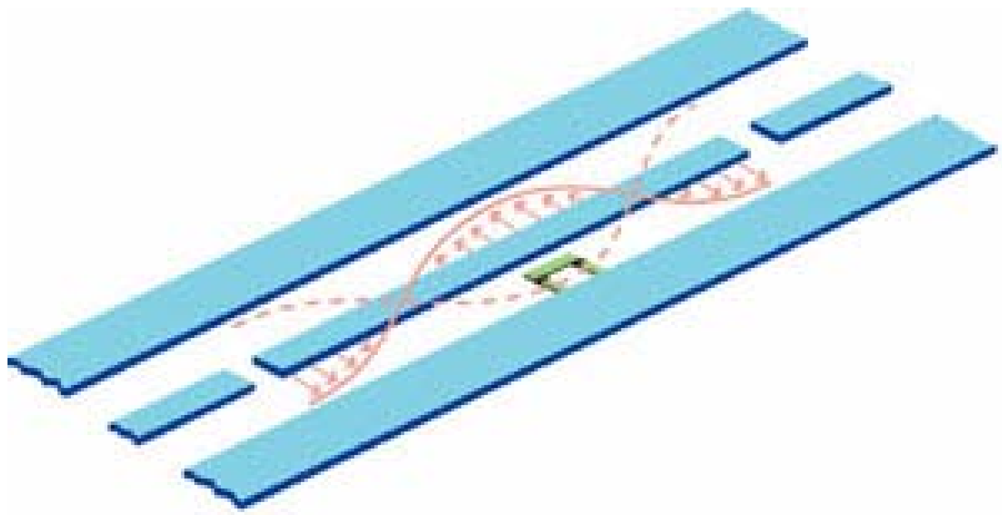
Cavity QED on a Chip



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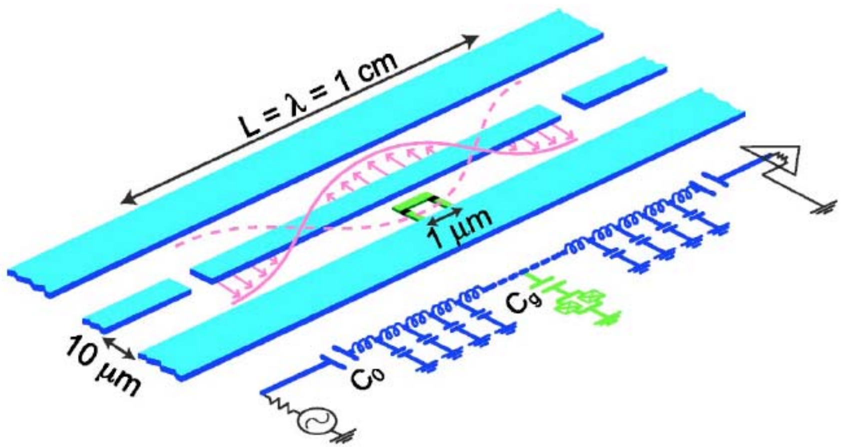
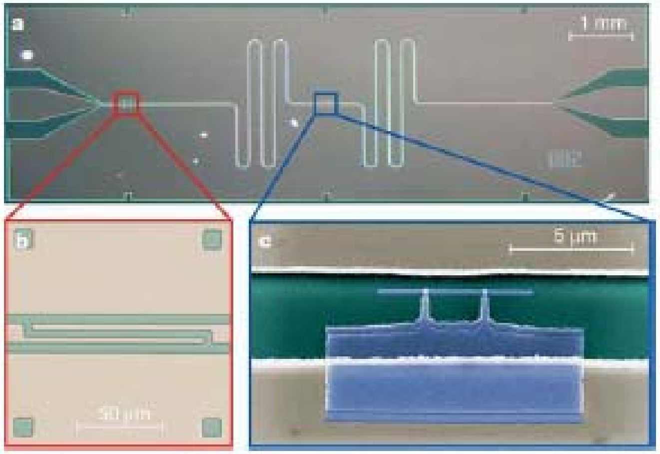
Experimental System - Numbers to keep in mind

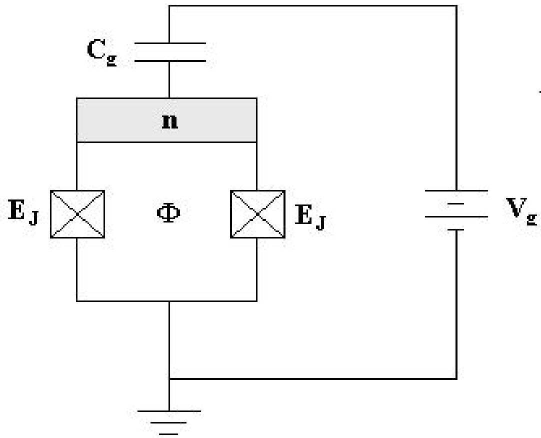




Parameter	Symbol	3D optical	3D microwave	1D circuit
Resonance or transition frequency	$\omega_r/2\pi$, $\Omega/2\pi$	350 THz	51 GHz	10 GHz
Vacuum Rabi frequency	g/π, g/ω_r	220 MHz, 3×10^{-7}	47 kHz, 1×10^{-7}	100 MHz, 5×10^{-3}
Transition dipole	d/ea_0	~1	1×10^{3}	2×10^{4}
Cavity lifetime	$1/\kappa, Q$	10 ns, 3×10^{7}	1 ms, 3×10^{8}	160 ns, 10^{4}
Atom lifetime	$1/\gamma$	61 ns	30 ms	2 μs
Atom transit time	t_{transit}	$\geqslant50$ μs	100 μs	∞
Critical atom number	$N_0=2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	$\leqslant6\times10^{-5}$
Critical photon number	$m_0=\gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	$\leqslant1\times10^{-6}$
Number of vacuum Rabi flops	$n_{\text{Rabi}}=2g/(\kappa+\gamma)$	~10	~5	$\sim10^{2}$

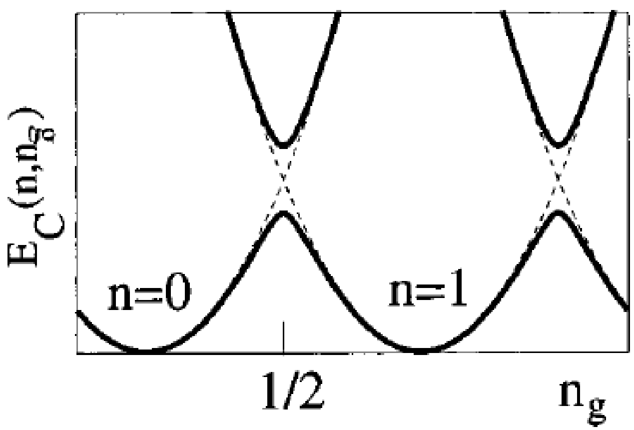
Superconducting Qubit

Charge Type



$$H_Q = 4E_c \sum_N (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \sum_N (|N+1\rangle\langle N| + \text{H.c.})$$

Where: $N_g = C_g V_g / 2e$



Two Level System

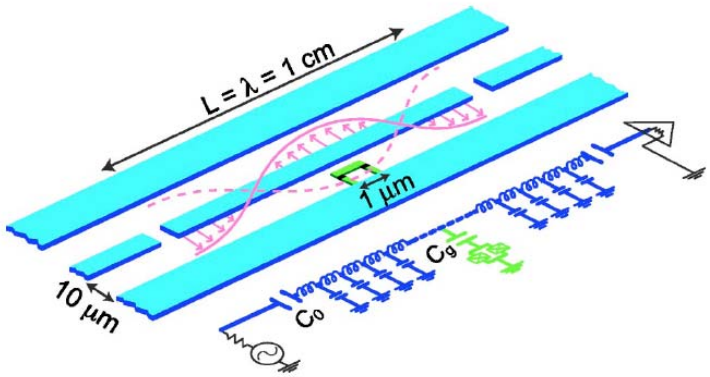
$$H_Q = -\frac{E_{\text{el}}}{2}\overline{\sigma^z} - \frac{E_J}{2}\overline{\sigma^x}$$

Qubit + Cavity: Jaynes-Cummings

$V_g \rightarrow V_g + V_{AC}$

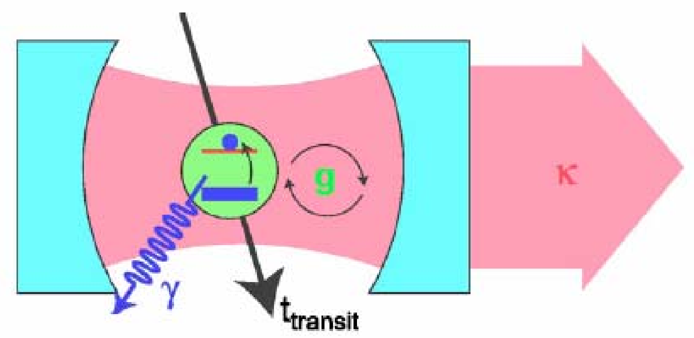
$$\hat{V}_{AC}(t) = \sqrt{\frac{\hbar\omega}{Lc}}[\hat{a}(t) + \hat{a}^\dagger(t)].$$

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z - e\frac{C_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_r}{Lc}}(a^\dagger + a)$$
$$\times[1 - 2N_g - \cos(\theta)\sigma^z + \sin(\theta)\sigma^x].$$



Where: $\Omega = \sqrt{E_J^2 + [4E_C(1-2N_g^{dc})]^2}/\hbar$

$\theta = \arctan[E_J/4E_C(1-2N_g^{dc})]$



Setting N_g =1/2:

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a)$$

Readout Scheme:

Dispersive Regime:

$$UHU^{\dagger} \approx \hbar\left[\omega_{\mathrm{r}} + \frac{g^2}{\Delta}\sigma^z\right]a^{\dagger}a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma^z$$

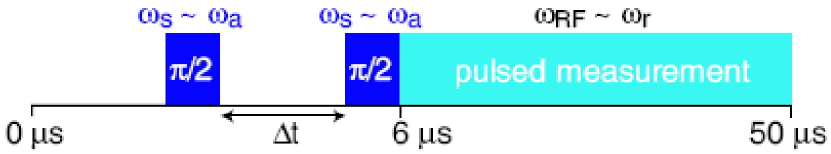
Phase Measurement (QND Measurement for the “atom”):

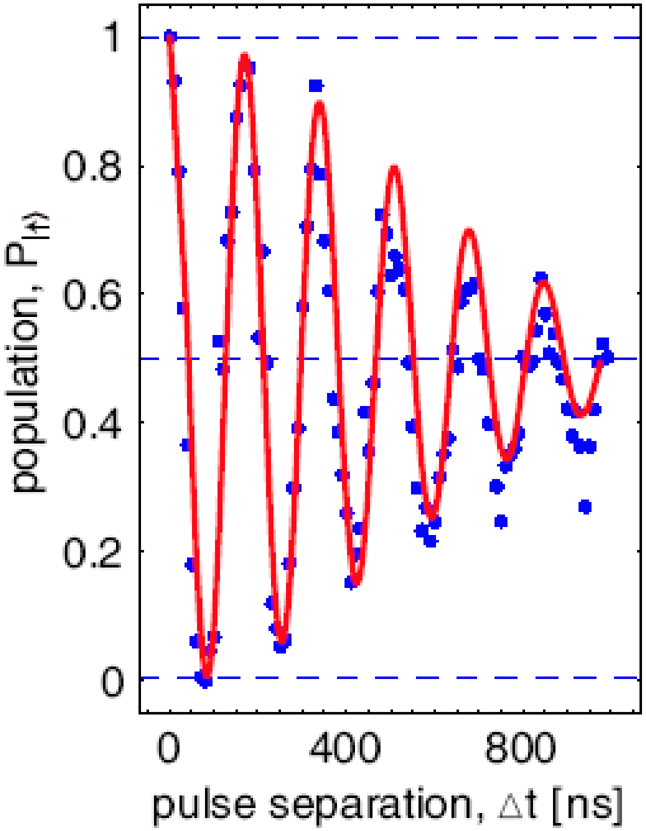
External Driving Field $\omega_{\mu w} = \omega_r$ $$H_{\mu w}(t) = \hbar\varepsilon(t)(a^{\dagger}e^{-i\omega_{\mu\mathrm{w}}t} + ae^{+i\omega_{\mu\mathrm{w}}t})$$

$$(a|e\rangle + b|g\rangle) \otimes |\alpha\rangle \rightarrow a|e, e^{i\phi}\alpha\rangle + b|g, e^{-i\phi}\alpha\rangle$$

where $\phi = g^2/\kappa\Delta$

•Example: Ramsey Fringes.





$T_2 = 500$ ns.

References :

- Cavity quantum eletrodynamics for superconducting eletrical circuits: An architeture for quantum computation. PRA **69**, 062320 (2004).
- Prospects for Strong Cavity Quantum Eletrodynamics with Superconducting Circuits. Cond-mat/0310670.
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- Approaching Unit Visibility for Control of a Superconducting Qubit with a Dispersive Readout. Cond-mat/0502645.
- Schoelkopf’s group at Yale: http://www.eng.yale.edu/rslab/