Dissipation and Quantum Operations

Ignacio García-Mata

Dto. de Física, Lab. TANDAR, Comisión Nacional de Energía Atómica
Buenos Aires, Argentina

International Summer School on Quantum Information,
MPI PKS Dresden 2005

Outline

Outline

Intro.

- Motivations
  - Understanding noise and (trying to) control it.
  - Something to say about how the classical world emerges from the Quantum.
- Tools
  - Quantum Channels (q. operations, superoperators, QDS's . . .) on an N dimensional Hilbert space, $N \times N$ dimensional phase space ($\hbar = 1/(2\pi N)$), Kraus operators. . .

Outline

Open Systems

- Markov (local in time) assumption
- Lindblad Master equation

$$\dot{\rho} = \mathcal{L}(\rho) = -i[H, \rho] + \frac{1}{2}\sum_k \left\{ [L_k \rho, L_k^\dagger] + [L_k, \rho L_k^\dagger] \right\}$$

- CPTP Quantum Dynamical Semi-group (channel, superoperator, quantum operation,... Lindblad (1976), Gorini et al. (1976))
- Kraus operator sum form

$$\mathbf{S}(\rho) = \sum_k M_k \rho M_k^\dagger$$

  where $\sum_k M_k^\dagger M_k = I$ for TP.
- We asume discrete time.

Outline

Open Systems in Phase Space

- Representation in phase space by the Wigner function.
- Master eq. $\rightarrow$ Fokker-Planck eq. for the Wigner function

$$\dot{W}(q,p) = [\text{Hamiltonian} + \text{non-Hermitian } L_k\text{'s}] + \text{diffusion}$$

- Phase space contraction associated to

$$\sum_k [L_k, L_k^\dagger] \xrightarrow[\hbar\to 0]{} -2\sum\{L_k(q,p), L_k^*(q,p)\} =$$

- Dissipation due to non-Hermitian $L_k's \Longrightarrow \mathcal{L}(I) \neq 0$
- Non-unital dynamics

$$\mathbf{S}(I) \neq I$$

Open Systems in Phase Space

- Representation in phase space by the Wigner function.
- Master eq. $\rightarrow$ Fokker-Planck eq. for the Wigner function

$$\dot{W}(q,p) = [\text{Hamiltonian} + \text{non-Hermitian } L_k\text{'s}] + \text{diffusion}$$

- Phase space contraction associated to

$$\sum_k [L_k, L_k^\dagger] \xrightarrow[\hbar\to 0]{} -2\sum\{L_k(q,p), L_k^*(q,p)\} = \mathrm{div}(\vec{\mathrm{drift}}) < 0$$

- Dissipation due to non-Hermitian $L_k's \Longrightarrow \mathcal{L}(I) \neq 0$
- Non-unital dynamics

$$\mathbf{S}(I) \neq I$$

Outline

Quantum Operations

- So we have a map like

$$\rho' = \mathbf{S}(\rho) = \sum_\alpha c_\alpha T_\alpha \rho T_\alpha^\dagger \;\text{ with } c_\alpha \geq 0$$

TP if $\sum_\alpha c_\alpha T_\alpha^\dagger T_\alpha = I$ Unital if $\sum_\alpha c_\alpha T_\alpha T_\alpha^\dagger = I$

- Unital $\equiv$ Diffusion, Dephasing, pure decoherence ...
- Bistochastic maps usually Random Unitary Processes

- Non-unital $\equiv$ Dissipation (+ decoherence...)

Quantum Operations

- So we have a map like

$$\rho' = \mathbf{S}(\rho) = \sum_\alpha c_\alpha T_\alpha \rho T_\alpha^\dagger \quad \text{with } c_\alpha \geq 0$$

TP if $\sum_\alpha c_\alpha T_\alpha^\dagger T_\alpha = I$ Unital if $\sum_\alpha c_\alpha T_\alpha T_\alpha^\dagger = I$

- Unital $\equiv$ Diffusion, Dephasing, pure decoherence ...
- Bistochastic maps usually Random Unitary Processes

- Non-unital $\equiv$ Dissipation (+ decoherence...)

Quantum Operations

- So we have a map like

$$\rho' = \mathbf{S}(\rho) = \sum_\alpha c_\alpha T_\alpha \rho T_\alpha^\dagger \quad \text{with } c_\alpha \geq 0$$

TP if $\sum_\alpha c_\alpha T_\alpha^\dagger T_\alpha = I$

Unital if $\sum_\alpha c_\alpha T_\alpha T_\alpha^\dagger = I$

- Unital $\equiv$ Diffusion, Dephasing, pure decoherence ...
- Bistochastic maps usually Random Unitary Processes

  See for example Bianucci et al, PRE 2002; Nonnenmacher, Nonlinearity 2003, IGM et al., PRL 2003; IGM & Saraceno, PRE 2004; Aolita ("boludo") et al. PRA 2004; MMO

- Non-unital $\equiv$ Dissipation (+ decoherence...)

Quantum Operations

- So we have a map like

$$\rho' = \mathbf{S}(\rho) = \sum_\alpha c_\alpha T_\alpha \rho T_\alpha^\dagger \text{ with } c_\alpha \geq 0$$

TP if $\sum_\alpha c_\alpha T_\alpha^\dagger T_\alpha = I$

Unital if $\sum_\alpha c_\alpha T_\alpha T_\alpha^\dagger = I$

- Unital $\equiv$ Diffusion, Dephasing, pure decoherence ...
- Bistochastic maps usually Random Unitary Processes

- Non-unital $\equiv$ Dissipation ($+$ decoherence...)

Dissipative Quantum Operations

- Non-unital $\equiv$ Dissipation ($+$ decoherence...)

Quantities

$$\eta = \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N) - I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad \text{non-unitality ("distance")}$$

$$= \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N))^2 - (I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad \text{("}\partial_t\text{" of purity)}$$

$$= \frac{1}{N}\,\mathrm{Tr}\left[\Big(\sum_k [M_k, M_k^\dagger]\Big)^2\right] \stackrel{\mathrm{def}}{=} \frac{1}{N}\,\mathrm{Tr}[\Gamma^2]$$

In the classical limit η is related to the integral over phase space of the Poisson brackets of $L_k(q,p)$'s $\longrightarrow$ global dissipation parameter.

Dissipative Quantum Operations

- Non-unital $\equiv$ Dissipation ($+$ decoherence...)

Quantities

$$
\begin{aligned}
\eta &= \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N) - I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad \text{non-unitality (“distance”)} \\
&= \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N))^2 - (I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad (\text{“}\partial_t\text{” of purity}) \\
&= \frac{1}{N}\,\mathrm{Tr}\left[(\sum_k [M_k, M_k^\dagger])^2\right] \stackrel{\text{def}}{=} \frac{1}{N}\,\mathrm{Tr}[\Gamma^2]
\end{aligned}
$$

In the classical limit η is related to the integral over phase space of the Poisson brackets of $L_k(q,p)$'s $\longrightarrow$ global dissipation parameter.

Dissipative Quantum Operations

- Non-unital $\equiv$ Dissipation ($+$ decoherence...)

Quantities

$$\eta = \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N) - I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad \text{non-unitality (“distance”)}$$

$$= \frac{\mathrm{Tr}\left[(\mathbf{S}(I/N))^2 - (I/N)^2\right]}{\mathrm{Tr}[(I/N)^2]} \quad (\text{“}\partial_t\text{” of purity})$$

$$= \frac{1}{N}\,\mathrm{Tr}\left[\Big(\sum_k [M_k, M_k^\dagger]\Big)^2\right] \stackrel{\mathrm{def}}{=} \frac{1}{N}\,\mathrm{Tr}[\Gamma^2]$$

In the classical limit η is related to the integral over phase space of the Poisson brackets of $L_k(q,p)$'s $\longrightarrow$ global dissipation parameter.

Dissipative Quantum Operations

Definition

$$\Gamma = \sum_k [M_k, M_k^\dagger]$$

- Hermitian and traceless
- Representation in phase space (Wigner or Husimi) is real and gives picture of local contraction regions ($\Gamma(q,p) < 0$)

Dissipative Quantum Operations

Definition

$$\Gamma = \sum_k [M_k, M_k^\dagger] \xrightarrow[\text{"}\hbar \to 0\text{"}]{} \propto \mathrm{div}(\vec{\mathrm{drift}})$$

- Hermitian and traceless
- Representation in phase space (Wigner or Husimi) is real and gives picture of local contraction regions ($\Gamma(q,p) < 0$)

Outline

Amplitude damping channel

1-qubit Amplitude Damping channel (Preskill, Nielsen & Chuang)

- Kraus operators

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}^{\mathrm{AD}}(\rho) = \begin{pmatrix} \rho_{00} + p\rho_{11} & \sqrt{1-p}\,\rho_{01} \\ \sqrt{1-p}\,\rho_{10} & (1-p)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}^{\mathrm{AD}}(I) = \begin{pmatrix} 1+p & 0 \\ 0 & (1-p) \end{pmatrix} \neq I$$

- $\mathbf{S}^{\mathrm{AD}}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\qquad \eta = p^2 \qquad \Gamma = p\sigma_z$

Amplitude damping channel

1-qubit Amplitude Damping channel (Preskill, Nielsen & Chuang)

- Kraus operators

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}^{\mathrm{AD}}(\rho) = \begin{pmatrix} \rho_{00} + p\rho_{11} & \sqrt{1-p}\,\rho_{01} \\ \sqrt{1-p}\,\rho_{10} & (1-p)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}^{\mathrm{AD}}(I) = \begin{pmatrix} 1+p & 0 \\ 0 & (1-p) \end{pmatrix} \neq I$$

- $\mathbf{S}^{\mathrm{AD}}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\quad \eta = p^2 \quad \Gamma = p\sigma_z$

Amplitude damping channel

1-qubit Amplitude Damping channel (Preskill, Nielsen & Chuang)

- Kraus operators

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}^{\mathrm{AD}}(\rho) = \begin{pmatrix} \rho_{00} + p\rho_{11} & \sqrt{1-p}\,\rho_{01} \\ \sqrt{1-p}\,\rho_{10} & (1-p)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}^{\mathrm{AD}}(I) = \begin{pmatrix} 1+p & 0 \\ 0 & (1-p) \end{pmatrix} \neq I$$

- $\mathbf{S}^{\mathrm{AD}}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\quad \eta = p^2 \quad \Gamma = p\sigma_z$

1-qubit Amplitude Damping channel (Preskill, Nielsen & Chuang)

- Kraus operators

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}^{\mathrm{AD}}(\rho) = \begin{pmatrix} \rho_{00} + p\rho_{11} & \sqrt{1-p}\,\rho_{01} \\ \sqrt{1-p}\,\rho_{10} & (1-p)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}^{\mathrm{AD}}(I) = \begin{pmatrix} 1+p & 0 \\ 0 & (1-p) \end{pmatrix} \neq I$$

- $\mathbf{S}^{\mathrm{AD}}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\qquad \eta = p^2 \qquad \Gamma = p\sigma_z$

Simple Model

- Superoperators of the form $\mathbf{S}_2 = (1-\epsilon)I + \epsilon \mathbf{S}_1$
- Propose

$$\mathbf{S}_{\epsilon\alpha}(\rho) = (1-\epsilon)\rho + \epsilon \sum_{ij} P_{[\alpha i],i}\, \rho P^{\dagger}_{[\alpha i],i} \quad 0 < \alpha < 1$$

where $P_{ij} \stackrel{\text{def}}{=} |i\rangle\langle j|$ and $i = 0, 1, \ldots, N-1$ labels momentum eigenbasis ($p_i = i/N$) and $[\alpha i] = int(\alpha i)$.

- $\alpha = 1$: Unital, Generalized Phase Damping channel (See Aolita et al., PRE 2004).
- $\alpha < (N-1)/N$ Non-Unital (AD channel-like). Invariant state: $\mathbf{S}(|0\rangle\langle 0|) = |0\rangle\langle 0| \; \forall\alpha$

$$\eta = \epsilon^2 \left(\frac{1-\alpha}{\alpha}\right), \quad \Gamma = \epsilon(\sum_i |\alpha i\rangle\langle\alpha i| - I)$$

$\alpha \sim 0$ “mimics” classical friction

$$\begin{aligned} \langle q\rangle' &= \langle q\rangle \\ \langle p\rangle' &= (1-\epsilon)\langle p\rangle \end{aligned}$$

$N = 2$ case ("1 qubit")

In this case $N = 2$ and the minimum η is attained for $\alpha \leq 1/2$ $(= (N-1)/N)$.

- Kraus operators

$$M_0 = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & \sqrt{1-\epsilon} \end{pmatrix}, \quad M_1 = \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & \sqrt{\epsilon} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}_{\epsilon,\alpha}(\rho) = \begin{pmatrix} \rho_{00} + \epsilon\rho_{11} & (1-\epsilon)\,\rho_{01} \\ (1-\epsilon)\,\rho_{10} & (1-\epsilon)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}_{\epsilon,\alpha}(I) = \begin{pmatrix} 1+\epsilon & 0 \\ 0 & (1-\epsilon) \end{pmatrix} \neq I$$

- $\mathbf{S}_{\epsilon,\alpha}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\qquad \eta = \epsilon^2$ $\qquad \Gamma = \epsilon(P_{00} - P_{11}) = \epsilon\sigma_z$

$N = 2$ case ("1 qubit")

In this case $N = 2$ and the minimum η is attained for $\alpha \leq 1/2$ $(= (N-1)/N)$.

- Kraus operators

$$M_0 = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & \sqrt{1-\epsilon} \end{pmatrix}, \quad M_1 = \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & \sqrt{\epsilon} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}_{\epsilon,\alpha}(\rho) = \begin{pmatrix} \rho_{00} + \epsilon\rho_{11} & (1-\epsilon)\,\rho_{01} \\ (1-\epsilon)\,\rho_{10} & (1-\epsilon)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}_{\epsilon,\alpha}(I) = \begin{pmatrix} 1+\epsilon & 0 \\ 0 & (1-\epsilon) \end{pmatrix} \neq I$$

- $\mathbf{S}_{\epsilon,\alpha}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\qquad \eta = \epsilon^2$ $\qquad \Gamma = \epsilon(P_{00} - P_{11}) = \epsilon\sigma_z$

$N = 2$ case ("1 qubit")

In this case $N = 2$ and the minimum η is attained for $\alpha \leq 1/2$ $(= (N-1)/N)$.

- Kraus operators

$$M_0 = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & \sqrt{1-\epsilon} \end{pmatrix}, \quad M_1 = \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & \sqrt{\epsilon} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}_{\epsilon,\alpha}(\rho) = \begin{pmatrix} \rho_{00} + \epsilon\rho_{11} & (1-\epsilon)\,\rho_{01} \\ (1-\epsilon)\,\rho_{10} & (1-\epsilon)\rho_{11} \end{pmatrix}$$

- Non-unital because

$$\mathbf{S}_{\epsilon,\alpha}(I) = \begin{pmatrix} 1+\epsilon & 0 \\ 0 & (1-\epsilon) \end{pmatrix} \neq I$$

- $\mathbf{S}_{\epsilon,\alpha}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\qquad \eta = \epsilon^2$ $\qquad \Gamma = \epsilon(P_{00} - P_{11}) = \epsilon\sigma_z$

$N = 2$ case ("1 qubit")

In this case $N = 2$ and the minimum η is attained for $\alpha \leq 1/2$ $(= (N-1)/N)$.

- Kraus operators

$$M_0 = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & \sqrt{1-\epsilon} \end{pmatrix}, \quad M_1 = \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & \sqrt{\epsilon} \\ 0 & 0 \end{pmatrix}$$

- The density matrix transforms as

$$\mathbf{S}_{\epsilon,\alpha}(\rho) = \begin{pmatrix} \rho_{00} + \epsilon\rho_{11} & (1-\epsilon)\,\rho_{01} \\ (1-\epsilon)\,\rho_{10} & (1-\epsilon)\rho_{11} \end{pmatrix}$$

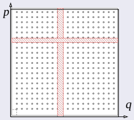
- Non-unital because

$$\mathbf{S}_{\epsilon,\alpha}(I) = \begin{pmatrix} 1+\epsilon & 0 \\ 0 & (1-\epsilon) \end{pmatrix} \neq I$$

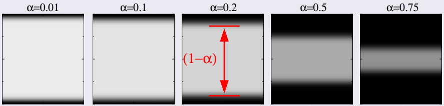
- $\mathbf{S}_{\epsilon,\alpha}(|0\rangle\langle 0|) = |0\rangle\langle 0|$ $\eta = \epsilon^2$ $\Gamma = \epsilon(P_{00} - P_{11}) = \epsilon\sigma_z$

Some figures

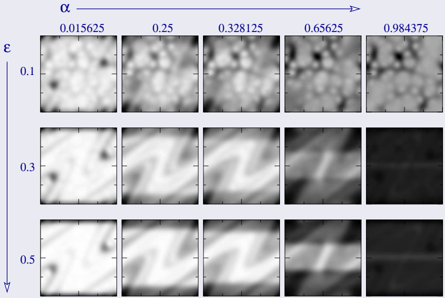
$\Gamma(q,p)$ (N dim. system, quantized torus phase space, Husimi rep....)



$$\Gamma(q,p) =$$

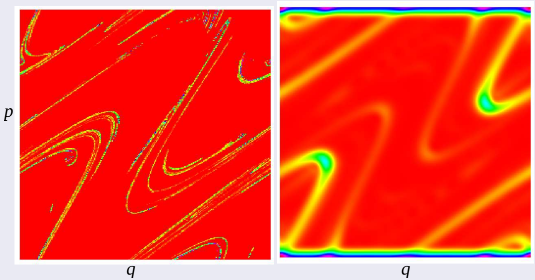


Composition with a unitary map $\mathbf{S}_{\rm tot}(\rho) = \mathbf{S}_{\epsilon\alpha}(U_{\rm map}\rho U^{\dagger}_{\rm map})$



Another figure ($\alpha \sim 0$ case)

classical and quantum invariant states



classical quantum

Summary

- Remark differences between unital and nonunital quantum operations.
- Independent of master eq. approach.
- Give phase space interpretation.
- Simple example illustrates main features.

Collaboration

- Buenos Aires (Quantum Chaos Group, CNEA):
  - Marcos Saraceno
  - María Elena Spina
- Como, Italy (Universitá degli Studi dell'Insubria):
  - Gabriel Carlo

Propaganda

quant-ph/0508190

Collaboration

- Buenos Aires (Quantum Chaos Group, CNEA):
  - Marcos Saraceno
  - María Elena Spina
- Como, Italy (Universitá degli Studi dell'Insubria):
  - Gabriel Carlo

Propaganda

quant-ph/0508190

Muchas Gracias