Ruelle resonances in kicked quantum spin chain

Tomaž Prosen

Physics Department, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1111 Ljubljana, Slovenia

Abstract

We study exponential decay of high temperature time correlation functions in a non-integrable quantum spin chain problem, namely Ising spin 1/2 chain kicked with tilted homogeneous magnetic field. For this purpose we define a master propagator over a suitable banach space of quantum observables (quantum many-body analogue of Perron-Frobenius operator) whose leading eigenvalue determines the asymptotic decay of correlations. This is demonstrated with explicit calculation for which a fast algorithm for the construction of the master propagator is developed.

1 Introduction

It is known that relaxation to equilibrium of deterministic dynamical systems can often be treated in terms of the spectral problem for a non-unitary master operator. This is an established theory in the context of classical dynamical systems [1–3] where the master operator, propagating the phase space densities, is called Perron-Frobenius operator. In quantum mechanics, the relaxation to equilibrium (quantum mixing property) can only take place with infinitely many degrees of freedom, a situation which prohibits analytic calculations for a generic nontrivial example. We note that there exists very general and abstract procedure, namely the so-called recurrence-relation method [4], which facilitates the study of time correlation functions in classical and quantum mechanics by means of tridiagonal matrix of a transfer operator in a dynamically generated operator basis.

In classical hamiltonian mechanics, irreversibility emerges as a consequence of complexity growth taking place in the course of time evolution at smaller and smaller scales, so effective non-unitarity of time evolution is obtained by *coarse graining* at arbitrarily small scale. This view can also be adopted in semi-classical mechanics provided the coarse graining is performed on a scale

Preprint submitted to Elsevier Science *3 March 2003*

larger than the Planck cell size [5,6]. In this paper we go further and address the following simple question: "Can one define a non-unitary evolution operator for an infinite conservative closed quantum many-body system without reference to any classical concept?" In order to do so, we study a particular but hopefully generic case of a simple non-integrable spin 1/2 chain, where the concept of coarse graining is substituted by a projection onto subspace of local observables of finite order (finite interaction range). A strong motivation and inspiration for such a project was provided by the evidence of exponential decay of high (or infinite) temperature correlation functions which has been found in several different models of *non-integrable* interacting quantum many-body systems [7–9]. These findings suggest existence of well and robustly defined leading eigenvalue of a certain transfer operator. In contrast we note that for *integrable* interacting quantum many-body systems, high temperature correlations typically exhibit asymptotic power-law decay [10,11].

The central result of this paper is an effective numerical algorithm for explicit construction of time automorphism on a rich operator algebra and very suggestive convergence of Ruelle resonances (i.e. point spectrum of the 'coarse grained' time automorphism) for the Ising spin 1/2 chain kicked periodically with a tilted magnetic field. Some different aspects of this study were already published in a compact form [12]. Perhaps it should also be mentioned that such kicked quantum spin chains provide interesting toy models of scalable quantum computation [8].

2 Operator algebra and notation

We want to analyze dynamics of a non-integrable quantum many-body system over a closed but sufficiently large set of observables. Let us take a one-dimensional lattice — a chain of spins 1/2 which are described by Pauli variables $\sigma_s(x)$, $x \in \mathbb{Z}$, $s \in \{1,2,3\}$. We start by considering a space of local observables spanned by clusters

$$G_c(x) = \sigma_{c_0}(x)\sigma_{c_1}(x+1)\cdots\sigma_{c_{l-1}}(x+l-1), \tag{1}$$

where integer $c = (c_0 c_1 \ldots c_{l-1}) = \sum_{k=0}^{l-1} c_k 4^k$ is a *code* with base-4 digits $c_k \in \{0,1,2,3\}$. We put formally $\sigma_0(x) \equiv \mathbb{1}$, so digit $c_k = 0$ represents an empty space. In order to have unique coding we require the first and the last digit to be nonzero $c_0, c_l \neq 0$, hence $c \neq 0 \pmod 4$. We shall use $l(c)$ to denote the length of the code c, i.e. the position of its most significant nonzero digit. We note that operators $G_c(x)$ have unit operator norm $||G_c(x)|| = 1$ and span, after norm closure, the so-called quasi-local C^* operator algebra $\mathfrak{A}$ [13]. Furthermore, we shall also consider the space of *translationally invariant* (TI) observables $\mathfrak{T}$ which is spanned by the basis

$$Z_c = \sum_{x\in\mathbb{Z}} G_c(x) \tag{2}$$

Then we define an *inner product* of two TI operators, $\Omega,\Theta \in \mathfrak{T}$, $\Omega = \sum_c \omega_c Z_c$, $\Theta = \sum_c \theta_c Z_c$, as

$$(\Omega|\Theta) = \sum_c \omega_c^* \theta_c, \tag{3}$$

so operators Z_c constitute an orthonormal basis. We note that TI operators can be interpreted also as TI linear functionals over quasi-local algebra $\mathfrak{A}$, or as density matrices of quantum states (if positive), namely to each $\Omega \in \mathfrak{T}$ one can assign a functional $\omega(A) = \lim_{L\to\infty} 2^{-L}\,\mathrm{tr}_L(\Omega A)$ where tr_L denotes the trace over a finite portion of the chain of size L covering an arbitrary subset of $\mathbb{Z}$ as $L \to \infty$ [the so-called *limit by inclusion*]. In particular, it is useful to consider a *tracial state* $\langle A\rangle = \lim_{L\to\infty} 2^{-L}\,\mathrm{tr}_L A$ (*infinite temperature* Gibbs state) which defines a natural inner product in $\mathfrak{A}$ w.r.t. which $G_c(x)$ is an orthonormal basis, namely $\langle G_c^*(x) G_{c'}(x')\rangle = \delta_{xx'}\delta_{cc'}$ if $c, c' \neq 0 \pmod 4$.

3 Explicit kicked Ising dynamics on the operator algebra

We consider the simplest non-trivial, non-integrable quantum many-body dynamics that we can think of, namely the nearest neighbour Ising interaction periodically kicked with homogeneous tilted magnetic field, the so-called Kicked Ising (KI) model [8] with periodically time-dependent hamiltonian

$$H_{\mathrm{KI}}(t) = \sum_{x\in\mathbb{Z}} \{J\sigma_1(x)\sigma_1(x+1) + \sum_{n\in\mathbb{Z}} \delta(t-n)(h_x\sigma_1(x) + h_z\sigma_3(x))\}, \tag{4}$$

generating a unitary Floquet map

$$U = \exp(-\mathrm{i}H_1)\exp(-\mathrm{i}H_0), \quad H_0 = h_x Z_{(1)} + h_z Z_{(3)}, \quad H_1 = J Z_{(11)}. \tag{5}$$

KI is *completely integrable* in cases of kicking with longitudinal field $h_z = 0$ (trivially) or transverse field $h_x = 0$ (Wigner-Jordan mapping to spinless fermions [11]). However, if the magnetic field is tilted in the plane $x - z$ then the model is non-integrable and can exhibit truly mixing dynamics in the thermodynamic limit [8]. It seems particularly interesting that certain remnants of integrability, like divergent transport coefficients or non-vanishing time-averaged autocorrelation functions [14], appear to survive sufficiently small but finite amount of perturbation (e.g. tilt) even in the thermodynamic limit [8,7].

Our central interest here is to study time-automorphism (TA) induced by (4,5) on C^* algebra, $\mathbf{T} : \mathfrak{A} \to \mathfrak{A}$, or on the space of TI observables, $\hat{\mathbf{T}} : \mathfrak{T} \to \mathfrak{T}$:

$$\hat{\mathbf{T}}\Omega = U^\dagger \Omega U = \exp(\mathrm{i}\,\mathrm{ad}H_0)\exp(\mathrm{i}\,\mathrm{ad}H_1)\Omega, \quad (\,\mathrm{ad}H)\Omega := H\Omega - \Omega H. \tag{6}$$

KI model has a unique feature, namely the application of TA on any element of $\mathfrak{A}$, or $\mathfrak{T}$, can be computed *explicitly* and *efficiently*. This is in contrast with a general quantum lattice model where even the existence of TA is a difficult mathematical problem [13]. It is this feature which makes KI model a particularly useful toy for exploring relaxation and nonequilibrium quantum statistical mechanics.

Let us write the action of TA on the local operator basis $G_k(x)$ in terms of a matrix $W_{k,k'}$

$$\mathbf{T}G_k(x) = \sum_{k'} W_{k,k'} G_{k'}(x). \tag{7}$$

Note that indices k, k' run over all non-negative integers (and not like index $c \neq 0 \pmod 4$), so the expression (7) is not optimized in the sense of nonuniqueness $G_{4k}(x) = G_k(x+1)$. We shall now show that the matrix $W_{k,k'}$ can be constructed explicitly by recursive application of the homomorphism property

$$\mathbf{T}G_{(k_0,k_1\ldots k_{l-1})}(x) = \mathbf{T}G_{(k_0,k_1\ldots k_{l-2})}(x)\mathbf{T}\sigma_{k_{l-1}}(x+l). \tag{8}$$

The key ingredient of our method is the *locality* of TA, namely *application of TA on a local variable can only affect neighbouring sites in one time step.* For later convenience we express this property most generally in terms of a 64×64 matrix $\Lambda_{z,t}$, $z = z_0 + 4z_1 + 4^2 z_2$, $t = t_0 + 4t_1 + 4^2 t_2$, as

$$\sigma_{z_0}(x-1)\sigma_{z_1}(x)\mathbf{T}\sigma_{z_2}(x) = \sum_{t=0}^{63} \Lambda_{z,t}\,\sigma_{t_0}(x-1)\sigma_{t_1}(x)\sigma_{t_2}(x+1). \tag{9}$$

The matrix elements $\Lambda_{z,t}$ can be easily computed for KI model, and we do not display them here due to lack of space. We note however an important general relation following from eq. (9):

$$\text{if} \quad 0 \le u < 16 \qquad \text{then} \qquad \Lambda_{u,t} = \delta_{u,t}. \tag{10}$$

Plugging eq. (7) to eq. (8) and using eq. (9) one obtains a recursion relation for a finite $4^{r+1} \times 4^{r+3}$ dimensional block $\mathrm{W}^{(r+1)}$ of the matrix $W_{k,k'}$ in terms of its $4^r \times 4^{r+2}$ sub-block $\mathrm{W}^{(r)}$, namely for non-negative integers $k < 4^r, k' < 4^r, s < 4, u < 16, t < 64$ we have

$$W^{(r+1)}_{k+4^r s, k'+4^r t} = \sum_{u=0}^{15} W^{(r)}_{k,k'+4^r u} \Lambda_{u+16s,t} \tag{11}$$

with the *initial condition*

$$W^{(1)}_{s,t} = \Lambda_{16s,t}. \tag{12}$$

We stress that the consistency condition $W^{(r+1)}_{k,k'} = W^{(r)}_{k,k'}$, $0 \le k < 4^r$, $0 \le k' < 4^{r+2}$, follows from eq. (10). Dividing the matrix $\mathrm{W}^{(r)}$ into 4×64 smaller square blocks of size $4^{r-1} \times 4^{r-1}$ written as $\mathrm{W}^{(r)}_{s,t}$ we can rewrite eq. (11) in an appealing matrix form

$$\mathrm{W}^{(r+1)}_{st} = \sum_{u=0}^{15} \begin{pmatrix} \mathrm{W}^{(r)}_{0,4u} & \mathrm{W}^{(r)}_{0,4u+1} & \mathrm{W}^{(r)}_{0,4u+2} & \mathrm{W}^{(r)}_{0,4u+3} \\ \mathrm{W}^{(r)}_{1,4u} & \mathrm{W}^{(r)}_{1,4u+1} & \mathrm{W}^{(r)}_{1,4u+2} & \mathrm{W}^{(r)}_{1,4u+3} \\ \mathrm{W}^{(r)}_{2,4u} & \mathrm{W}^{(r)}_{2,4u+1} & \mathrm{W}^{(r)}_{2,4u+2} & \mathrm{W}^{(r)}_{2,4u+3} \\ \mathrm{W}^{(r)}_{3,4u} & \mathrm{W}^{(r)}_{3,4u+1} & \mathrm{W}^{(r)}_{3,4u+2} & \mathrm{W}^{(r)}_{3,4u+3} \end{pmatrix} \Lambda_{u+16s,t}. \tag{13}$$

The relation (11,13) may be interpreted as a kind of *renormalization group* flow construction of TA. Note again that indices k, k' in $W_{k,k'}$ are arbitrary integers, i.e. they may also be divisible by 4. However, application of TA on TI basis Z_c results in a different matrix $T_{c,c'}$

$$\hat{\mathbf{T}} Z_c = \sum_{c'} T_{c,c'} Z_{c'} \tag{14}$$

where indices c, c' should have the least important digit different from zero $c, c' \neq 0 \pmod 4$. Summation over x and careful bookkeeping of indices reveals the following simple relation between the two TA-matrices

$$T_{c,c'} = \sum_{k=0}^{r+2} W_{c,4^k c'} \tag{15}$$

where r is the smallest integer satisfying $4^r > c$. The matrix $T_{c,c'}$ is block-banded, namely it satisfies

$$T_{c,c'} = T_{c',c} = 0 \qquad \text{if} \quad c > 16c'. \tag{16}$$

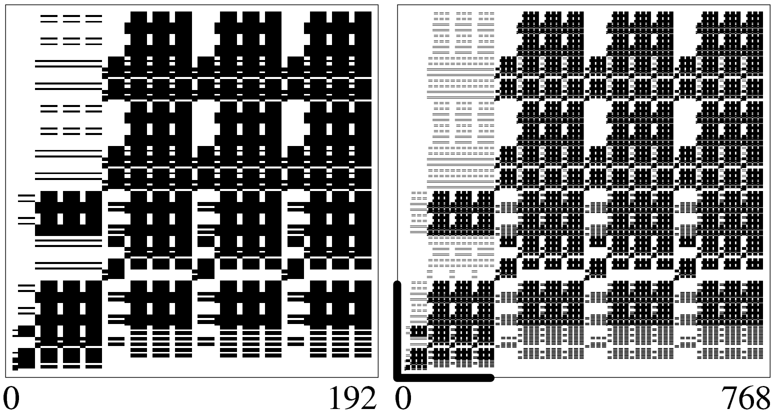

Fig. 1. Non-vanishing matrix elements (black) of TA-matrix $\mathrm{T}^{(r)} = \{T_{c,c'}; 0 \le c, c' < 4^{r+1}\}$ of a non-integrable KI model for $r = 4$ (left), 5 (right) in the code-ordered basis.

From the recursive construction (11,13) it also follows that the matrix $W_{k,k'}$, or $T_{c,c'}$, should have a fractal structure upon multiplication of the row and column indices by factor 4. This is clearly demonstrated in figure 1 for the non-integrable KI model (in case of integrability, if either $h_x = 0$ or $h_z = 0$, additional matrix elements are vanishing).

Let us group all the TI-operators of order r or less to constitute a linear subspace $\mathfrak{T}_r = \{\sum_c^{l(c)\le r} \omega_c Z_c\}$, so that we have an inclusion sequence $\mathfrak{T}_1 \subset \mathfrak{T}_2 \subset \cdots \subset \mathfrak{T}$. We note that the condition (16) results in

$$\hat{\mathbf{T}}\mathfrak{T}_r \subset \mathfrak{T}_{r+1} \tag{17}$$

meaning that we have explicitly constructed TA on an inclusive sequence of sets covering the entire space $\mathfrak{T}$ of TI observables. There is of course a delicate mathematical issue on the proper metric on the infinite dimensional space $\mathfrak{T}$ which determines the spectral problem for the operator $\hat{\mathbf{T}}$. This shall be discussed later.

From the practical point of view it is more important to be able to apply TA on a given vector $\Omega = \sum_c^{l(c)\le r} \omega_c Z_c \in \mathfrak{T}_r$ of $N = 3\cdot 4^{r-1}$ coefficients ω_c, as $\hat{\mathbf{T}}\Omega = \sum_c^{l(c)\le r+1} \omega'_c Z_c$ giving $4N$ coefficients ω'_c, than to compute the whole matrix $T_{c,c'}$. We shall now outline a *fast algorithm* which produces $\omega_c \to \omega'_c$ in $O(r4^r) = O(N \log N)$ operations, using the recursive construction of the TA. The idea is simply to write $\omega'_c = \sum_{c'} \omega_{c'} T_{c',c}$ and apply eqs. (15) and (11).

Algorithm:

(1) Set initial vector: $\omega_c,\ \ 0 \le c < 4^r,\ c \ne 0 \pmod 4$
(2) Offset the code for two digits: $y_k^{(0)} = \sum_c \omega_c \delta_{k,16c}$, for all k, $0 \le k < 4^{r+2}$.

(3) For $l = 1, 2, \ldots r$:

$$y^{(l)}_{k+4^{l-1}t+4^{l+2}v} = \sum_{t'=0}^{63} y^{(l-1)}_{k+4^{l-1}t'+4^{l+2}v} \Lambda_{t',t}, \tag{18}$$

with $0 \le k < 4^{l-1}, 0 \le v < 4^{r-l}$.

(4) Apply translational invariance to yield the final vector:

$$\omega'_c = \sum_{k=0}^{r+1} y^{(r)}_{4^k c}, \quad 0 \le c < 4^{r+1},\ c \ne 0 \pmod 4. \tag{19}$$

4 Ruelle resonance spectrum and correlation decay

Relaxation to equilibrium is characterized by the decay of time correlation functions, $C_{\Omega\Theta}(t) = (\Omega|\hat{\mathbf{T}}^t\Theta)$, which are through the Fourier transform related to the spectrum of TA $\hat{\mathbf{T}}$. (Note that one does not need to subtract product of averages from the correlation function since the trivial identity observable is *not* included in the space $\mathfrak{T}$, hence $\langle\Omega\rangle = 0$ for all $\Omega \in \mathfrak{T}$.) However, the spectrum of $\hat{\mathbf{T}}$ depends crucially on the metric of the space $\mathfrak{T}$, namely $\hat{\mathbf{T}}$ is *unitary* with the spectrum lying on the unit circle if the Hilbert space metric (3) is used. This would exclude possibility of mixing for any finite operator space. But since the dimensionality of $\mathfrak{T}$ is infinite, the correlations may still decay due to transport of quantum amplitudes to modes Z_c of higher and higher orders $l(c)$. This process may, physically speaking, be captured by a kind of "coarse graining", or mathematically speaking, by Ulam's finite rank approximation. Thus we define a projection operator $\hat{\mathbf{P}}_r : \mathfrak{T} \to \mathfrak{T}_r$ which simply truncates to order r, $\hat{\mathbf{P}} \sum_c \omega_c Z_c = \sum_c^{l(c)\le r} \omega_c Z_c$. Heuristically speaking, this projection corresponds to "coarse graining" of quantum observables with "resolution" $1/r$. We define a sequence of finite-rank truncated (coarse grained) transfer operators

$$\hat{\mathbf{T}}^{(r)} = \hat{\mathbf{P}}_r \hat{\mathbf{T}} \hat{\mathbf{P}}_r \tag{20}$$

which are represented in basis Z_c by truncated $3 \cdot 4^r$ dimensional square matrices $\mathrm{T}^{(r)} = \{T_{cc'}; 0 \le c, c' < 4^{r+1},\ c, c' \ne 0 \pmod 4\}$. The relaxation or decay of time correlations is then expected to be approximately described in terms of spectra of *non-unitary matrices* $\mathrm{T}^{(r)}$, with increasing accuracy as $r \to \infty$. *Ruelle resonances* are defined, following the idea of refs. [3,5], in terms of "frozen" eigenvalues, i.e. those eigenvalues (and the corresponding eigenvectors) which converge as $r \to \infty$.

Let e^{-q_n} be the converged (frozen) eigenvalues of $\hat{\mathbf{T}}^{(r)}$, and $\{\Theta_n^{\rm R}\}$, $\{\Theta_n^{\rm L}\}$ the corresponding *right* and *left* eigenvectors, respectively. Then for arbitrary pair $X, \Omega \in \mathfrak{T}$, the time correlation function $(X|\Omega(t)) = (X|\hat{\mathbf{T}}^t\Omega)$ has the asymptotic behaviour (see e.g. [2])

$$(X|\Omega(t)) \to \sum_n w_n e^{-q_n t}, \quad w_n = \frac{(X|\Theta_n^{\rm R})(\Theta_n^{\rm L}|\Omega)}{(\Theta_n^{\rm L}|\Theta_n^{\rm R})}. \tag{21}$$

The above relation is the contribution of the point spectrum and is exact if the spectrum is pure-point. However, one may quite typically have various singular components and branch cuts which may also be taken into account [2]. Note an interesting point here, namely denominator $(\Theta_n^{\rm L}|\Theta_n^{\rm R})$ is finite although both vectors should have infinite l^2 norm $(\Theta_n^{\rm L}|\Theta_n^{\rm L}) = \infty$, $(\Theta_n^{\rm R}|\Theta_n^{\rm R}) = \infty$, for any eigenvalue away from the unit circle.

There is a simple relation between Ruelle resonance spectrum and ergodic properties of dynamics: (i) If there is a *spectral gap*, i.e. all $|e^{-q_n}| \le \exp(-\lambda) < 1$, then dynamics is *exponentially mixing*, $|(X|\Omega(t))| \le C\exp(-\lambda t)$. (ii) If some eigenvalues are on the unit circle, meaning that the corresponding eigenvector coefficients should be in l^2, then the system is *non-mixing* since there are correlation functions which do not decay. (iii) If some eigenvalues are at 1 then the system is *non-ergodic* since the correlation functions may have non-vanishing time-averages. If Q_n is a complete set of orthonormalized eigenvectors corresponding to eigenvalue 1, $(Q_n|Q_m) = \delta_{n,m}$ (and note that since we are on the unit circle: $Q_n^{\rm R} = Q_n^{\rm L}$) then

$$D_\Omega := \overline{C_{\Omega\Omega}(t)} = \sum_n |(\Omega|Q_n)|^2. \tag{22}$$

The latter (iii) happens in generic completely integrable quantum lattices, where Q_n correspond to an infinite sequence of conservation laws [15]. Furthermore, we have a strong numerical evidence that also in certain non-integrable quantum lattices[7], and also in KI model [8], one has a regime where few normalizable ('pseudo-local') but not local (like in integrable models) conservation laws exist. This we call the regime of *intermediate* dynamics and is characterized by a nonvanishing *stiffness* $D_\Omega \neq 0$ signalling *ballistic transport*.

Note that the trivial invariant observable, namely the identity $Q_0 = 1$, $\hat{\mathbf{T}}Q_0 = Q_0$, an analogy of a uniform density with a 'special' eigenvalue 1 which is always there, does not belong to our operator space $\mathfrak{T}$. We also note that there may be an intermediate situation, between (i) and (ii), where the spectral gap vanishes but there is still no point eigenvalue on the unit circle, for example if we have a branch cut touching the unit circle. This situation is consistent with mixing and power-law decay of correlations, and happens quite often in classical dynamical systems as a result of intermittent behaviour, e.g. due to marginally stable orbits. We have not yet studied a possibility of such behaviour in our model.

We have systematically scanned the parameter space (J, h_x, h_z) of the KI model and found all qualitatively different constellations of frozen eigenvalues of the truncated TA matrices $\mathrm{T}^{(r)}$ corresponding to mixing, non-mixing but ergodic, and non-ergodic motions. It is particularly interesting to study the transition from non-ergodic to ergodic and mixing motion when the leading Ruelle resonance leaves the unit circle. We speculate that this has characteristics of a quantum phase transition and will be reported in forthcoming publications. For the purpose of this presentation we show in figure 2 the spectra of three consecutive truncated TA matrices $\mathrm{T}^{(r)}$, $r = 5, 6, 7$, for two representative cases: (a) strongly nonintegrable $J = 0.7, h_x = 0.5, h_z = 1.1$, and (b) integrable $J = 0.7, h_x = 0.0, h_z = 1.1$. It is seen that the leading few eigenvalues have clearly converged, in particular in the non-integrable case (a) where they are nicely isolated. In the integrable case (b)we find more regular structure of the spectrum and multiply degenerate eigenvalue 1 corresponding to local invariants of motion. We have also observed interesting concentrations of the interior eigenvalues along circles. The radia of these circles appear to be sensitive functions of systems parameters, for example in the case (b) we find condensation of eigenvalues around the circle with radius $\approx 1/2$. This phenomenon is not theoretically understood.

The numerical method described in the previous section can be used to efficiently compute the correlation function $C_{\Omega\Theta}(t)$, and in particular its coarse grained approximation $C^{(r)}_{\Omega\Theta}(t) = (\Omega|[\hat{\mathbf{T}}^{(r)}]^t\Theta)$ which can be computed, for t time steps, in $O(tr4^r)$ operations. One simply needs to apply the above algorithm iteratively followed each time by the projection master operator $\hat{\mathbf{P}}^{(r)}$. We note that in the particular model studied, this method is typically much more efficient and accurate than the numerically exact calculation of time-correlations on finite systems, in particular when the leading Ruelle resonances are well isolated so they freeze for relatively small order of truncation r. In such cases, dynamics on a truncated operator algebra gives relatively good approximation of dynamics on the infinite system.

In figure 3 we compare the time autocorrelation function of the magnetization $M = \sum_{x\in Z}\sigma_3(x) = Z_{(3)}$, computed in three different ways: (1) from exact time evolution $C_L(t) = \frac{1}{L}\langle MU_L^{-t}MU_L^t\rangle$ on a finite lattice of length L with periodic boundary conditions, (2) iteration of truncated TA matrix on infinite lattice $C^{(r)}(t) = (M|\hat{\mathbf{T}}_r^t M)$, and (3) asymptotics based on (few) leading eigenvalue resonance(s) (using formula (21) in terms of q_n and w_n.).

We note that the leading eigenvalue and eigenvector of truncated TA $\hat{\mathbf{T}}^{(r)}$ can most efficiently be computed using our Algorithm as a key step of an iterative *power-method*. In this way we were able to perform calculations of the leading

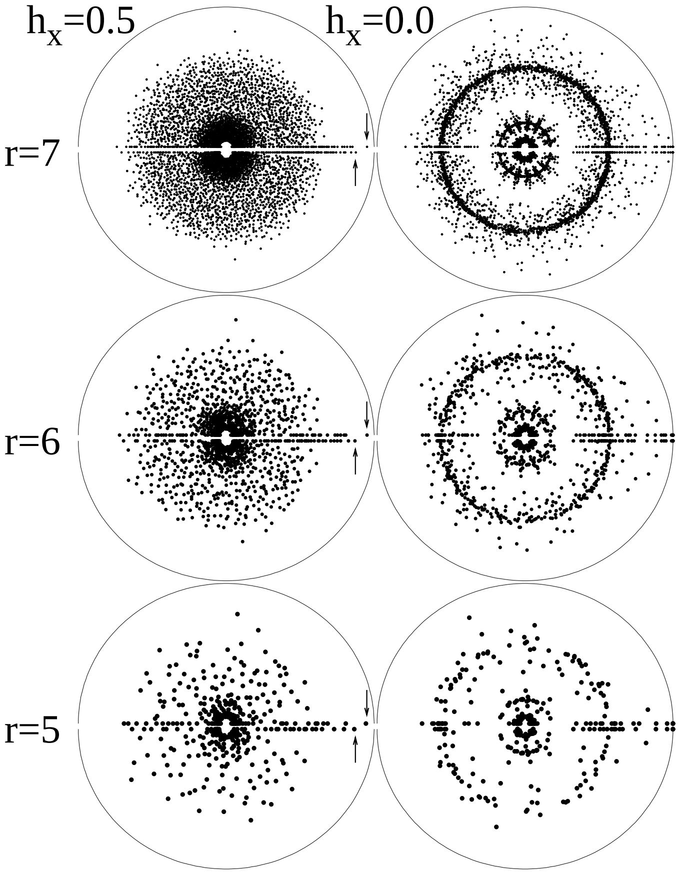

Fig. 2. The spectra of truncated transfer operators $\hat{\mathbf{T}}^{(r)}$, for $r = 5, 6, 7$ in strongly non-integrable (left) and integrable (right) case (see text), lying inside complex unit circle (thin arcs). The points in upper/lower unit semi-disks correspond to positive/negative parity $\hat{\mathbf{R}}Z_{(c_0,c_1,\ldots,c_{l-1})} = Z_{(c_{l-1},\ldots,c_1,c_0)}$ eigenvectors. Arrows point at converged positions of the leading eigenvalue e^{-q_1}.

Ruelle resonances up to $r = 12$ (see figure 3) in contrast to full diagonalization of matrices $\mathrm{T}^{(r)}$ (as used to determine the whole spectra shown in figure 2) which were feasible only up to $r = 7$. It is very interesting to observe the structure of the eigenvector coefficients $v_c^{\mathrm{L,R}} = (Z_c|\Theta_n^{\mathrm{L,R}})$ corresponding to the leading eigenvalue. Numerical results (see ref. [12]) strongly suggest self-similar behaviour upon multiplying the code c by 4 which is a consequence of the fractal structure of the transfer matrix (figure 1). It is hoped that this scaling structure of resonance eigenvectors could be quantitatively understood using an appropriate renormalization group picture.

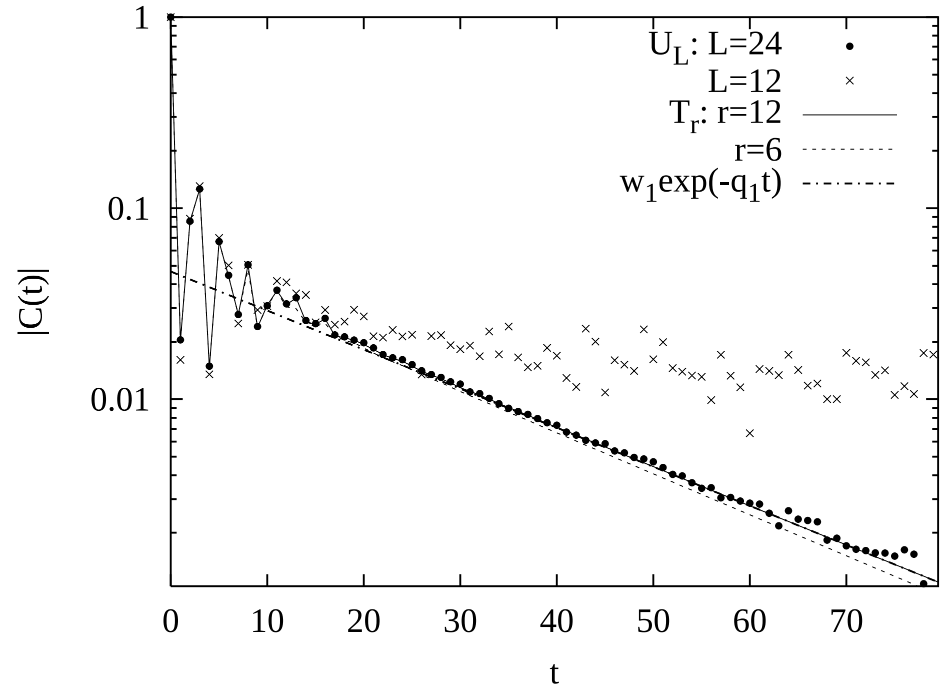

Fig. 3. Correlation function of the magnetization $C(t) = (M|M(t))$, in the mixing case $h_x = 0.5$, computed from finite system dynamics for different sizes L (symbols), and from truncated adjoint propagators of infinite systems (curves) for different orders r.

5 Conclusion

An appropriate quantum Perron-Frobenius transfer propagator has been defined for a generic quantum many-body system, namely kicked Ising spin chain, whose lack of unitarity provides a mechanism of quantum dissipation and relaxation to equilibrium in such an isolated conservative system. In the particular model we have proved the existence and given efficient algorithm for explicit construction of the time-automorphism in the quasi-local C^* algebra of quantum observables and in the Lie algebra of translationally invariant observables. We have studied the spectrum of the transfer operator numerically and showed that the leading eigenvalue provides numerically accurate rate of correlation decay.

We believe that these are generic results and that our method could also be somehow translated to time-independent quantum lattices, where the mathematics of existence and construction of time evolution is more difficult.

Useful discussions with F Leyvraz, J L Lebowitz, and T H Seligman are gratefully acknowledged. The work has been financially supported by the Ministry of Education, Science and Sports of Slovenia, and in part by the US ARO grant DAAD19-02-1-0086.

References

[1] D. Ruelle, *J. Stat. Phys.* **44**, 281 (1986); M. Pollicot, *Invent. Math.* **85**, 147 (1986); M. Blank, G. Keller and C. Liverani, *Nonlinearity* **15**, 1905 (2002).

[2] P. Gaspard, *Chaos, Scattering and Statistical Mechanics* (Cambridge University Press, 1998).

[3] J. Weber, F. Haake, and P. Šeba, *Phys. Rev. Lett.* **85**, 3620 (2000); J. Weber, F. Haake, P. A. Braun, C. Manderfeld, and P. Šeba, **J. Phys. A 34**, 7195 (2001).

[4] M. H. Lee, *Phys. Rev. Lett.* **49**, 1072 (1982); M. H. Lee, *Phys. Rev.* **B 26**, 2547 (1982); H. Mori, *Prog. Theor. Phys.* **34**, 399 (1965).

[5] C. Manderfeld, J. Weber, F. Haake, J. Phys. A: Math. Gen. **34**, 9893 (2001).

[6] S. Nonnenmacher, *e-print* `arXiv:nlin.CD/0301014`.

[7] T. Prosen, *J. Phys.* **A 31**, L645 (1998); T. Prosen, *Phys. Rev. Lett.* **80**, 1808 (1998); T. Prosen, *Phys. Rev.* **E 60**, 3949 (1999).

[8] T. Prosen, *Phys. Rev.* **E 65**, 036208 (2002); T. Prosen and T. H. Seligman, *J. Phys.* **A 35**, 4707 (2002).

[9] B. V. Fine, *e-prints* `arXiv:condmat/9911229`; `arXiv:condmat/9911230`.

[10] K. Fabricius, U. Löw and J. Stolze, *Phys. Rev. B* **55**, 5833 (1997); K. Fabricius and B. M. McCoy, *Phys. Rev. B* **57**, 8340 (1998).

[11] T. Prosen, *Prog. Theor. Phys. Suppl.* **139**, 191 (2000).

[12] T. Prosen, *J. Phys.* **A 35**, L737 (2002).

[13] O. Bratelli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics I,II*, (Springer-Verlag, 1979, 1981); B. Simon, *The Statistical Mechanics of Lattice Gases*, (Princeton University Press, 1993).

[14] X. Zotos, F. Naef and P. Prelovšek, *Phys. Rev.* **B 55**, 11029 (1997).

[15] M. P. Grabowski and P. Mathieu, *Ann. Phys., NY* **243**, 299 (1995).