Quantum simulator for the $O(3)$ nonlinear sigma model

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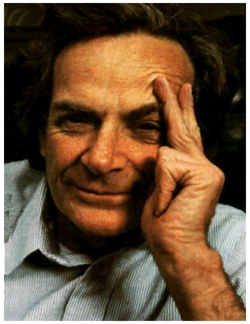
Introduction



- 1982, Richard Feynman

  Computers based on the laws of quantum physics instead of classical physics can be used for modeling quantum systems.

Introduction

- Quantum systems can be simulated with polynomial effort just with using another quantum system.
- For simulating a spin-$\frac{1}{2}$ quantum system with n spins:

  1. $\mathcal{O}(\,2^n\,)$ bits, with classical computer.
  2. $\mathcal{O}(\,n\,)$ qubits, with quantum computer.

$$|\psi>= C_{00\cdots 00}\,|\uparrow\uparrow\cdots\uparrow\uparrow> + \cdots + C_{11\cdots 11}\,|\downarrow\downarrow\cdots\downarrow\downarrow>$$

Motivation

Simulating strongly interacting quantum field theory, Quantum Chromo-dynamics:

- Strong interactions between gluons and quarks
- Non-perturbative theory
- Re-normalizable in $3+1$ dimension
- Asymptotic freedom: $g^2\,(p^2 \gg \Lambda^2_{QCD}) \propto \dfrac{1}{\ln(p^2/\Lambda^2_{QCD})}$
- Non-vanishing vacuum condensates / trace anomaly:
  $$< \hat{\mathcal{L}} > \neq 0 \quad , \quad < \hat{T}^{\nu}_{\nu} > \neq 0$$
- The theory exhibits instantons

Motivation

- Quantum Chromo-dynamics is very complicated:

$$\mathcal{L} = -\frac{1}{4} F^{\alpha}_{\mu\nu}(x)\ F^{\mu\nu}_{\alpha}(x) + \cdots$$

- Nonlinear sigma model is strongly interacting quantum field theory (Nontrivial theory $\rightarrow$ Toy model for Quantum Chromo-dynamics).

- Universal quantum computers are not available (yet).

Motivation

- One possibility:

  Designing special quantum system in the laboratory as a special quantum computer.

- Simulating the quantum dynamics of the $O(3)$ nonlinear sigma model with construction of a laboratory system based on present-day technology.

The Model

- $1+1$ dimensional $O(N)$ nonlinear sigma model

- The model is described by:

$$\mathcal{L} = \frac{\hbar}{2c}\,\partial_\nu \boldsymbol{\sigma} \cdot \partial^\nu \boldsymbol{\sigma} = \frac{\hbar}{2c}\left[\,(\partial_t \boldsymbol{\sigma})^2 - c^2\,(\partial_x \boldsymbol{\sigma})^2\,\right]$$

$$\boldsymbol{\sigma} = (\,\sigma_1\,,\,\sigma_2\,,\,\cdots\,,\,\sigma_N\,) \in \mathbb{R}^N$$

- The interaction is introduced by the constraint:

$$\boldsymbol{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \cdots \sigma_N^2 = \frac{N}{g^2}$$

The Model

Many properties of sigma model are similar to Quantum Chromo-dynamics $\longrightarrow$ Toy model:

- Re-normalizable in $1+1$ dimension
- Asymptotic freedom:

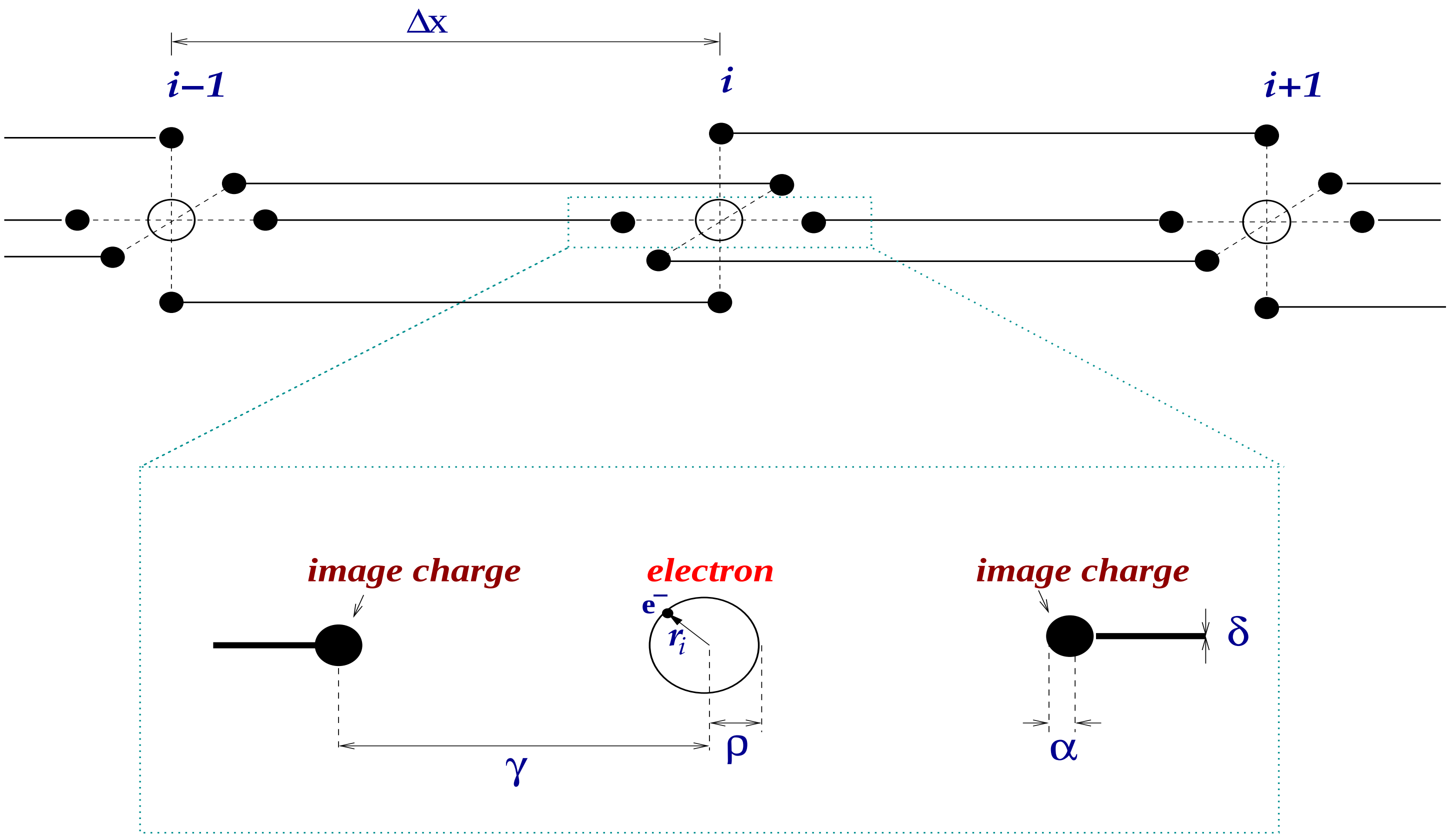
$$g^2\,(p^2 \gg \Lambda^2) \;\propto\; \frac{1}{\ln(p^2/\Lambda^2)}$$

- Non-vanishing vacuum condensates / trace anomaly:

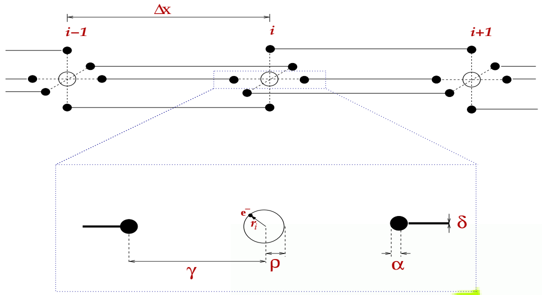
$$< \hat{\mathcal{L}} > \neq 0 \quad , \quad < \hat{T}^{\nu}_{\nu} > \neq 0$$

- The model exhibits instantons (mapping of $\mathbb{S}_2$ onto $\mathbb{R}^2$)

The Analogue



$$\lambda \gg \Delta x \gg \gamma \gg \rho \ , \ \alpha \gg \delta$$



The Analogue

- The total Lagrangian for the system of electrons:

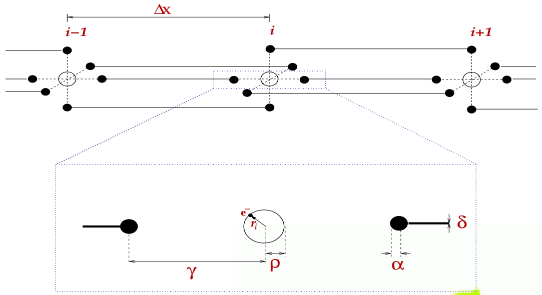
$$\mathcal{L} = \sum_i \left[\frac{m}{2} \dot{\boldsymbol{r}}_i^{\,2} - V(\boldsymbol{r}_{i+1}, \boldsymbol{r}_i) \right]$$

$$V(\boldsymbol{r}_{i+1}, \boldsymbol{r}_i) = \frac{e^2 \alpha^2}{4\pi \epsilon_0 \gamma^4} \; \frac{(\boldsymbol{r}_{i+1} - \boldsymbol{r}_i)^2}{4\alpha + \Delta x / \ln(\Delta x/\delta)}$$

- The continuum limit of $\mathcal{L}$ generates the Lagrangian of the $O(3)$ nonlinear sigma model:

$$\boldsymbol{\sigma} = \frac{\sqrt{3}}{g_{\rm eff}} \frac{\boldsymbol{r}_i}{\rho} \qquad \text{(For } N = 3 \text{)}$$

$$\sum_i \Delta x \rightarrow \int dx \qquad \text{(For } \lambda \gg \Delta x \text{)}$$



The Analogue

- The effective Lagrangian for this model:

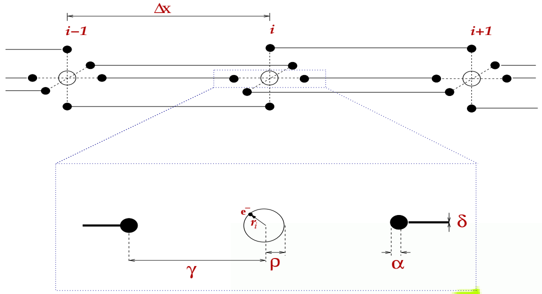
$$\mathcal{L}_{\rm eff} = \frac{m\,\rho^2\,g^2}{2N\Delta x}\left[(\partial_t\boldsymbol{\sigma})^2 - \frac{e^2}{4\pi\epsilon_0 m}\;\frac{2\alpha^2\,\Delta x^2/\gamma^4}{4\alpha+\Delta x/\ln(\Delta x/\delta)}\,(\partial_x\boldsymbol{\sigma})^2\right]$$

- The Lagrangian for sigma model:

$$\mathcal{L} = \frac{\hbar}{2c}\,\partial_\nu\boldsymbol{\sigma}\cdot\partial^\nu\boldsymbol{\sigma} = \frac{\hbar}{2c}\left[(\partial_t\boldsymbol{\sigma})^2 - c^2\,(\partial_x\boldsymbol{\sigma})^2\right]$$

$\Longrightarrow$ effective propagation speed:

$$c_{\rm eff} = c_0\,\sqrt{\frac{e^2}{4\pi\,\epsilon_0\,m\,c_0^2}\,\frac{2\alpha^2\,\Delta x^2/\gamma^4}{4\alpha+\Delta x/\ln(\Delta x/\delta)}}$$



The Analogue

- The effective coupling for $N = 3$:

$$g_{\text{eff}} = \sqrt{3}\,\frac{\gamma}{\rho}\,\left(\frac{4\pi\,\epsilon_0\,\hbar^2}{m\,e^2}\;\frac{4\alpha + \Delta x/\ln(\Delta x/\delta)}{2\alpha^2}\right)^{1/4}$$

determines the value of the running coupling at a length scale corresponding to the lattice spacing Δx.

- In analogy to Quantum Chromo-dynamics (Λ_{QCD}) :

$$g^2_{\text{eff}}\;(p^2 \gg \Lambda^2_\sigma) \;\propto\; \frac{1}{\ln(\,p^2\,/\,\Lambda^2_\sigma\,)}$$

- The condition for scale:

$$p = \frac{1}{\Delta x} \quad \Rightarrow \quad \Lambda_\sigma\,\Delta x \;\ll\; 1$$

Disturbances

- Interaction energies of the disturbances must be much smaller than the energy gap between the ground state and the first excited state.
- For the nonlinear sigma model, this gap is determined by the induced scale Λ_σ:

$$\text{Energies of all perturbations} \ll \hbar\, c_{\text{eff}}\, \Lambda_\sigma$$

- The temperature must be small enough :

$$k_B T \ll \hbar\, c_{\text{eff}}\, \Lambda_\sigma$$

Disturbances

- The kinetic terms due to inductances of the wires are negligible for a sufficiently large slow-down:

$$LI^2 \ll m\,\dot{\boldsymbol{r}}^2 \quad \longrightarrow \quad 4\alpha\left(\frac{c_{\rm eff}}{c_0}\right)^2\left(\ln\frac{\Delta x}{\delta}\right)/\Delta x \ll 1$$

- Spin-spin and spin-orbit couplings are negligible.

Phase Diagram

- Fixing the electron spins by an external magnetic field?
- Phase diagram in terms of the temperature T and chemical potential $\mu \quad \rightarrow \quad$ Particle numbers N?
- Noether charge of $O(3)$ symmetry:

$$Q = \frac{1}{c_{\text{eff}}} \boldsymbol{n} \cdot \int dx \, \boldsymbol{\sigma} \times \dot{\boldsymbol{\sigma}} = \sum_i \boldsymbol{n} \cdot \frac{\boldsymbol{L}_i^e}{\hbar}$$

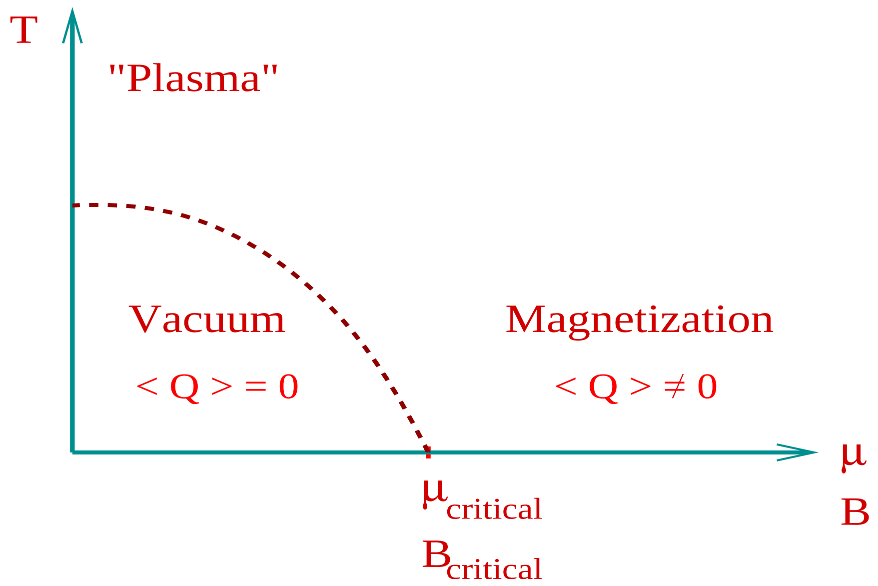
- Grand-canonical Hamiltonian:

$$\hat{H}_{gc} = \hat{H}_0 + \mu_N \hat{N} = \hat{H}_0 + \mu_Q \hat{Q}$$

Phase Diagram

External magnetic field is equivalent to chemical potential:

$$\dot{\boldsymbol{r}} \cdot \boldsymbol{A} = \frac{\boldsymbol{B}\cdot(\dot{\boldsymbol{r}}\times\boldsymbol{r})}{3} \qquad \Rightarrow \qquad \mu_{\rm eff} = \frac{e\,\hbar}{3m}\,B$$



$$\mu_{\rm crit} = \hbar\,c_{\rm eff}\,\Lambda_\sigma$$

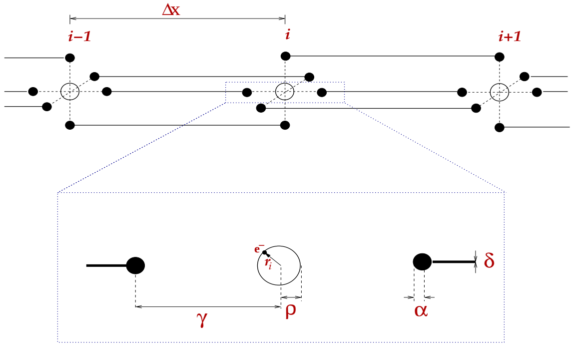
$$B_{\rm crit} = \mathcal{O}(m\,c_{\rm eff}\,\Lambda_\sigma/e)$$

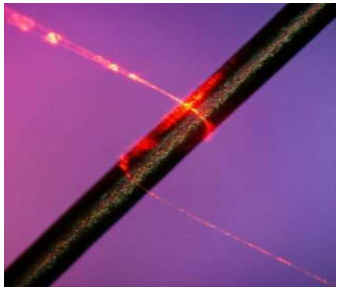
$$\boldsymbol{m}_s \cdot \boldsymbol{B} = \mathcal{O}(\hbar\,c_{\rm eff}\,\Lambda_\sigma)$$

$$k_B T \ll \hbar\,c_{\rm eff}\,\Lambda_\sigma$$

One can fix the electron spins with a small external magnetic field: $B \ll B_{\rm crit}$

Experimental Parameters





$\delta = 100\ nm, \qquad \rho = 400\ nm$

$\alpha = 500\ nm, \qquad \gamma = 2.5\ \mu m$

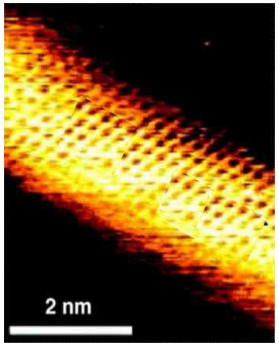
$\Delta x = 12.5\ \mu m$

Silica nanowire
$50\ nm$ thickness
Harvard University

- We obtain: $g_{\rm eff} = \mathcal{O}(1), \quad c_{\rm eff} \approx 10^4\ m/s$
  $\Lambda_\sigma^{-1} \approx 125\ \mu m, \quad \hbar c_{\rm eff}\Lambda_\sigma \approx 600\ \mu K$
- Present limit for solids: $T = \mathcal{O}(10\ \mu K)$

Experimental Parameters

- $\delta = 1\ nm,$ $\rho = 12\ nm$
  $\alpha = 5\ nm,$ $\gamma = 25\ nm$
  $\Delta x = 125\ nm$



Silica nanowire
$3\ nm$ thickness
City University
of Hong-Kong

- We obtain:
  $c_{\text{eff}} \approx 10^5\, m/s,$ $g_{\text{eff}} = \mathcal{O}(1)$
  $\hbar c_{\text{eff}} \Lambda_\sigma = \mathcal{O}(1\ K)$

- This range provides a wide window of opportunity.

Conclusions

- Constructing a quantum simulator for the $O(3)$ nonlinear sigma model (with present-day technology).
- Proof of principle and modeling feasibility $\longrightarrow$ Quantum Chromo-dynamics?
- Direct access to the quantum state and investigation of the strong entanglement.
- Non-perturbative properties $\rightarrow$ instanton density.
- Creating particles $\longrightarrow$ S-matrix.