Measuring chaotic scattering with canonically deformed detectors

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Introduction (First classical system)

Hamiltonian

We study a scattering system with one degree of freedom and a periodic time dependence given by the Hamiltonian

$$H(q,p,t) = \frac{p^2}{2} + A\,V(q)\sum_{n=-\infty}^{\infty}\delta(t-n).$$

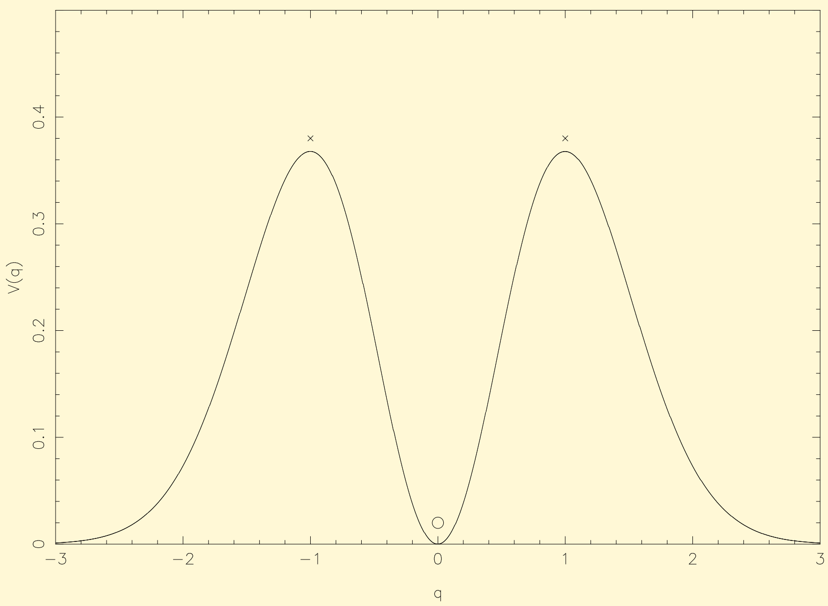
It is a periodically kicked system. Here q is the position and p is the momentum of the particle at time t. $V(q)$ indicates the potential function of the kick and A its strength. The dynamics can be represented by the stroboscopic map taken at times $t = 1/2$, *i.e.* always in the middle between two consecutive kicks. The map is given by

$$p_{n+1} = p_n - AV'(q_n + p_n/2)$$
$$q_{n+1} = q_n + \tfrac{1}{2}(p_n + p_{n+1})$$

Potential 1

This potential will be useful to explain some of the characteristics of these type of systems

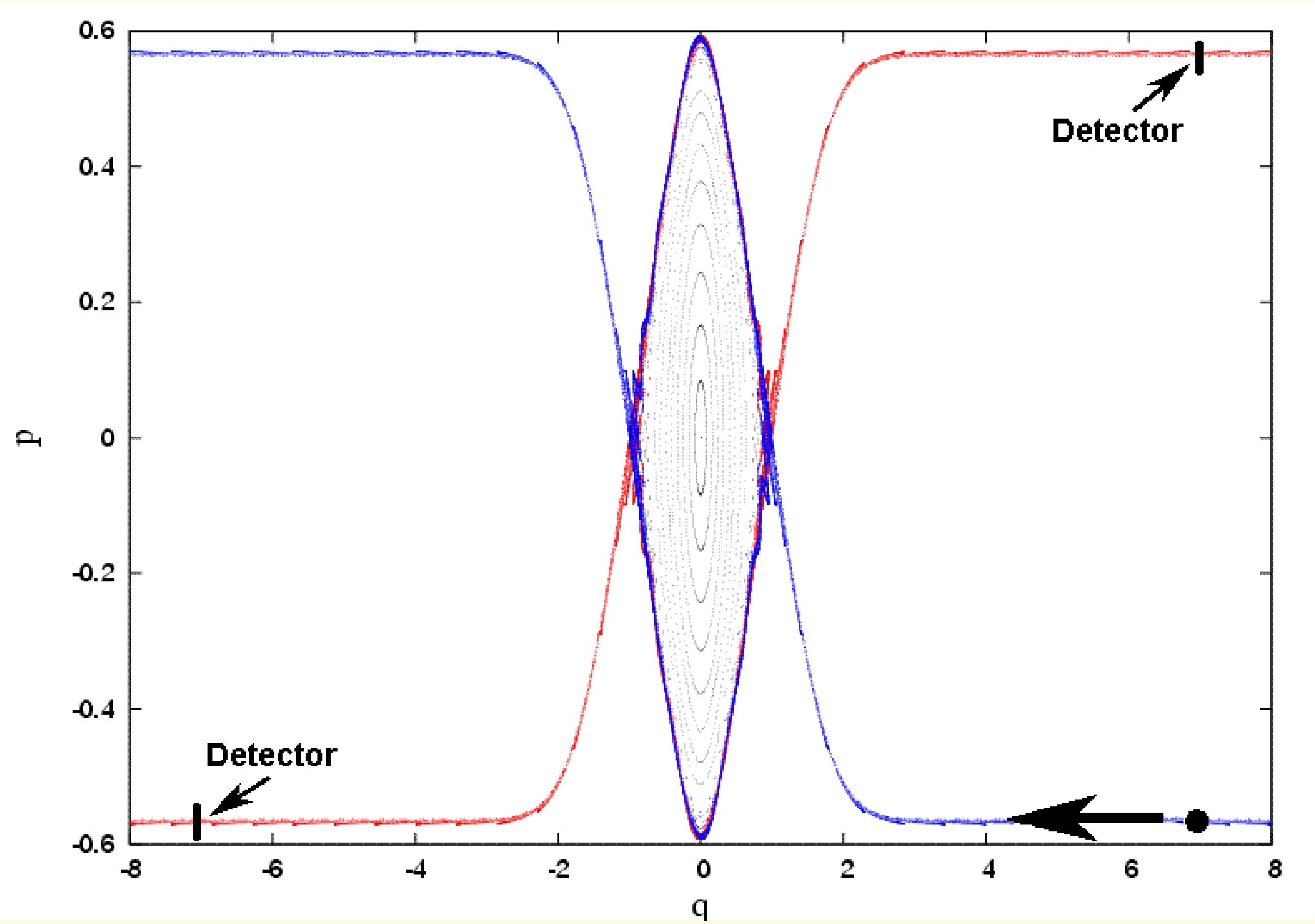
$$V_1(q) = q^2 e^{-q^2}$$



With this potential we have three fixed points in the Poincaré map: an elliptic one, between two hyperbolic.

Poincaré section

The Poincaré section generated with the stroboscopic map and the previous potential

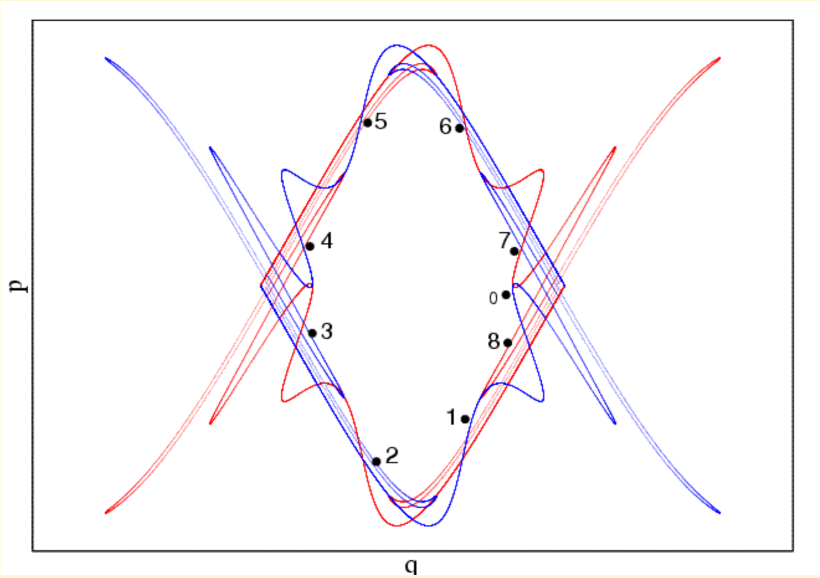


we measure the time when the outgoing particle passes a certain position (detector).

Orbital period

In previous work [a] a relation is established between the degree of development of the system, represented by a parameter γ and the orbital period τ of the trajectories around the center fixed point in the Poincaré map

$$\tau = -2\log_3(\gamma) + 3/2$$

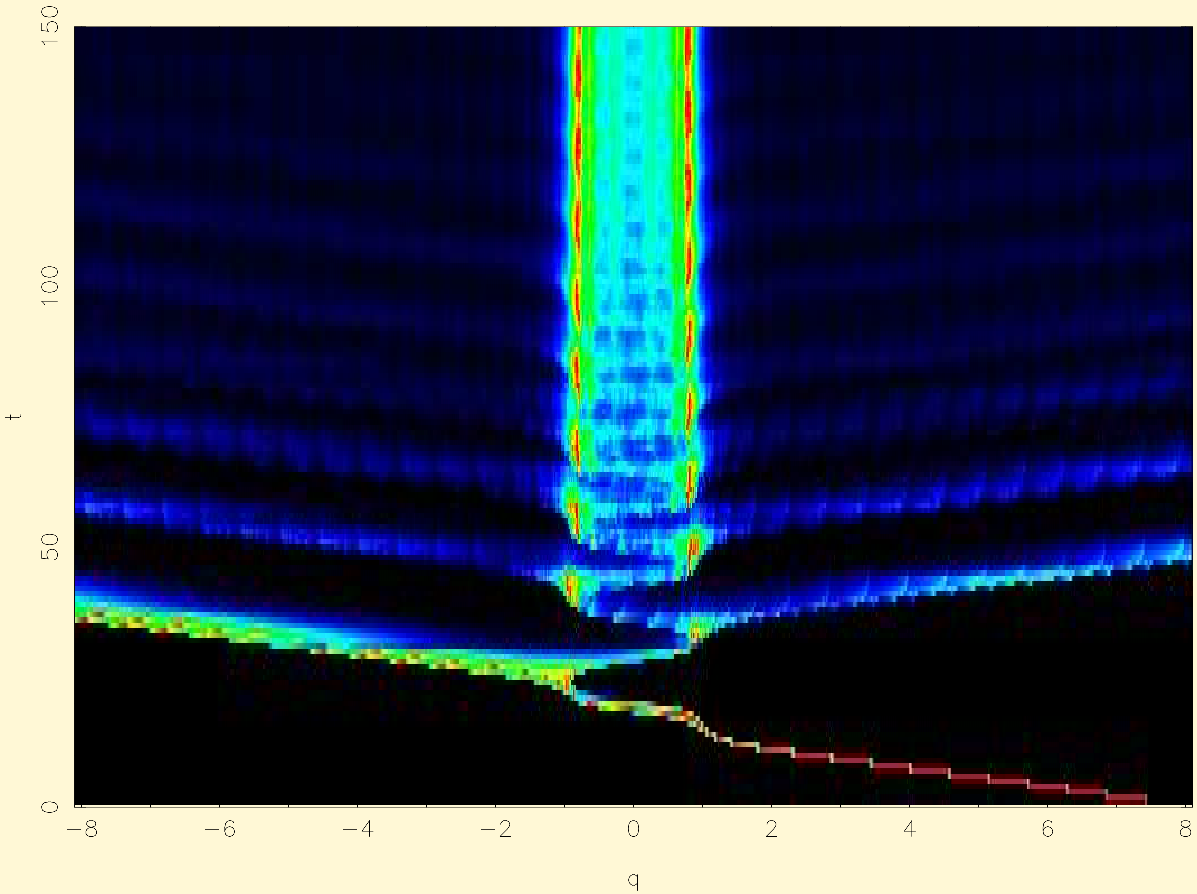


The figure shows an example of a typical trajectory. In this case $\gamma = 3^{-3}$. It can be observed that $7 < \tau < 8$.

Remark: the degree of development of the system we study corresponds to $\gamma = 3^{-7}$.

[a] C. Jung, C. Mejia-Monasterio, O. Merlo y T.H. Seligamn, *New. J. Phys.* **6**, 48 (2004)

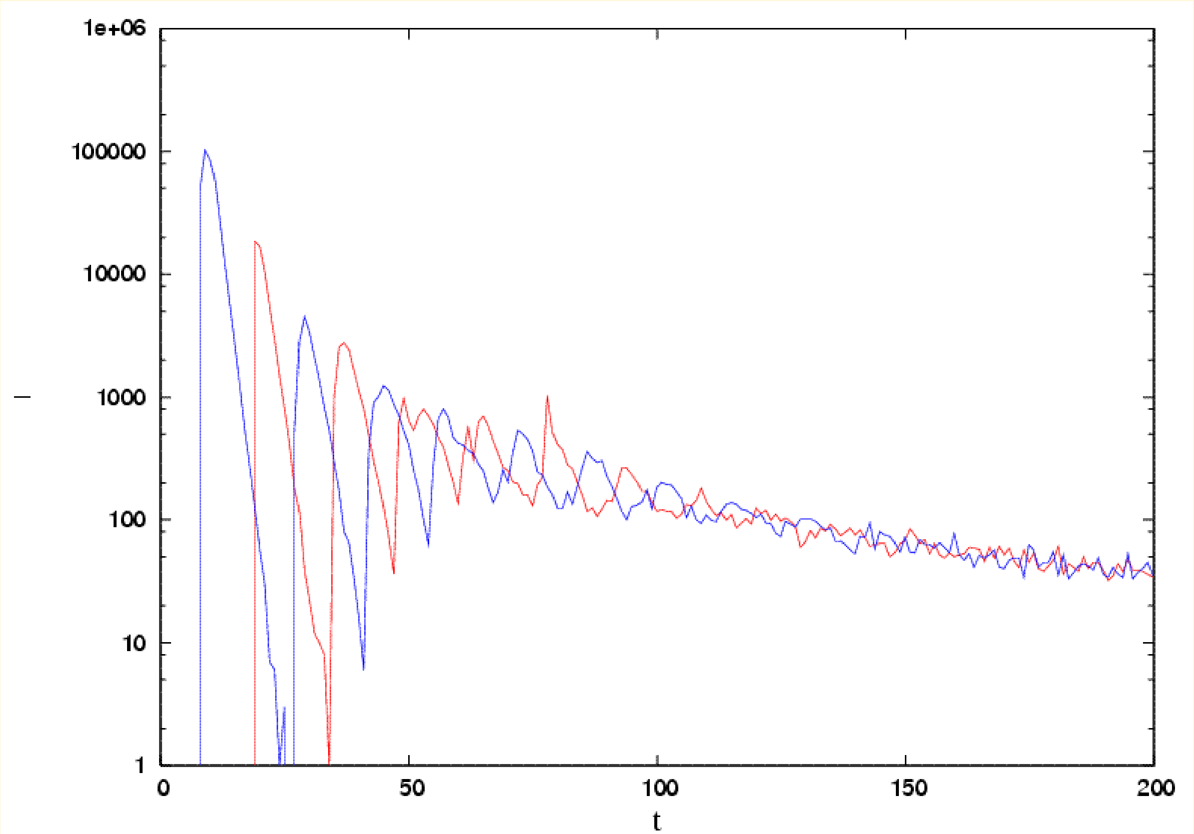
Classical density distribution



number of particles around q at time t.

Signal

The signal I(t) obtained by the detectors in the asymptotic region

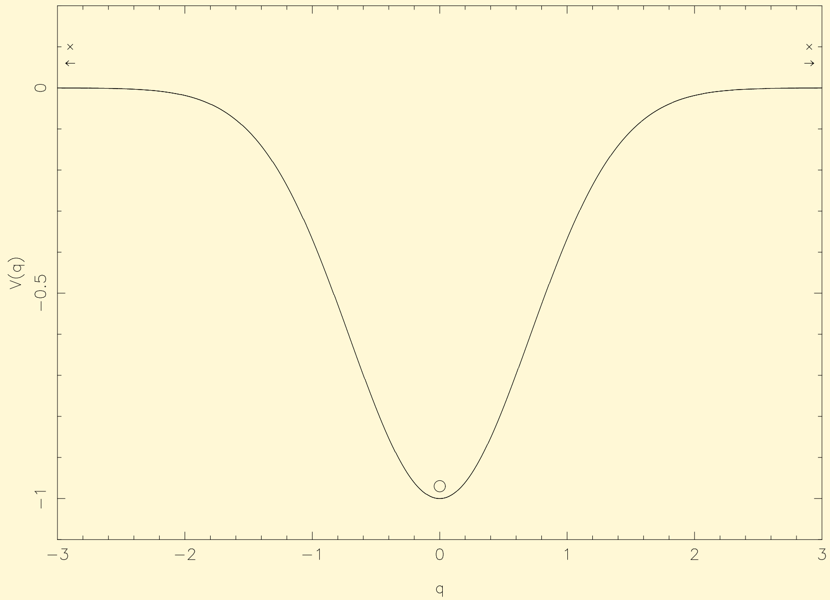


Blue one comes from the detector in the left and the red one from the detector in the right. This self-pulsing effect of the outgoing trajectories is known as *scattering echoes*.

Second classical system

The main part of this job was the study of a purely attractive potential. For this purpose we used

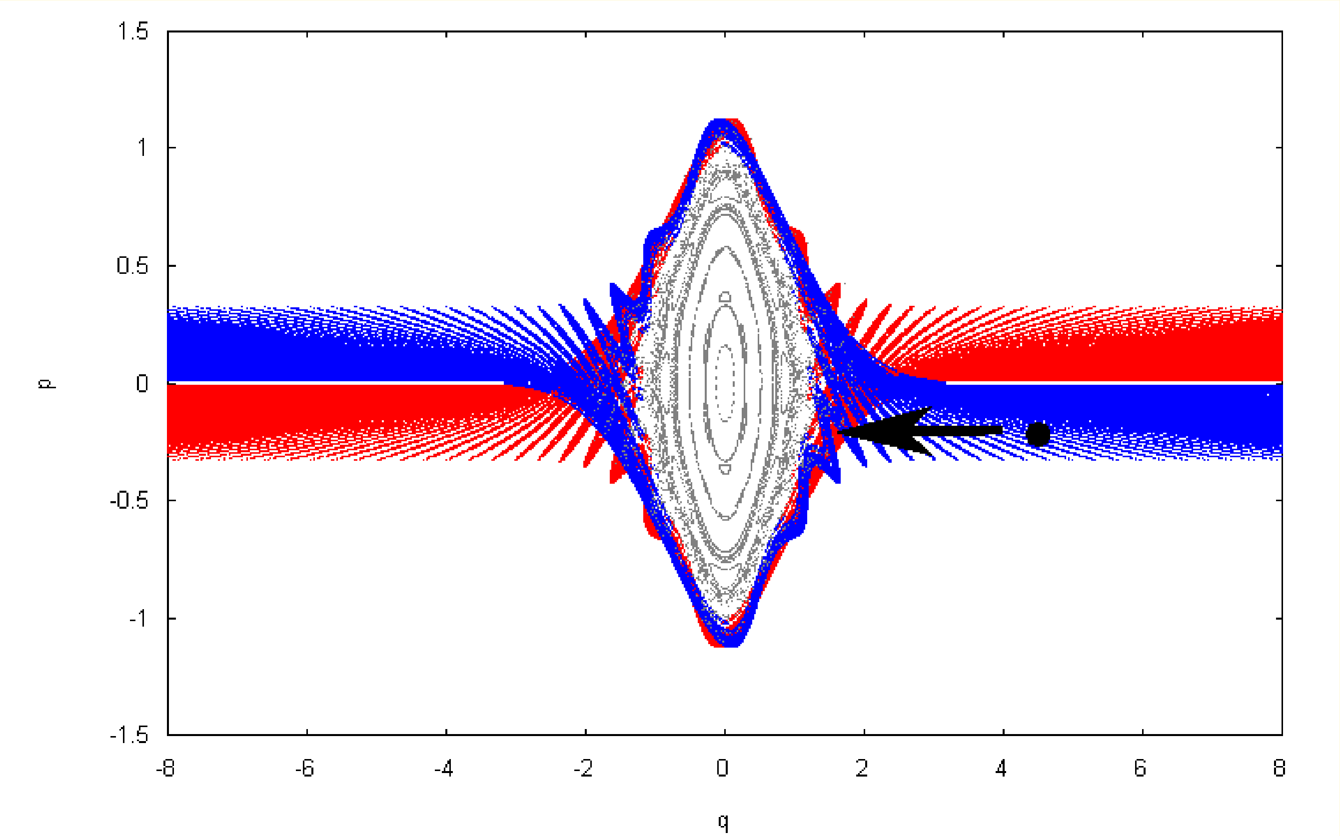
$$V_2(q) = -e^{-q^2}$$



The main difference of this potential si that the outer fixed points sit at infinity.

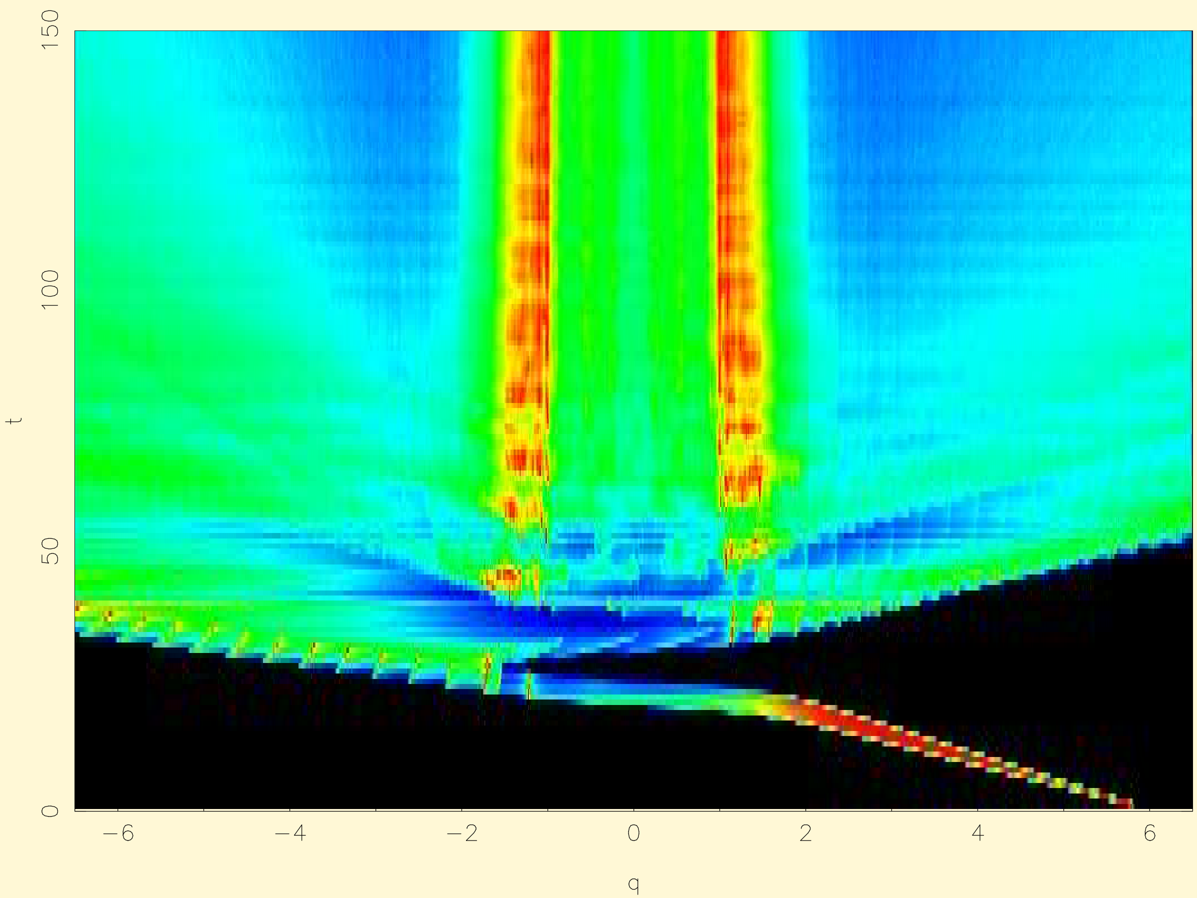
Poincaré section

The Poincaré section generated with the stroboscopic map and the previous potential



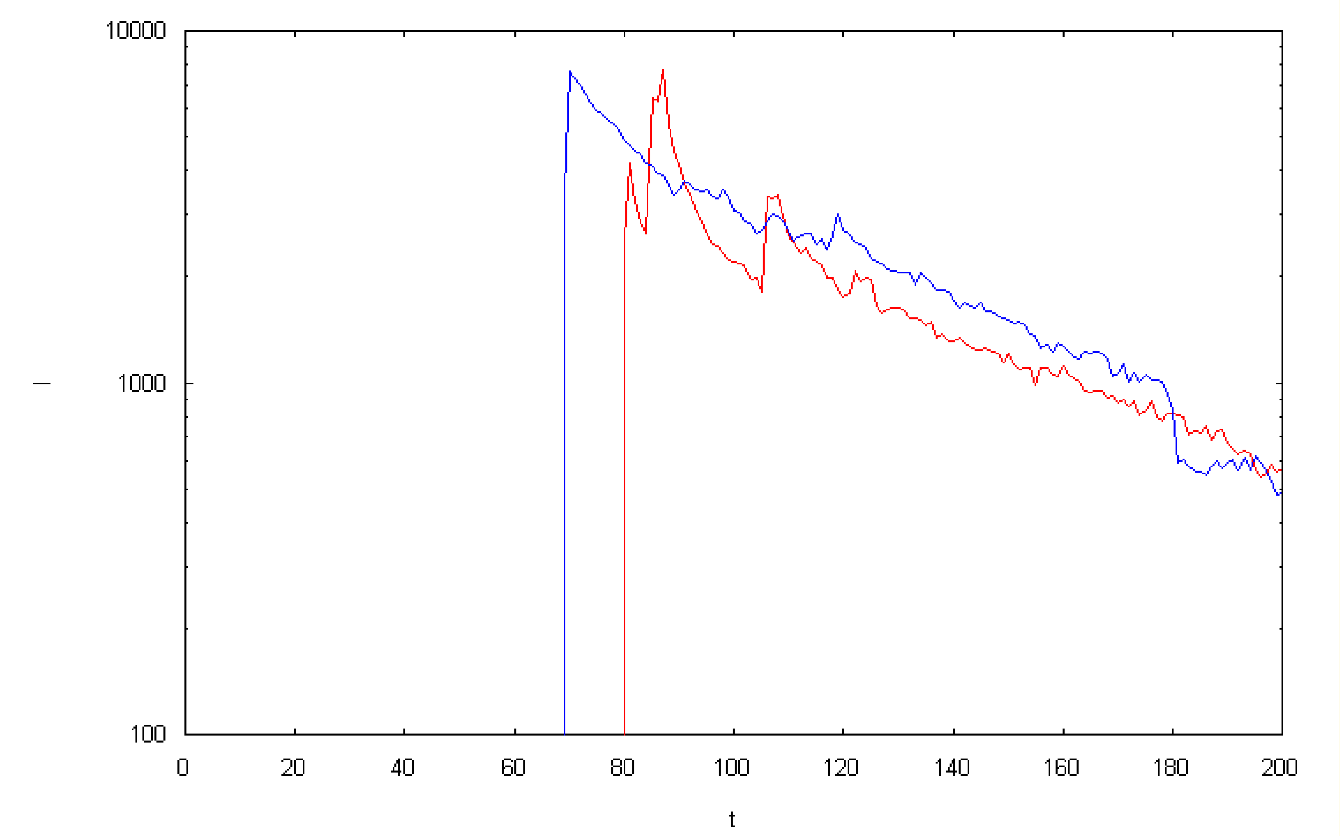
Notice how the manifolds reach values of cero momentum.

Classical density distribution



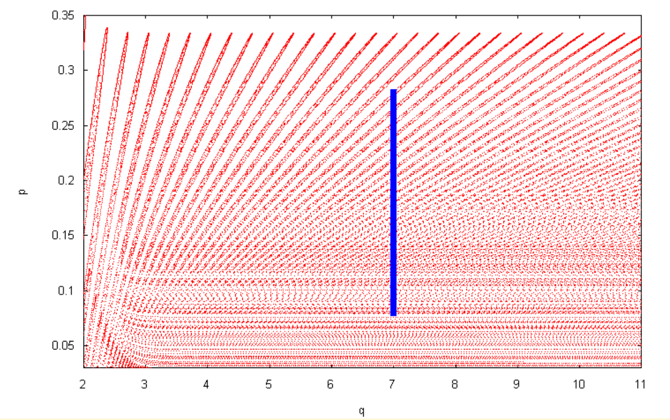
number of particles around q at time t.

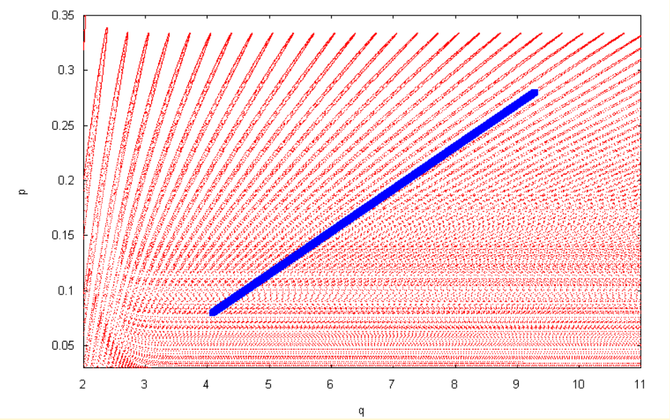
Signal



The signal corresponds to the time when the outgoing particle passes a certain position (detector). A new kind of experiment should be performed.

Solution





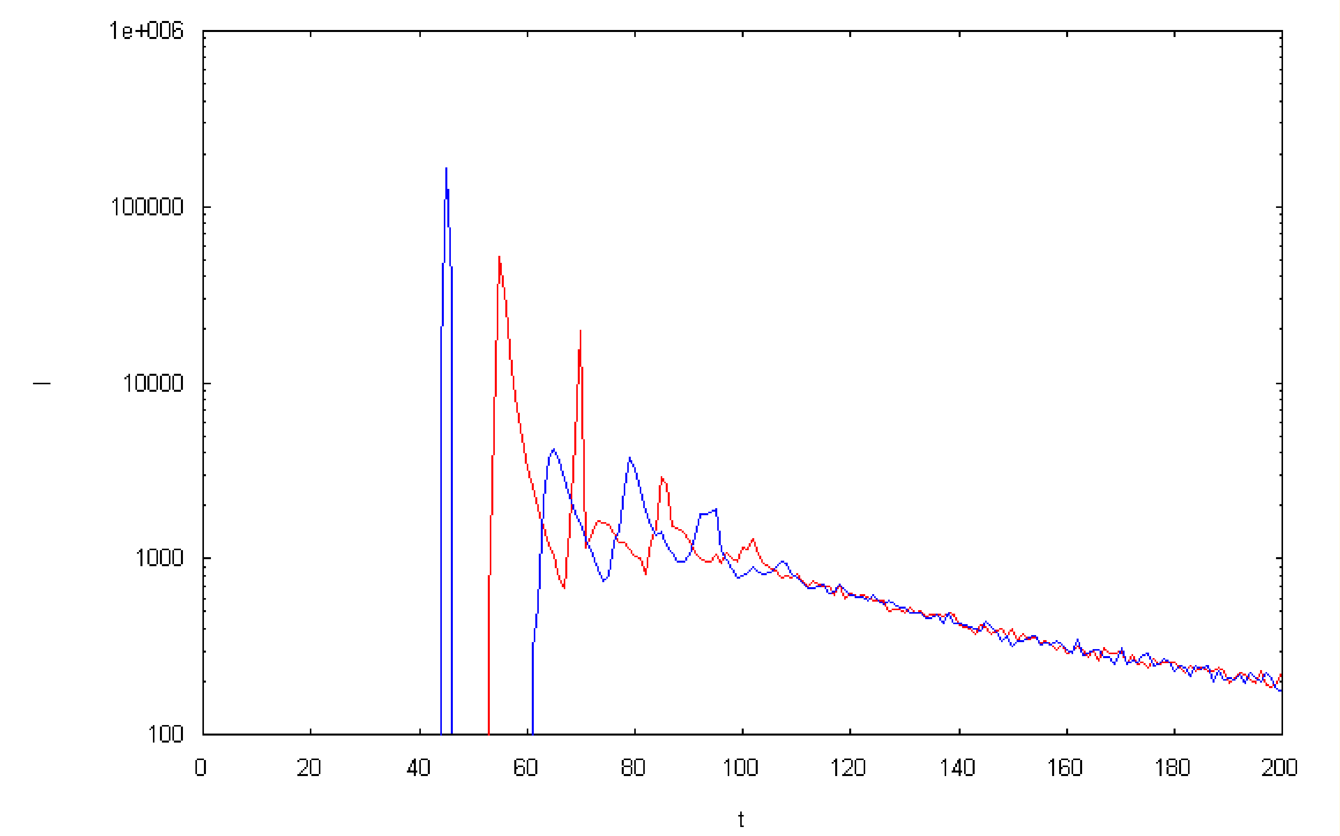
To deal with this problem, we propose a detector, whose sensitivity depends of an appropriate combination of position and momentum. In the present case we choose

$$P = p$$
$$Q = q + f(p)$$

we used $f(p) = 2\lambda p$, a linear function which was found to work almost as well as a nonlinear one[a].

[a] C. Jung, T.H. Seligman and J.M. Torres, *JNMP*, **12**, 404 (2005)

Signal with the transformed detector



Signals corresponding to the canonically transformed detectors. (CTD).

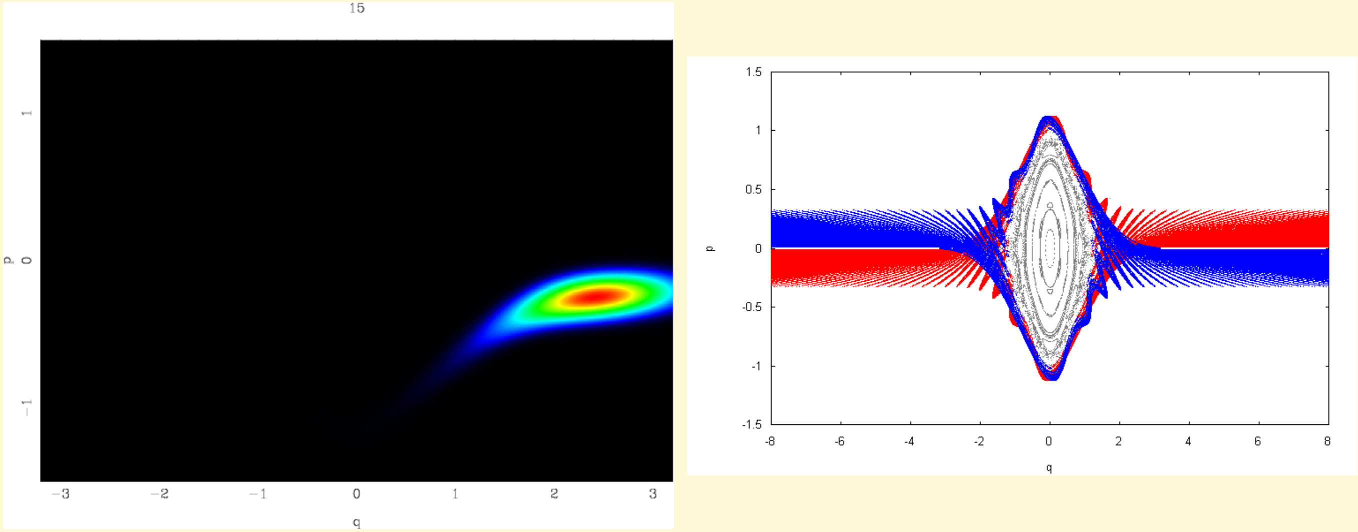
Quantum system

The dynamics

For the quantum system, we use a wave packet dynamics, with a minimum uncertainty wave packet $\Psi(q, t = 0)$ as initial condition. The unitary time evolution operator for kicked systems is used

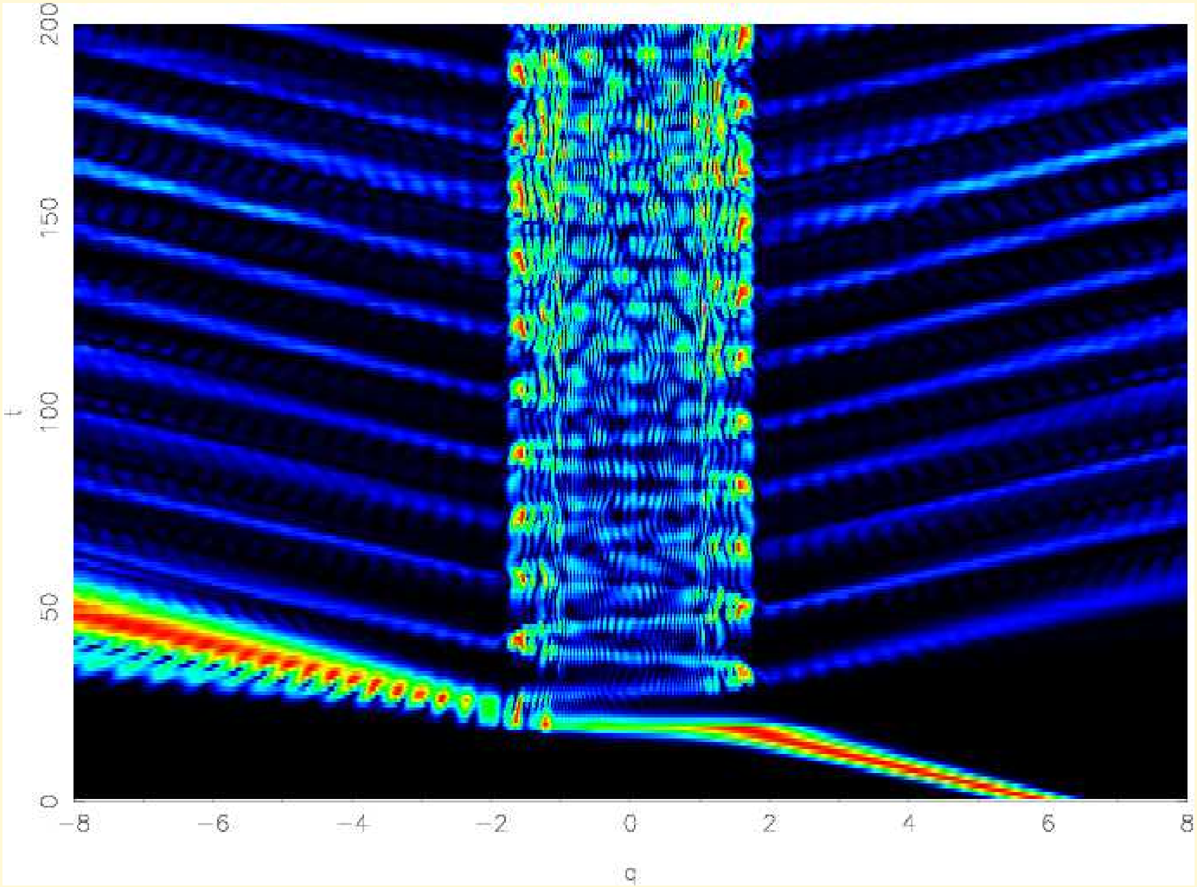
$$\hat{U} = e^{-ip^2/4\hbar} e^{-iV(q)/\hbar} e^{-ip^2/4\hbar}$$

The potential $V(q) = \exp -q^2$ is used (second classical system).



Husimi distribution of the incident wave packet. At the right the corresponding Poincaré section of the analog classical system.

Quantum density distribution

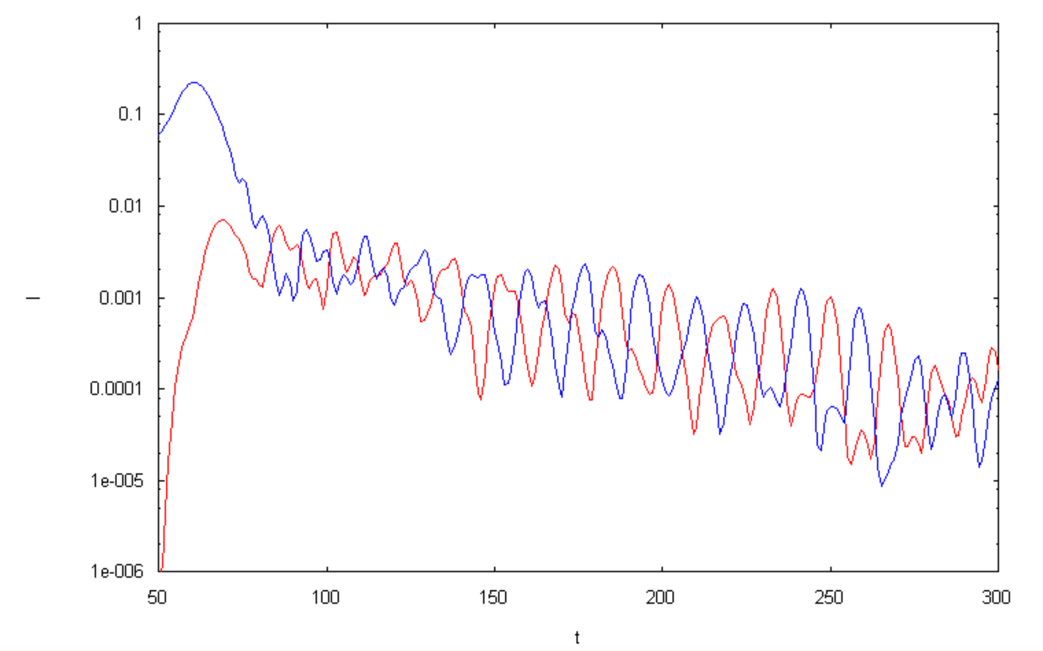


$$|\Psi(q,t)|^2$$

Signal

To measure the quantum scattering echoes we use the overlap of the evolved $\Psi(q,t)$ with a minimum uncertainty wave packet $\psi_d(q)$ placed in the asymptotic region

$$I(t) = |\int_{-\infty}^{\infty} \psi_d^*(q)\Psi(q,t)dq|^2$$



Signals I(t) corresponding to the overlap with $\psi_d(q)$ as a minimum uncertainty wave packet.

Canonically deformed wave packet

We introduce another way to perform the measurement by applying the unitary representation of the canonical transformation to the minimum uncertainty wave packet to obtain the wave packet to be used as a detector in the new position coordinates

$$\psi_{ctd}(Q) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{-iG(p,Q)/\hbar} dp$$

where $\phi(p)$ is a minimum uncertainty wave packet in momentum space and

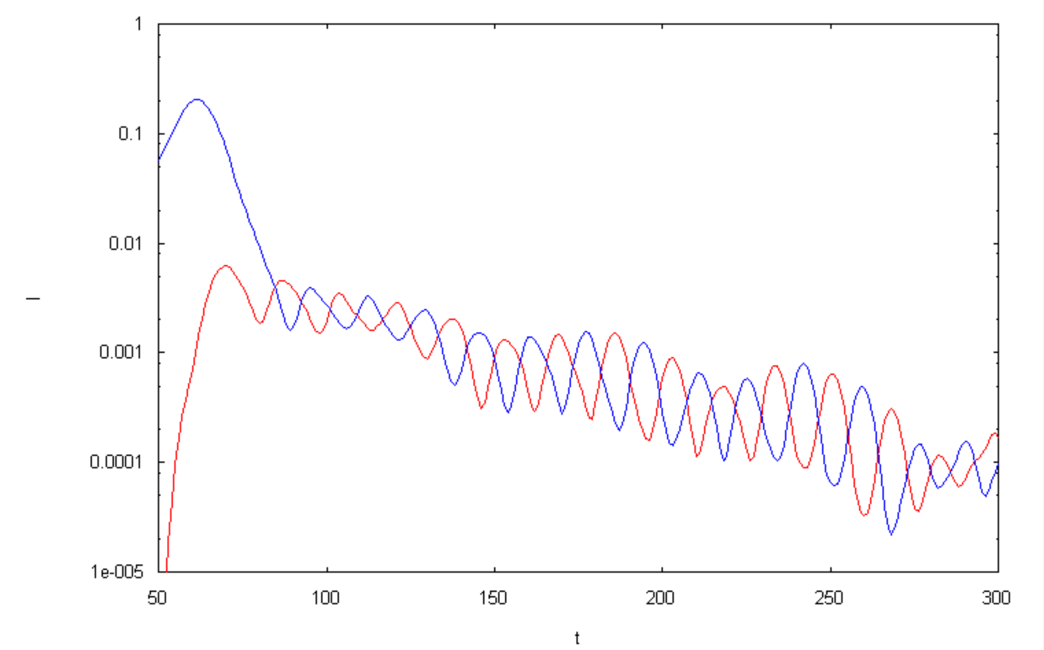
$$G(p,Q) = -Qp + \int^{p} f(s)ds$$

is the generating function of the canonical transformation used in the classical system. For the linear transformation we have simply the form $G(p,Q) = -Qp + \lambda p^2$.

Signal

Now we use the overlap of the evolved $\Psi(q,t)$ with the deformed wave packet $\psi_{ctd}(q)$ placed in the asymptotic region

$$I(t) = \left| \int_{-\infty}^{\infty} \psi^*_{ctd}(q)\Psi(q,t)dq \right|^2$$



Signals I(t) corresponding to the overlap with with $\psi_d(q)$ as a canonically deformed wave packet.

Conclusions

Conclusions

1. The use of a canonically transformed detector in classical mechanics allowed us to measure the scattering echoes that couldn't be measure in the usual way.
2. In quantum mechanics the appearance of the echoes is more evident.
3. We show that the overlap with the canonically deformed wave packet can be used to get a better signal of the echoes.

Thank you!