Charged Composite Complexes in Landau Levels: Magnetic Translations and Coherent States

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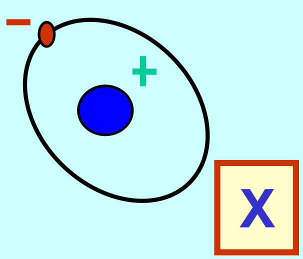

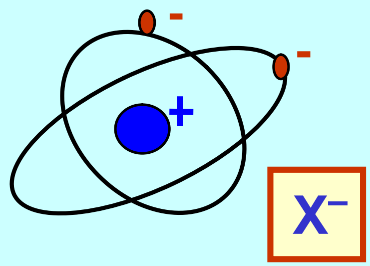

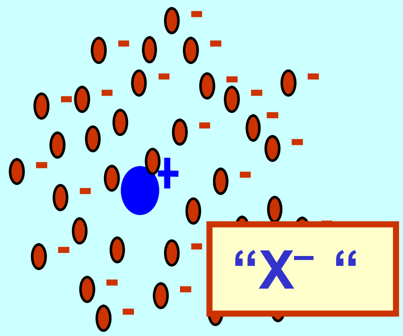

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Outline

❖ **Motivation** **X^- , *charged* e-h complexes in B**

➢ **Charged 2D magnetoexcitons X^-**

Magnetic Translations **Classification of states**

Dark and Bright X^- states

Relation to the "Hidden Symmetry"

➢ **"Applications":** **X^- Internal transitions** $\nu_e \to 0$

$\nu_e = 1, 2$

➢ **Coherent States and Symmetry Driven Squeezing**

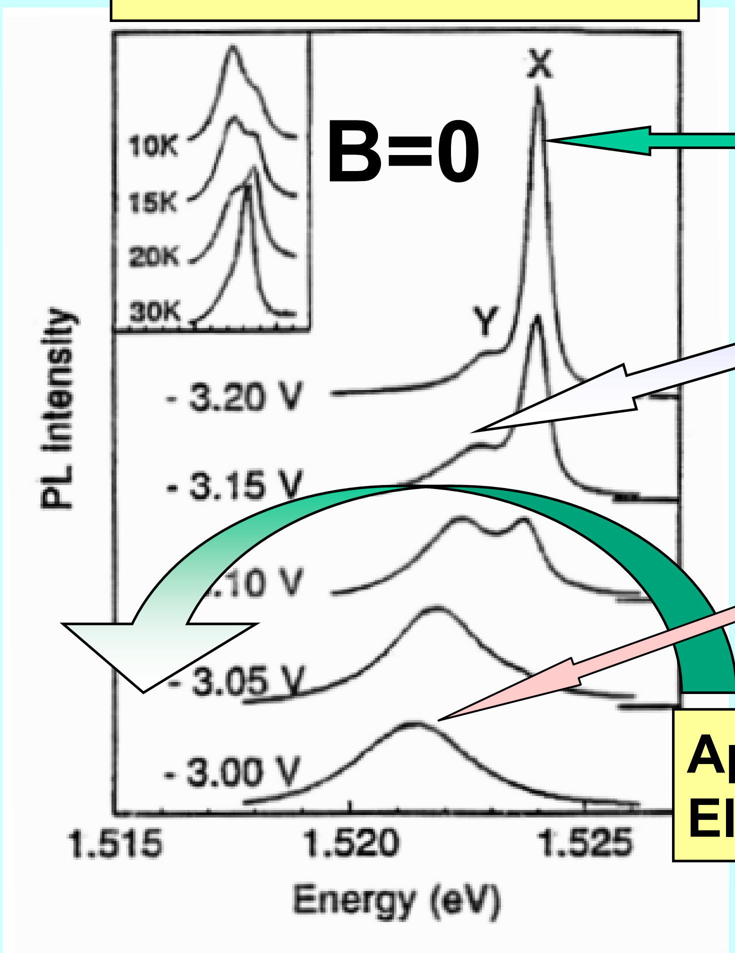
❖ **Summary**

Experiment: Motivation

Negatively Charged Excitons and MIT

Metal-Insulator Transition

Single Quantum Well



Free Exciton

Negatively Charged Exciton

X_s^- singlet

Fermi-Edge Singularity

Apply Gate Voltage to Change Electron Density in Well

Finkelstein et al. PRL 1995

Huard et al. PRL 2000

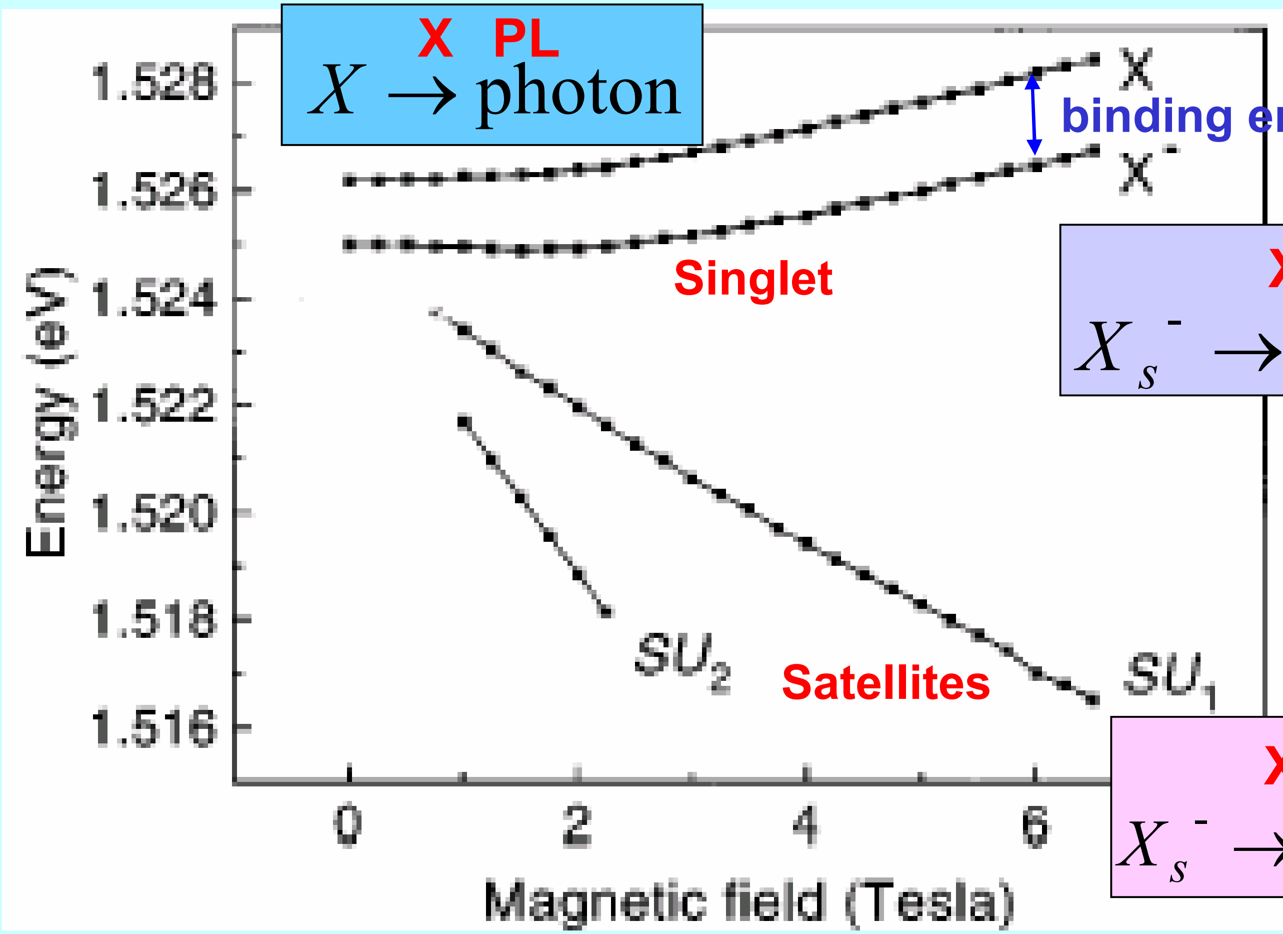
X^- Photoluminescence and Shake-Ups (SU_n)

Depleted 2DEG

GaAs QW

X PL

$$X \rightarrow \text{photon}$$



X⁻ PL

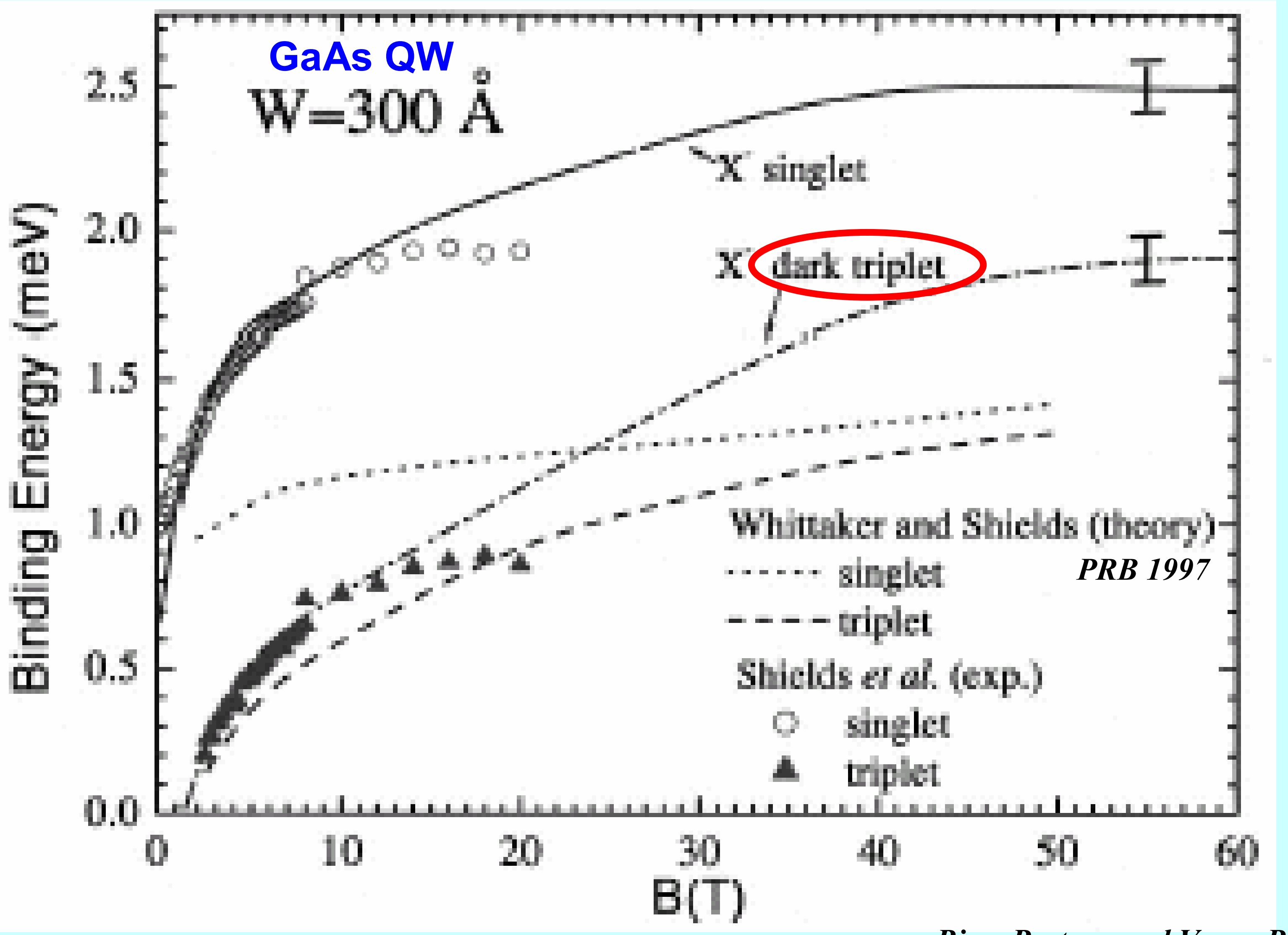
$$X_s^- \rightarrow e_0^- + \text{photon}$$

Finkelstein et al. 1996

X⁻ SU$_n$

$$X_s^- \rightarrow e_n^- + \text{photon}$$

Singlet and Triplet X^- States



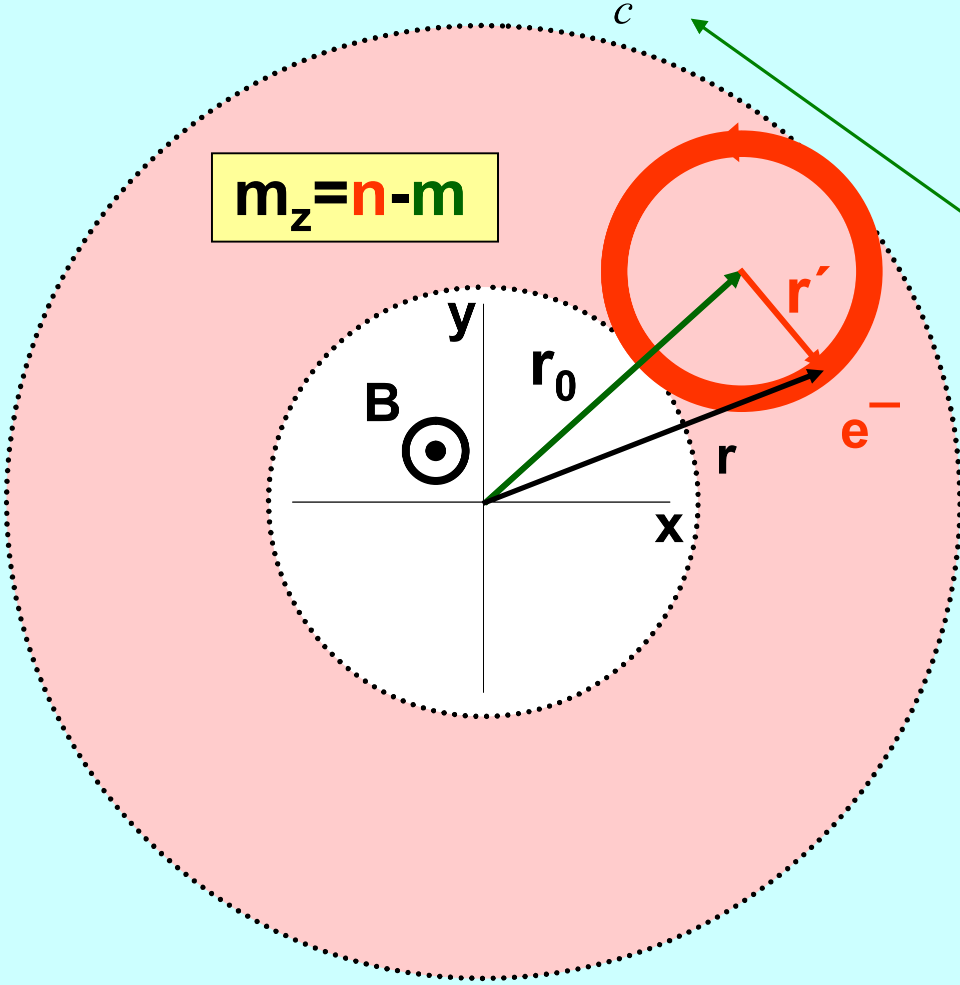
Riva, Peeters, and Varga PRB 2001

Magnetic Translations

Orbit Radius and Orbit Center

$$H = \frac{\hat{\boldsymbol{\pi}}^2}{2M} \qquad \hat{\boldsymbol{\pi}} = \mathbf{p} + e\mathbf{A}/c \qquad \hat{\boldsymbol{\pi}} = M\mathbf{v}$$

$$\dot{\boldsymbol{\pi}} = -\frac{e}{c}\mathbf{v}\times\mathbf{B}$$



Kinematic Momentum Operator $\hat{\boldsymbol{\pi}}$

LL #: n=0, 1, 2, ...

Orbit radius, Energy

$$\hat{\boldsymbol{\pi}}^2 \Leftrightarrow r'^2 = (2n+1)l_B^2$$

$$\hat{\mathbf{K}} = \hat{\boldsymbol{\pi}} + \frac{e}{c}\mathbf{r}\times\mathbf{B}$$

Magnetic Translation Operator

Oscillator #: m=0, 1, 2, ...

Orbit center, Degeneracy

$$\hat{\mathbf{K}}^2 \Leftrightarrow r_0^2 = (2m+1)l_B^2$$

$$l_B = \sqrt{\frac{\hbar c}{eB}}$$

Magnetic Translations: Single particle   -e < 0

Operator of finite Magnetic Translations (MTs):

$$\hat{T}(\mathbf{a}) = \exp\left(i\,\frac{\hat{\mathbf{K}}\cdot\mathbf{a}}{\hbar} \right)$$

Generator of MTs:

$$\hat{\mathbf{K}} = \hat{\boldsymbol{\pi}} - \frac{(-e)}{c}\mathbf{r}\times\mathbf{B} = -i\hbar\nabla - \frac{e}{2c}\mathbf{B}\times\mathbf{r}$$

Finite MTs:

$$\hat{T}(\mathbf{a})\Psi(\mathbf{r}) = \exp\left(-i\,\frac{e(\mathbf{B}\times\mathbf{r})\cdot\mathbf{a}}{2\hbar c} \right)\Psi(\mathbf{r}+\mathbf{a})$$

Non-commutative (Non-Abelian) Group:

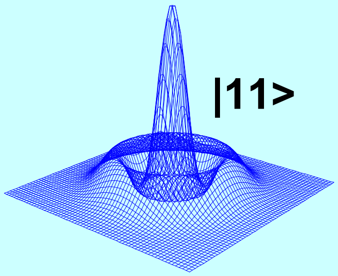
$$\hat{T}(\mathbf{a})\hat{T}(\mathbf{b}) = \exp\left(-i\,\frac{(\mathbf{a}\times\mathbf{b})\cdot\hat{z}}{2l_B^{\;2}} \right)\hat{T}(\mathbf{a}+\mathbf{b}) \neq \hat{T}(\mathbf{b})\hat{T}(\mathbf{a})$$

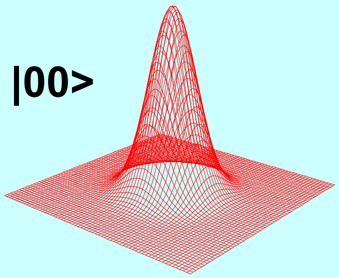
Ray representation (an extra phase factor)

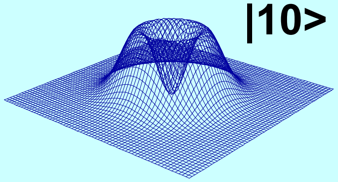
Zak PRB 1964   *Brown PRB 1964*

Raising and Lowering Operators I

$m_z = n - m$







Non-commutative algebras

$$[\hat{K}_x, \hat{K}_y] = ie\hbar B_z / c = -[\hat{\pi}_x, \hat{\pi}_y]$$

Like for momentum-coordinate

$$[\hat{p}, \hat{q}] = -i\hbar$$

2D e⁻ in B = *two* oscillators:

Lippmann and Johnson 1949

Malkin and Man'ko 1968

$$\hat{\pi}_+ = \hat{\pi}_x + i\,\hat{\pi}_y$$

$$\hat{K}_- = \hat{K}_x - i\hat{K}_y$$

$$|nm\rangle = (A^+)^n (B^+)^m |00\rangle / \sqrt{n!m!}$$

Raising and Lowering Operators II

Hamiltonian

$$H = \frac{\hat{\boldsymbol{\pi}}^2}{2M} = \hbar\omega_c(A^+A + \tfrac{1}{2})$$

Generator of MTs squared

$$\hat{\mathbf{K}}^2 = \hat{K}_x{}^2 + \hat{K}_y{}^2 = l_B{}^{-2}(B^+B + \tfrac{1}{2})$$

Operator of Orbital Angular Momentum Projection

$$\hat{L}_z = (A^+A - B^+B)$$

Common eigenstates

$$|nm\rangle = (A^+)^n (B^+)^m |00\rangle / \sqrt{n!m!}$$

Composite Complexes: Magnetic Translations

$$\hat{\mathrm{H}} = \sum_j \frac{\hat{\boldsymbol{\pi}}_j^{\,2}}{2m_j} + \sum_{i \neq j} U(\mathbf{r}_i - \mathbf{r}_j)$$

$$\hat{\pi}_{\mathrm{j}} = -\mathrm{i}\hbar\nabla_{\mathrm{j}} - \frac{\mathrm{e_j}}{\mathrm{c}}\vec{\mathrm{A}}(\vec{\mathrm{r}}_{\mathrm{j}})$$

Exact Symmetry – Magnetic Translations: $[\hat{\mathrm{H}}, \hat{\mathbf{K}}] = 0$

Generator of MTs for the whole system

$$\hat{\mathbf{K}} = \sum_{\mathrm{j}} (\hat{\boldsymbol{\pi}}_{\mathrm{j}} - (\mathrm{e_j}/\mathrm{c})\vec{\mathrm{r}}_{\mathrm{j}} \times \vec{\mathrm{B}})$$

$$[\hat{\mathrm{K}}_{\mathrm{x}}, \hat{\mathrm{K}}_{\mathrm{y}}] = \frac{i\hbar B_z}{c} Q$$

Total charge

$$Q = \sum_i e_i$$

Composite Complexes: Magnetic Translations

$$[\hat{\mathrm{K}}_{\mathrm{x}},\hat{\mathrm{K}}_{\mathrm{y}}]=\frac{i\hbar B_z}{c}Q$$

Neutral Systems

$$Q=\sum_i e_i=0$$

Hydrogen atom H
Exciton X

Lamb 1952   Gor'kov & Dzyaloshinskii 1967

$$[\hat{\mathrm{H}},\hat{\mathbf{K}}]=0$$

Charged Systems

$$Q=\sum_i e_i\neq 0$$

Hydrogen ion H^-
Trions X^- , X^+

Avron, Herbst & Simon 1978

Commutative MTs

QuasiMomentum

$$\hat{\mathbf{K}}\rightarrow\mathbf{K}=(K_x,K_x)$$

Continuous quantum #

Continuous spectra
(magnetoexciton bands)

Non-commutative MTs

Oscillator quantum #

$$\hat{\mathbf{K}}^2\Leftrightarrow k=0,1,\ldots$$

"cyclotron orbit center"

Continuous+Discrete spectra

X^- : Symmetries and Optical Selection Rules

$$\hat{H} = \sum_{i=1,2} \frac{\hat{\pi}_{e_i}^2}{2m_e} + \frac{\hat{\pi}_h^2}{2m_h} + \frac{e^2}{\varepsilon|\vec{r}_1 - \vec{r}_2|} - \sum_{i=1,2} \frac{e^2}{\varepsilon|\vec{r}_i - \vec{r}_h|}$$

$$\hat{\pi}_j = -i\hbar\nabla_j - \frac{e_j}{c}\vec{A}(\vec{r}_j)$$

Axial Symmetry: $[\hat{H}, \hat{L}_z] = 0$

$$\hat{L}_z = \sum_j (\vec{r}_j \times -i\hbar\nabla_j)_z$$

Total angular momentum projection M_z

$\Delta M_z = \pm 1$ FIR $\sigma^{\pm}$

$\Delta M_z = 0$ PL

Exact Symmetry – Magnetic Translations: $[\hat{H}, \hat{\mathbf{K}}] = 0$

$$\hat{\mathbf{K}} = \sum_j (\hat{\boldsymbol{\pi}}_j - (e_j/c)\vec{r}_j \times \vec{B})$$

$$[\hat{K}_x, \hat{K}_y] = \frac{i\hbar B_z}{c} Q$$

total charge $Q = \sum_i e_i \neq 0$

$\Delta k = 0$

Conservation of k

oscillator quantum #

$\hat{\mathbf{K}}^2 \Leftrightarrow k = 0, 1, \ldots$

"cyclotron orbit center"

Avron, Herbst & Simon 1978

ABD & Sivachenko 1999

2D Magneto-X^- : Quantum Numbers

$n_e = 0$; $n_h = 0$

2D + High B

$X_{00} + e_0$

Continuum

$$E_0 = \sqrt{\frac{\pi}{2}}\frac{e^2}{\varepsilon l_B} \propto \sqrt{B}$$

X^-_{t00}

k = 0 1 2

M_z =-1 -2 -3

$$E_b = 0.043\,E_0$$

The only FAMILY of (triplet) bound states

Parent State

$$|k=0\,M_z\rangle$$

Daughter States

$$|k\,M_z-k\rangle = (\hat{K}_-)^k\,|0\,M_z\rangle / \sqrt{k!}$$

raising operator : $\hat{K}_- = \hat{K}_x - i\hat{K}_y$ $\quad [\hat{L}_z, \hat{K}_-] = -\hat{K}_-$

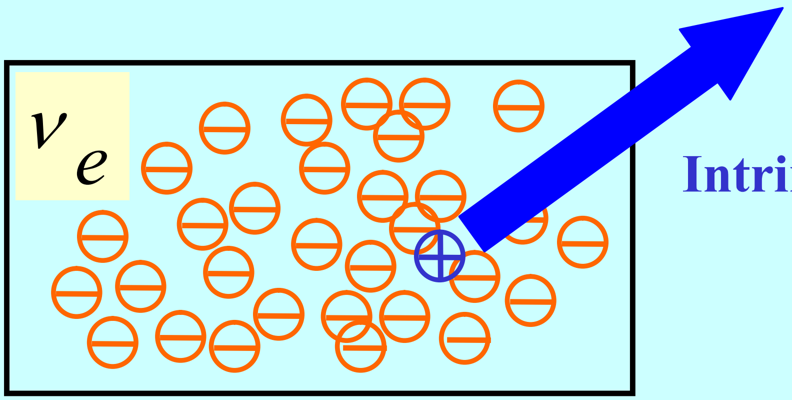
DARK in PL!

“Hidden Symmetry”

Magnetic translations

Magneto-PL: Probing 2DEG

Carries information about e-e correlations **(?)**



Intrinsic PL

Luminescence Operator

$$\nu_e \le 2$$

$$\hat{L}_{PL} = p_{cv} \int d\mathbf{r}\, \hat{\Psi}_e(\mathbf{r}) \hat{\Psi}_h(\mathbf{r}) \mapsto p_{cv} Q_0$$

$$Q_0 = \sum_m \hat{a}_m \hat{b}_m$$

$\mathbf{K} = 0$ neutral magnetoexciton (MX) in zero LL's

Magneto-PL and the "Hidden Symmetry"

Symmetric e-h systems

$$U_{ee} = U_{hh} = -U_{eh}$$

$$\varphi_h = \varphi_e^*$$ **LL degeneracy**

Lerner & Lozovik 1981, 1982

ABD& Lozovik 1983, 1984

Apalkov & Rashba 1992

MacDonald, Rezayi & Keller 1992

Exact Quantum Equation of Motion

$$[H_{\text{int}}, Q_0] = -E_0 Q_0$$

Ideal Gas of Composite Bosons

$$E_0 = \sqrt{\frac{\pi}{2}} \frac{e^2}{\varepsilon l_B} \propto \sqrt{B}$$

$$\hat{L}_{PL} = p_{cv} Q_0$$

$$\nu_e \leq 2$$

In symmetric QWs in high fields:

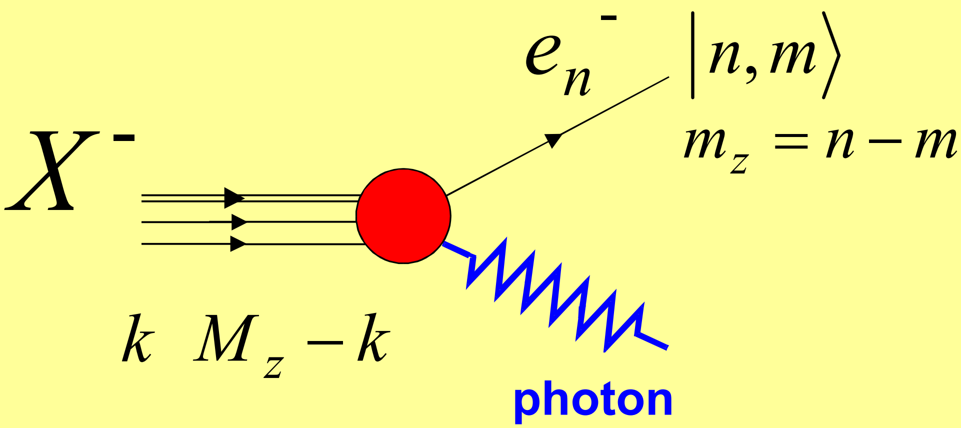
Recombination Energy= $E_{\text{gap}} - E_0$

$-1.043 E_0$ X_t^-

2D + High B: DARK

Palacios, Yoshioka & MacDonald 1996

X^- : *Exact* Magneto-PL Selection Rules



$$k = m$$

Orbit center conserved

$$M_z - k = m_z = n - m$$

Angular momentum conserved

X^- PL $n = M_z$

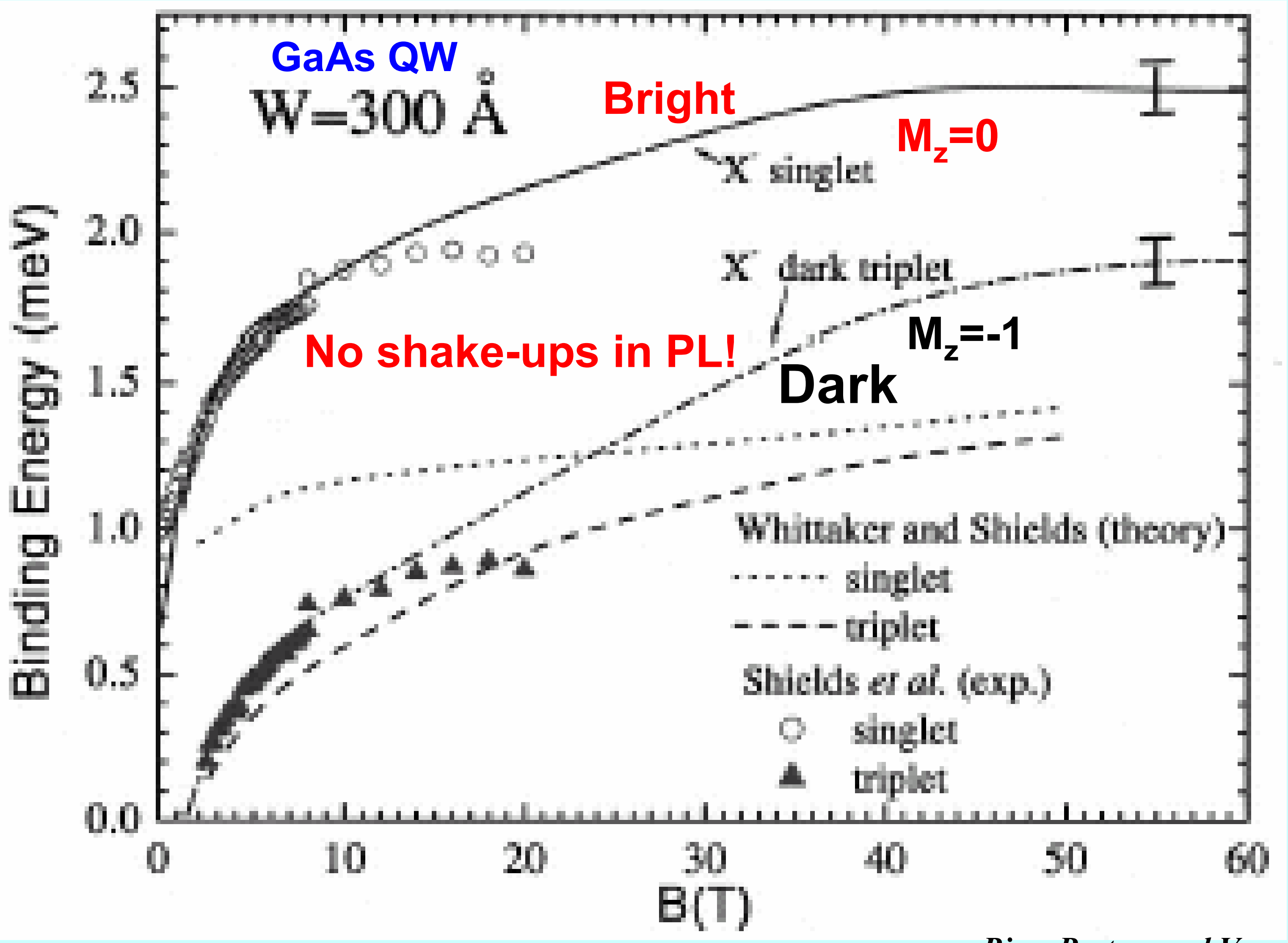
$$X^- \to e_n^- + \text{photon}$$

Final e- state: unique LL #

- **Dark X^- States** $M_z < 0$
- **No Shake-Ups in X^- PL** $n = 1,\ 2,\ \ldots$

ABD & Sivachenko PRL 2000

Bright and Dark X^- States

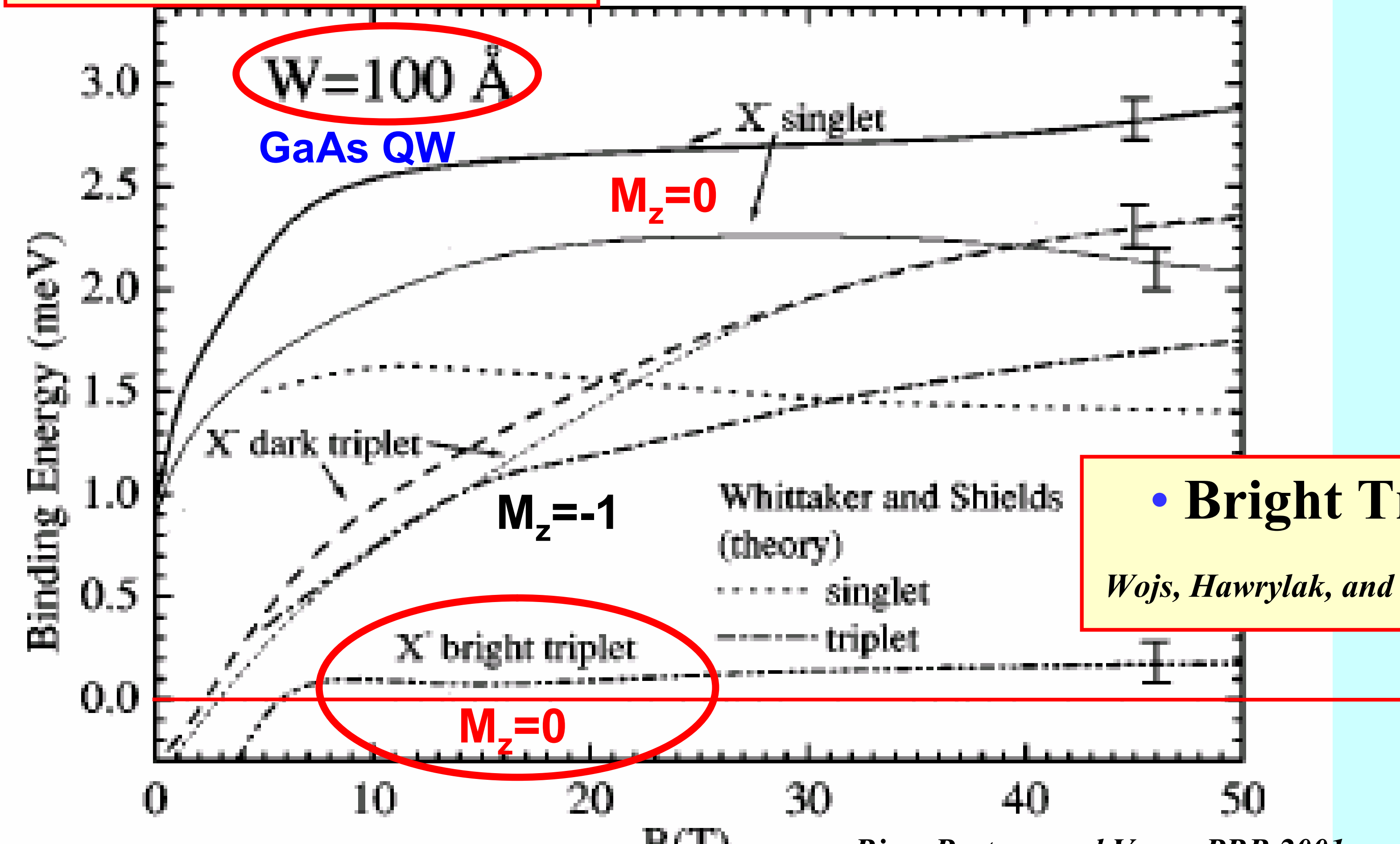


Riva, Peeters and Varga 2001

Why are X⁻ Dark Triplet and Shake-Ups Observed?

- **Disorder**
  (X⁻ localization?)
  ABD and Sivachenko PRL 2000

- **Scattering (2DEG)?**
  Sanvitto et al. PRL 2002



Riva, Peeters, and Varga PRB 2001

- **Bright Triplet?**
  Wojs, Hawrylak, and Quinn PRB 2001

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Charged Excitons in the Fractional Quantum Hall Regime

G. Yusa, H. Shtrikman, and I. Bar-Joseph
Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel
(Received 12 March 2001; published 1 November 2001)

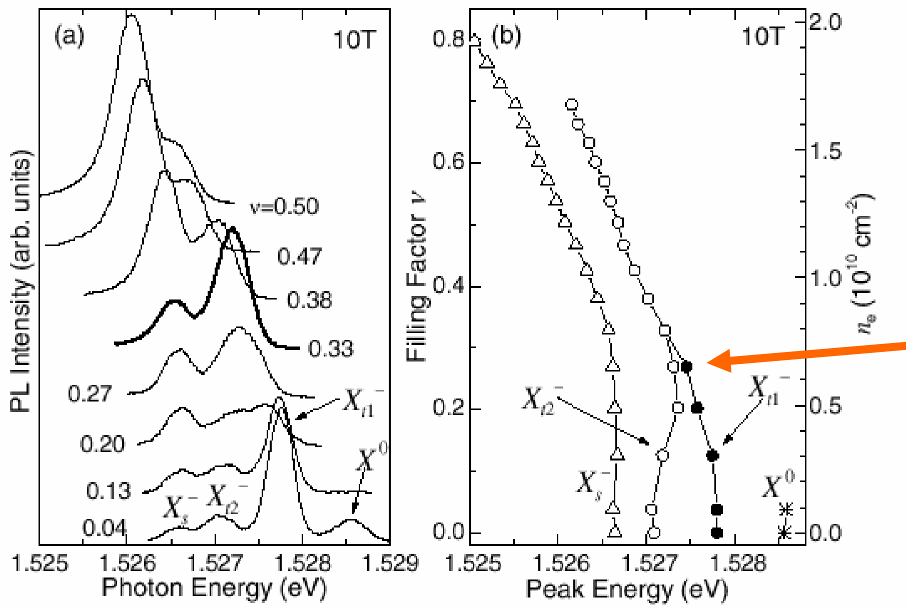


FIG. 3. (a) The PL spectra at 10 T for $0.04 < \nu < 0.50$. (b) The peak energies as a function of ν.

Finite filling factors ν

Dark and Bright Triplet States Merge?

Ashkinadze *et al.* PRB 69, 115303 (2004)

$\nu < 1$: Multiple (4) peaks

Possible relevance of disorder

Another “applications”

Combined Exciton-Cyclotron Resonance (ExCR)

Depleted 2DEG: PLE spectra

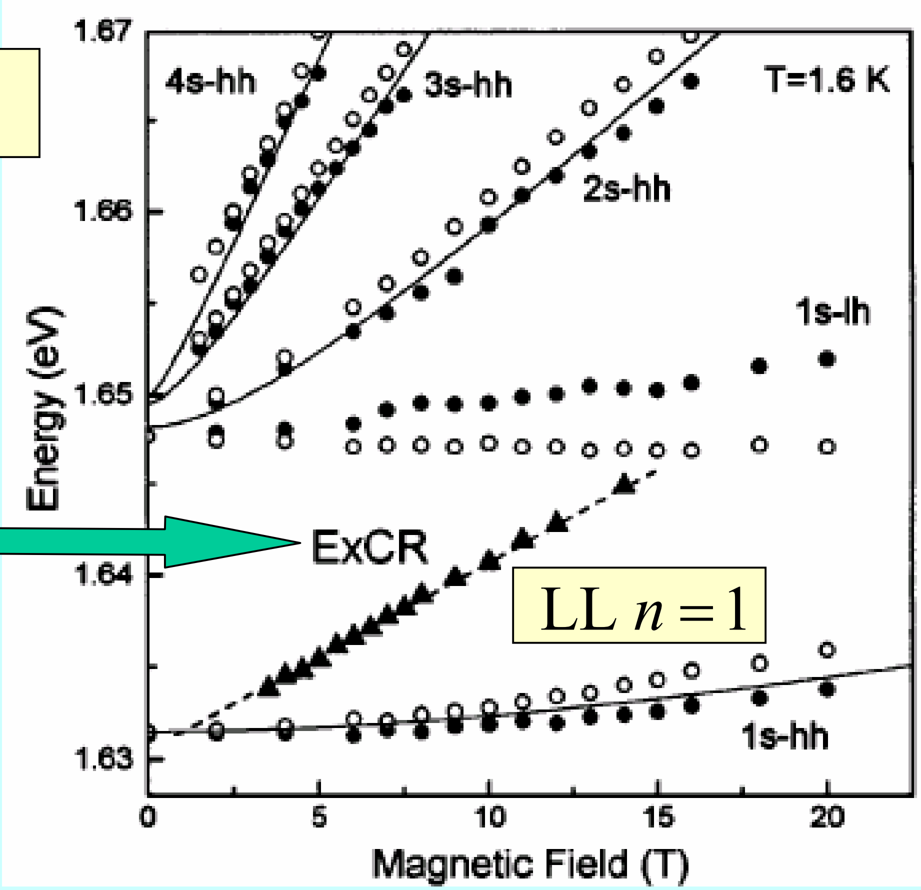
$n_e = 5\times10^{10}\,\mathrm{cm}^{-2}$ **Spin-polarized**

CdTe/CdMgTe 75 Å QW

Yakovlev, Kochereshko, Suris et al. PRL 1997

$e_0^- + \text{photon} \rightarrow X + e_n^-$

2e-h continuum



Creation of X⁻ in (higher) LLs?

$e_0^- + \text{photon} \rightarrow X^-$

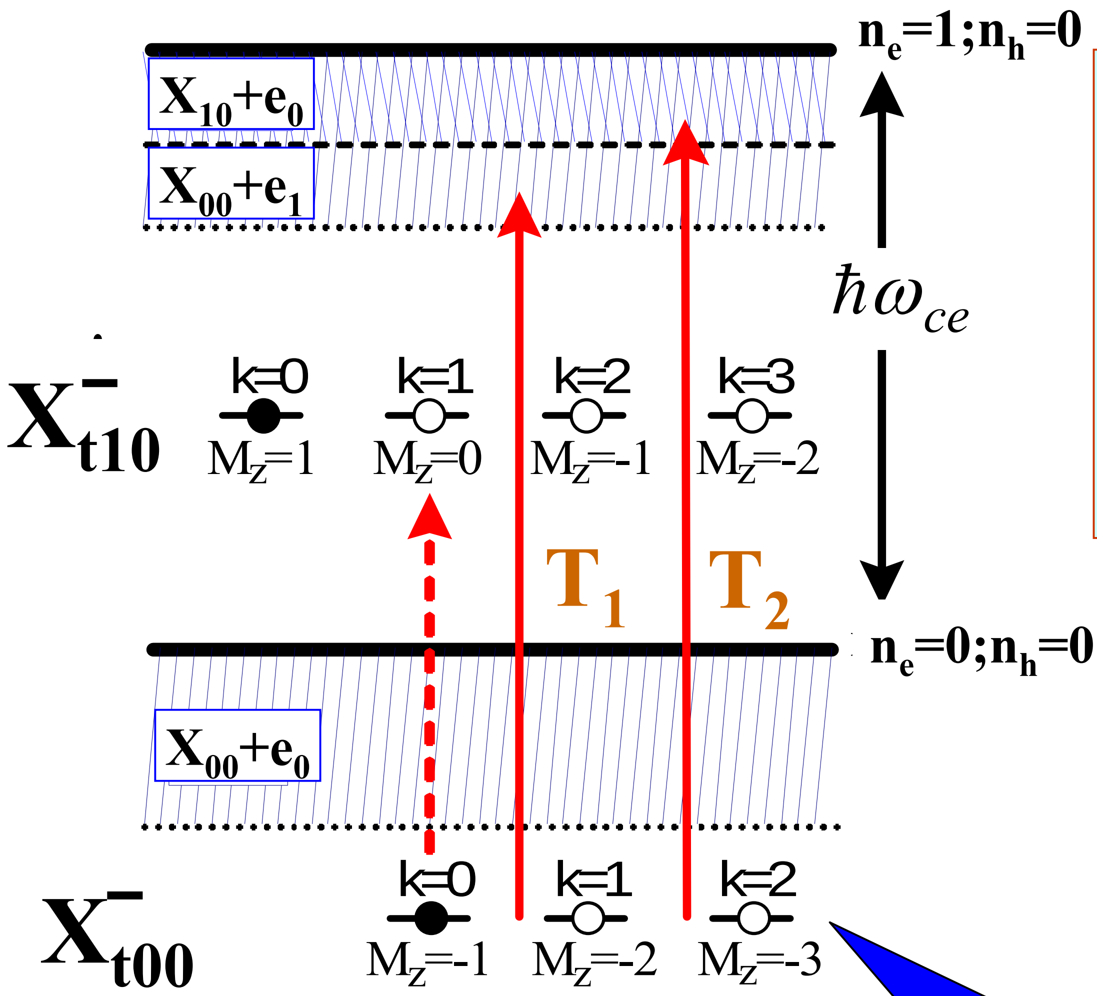
bound

Charged e-h complexes in B

ABD **PRB 64, 241101 (2001);**
PRB 69, 115332 (2004)

Shake-ups in 2DEG Magneto-Photoabsorption

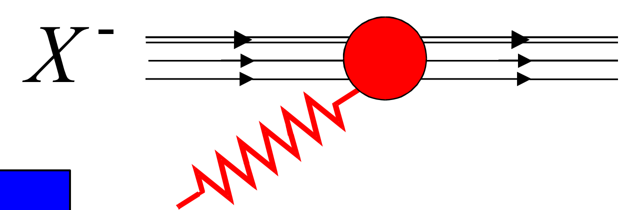
X^- Internal Transitions : Theory



- **Bound-to-bound:**
  - D⁻- equivalent triplet trans.
  - (forbidden) $X^-_{t00} \rightarrow X^-_{t00}$
- T_1 **Bound-to-continuum**
  - $X^-_{t00} \rightarrow X_{00} + e_1$
- T_2 **Bound-to-continuum**
  - $X^-_{t10} \rightarrow X_{10} + e_0$

Selection Rules

$\Delta k = 0; \Delta M_z = +1$



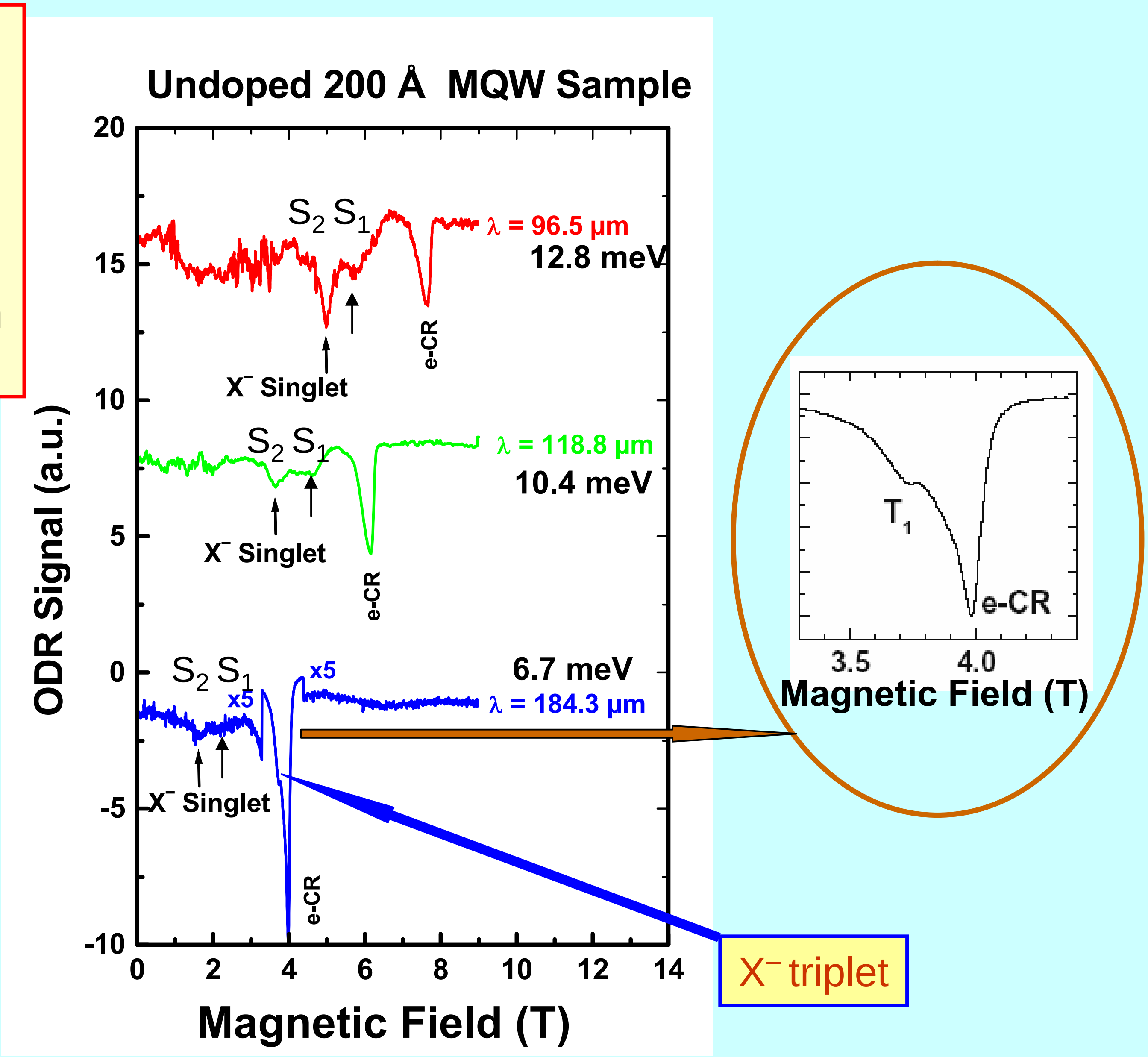
FIR σ^+ photon

Triplet, 2D, High B

k = Magnetic Translation Q #
(orbit center)

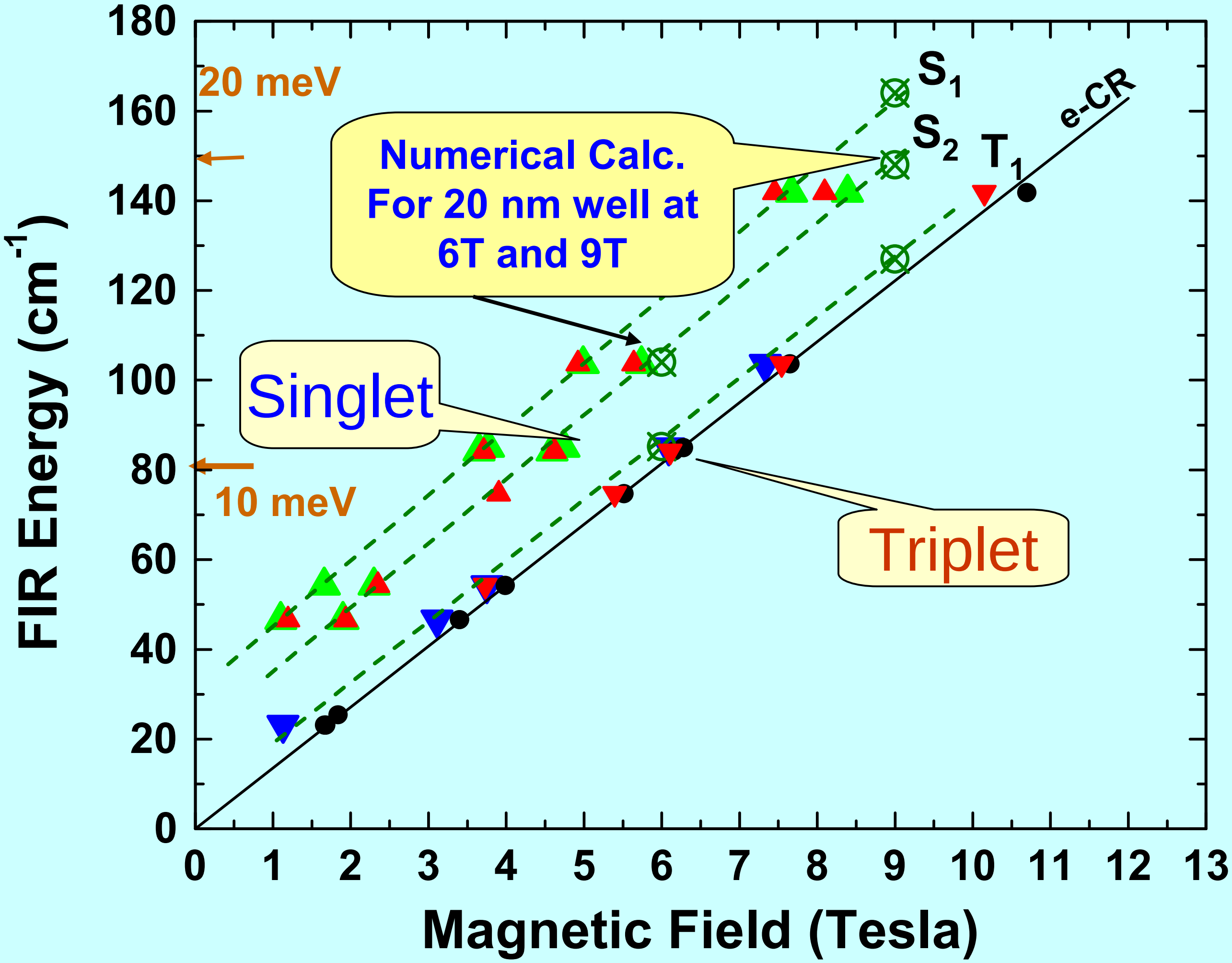
X^- Internal Transitions: Experiment

- **Optically Detected Resonance (ODR) spectroscopy:** changes in PL feature(s) due to resonant absorption of FIR radiation



Nickel et al. PRL 2002

Internal transitions of X^- - Summary Plot



Data from three, 20 nm well-width undoped samples

Excellent agreement with theory

- **Use internal transitions as probe for "X^-" and effects of excess electrons**

Nickel *et al.*, PRL <u>88</u>, 056801 (2002)

Keeping the symmetries intact

Laughlin's State: electrons in Zero LL

N-electron state in lowest LL:

$$\prod_{i<j} (z_i^* - z_j^*)^m \exp\left(-\frac{\sum_i \mathbf{r}_i^2}{4 l_B^2} \right)$$

A uniform polynomial in z_i^* of degree $m(N-1)$ $z^* = x - iy$

Axial Symmetry Satisfied: $M_z = -m(N-1)$

Permutational Symmetry Satisfied: m is odd

For a large disk: a uniform state with electron filling factor $\nu = \frac{1}{m}$

X^- : Two electrons + one hole in Zero LL

Possible basis states:

$$\left\langle \mathbf{r}_{e1}\, \mathbf{r}_{e2}\, \mathbf{r}_h \middle| X^- \right\rangle = Z_h^{\,l} (Z_{e1}^{\,*} - Z_{e2}^{\,*})^m \exp\left(-\frac{\mathbf{r}_{e1}^{\,2} + \mathbf{r}_{e2}^{\,2} + \mathbf{r}_h^{\,2}}{4 l_B^{\,2}} \right)$$

Axial Symmetry Satisfied: Total Angular Momentum Projection is fixed

$$M_z = l - m$$

Permutational Symmetry Satisfied:

Electron Singlet : m is even

Electron Triplet : m is odd

What about translations?

Charged Trions X^- in Zero LL

Basis states compatible with all symmetries:

$$\left\langle \mathbf{r}_{e1}\mathbf{r}_{e2}\ \mathbf{r}_h \middle| k=0, M_z = l-m \right\rangle =$$

$$= Z_h^{\ l}\left(Z_{e1}^{\ *} - Z_{e2}^{\ *}\right)^m \exp\left(-\frac{\mathbf{r}_{e1}^{\ 2} + \mathbf{r}_{e2}^{\ 2} + \mathbf{r}_h^{\ 2} - (z_{e1}^{\ *} + z_{e2}^{\ *}) z_h}{4 l_B^{\ 2}} \right)$$

Axial Symmetry Satisfied:
Total Angular Momentum Projection is fixed

$$M_z = l - m$$

Permutational Symmetry Satisfied:

Electron Singlet : m is even

Electron Triplet : m is odd

Magnetic Translations Satisfied:

Oscillator quantum # : k is fixed $(=0)$

Composite Charged Complex: Ladder Operators

MT operator

$$\hat{\mathbf{K}} = \sum_{\mathrm{j}} (\hat{\boldsymbol{\pi}}_{\mathrm{j}} - (\mathrm{e}_{\mathrm{j}}/\mathrm{c})\vec{\mathrm{r}}_{\mathrm{j}} \times \vec{\mathrm{B}})$$

$$[\hat{\mathrm{K}}_{\mathrm{x}}, \hat{\mathrm{K}}_{\mathrm{y}}] = \tfrac{i\hbar B}{c} Q < 0$$

Negatively Charged

$$\hat{k}_{\pm} = \tfrac{1}{\sqrt{2}\hbar}(\hat{K}_x \pm i\hat{K}_y)\tilde{l}_B$$

Bose Ladder Operators for the whole system

$$[\hat{k}_{+}, \hat{k}_{-}] = 1$$

Lowering

Raising

$$\tilde{l}_B = \sqrt{\frac{\hbar c}{|Q|B}}$$

Magnetic length

$$\mathbf{X}^{-}$$

$$\hat{k}_{+} = B_e(\mathbf{r}_1) + B_e(\mathbf{r}_2) - B_h{}^{+}(\mathbf{r}_h)$$

$$\hat{k}_{-} = B_e{}^{+}(\mathbf{r}_1) + B_e{}^{+}(\mathbf{r}_2) - B_h(\mathbf{r}_h)$$

Electron vs Hole States in B

Electron Raising intra-LL operator: $B_e^{\,+}(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\frac{z^*}{2l_B} - 2l_B\frac{\partial}{\partial z}\right)$

Hole Raising intra-LL operator: $B_h^{\,+}(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\frac{z}{2l_B} - 2l_B\frac{\partial}{\partial z^*}\right)$

$\mathbf{X}^-$

Lowering $\hat{k}_+ = B_e(\mathbf{r}_1) + B_e(\mathbf{r}_2) - B_h^{\,+}(\mathbf{r}_h)$

Raising $\hat{k}_- = B_e^{\,+}(\mathbf{r}_1) + B_e^{\,+}(\mathbf{r}_2) - B_h(\mathbf{r}_h)$

How to Handle $\hat{k}_+$?

Lowering!? $\hat{k}_+\left|0\right\rangle \neq 0$

$$\hat{k}_+ = B_e(\mathbf{r}_1) + B_e(\mathbf{r}_2) - B_h{}^+(\mathbf{r}_h)$$

$$\underset{}{\left|0\right\rangle} = \left|\underset{e_1}{00},\underset{e_2}{00},\underset{h}{00}\right\rangle$$

Vacuum

$$\left\langle \mathbf{r}_1\mathbf{r}_2\mathbf{r}_h \middle| 0\right\rangle = \exp\left(-\frac{\mathbf{r}_1{}^2 + \mathbf{r}_2{}^2 + \mathbf{r}_h{}^2}{4l_B{}^2}\right)$$

The old vacuum does not have a specific value of the Oscillator Q. #
=> Is not compatible with MTs

Solve for the new vacuum

$$\hat{k}_+\left|\widetilde{0}\right\rangle = 0$$

and find its coordinate representation

How to Handle the Ladder Operators?

Lowering $\hat{k}_+ \left|\widetilde{0}\right\rangle = 0$

$$\hat{k}_+ = B_e(\mathbf{r}_1) + B_e(\mathbf{r}_2) - B_h^{\ +}(\mathbf{r}_h)$$

Mixture of raising and lowering operators.

Bogoliubov Transformation for Bosons?

Step I: Orthogonal coordinate transformation

$$\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_h\} \rightarrow \left\{\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{\sqrt{2}}, \mathbf{r} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}, \mathbf{r}_h\right\}$$

2e: center-of-charge relative

$$\hat{k}_+ = \sqrt{2} B_e(\mathbf{R}) - B_h^{\ +}(\mathbf{r}_h)$$

$$\hat{k}_- = \sqrt{2} B_e^{\ +}(\mathbf{R}) - B_h(\mathbf{r}_h)$$

Bogoliubov Transformation for Bosons

$$u = \sqrt{2} \qquad v = 1$$

$$u^2 - v^2 = 1$$

$$\hat{k}_+ = \sqrt{2} B_e(\mathbf{R}) - B_h^{\,+}(\mathbf{r}_h)$$

Step2: Diagonalization

$$u = \cosh\Theta$$

$$v = \sinh\Theta$$

$$\hat{k}_+ = S B_e(\mathbf{R}) S^+$$

$$S = \exp\left\{\Theta\left(B_h^{\,+}(\mathbf{r}_h) B_e^{\,+}(\mathbf{R}) - \mathrm{H.c.}\right)\right\}$$

$$\tanh\Theta = v / u$$

A new vacuum

$$|0\rangle \rightarrow \left|\widetilde{0}\right\rangle = S|0\rangle$$

$$\hat{k}_+\left|\widetilde{0}\right\rangle = SB(\mathbf{R})S^+S|0\rangle = SB(\mathbf{R})|0\rangle = 0$$

A coherent state with built-in R-r_h correlations

Squeezed Coherent States

A new vacuum: a two-mode squeezed state

$$\left|\tilde{0}\right\rangle = S\left|0\right\rangle = \frac{1}{\cosh\Theta}\exp\left\{ \tanh\Theta \; B_h^{\,+}(\mathbf{r}_h) B_e^{\,+}(\mathbf{R}) \right\}\left|0\right\rangle$$

$$\tanh\Theta = 1/\sqrt{2}$$

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{\sqrt{2}}$$

Coordinate representation

$$\left\langle \mathbf{r}\mathbf{R}\mathbf{r}_h \middle| \tilde{0} \right\rangle = \exp\left\{ -\frac{\mathbf{r}^2 + \mathbf{R}^2 + \mathbf{r}_h^{\,2} - \sqrt{2} Z^* z_h}{4 l_B^{\,2}} \right\}$$

$$\mathbf{r} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}$$

$$\left\langle \tilde{0} \middle| \mathbf{R}\cdot\mathbf{r}_h \middle| \tilde{0} \right\rangle = \sqrt{8} l_B^{\,2} \neq 0$$

A coherent state with built-in **R-r$_h$** correlations

$$\left\langle \tilde{0} \middle| (\mathbf{R} + \mathbf{r}_h)^2 \middle| \tilde{0} \right\rangle = 4(2+\sqrt{2}) l_B^{\,2}$$

$$\left\langle \tilde{0} \middle| (\mathbf{R} - \mathbf{r}_h)^2 \middle| \tilde{0} \right\rangle = 4(2-\sqrt{2}) l_B^{\,2}$$

Squeezed oscillator states =
squeezing in real space in B!

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2D, high-B limit: Triplet X^- in Zero LL

The only state bound in zero LL: Triplet with M_z=-1

The simplest state compatible with all symmetries:

$$\left\langle \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_h \middle| \widetilde{0} \right\rangle$$

$$\left\langle \mathbf{r}_{e1}\, \mathbf{r}_{e2}\, \mathbf{r}_h \middle| M_z = -1, T \right\rangle =$$

$$= (Z_{e1}^* - Z_{e2}^*) \exp\left(-\frac{\mathbf{r}_{e1}^2 + \mathbf{r}_{e2}^2 + \mathbf{r}_h^2 - (z_{e1}^* + z_{e2}^*) z_h}{4 l_B^2} \right)$$

A two-mode squeezed state with built-in (symmetry driven) e-h correlations **=> Already ensures binding**

Total Coulomb Interaction Energy

$$E = \left(\frac{\sqrt{2}}{4} - \frac{\sqrt{5}}{6} \right) E_0 = -1.007 E_0 < -E_0$$

Binding energy E_b=0.007E_0
17% of numerically exact value 0.043E_0

Variational functions
With four-mode squeezing

***ABD* JETP Lett. 74, 318 (2001)**

Keeping Magnetic and Axial Symmetries

Bogoliubov transformations effectively generate new charged particles in a magnetic field with coordinates

$$\boldsymbol{\rho}_1 = \sqrt{2}\mathbf{R} - \mathbf{r}_h \qquad \boldsymbol{\rho}_2 = \sqrt{2}\mathbf{r}_h - \mathbf{R} \quad \text{and} \quad \mathbf{r} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}$$

$$SB_e(\mathbf{R})S^+ = B_e(\boldsymbol{\rho}_1) \qquad SB_h(\mathbf{r}_h)S^+ = B_e(\boldsymbol{\rho}_2)$$

The interaction Hamiltonian becomes

$$H = H_{ee} + H_{eh} = \frac{e^2}{\sqrt{2}r} - \frac{\sqrt{2}e^2}{|\boldsymbol{\rho}_2 - \mathbf{r}|} - \frac{\sqrt{2}e^2}{|\boldsymbol{\rho}_2 + \mathbf{r}|}$$

H does not depend on ρ_1

A two-particle problem?

- **All symmetries maintained**
- **Variables separate**
- **Built-in correlations (squeezing)**
- **Fast (exponential) convergence**
- **Complicated Coulomb matrix elements**

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and to be published

Summary

- **Magnetic Translations for Charged e-h complexes:**
  classification of states, exact selection rules, squeezing ..

- **Dark triplet X^- states**
  relevance of scattering (disorder? 2DEG?)

- **Internal transitions of isolated X^- in B**
  observed experimentally (ODR),
  are in excellent agreement with theory

- **Theory of Shake-ups in depleted 2DEG magneto-photoabsorption**

- **"X^--like"= Many electrons + X^-**
  observed e-CR and additional blue-shifted resonances
  when $\nu < 2$

Collective response of "many electron/few hole" system

Magnetoplasmon bound to mobile VB hole