Vacuum Entanglement

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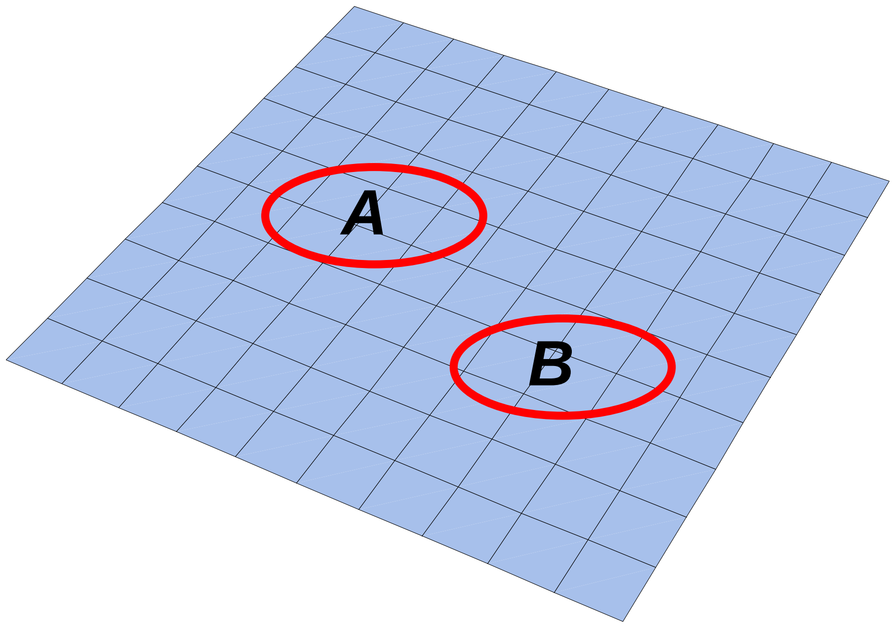
International summer school on
Quantum Information
August 29 – September 30,2005

Vacuum Entanglement



Motivation:

QI: natural set up to study Ent
causal structure ! LO.
many body Ent.

Q. Phys.: Can Ent. shed light on "quantum effects"? (low temp. Q. coherences, Q. phase transitions, DMRG, Entropy Area law.)

Background

Continuum results:

BH Entanglement entropy:

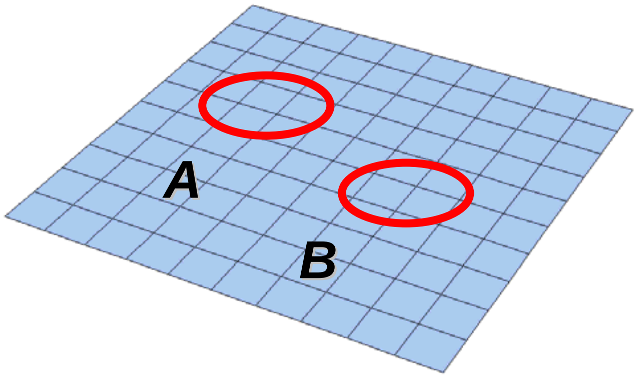
Unruh (76), Bombelli et. Al. (86), Srednicki (93), Callan & Wilczek (94) .

Algebraic Field Theory:

Summers & Werner (85), Halvarson & Clifton (00),Verch & Werner (2004).

Discrete models:

Spin chains: *Wootters (01), Nielsen (02), Latorre et. al. (03).*



(I) Are A and B entangled?

Yes, for arbitrary separation.
("Atom probes”).

(II) Are Bell’s inequalities violated?

Yes, for arbitrary separation.
(Filtration, “hidden” non-locality).

(III) Where does it “come from”?

Localization, shielding.
(Harmonic Chain).

(IV) Can we detect it?

Entanglement Swapping.
(Linear Ion trap).

Outline :

(1). Field entanglement: local probes.

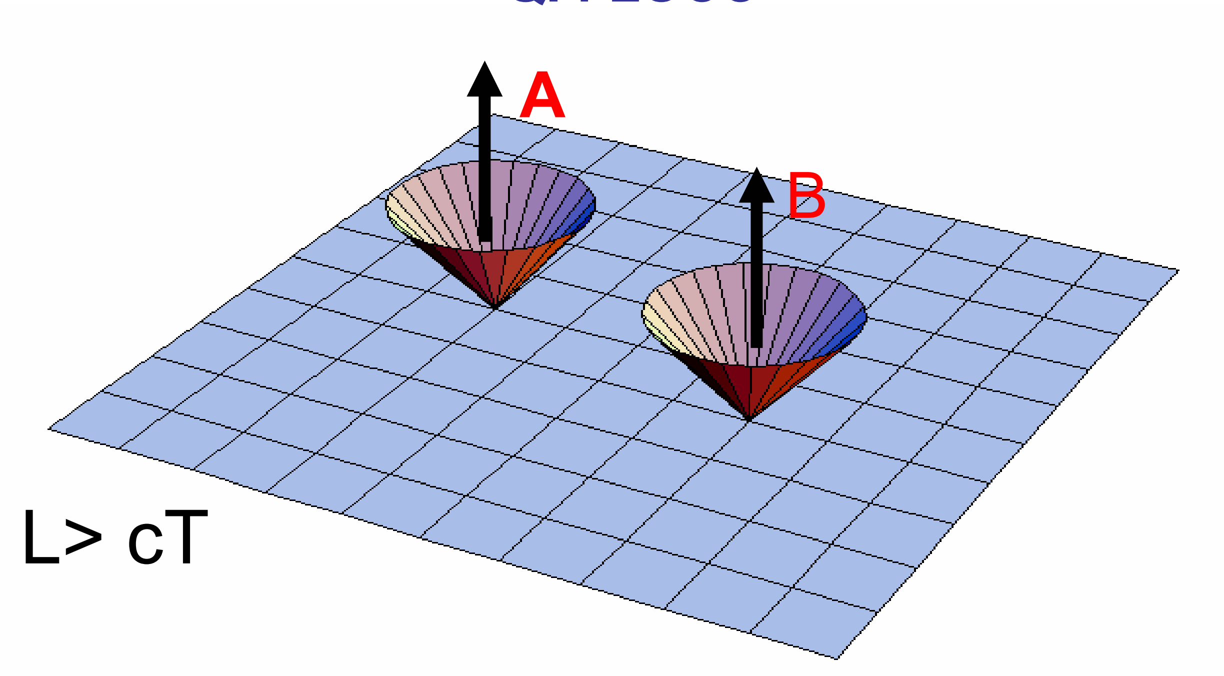
(2). Linear Ion trap: detection of ground state ent.

Probing Field Entanglement

RFT→ Causal structure

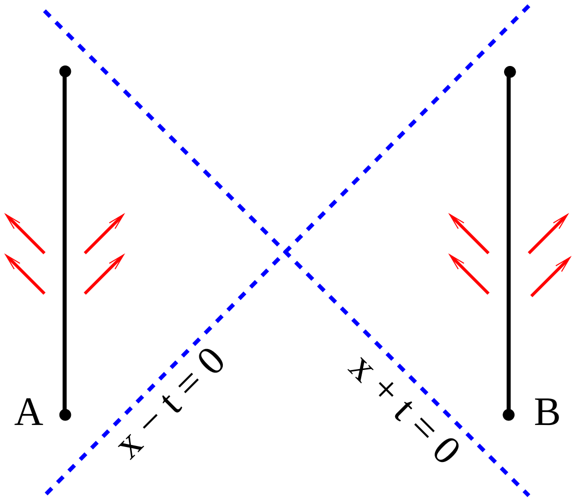
↕

QI : LOCC



A pair of causally disconnected localized detectors

Causal Structure + LO



For L>cT, we have $[\phi_A,\phi_B]=0$

Therefore $U_{INT}=U_A\ U_B$ → **LO**

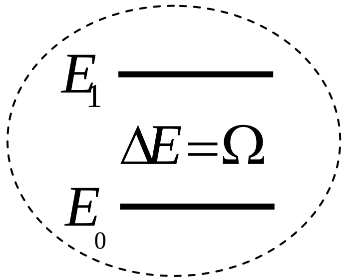
$\Delta\ E_{Total}$ =0, but

$\Delta\ E_{AB}$ >0. (Ent. Swapping)

Vacuum ent ! Detectors' ent.

Lower bound.

Field – Detectors Interaction



Two-level system

Interaction:

$$H_{INT}=H_A+H_B$$

$$H_A=\varepsilon_A(t)(e^{+i\Omega t}\,\sigma_A^{+}+e^{-i\Omega t}\sigma_A^{-})\,\phi(x_A,t)$$

↑ Window Function

Initial state:

$$|\Psi(0)\rangle = |\downarrow_A\rangle|\downarrow_B\rangle|0\rangle$$

Note: we do not use the rotating wave approximation.

Unruh (76), B. Dewitt (76), particle-detector models.

Probe Entanglement

$$\rho_{AB}^{(4\times4)} = Tr_F\,\rho^{(total)}$$

$$? \neq \sum_i p_i \rho_A^{2\times2} \rho_B^{2\times2}$$

Calculate to the second order (in ε) the final state, and evaluate the reduced density matrix.

Finally, we use Peres's (96) partial transposition criterion to check inseparability and use the Negativity as a measure.

$$U_{Interaction} = U_A \otimes U_B = (1 - i\varepsilon \int dt H_A - \frac{1}{2}\varepsilon^2 T \iint dt dt' H_A H_A)(\ldots)$$

$$\left|\Psi(T)\right\rangle = U_{Interaction} \left|\downarrow_A\right\rangle \left|\downarrow_B\right\rangle \left|0\right\rangle$$

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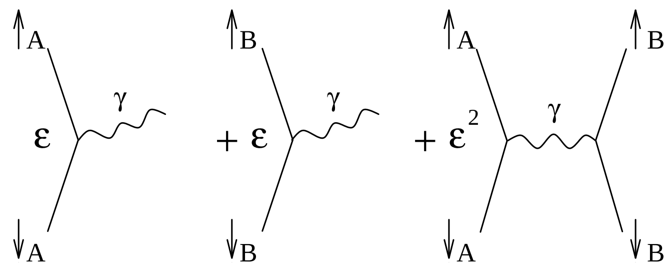
$$\begin{matrix} \downarrow\downarrow & \uparrow\uparrow & \downarrow\uparrow & \uparrow\downarrow \end{matrix}$$

$$\rho_{AB}(T) = \begin{bmatrix} 1 & \langle 0 | X_{AB} \rangle & & \\ \langle X_{AB} | 0 \rangle & \| X_{AB} \|^2 & & \\ & & \| E_A \|^2 & \langle E_A | E_B \rangle \\ & & \langle E_B | E_A \rangle & \| E_B \|^2 \end{bmatrix} + O(\varepsilon^5)$$

$$\left| X_{AB} \right\rangle = \Phi_A \Phi_B \left| 0 \right\rangle = \left| 0 \ or \ 2 \ photons \right\rangle$$

$$\left| E_A \right\rangle = \Phi_A \left| 0 \right\rangle = \left| 1 \ photon \right\rangle$$



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$$\rho_{AB}(T) = \begin{array}{c} \begin{array}{cccc} \downarrow\downarrow & \uparrow\uparrow & \downarrow\uparrow & \uparrow\downarrow \end{array} \\ \begin{bmatrix} 1 & \langle 0 | X_{AB} \rangle & & \\ \langle X_{AB} | 0 \rangle & \| X_{AB} \|^2 & & \\ & & \| E_A \|^2 & \langle E_A | E_B \rangle \\ & & \langle E_B | E_A \rangle & \| E_B \|^2 \end{bmatrix} \end{array} + O(\varepsilon^5)$$

P.T,

P.T.

$$|X_{AB}\rangle = \Phi_A \Phi_B |0\rangle = |0 \ or \ 2 \ photons\rangle$$

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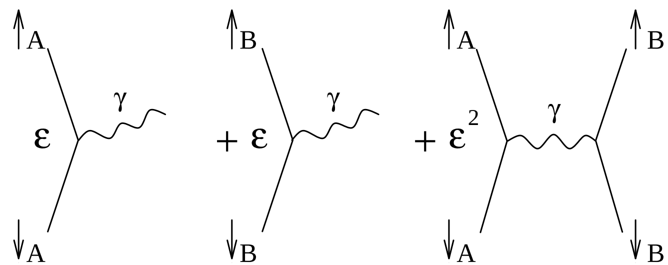
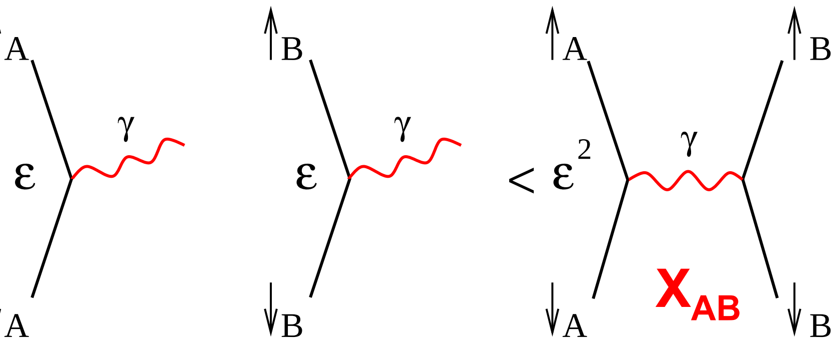


Emission < Exchange

$$\|E_A\|\|E_B\| < \left|\langle 0 | X_{AB} \rangle\right|$$



$$\int_0^\infty \omega d\omega [\tilde{\varepsilon}(\Omega+\omega)]^2 < \int_0^\infty \frac{d\omega}{L} Sin(\omega L) \varepsilon_A (\Omega+\omega) \varepsilon_B (\Omega-\omega)$$

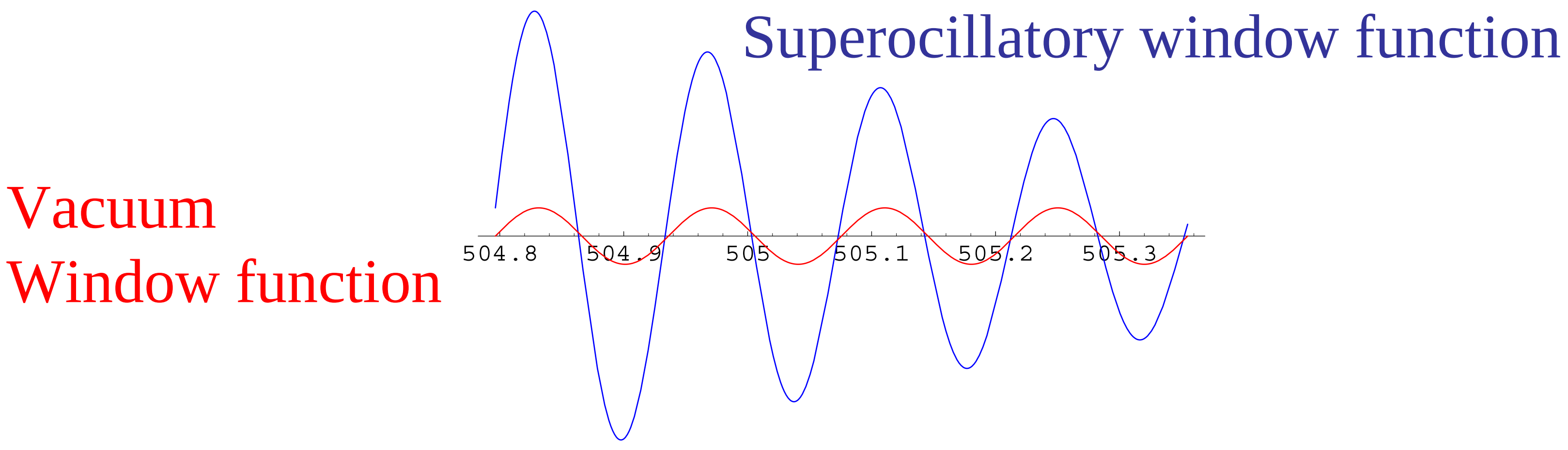
Off resonance

Vacuum "window function"

→ Superocillatory functions *(Aharonov (88), Berry(94)).*

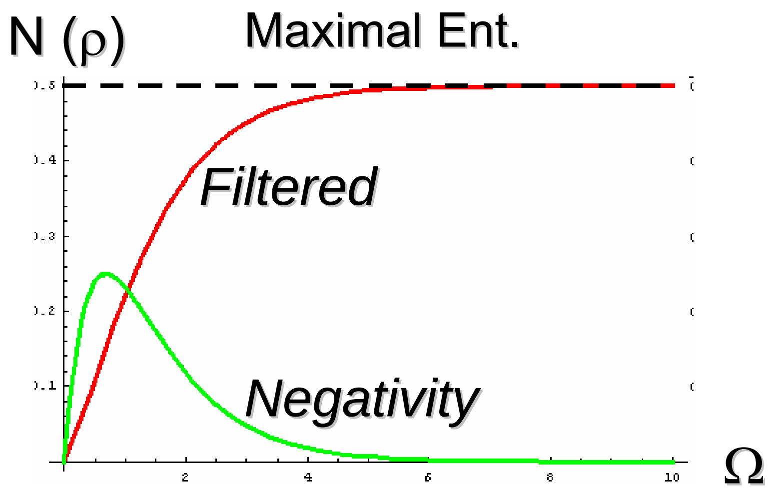
Entanglement for every separation

We can tailor a superoscillatory window function for every L to resonate with the vacuum "window function" $\sin(L\omega)$



→ Exchange term →exp(-f(L/T))

Bell's Inequalities

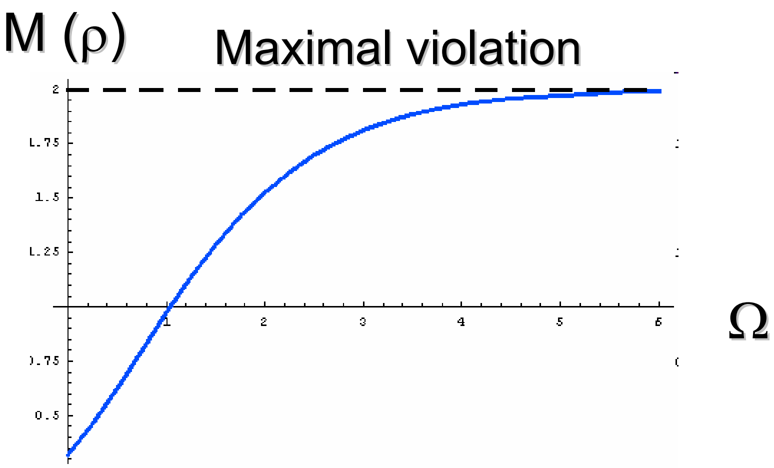


No violation of Bell's inequalities.

But, by applying local filters

$|\downarrow\downarrow>$ "+" h $X_{AB}|VAC>$ $|\uparrow\uparrow>$ "+"… $\rightarrow$

η^2 $|\downarrow>|\downarrow>$ "+" h $X_{AB}|VAC>|\uparrow>|\uparrow>$"+"…



CHSH ineq. Violated iff

M (ρ)>1, (Horokecki (95).)

"Hidden" non-locality.

Popescu (95). Gisin (96).

Summary (1)

1) Vacuum entanglement can be distilled!

2) Lower bound: E ¸ $e^{-(L/T)^2}$
(possibly $e^{-L/T}$)

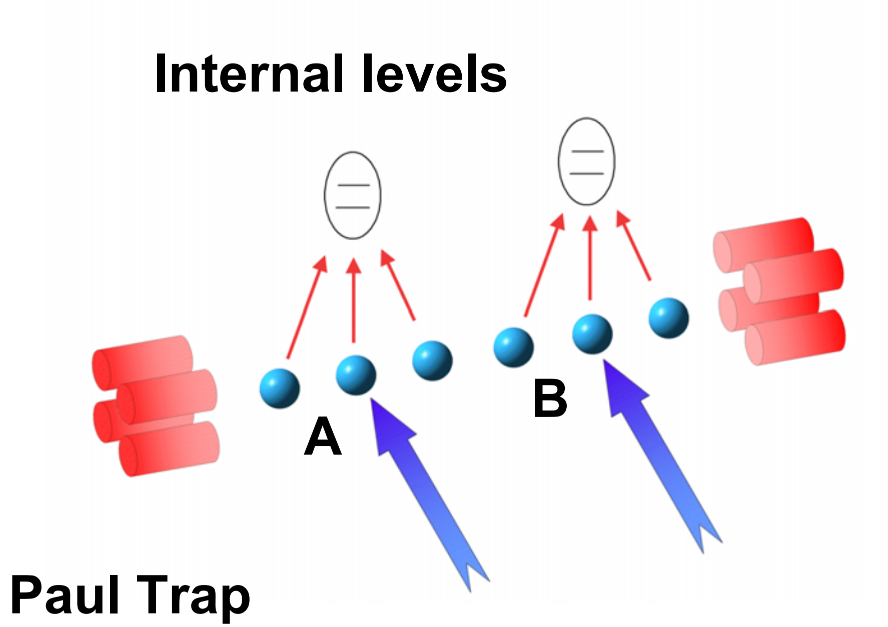
3) High frequency (UV) effect: $\Omega = L^2$.

4) Bell inequalities violation for arbitrary separation
maximal “hidden” non-locality.

Reznik, Retzker, Silman PRA 71, 042104

Can we detect Vacuum Entanglement?

Detection of Vacuum Entanglement in a Linear Ion Trap



$$H = H_0 + H_{int}$$

$$H_0 = \omega_z(\sigma_z^A + \sigma_z^B) + \sum \nu_n \, a_n^\dagger \, a_n$$

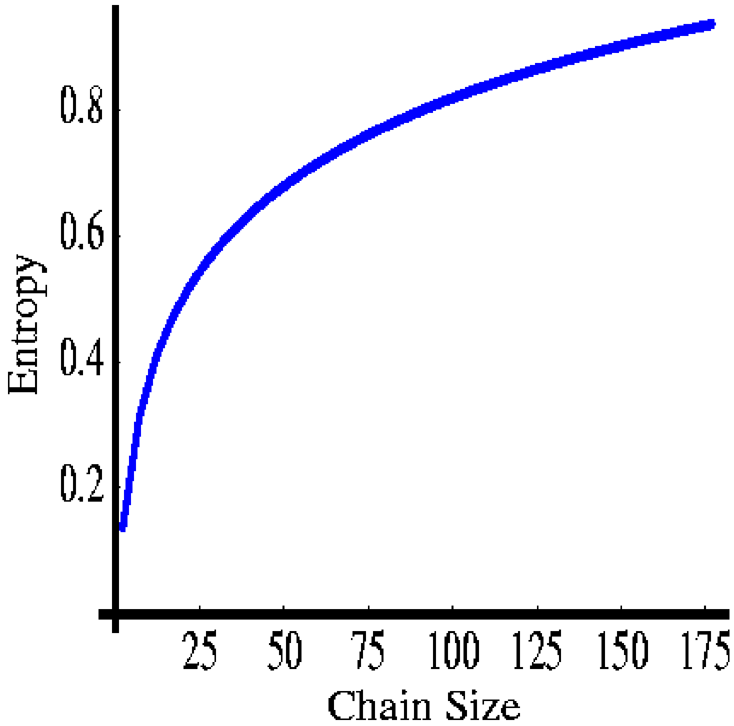
$$H_{int} = \Omega(t)(e^{-i\phi}\sigma_+^{(k)} + e^{i\phi}\sigma_-^{(k)})x_k$$

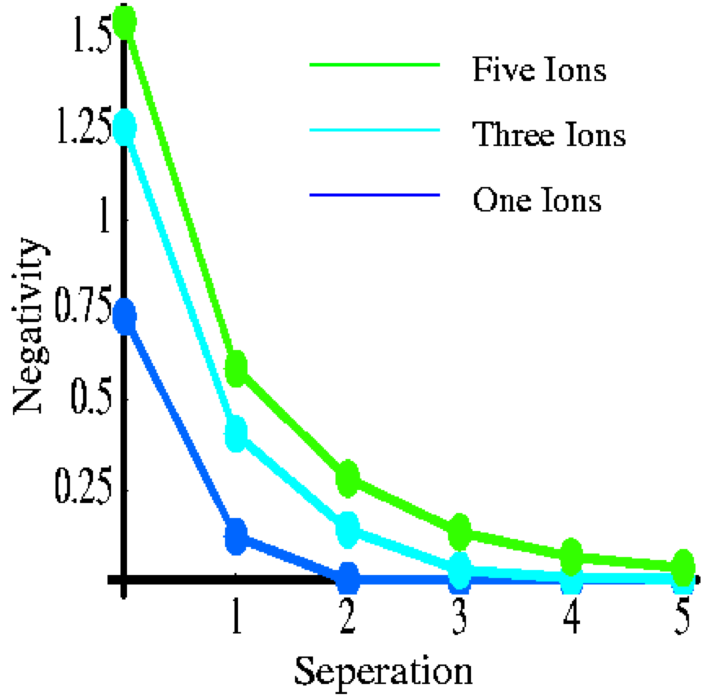
$$1/\omega_z << T << 1/\nu_0$$

A. Retzker, J. I. Cirac, B. Reznik PRL 94, 050504

Entanglement in a linear trap

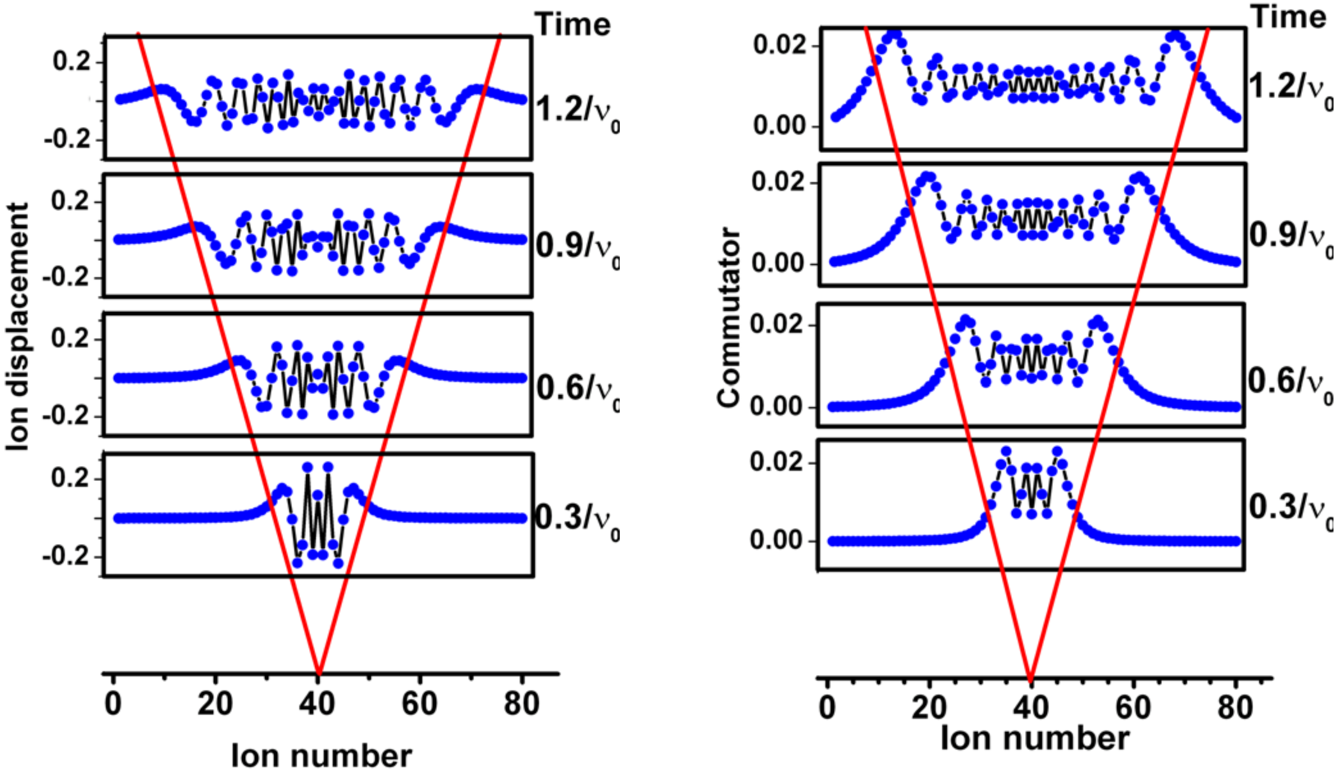
$$|vac\rangle = |\downarrow_c\rangle|\downarrow_d\rangle\sum_n e^{-\beta n}|n_A\rangle|n_B\rangle$$





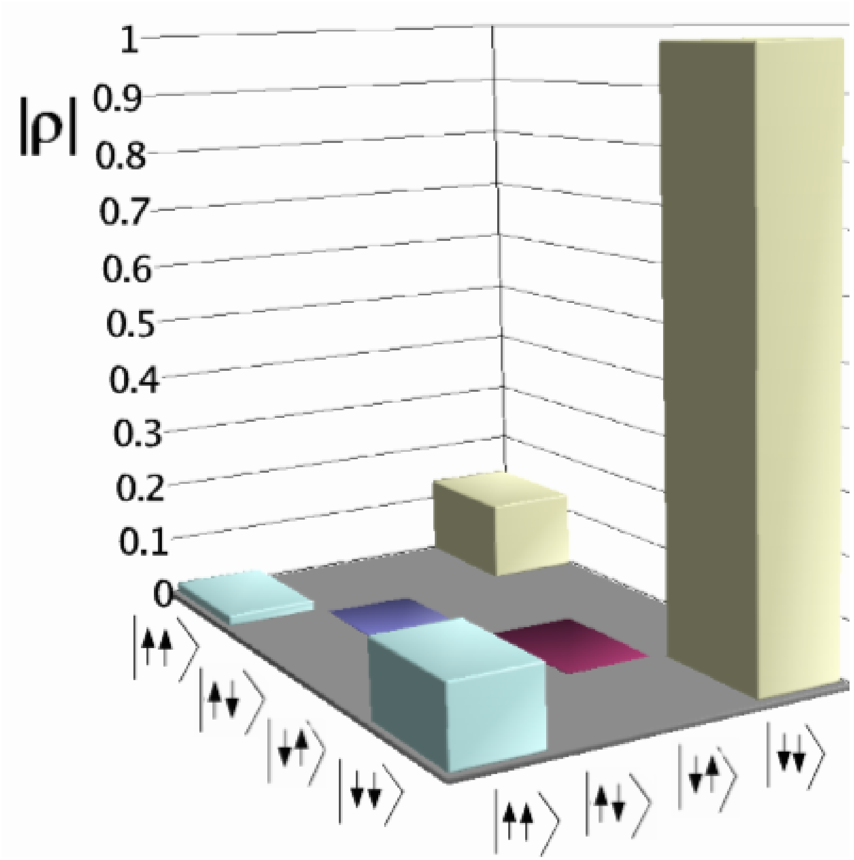
Entanglement between symmetric groups of ions as a function of the total number (left) and separation of finite groups (right).

Causal Structure



→ $U_{AB}=U_A \cdot U_B + O([x_A(0),x_B(T)])$

Two trapped ions



Final internal state

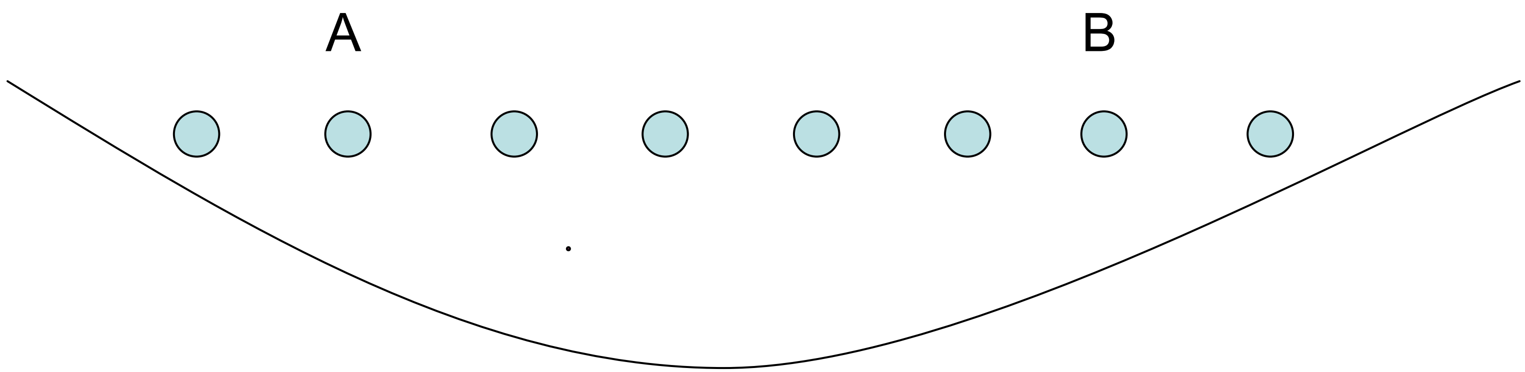
"Swapping" spatial internal states

$$|vac\rangle|\downarrow\downarrow\rangle \rightarrow |\chi\rangle\left(|\downarrow\downarrow\rangle + e^{-\beta}|\uparrow\uparrow\rangle\right)$$

$U=(e^{i\alpha\, x\, \sigma_x} - e^{i\beta\, p\, \sigma_y})-\ldots$

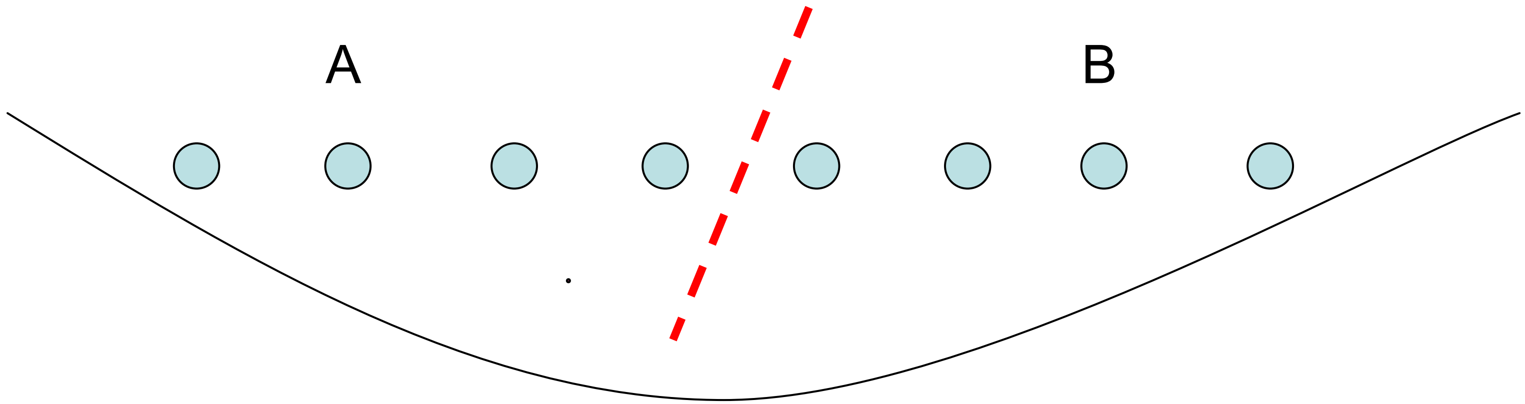
$E_{formation}(\rho_{final})$
accounts for 97% of the calculated
Entanglement: E(|vaci>)=0.136 e-bits.

Long Ion Chain



But how do we check that ent. is not due to “non-local” interaction?

Long Ion Chain



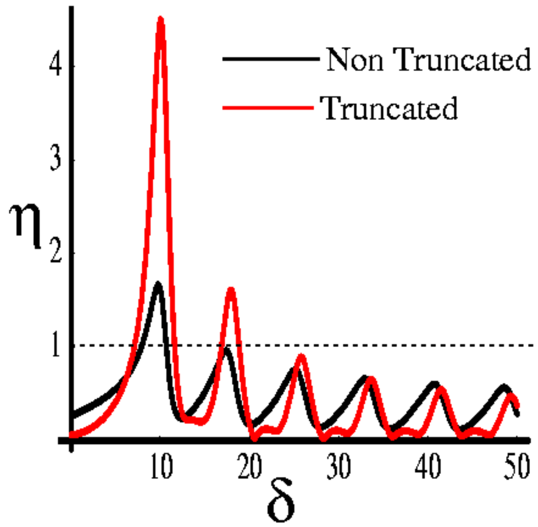
But how do we check that ent. is not due to “non-local” interaction?

H_{AB} $\quad$ $H_{truncated}=H_A \cdot H_B$

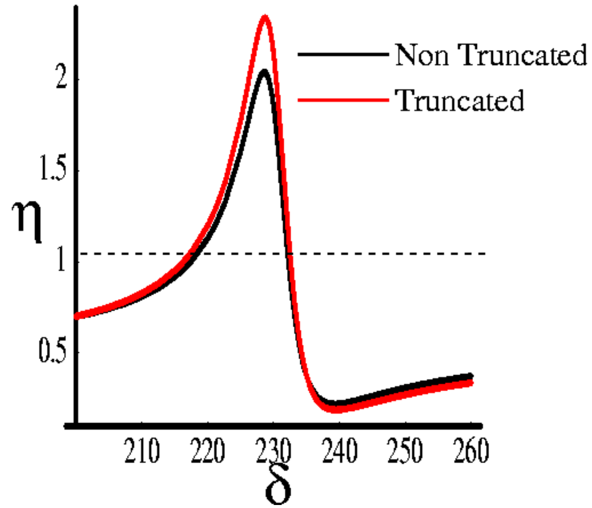
We compare the cases with a truncated and free Hamiltonians

Long Ion Chain

L=6,15, N=20



L=10,11   N=20



η=exchange/emission >1 , signifies entanglement.

δ denotes the detuning, L the locations of A and B.

Summary

Atom Probes:

Vacuum Entanglement can be "swapped" to detectors.

Bell's inequalities are violated ("hidden" non-locality).

Ent. reduces exponentially with the separation.

High probe frequencies are needed for large separation.

Linear ion trap:

-A proof of principle of the general idea is experimentally feasible for two ions.

-One can entangle internal levels of two ions without performing gate operations.