantum contribution before the Ehrenfest time and conserve the classical dynamics ?

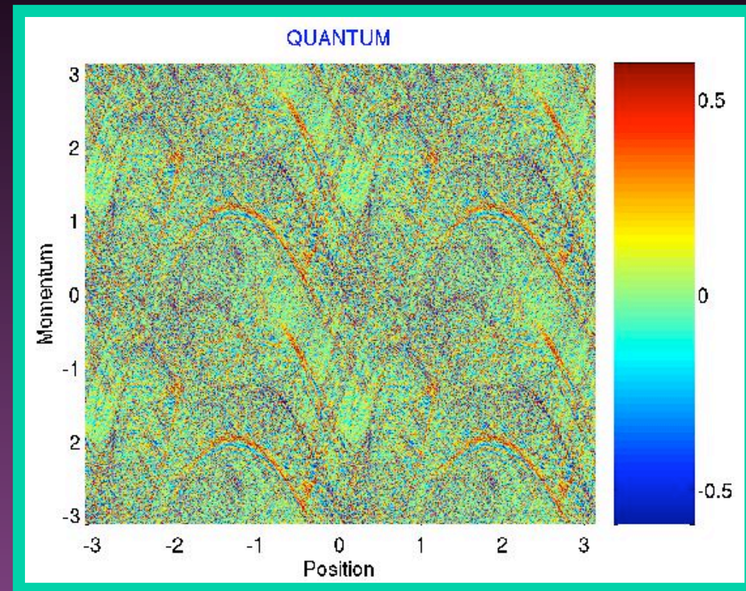
Heller & Tomsovic Physic Today 93

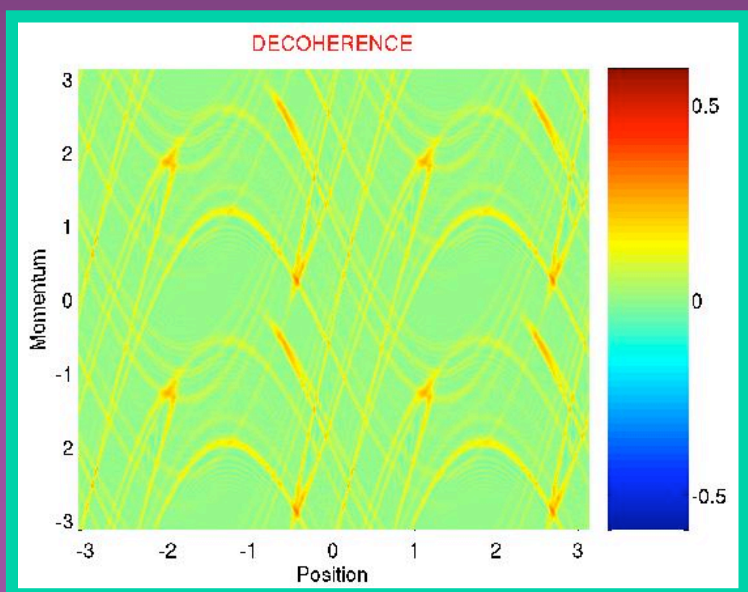
Casati & Chirikov Physica D 95

Ehrenfest time : $t_E = \lambda^{-1} \mathrm{Ln}(I/\hbar)$

Phase space representation



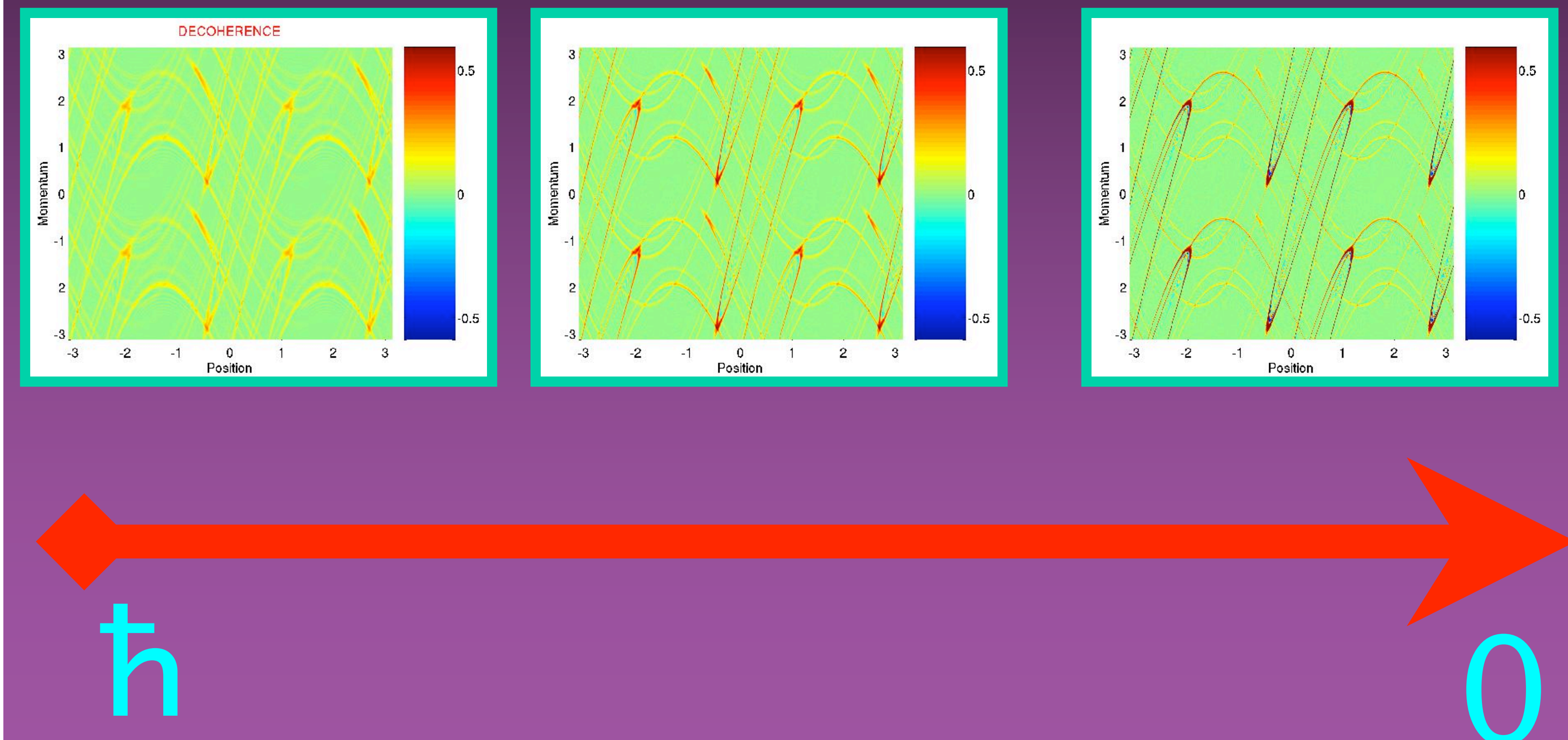




- ✓Better & better for $\hbar \to 0$
- ✓Classical mechanic out of QM
- ✓$\exp[-\lambda_1 t] \sim$

  Liouvillian evolution of ρ_1

$$W = \{W, \mathcal{H}\}_{PB} + \not{\mathcal{L}}_q$$


DECOHERENCE
Momentum
Position
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0

Conclusions

ENTANGLEMENT :

✔ Decay of purity results from a competition between :

✦ Classical Regime related to the Lyapunov exponent.

✦ Quantum Regime related to the Coupling.

DECOHERENCE :

✔ Classical dynamics is recovered for a semiclassical vanishing coupling for very general class of environment.