Multipartite Bell's Inequalities

Tomasz Paterek

Institute of Theoretical Physics and Astrophysics
University of Gdańsk
Poland

<u>Collaborators:</u>

Wieslaw Laskowski

Marek Zukowski

Caslav Brukner

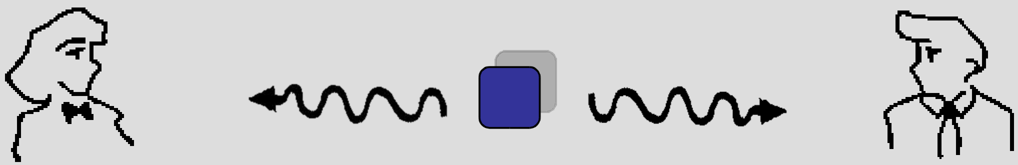
ISSQUI05, Dresden

Bell's Theorem

- ✓ Quantum Probabilities
- ✓ Underlying Theory
  reproduces quantum probabilities as averages over inaccessible variables
- ✓ Hidden Variable Theories
  hidden variables describe the properties of physical objects
- ✓ Bell's Theorem
  it is impossible to explain all quantum mechanical expectations
  with local hidden-variable theories

- ❑ A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
- ❑ J. S. Bell, Physics **1**, 195 (1964).

Clauser-Horne-Shimony-Holt



$A_1, A_2 = \pm 1$ $B_1, B_2 = \pm 1$

- ✓ realism
  undone measurements have well-defined, yet unknown, results
- ✓ locality
  no “action at a distance”
- ✓ For each experimental run:

$$A_1[B_1 + B_2] + A_2[B_1 - B_1] = \pm 2$$

- ✓ Average over many runs:

$$-2 \leq \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \leq 2$$

$$\left| E_{11} + E_{12} + E_{21} - E_{22} \right| \leq 2$$

❑ J. F Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

Quantum Prediction

- ✓ Take the state $|\psi^-\rangle = 1/\sqrt{2}\left[|0\rangle|1\rangle - |1\rangle|0\rangle\right]$
- ✓ Quantum correlation function:

$$E_{kl}^{QM} = -\cos\varphi_{kl}$$

- ✓ All measurements in one plane
  they are determined by the angle relative to some fixed axis

$$a_1 = 0 \qquad b_1 = \pi/4$$
$$a_2 = \pi/2 \qquad b_2 = -\pi/4$$

- ✓ CHSH expression:

$$|E_{11} + E_{12} + E_{21} - E_{22}| \le 2$$

$$\cos(\pi/4) + \cos(\pi/4) + \cos(\pi/4) - \cos(3\pi/4) = 2\sqrt{2}$$

Correlation Polytope

$$\begin{pmatrix} E_{11} \\ E_{12} \\ E_{21} \\ E_{22} \end{pmatrix} = \sum_{A_1, A_2, B_1, B_2 = \pm 1} P(A_1, A_2, B_1, B_2) \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \otimes \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

- ✓ Bell's inequalities = facets of the polytope
- ✓ N&S condition for local realistic model is a set of inequalities

Two Qubits

- ✓ For each experimental run:

$$\sum_{s_1,s_2=\pm1} S(s_1,s_2)(A_1+s_1A_2)(B_1+s_2B_2)=\pm4$$

- ✓ Average over many runs:

$$\left|\sum_{s_1,s_2=\pm1} S(s_1,s_2)[E_{11}+s_2E_{12}+s_1E_{21}+s_1s_2E_{22}]\right|\leq 4$$

- ✓ There are as many inequalities as different sign functions
- ✓ The complete set is equivalent to the *single* inequality:

$$\sum_{s_1,s_2=\pm1} \left|E_{11}+s_2E_{12}+s_1E_{21}+s_1s_2E_{22}\right|\leq 4$$

- ✓ Works for arbitrary number of qubits and two measurement settings

- ❑ H. Weinfurter and M. Zukowski, Phys. Rev. A **64**, 10102(R) (2001).
- ❑ R. F. Werner and M. W. Wolf, Phys. Rev. A **64**, 32112 (2001).
- ❑ M. Zukowski and C. Brukner, Phys. Rev. Lett. **88**, 210401 (2002).

Nonclassical States

- ✓ General N-qubit state:

$$\rho = \frac{1}{2^N} \sum_{x_1,\dots,x_N=0}^{3} T_{x_1 \dots x_N} \sigma_{x_1}^{1} \otimes \dots \otimes \sigma_{x_N}^{N}$$

- ✓ Quantum correlation function:

$$E_{k_1 \dots k_N}^{QM} = Tr[\rho(\vec{n}_{k_1} \cdot \vec{\sigma} \otimes \dots \otimes \vec{n}_{k_N} \cdot \vec{\sigma})] = \sum_{x_1,\dots,x_N=1}^{3} T_{x_1 \dots x_N} (\vec{n}_{k_1})_{x_1} \dots (\vec{n}_{k_N})_{x_N}$$

- ✓ Condition for the general inequality to hold:

$$\max \sum_{x_1,\dots,x_N=1}^{2} T_{x_1 \dots x_N}^{2} \leq 1$$

maximization is taken over all local measurement directions

- ✓ For more than 2 parties this condition is only the necessary one
  violation of it implies violation of the inequality

- ✓ There are pure entangled states which do not violate the inequality

❑ M. Zukowski, C. Brukner, W. Laskowski, and M. Wiesniak, Phys. Rev. Lett. **88**, 210402 (2002).

More Settings

✓ Three-particle 4x4x2 case

$$A_{12,S'} \equiv \sum_{s_1,s_2=\pm 1} S'(s_1,s_2)(A_1 + s_1 A_2)(B_1 + s_2 B_2) = \pm 4$$

$$A_{34,S''} \equiv \sum_{s_1,s_2=\pm 1} S''(s_1,s_2)(A_3 + s_1 A_4)(B_3 + s_2 B_4) = \pm 4$$

$$\sum_{s_1,s_2=\pm 1} S(s_1,s_2)(A_{12,S'} + s_1 A_{34,S''})(C_1 + s_2 C_2) = \pm 16$$

✓ There are $(2^4)^3$ inequalities
they are generated by one inequality with S,S′,S′′ non-factorable

✓ Incomplete set

✓ Tight inequalities

❑ W. Laskowski, TP, M. Zukowski, and C. Brukner, Phys. Rev. Lett. **93**, 200401 (2004).

More Nonclassical States

- ✓ N&S for the 4x4x2 inequality to hold:

$$\max \sum_{m=1}^{2} \sum_{k_m, l_m=1}^{2} T_{k_m l_m m}^2 \leq 1$$

- ✓ Necessary for the 2x2x2 case:

$$\max \sum_{k,l,m=1}^{2} T_{klm}^2 \leq 1$$

- ✓ New condition is more demanding
  the settings k_1, l_1 do not have to be equal to k_2, l_2

- ✓ The generalized GHZ states
  violate new inequality for all alpha and arbitrary number of particles

$$|\psi\rangle = \cos\alpha\, |0...0\rangle + \sin\alpha\, |1...1\rangle$$