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Qubits versus Bits for measuring an integral of a classical field

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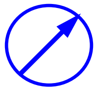
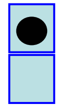
Phys. Rev. Lett. 92, 217902 (2004) *L.Vaidman Z.Mitrani*

QUBIT 

$\theta, \ \phi$

TO WRITE $\theta, \ \phi$

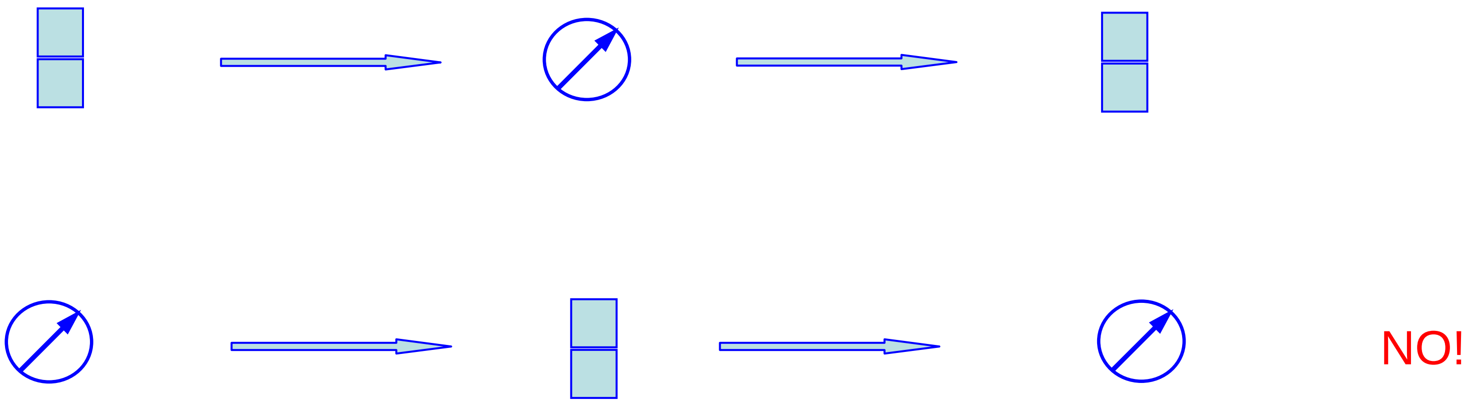
TO READ $0, \ 1$

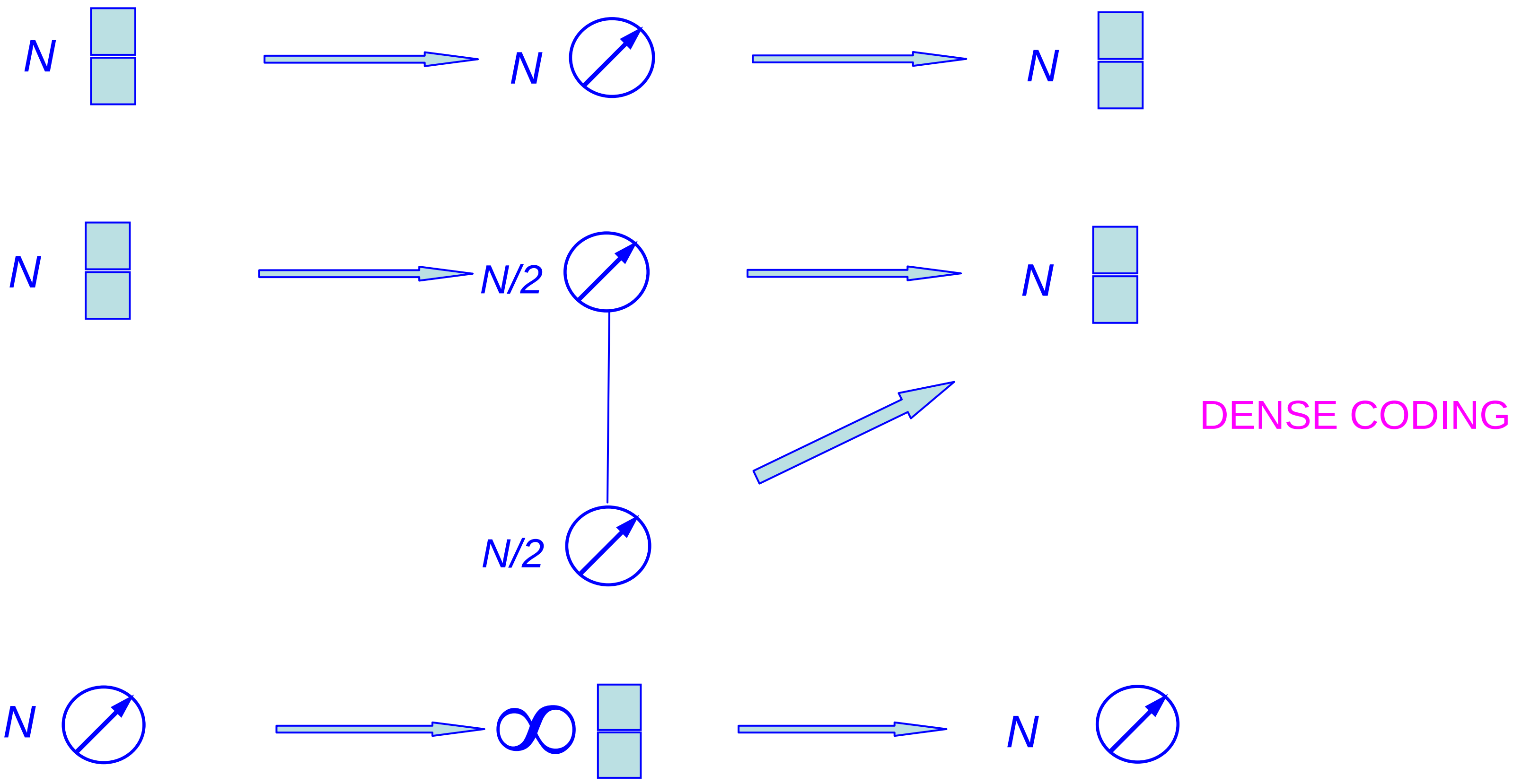
BIT 

{ 0 , 1 }

TO WRITE $0, \ 1$

TO READ $0, \ 1$





We can not store and retrieve more than one bit in a qubit **HOLEVO**

What can we do with a **Qubit** that we can not do with a **Bit**?

Measurement of the parity of the integral of a classical field

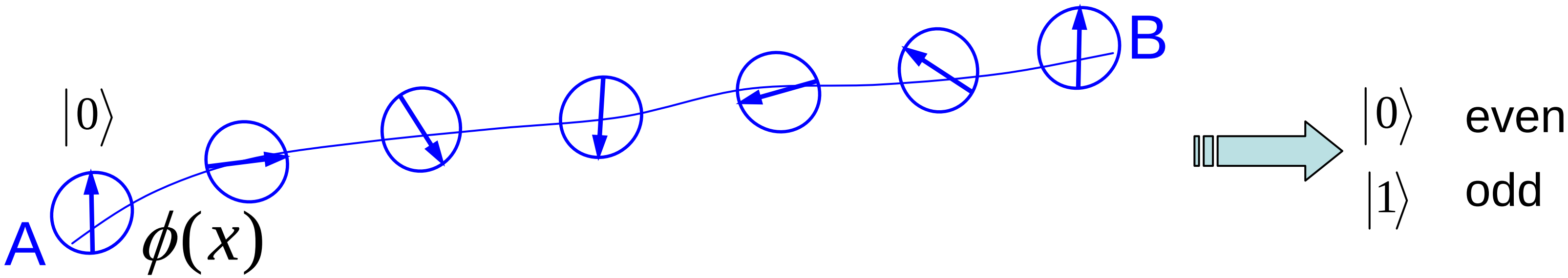
Galvao and Hardy, Phys. Rev. Lett. 90, 087902 (2003)

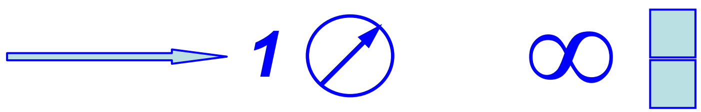
$$I = \int_A^B \phi(x)dx = m\alpha$$

, **α** - known real constant , **m** - integer.

Is **m** even or odd?

- qubit starts at $|0\rangle$.
- the field rotates the qubit by ***π*** per a “unit” of **α**.
- the qubit is measured.





Outline

- Define the task.
- Special Case – using qubits.
- Special Case – using bits.
- General Case – using qubits – first method.
- General Case – using qubits – second method (more efficient).
- Simulation results.
- Conclusions.

The Task – Measure the integral of a classical field.

- We are given N bits/qubits.
- The bits/qubits pass trough the field one at a time.
- We know that the integral ***I*** is of the order of ***M***.

$$I = \int_A^B \phi(x)\,dx$$

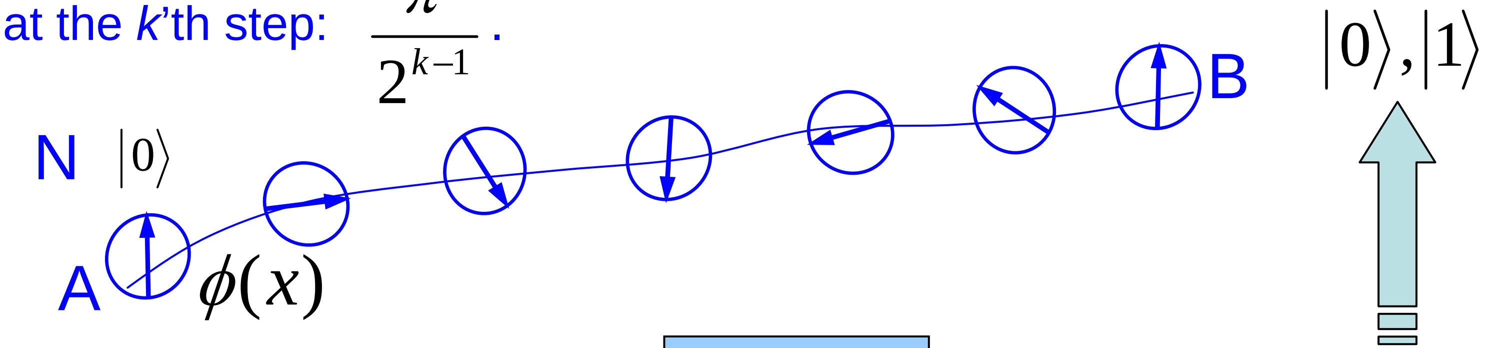
Measurement of the integral of a classical field -**special case**

$$I = \int_A^B \phi(x)dx = m\alpha$$

, **α** - known real constant , **m** - integer.

What is **m**?

- qubit starts at $|0\rangle$.
- at the fisrt step: qubit is rotated by ***π*** per a "unit" of **α**.
- at the second step: qubit is rotated by ***π/2*** per a "unit" of **α**.
- at the *k*'th step: $\dfrac{\pi}{2^{k-1}}$.



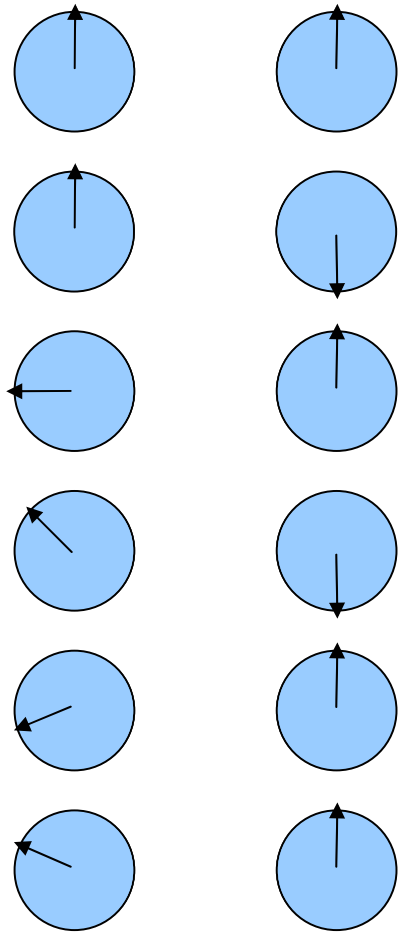
$$\Delta I = 0$$

$$\theta_k = \sum_{i=1}^{k-1} \frac{\pi d(i)}{2^{k-i}}$$

Example m=10:

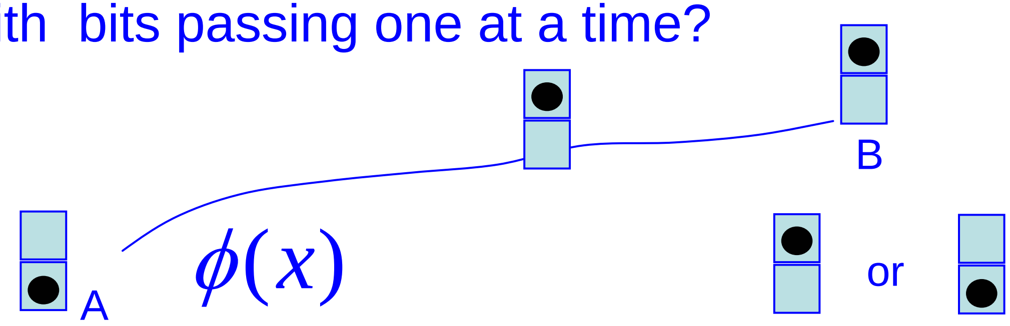
Correction Angle	Binary Digits
$\theta_1 = 0$	0
$\theta_2 = 0$	10
$\theta_3 = 2 \cdot \pi / 4$	010
$\theta_4 = 2 \cdot \pi / 8$	1010
$\theta_5 = 10 \cdot \pi / 16$	01010
$\theta_6 = 10 \cdot \pi / 32$	001010
From here on	…00001010

Start Position / End Position



What can we do with bits passing one at a time?

$$I = \int_A^B \phi(x)dx$$



- The bit starts at the state *0.*
- The probability to flip depends on the strength of the field: $dp_{flip} = \lambda\,\phi(x)\,dx$
- Once the bit has flipped to the state *1*, it won't flip back.
- We count the number of *1*'s.

$$dp_{flip} = \lambda\,\phi(x)\,dx$$

$$dp_{no-flip} = 1 - \lambda\,\phi(x)\,dx = e^{-\lambda\,\phi(x)\,dx}$$

$$p_{no-flip} = \prod dp = \prod e^{-\lambda\,\phi(x)\,dx} = e^{-\int \lambda\,\phi(x)\,dx} = e^{-\lambda I}$$

$$I = \frac{1}{\lambda}\ln\left(1 - \frac{N_{count}}{N}\right), \Delta I = \frac{1}{\lambda}\frac{1}{\sqrt{N}}\sqrt{e^{\lambda I} - 1}$$

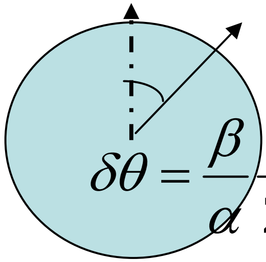
For a given *M:*

$$\Delta I \propto \frac{M}{\sqrt{N}}$$

General case – First method – measuring $\boldsymbol{\beta}$

$$I = \int_A^B \phi(x)\,dx = m\alpha + \beta$$

, **α** - chosen real constant , **m** - integer.



$$\delta\theta = \frac{\beta}{\alpha}\frac{\pi}{2^{k-1}}$$

$$\Psi = \cos(\frac{\delta\theta}{2})|0\rangle + \sin(\frac{\delta\theta}{2})|1\rangle$$

Worse than that, we can have errors in the binary digits that cause an additional shift in the final angle.

We can measure $\boldsymbol{\beta}$ ⇒ $\Delta\beta = \frac{1}{\sqrt{N_0}}$ ⇒ $\Delta\beta = \frac{1}{N_0}$ ⇒ $\Delta\beta = \frac{1}{N_0^{\,2}}$

The correction: $\theta_k = -\sum_{i=1}^{k-1}\frac{\pi d(i)}{2^{(k-i)}} - \frac{\pi\beta}{2^{k-1}\alpha}$

Probability of an error: $p < \frac{1}{3N_0^{\,2}}$ ⇒ $\alpha = \frac{M}{2^{N-N_0}}$

For a given *M:*

⇒ $$\Delta I \propto \frac{M}{2^{N-N_0} N_0}$$

General case – Second method – not measuring ***β***

The digital method works much better! → $\alpha = \frac{M}{2^N}$

- The qubit rotates by an angle: $\Theta_k = \frac{I\pi}{2^{k-1}\alpha}$.
- The qubit is then measured at the angle: $\tilde{\Theta}_k = \frac{\tilde{I}\pi}{2^{k-1}\alpha}$.

- The probability for the *k*'th digit to be correct is:

$$p_k = \cos^2 \frac{\Theta_k - \tilde{\Theta}_k}{2} = \cos^2 \frac{(I-\tilde{I})\pi}{2^k \alpha} = \cos^2 \frac{\chi\pi}{2^k}, \chi \equiv \frac{I-\tilde{I}}{\alpha}$$

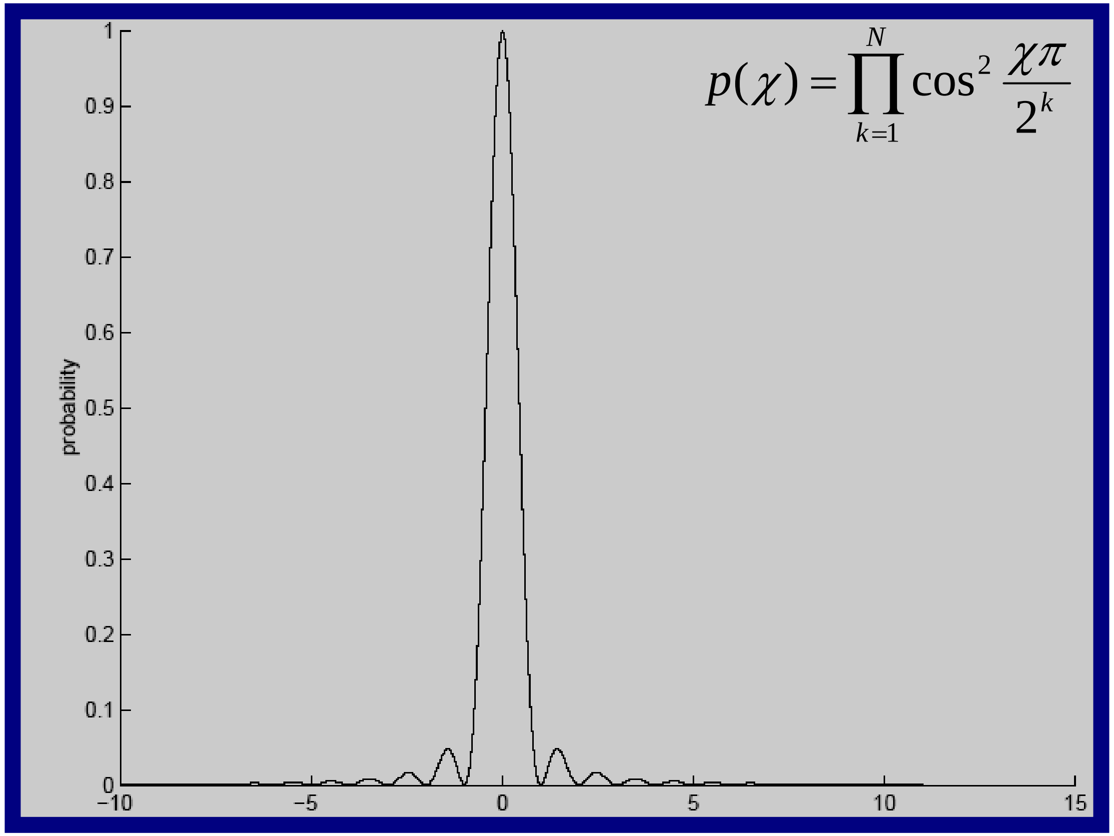
- The probability of an error χ is :

$$p(\chi) = \prod_{k=1}^{N} \cos^2 \frac{\chi\pi}{2^k}$$

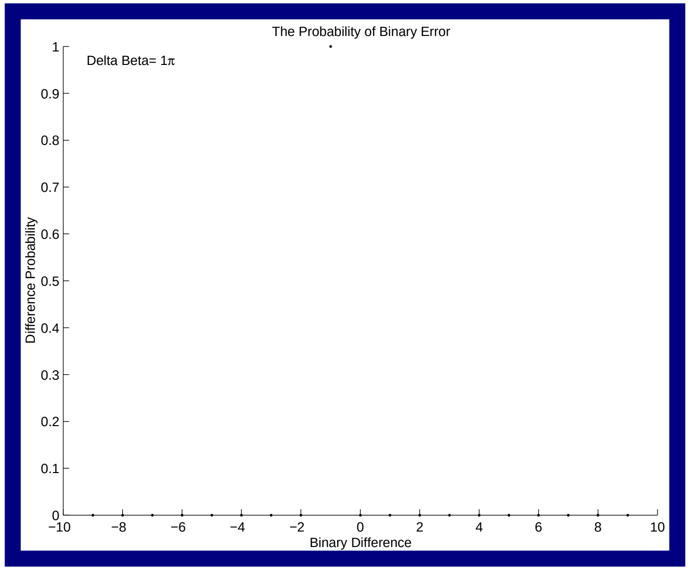
For a given *M:*

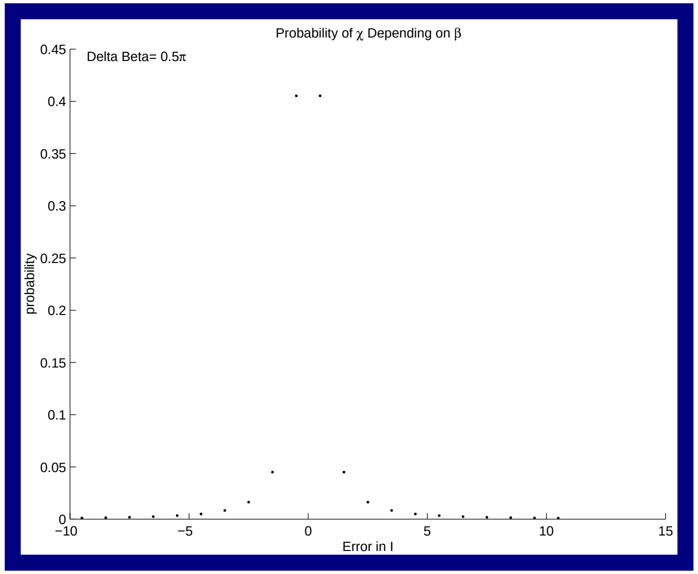
If $p(\chi)$ is small than ⇨ $\Delta I \propto \frac{M}{2^N}$

The probability distribution



$$p(\chi) = \prod_{k=1}^{N} \cos^2 \frac{\chi\pi}{2^k}$$



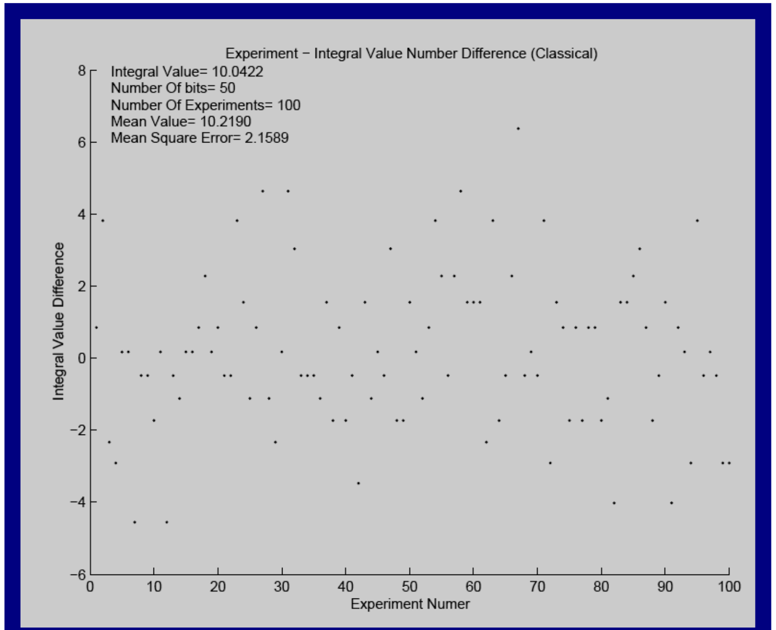


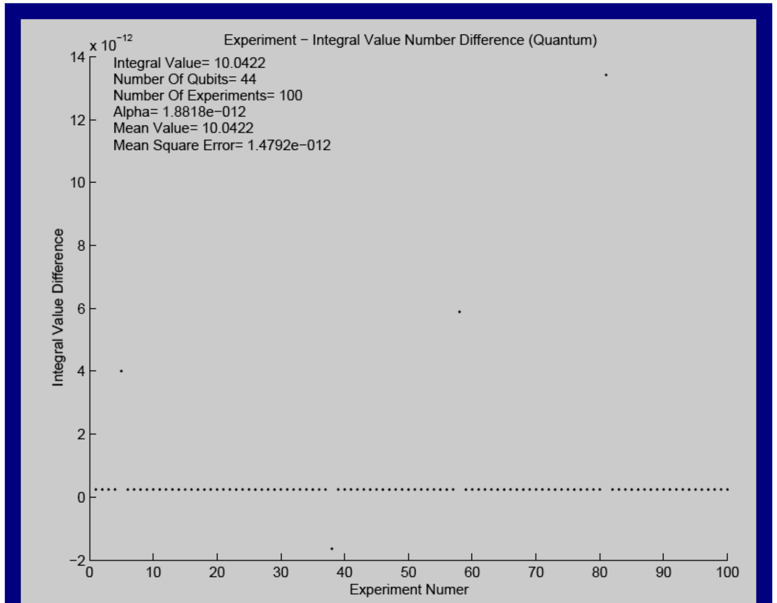
$$\chi = \frac{I - \tilde{I}}{\alpha} = (m - \tilde{m}) + \frac{\beta}{\alpha}$$

Simulation Results

n	$I = n\pi mod(10)$	quantum	classical
1	3.141592654	3.141592494	3.175583382
2	6.283185307	6.283185389	4.577585162
3	9.424777961	9.424777867	9.594107747
4	2.566370614	2.566370611	1.689440418
5	5.707963268	5.707963268	5.016553197
6	8.849555922	8.849555813	9.594105057
7	1.991148575	1.991148466	2.619198463
8	5.132741229	5.132741166	8.395441798
9	8.274333882	8.274333865	6.706033821
10	1.415926536	1.415926495	0.929766618

TABLE I: The results of simulation of classical and quantum measurements of ten values of I, $I_n = (n\pi)\text{mod}10$. Quantum method uses 30 qubits and classical method uses 30 bits.





Conclusions

- Special Case - qubits. $\Delta I = 0$
- Special/General Case - bits. $\Delta I \propto \frac{M}{\sqrt{N}}$
- General Case – measuring ***β*** . $\Delta I \propto \frac{M}{2^{N-N_0} N_0}$
- General Case – not measuring ***β*** . $\Delta I \propto \frac{M}{2^N}$

Exponential better precision can be achieved using qubits instead of bits for measuring the integral of a classical field.

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- Can be done with a single particle in a superposition of states.

Phys. Rev. A 71, 044303 (2005) *L.Vaidman, A.Kalev*

Will be nice if …

- Proof of the classical result to be the lower limit for bits.
- An experiment to show the quantum advantage.
- Use of this idea for other schemes/purposes.