Suppression of fluorescence in a lossless cavity

A survey of the work by Alsing et al, PRA'92

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What we will do

- We will review the paper by P.M. Alsing, D.A.Cardimona and H. J. Carmichael, Phys. Rev. A **45**, 1793 (1992) :

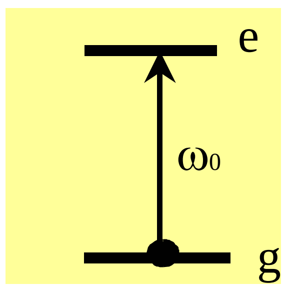
  Investigation of the behaviour of a two-level atom interacting with both
  (i) a single mode of the e.m. field in a lossless cavity and
  (ii) an external laser field.

- We will discuss possible extensions of this work.

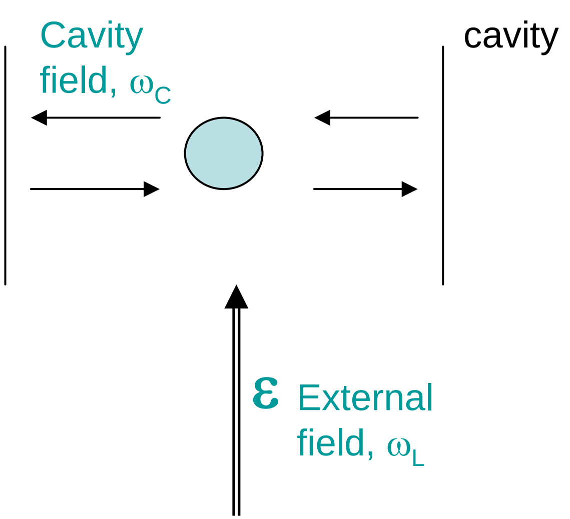
Outline:

- Description of the system
- Hamiltonian and evolution of the system
- Semiclassical equations
  - Atomic polarization
  - Mean intracavity field
- Steady-state solutions
  - In free space
  - In the cavity
- Conditions for a reduction in the fluorescence intensity
- Perspective: Cavity field state if the atomic motion and the spatial mode function are considered

DESCRIPTION OF THE SYSTEM



$\omega_0 = \omega_C = \omega_L$



1 two-level atom

2 single quantized cavity mode

3 Classical coherent external field

SYSTEM DYNAMICS

Interaction atom-cavity mode: Jaynes-Cummings Hamiltonian

$$\hat{H} = \underbrace{\frac{1}{2}\omega_0 \hat{\sigma}_z}_{\text{atom}} + \underbrace{\omega_0 \hat{a}^\dagger \hat{a}}_{\text{cavity}} + \underbrace{ig\left(\hat{a}^\dagger \hat{\sigma}_- - \hat{a}\hat{\sigma}_+\right)}_{\text{interaction}}$$

Master equation:

$$\frac{d\hat{\rho}}{dt} = -i\left[\hat{H}, \hat{\rho}\right] + \underbrace{\varepsilon\left[\hat{\sigma}_- e^{i\omega_0 t} - \hat{\sigma}_+ e^{-i\omega_0 t}, \hat{\rho}\right]}_{\text{interaction laser-atom}}$$

$$+ \underbrace{\frac{\gamma}{2}\left(2\hat{\sigma}_- \hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} - \hat{\rho}\hat{\sigma}_+ \hat{\sigma}_-\right)}_{\text{spontaneous emission}}$$

$$+ \underbrace{\kappa\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger \hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger \hat{a}\right)}_{\text{cavity decay}}$$

The atom is damped at rate γ by spontaneous emission

The cavity mode is damped at rate 2κ through the mirrors transmission

SEMICLASSICAL EQUATIONS

Mean intracavity field:

$$\alpha = \langle \hat{a} \rangle$$

$$\frac{d}{dt}\alpha = -\kappa\alpha + g(x)\,\nu$$

Atomic polarization:

$$\nu = \langle \sigma_- \rangle$$

$$\frac{d}{dt}\nu = -\frac{\gamma}{2}\nu - \left(g(x)\,\alpha + \varepsilon\right)$$

ATOMIC POLARIZATION

Steady-state solutions:
[below saturation]

In free space

$$\nu_f = -\frac{\varepsilon}{\gamma/2}$$

• Intensity of the fluorescence proportional to $(2\varepsilon/\gamma)^2$

In the cavity

$$\nu_c = \frac{-\dfrac{\varepsilon}{\gamma/2}}{1+\dfrac{2g^2}{\kappa\gamma}}$$

• Intensity propotional to

$$\left(\frac{\dfrac{2\varepsilon}{\gamma}}{1+\dfrac{2g^2}{\kappa\gamma}}\right)^2$$

Modification factor

$$\left(1+\frac{2g^2}{\kappa\gamma}\right)^{-2}$$

SUPPRESSION OF FLUORESCENCE

$$|\nu_c|^2 = \left(\frac{\frac{2\varepsilon}{\gamma}}{1 + \frac{2g^2}{\kappa\gamma}} \right)^2$$

when $\boxed{\frac{2g^2}{\kappa\gamma} >> 1}$

then $\Longrightarrow$ $$\nu_c = \frac{-\frac{\varepsilon}{\gamma/2}}{1+\frac{2g^2}{\kappa\gamma}} \longrightarrow 0$$

THE CAVITY FIELD

In the cavity, the steady-state solution of the mean intracavity field α is:

$$\alpha_c = \frac{g}{\kappa} v_c$$

and when

$$\frac{2g^2}{\kappa\gamma} >> 1$$

then $\Longrightarrow$

$$\alpha_c \rightarrow -\varepsilon / g$$

And this result $\left(\alpha_c \rightarrow -\varepsilon / g\right)$ means the situation where we have a suppression of the total field driving the atom:

$$\frac{d}{dt} v = -\frac{\gamma}{2} v - \underbrace{\left(g(x)\,\alpha + \varepsilon\right)}_{= \, 0}$$

SUMMARY

- One can achieve suppression of fluorescence in a cavity if

$$\frac{g^2}{\kappa\gamma} >> 1 \quad \text{(large cooperativity)}$$

- These results have been discussed in:

1 P.M. Alsing, D.A. Cardimona,H.J. Carmichael
Phys. Rev. A, **45**, 1793 (1992) [single atom, no motion, no spatial mode function]

2 S. Zippilli, G. Morigi, H.Ritsch, Phys. Rev. Lett. **93** (2004)
[generalization to spatial mode function, collective effects]

NEXT STEP

Atomic motion, spatial mode function

how does the cavity state changes?

$$\alpha_c = -\varepsilon / g(x) \qquad\qquad \nu_c = \frac{-\dfrac{\varepsilon}{\gamma/2}}{1+\dfrac{2g(x)^2}{\kappa\gamma}}$$

If $g(x) = g_0 \cos kx$ and $\cos kx = 0$

$\Longrightarrow$ This solution is NOT valid at the nodes of the cavity mode

- What happens if we move slowly (adiabatically) the atom from an antinode $(\cos kx = 1)$ to a node $(\cos kx = 0)$?