Rotating vortex dipoles in ferromagnets

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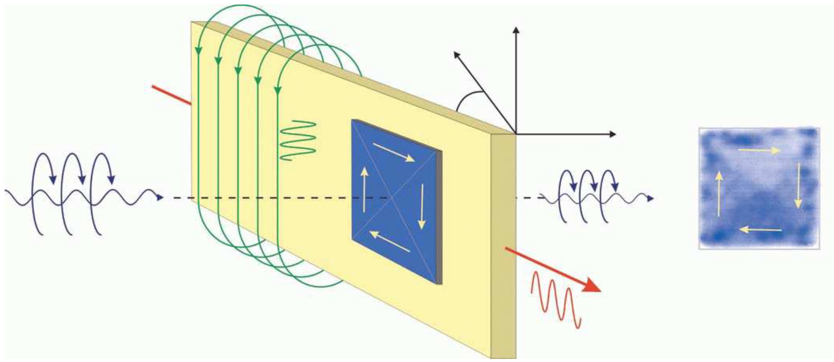
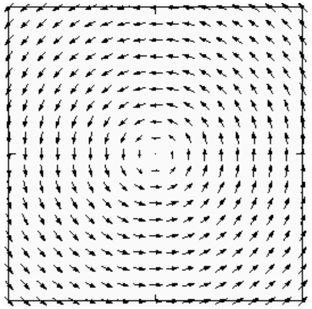
A vortex in a magnetic particle

[Waeyenberge *et al*, Nature (2006)]



Sample

Square NiFe elements: $1.5\mu\mathrm{m} \times 1.5\mu\mathrm{m} \times 50\mathrm{nm}$



Procedure

An ac current generates an in-plane magnetic field.

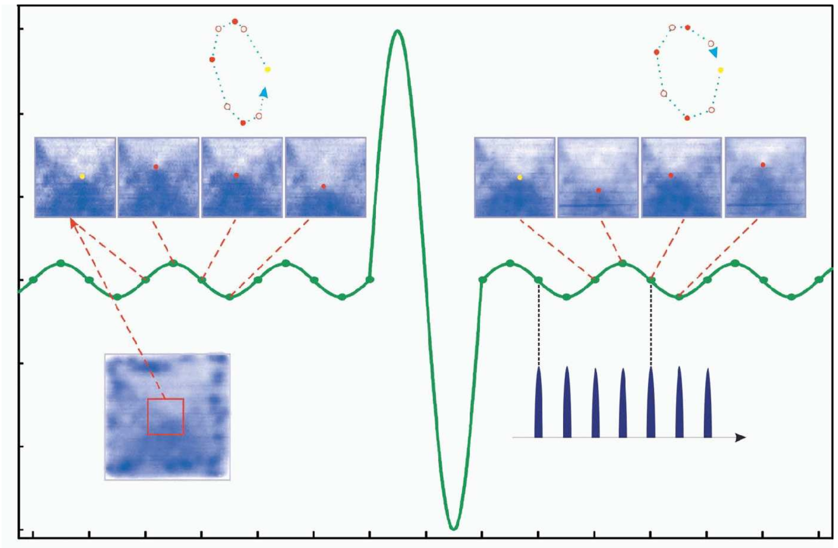
Observation Method

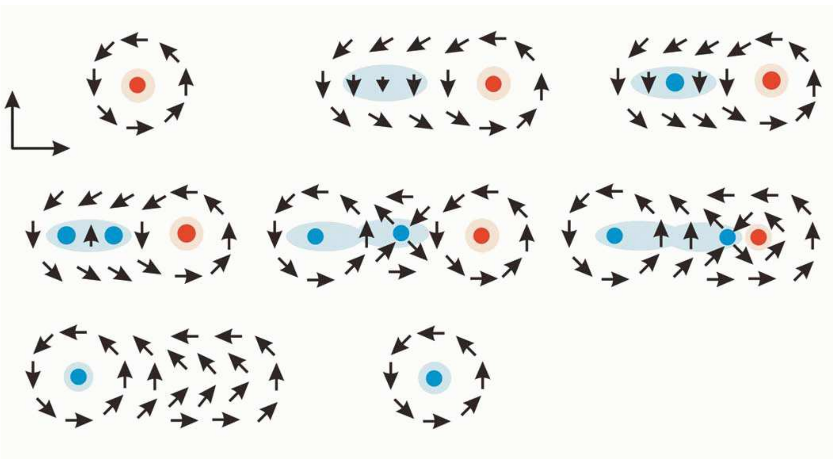
XMCD, resolution 30nm, 150ps.

Vortex polarity reversal

Procedure

- An ac current generates an alternating magnetic field (250 MHz, 0.1 mT).
- Add a "burst" of 1.5 mT, for one period.
- Result: you have obtained vortex core polarity switching!





Vortex polarity reversal II

[K. Yamada *et al*, Nature Materials (2007).]

Procedure

- An ac current is passed through the magnetic element.
- Check by MFM that you often obtain vortex core switching!

A ferromagnetic nanoelement: The model

The important energy terms are due to the exchange and anisotropy magnetic interactions:

$$E = E_{\mathrm{ex}} + E_{\mathrm{a}}, \qquad E_{\mathrm{ex}} = \frac{1}{2}\int (\boldsymbol{\nabla}\boldsymbol{m})^2\, d^2x, \quad E_{\mathrm{a}} = \frac{1}{2}\int (m_3)^2\, d^2x.$$

The equation of motion for the magnetisation $\boldsymbol{m}$ is the Landau-Lifshitz (LL) equation:

$$\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{m} \times \boldsymbol{f}, \qquad \boldsymbol{f} \equiv \Delta\boldsymbol{m} - m_3\, \hat{\boldsymbol{e}}_3, \qquad \boldsymbol{m}^2 = 1.$$

Note: The magnetostatic energy is very important in magnetic materials. But, it can be modeled (for some purposes) by the above anisotropy energy.

Vortices are created due to competition between exchange and magnetostatic (or anisotropy) interactions.

A magnetic vortex

is a magnetisation $[\boldsymbol{m} = (m_1, m_2, m_3)]$ configuration of the form:

$$m_1 + i\, m_2 = \sin\Theta(\rho)\, e^{iS(\phi-\phi_0)}, \quad m_3 = \lambda \cos\Theta(\rho).$$

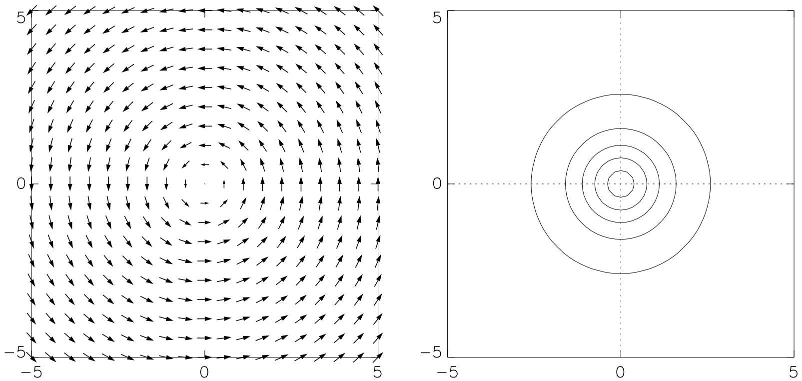
(ρ, ϕ): polar coordinates

$S = \pm 1, \pm 2, \ldots$ is the winding number (a topological invariant)

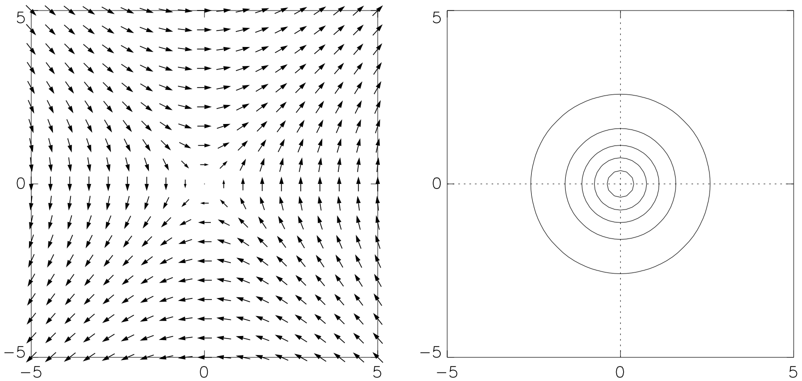
$\lambda = \pm 1$ is the vortex polarity

ϕ_0: constant.

Vortex $(S = 1)$



Antivortex $(S = -1)$



A further topological invariant (n is the corresponding topological density):

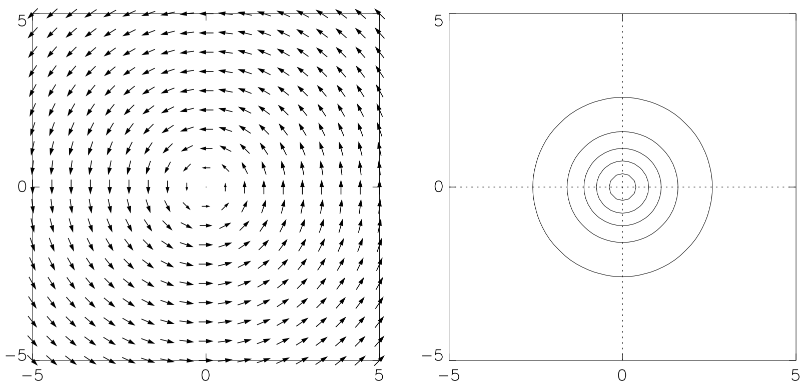
$$\mathcal{N} \equiv \frac{1}{4\pi} \int n \, d^2x, \quad n \equiv \frac{1}{2}\epsilon_{\mu\nu} \left(\partial_\nu \boldsymbol{m} \times \partial_\mu \boldsymbol{m}\right) \cdot \boldsymbol{m},$$

called the Skyrmion number.

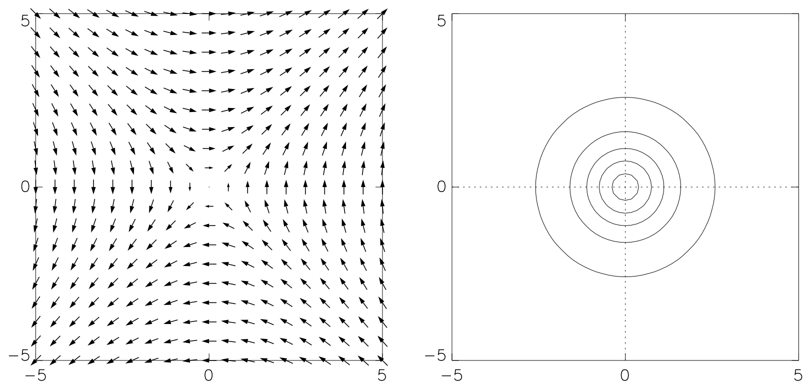
For a vortex:

$$\mathcal{N} = -\frac{1}{2} S \, \lambda.$$

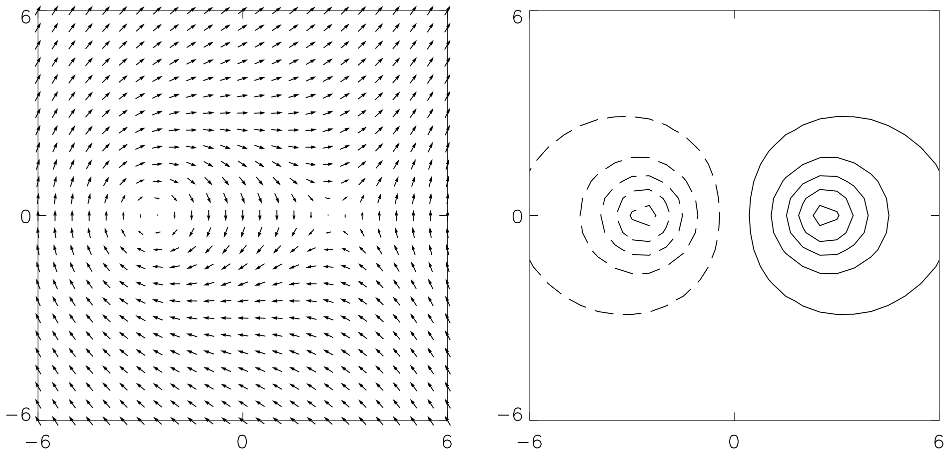
Vortex ($S = 1$, $\mathcal{N} = \mp 1/2$)



Antivortex ($S = -1$, $\mathcal{N} = \pm 1/2$)



A vortex-antivortex pair



Contour plots show the perpendicular component of the magnetization m_3.

Solid lines indicate $m_3 > 0$ (positive polarity),
dashed lines indicate $m_3 < 0$ (negative polarity)

A vortex $(S = 1,\ \lambda = -1)$ and an antivortex $(S = -1,\ \lambda = 1)$ form a pair
Vortex pair $\Rightarrow \mathcal{N} = 1$ (Skyrmion).

Position of the vortex dipole

The position of the vortex dipole can be defined as

$$R_x = \int x\, n\, dx dy, \quad R_y = \int y\, n\, dx dy.$$

The vortex $(S = 1, \lambda = 1)$ and the antivortex $(S = -1, \lambda = -1)$ have the same topological density distributions (n) and thus:
$(R_x, R_y) = (0, 0)$ for the vortex dipole.

But $(-R_y, R_x)$ is the conserved linear momentum associated with the LL equation
$\Rightarrow$ the vortex dipole is pinned in the magnet.

Dynamics of the vortex dipole

The angular momentum associated with the LL equation is

$$\ell = \frac{1}{2}\int \rho^2\, n\, dxdy,$$

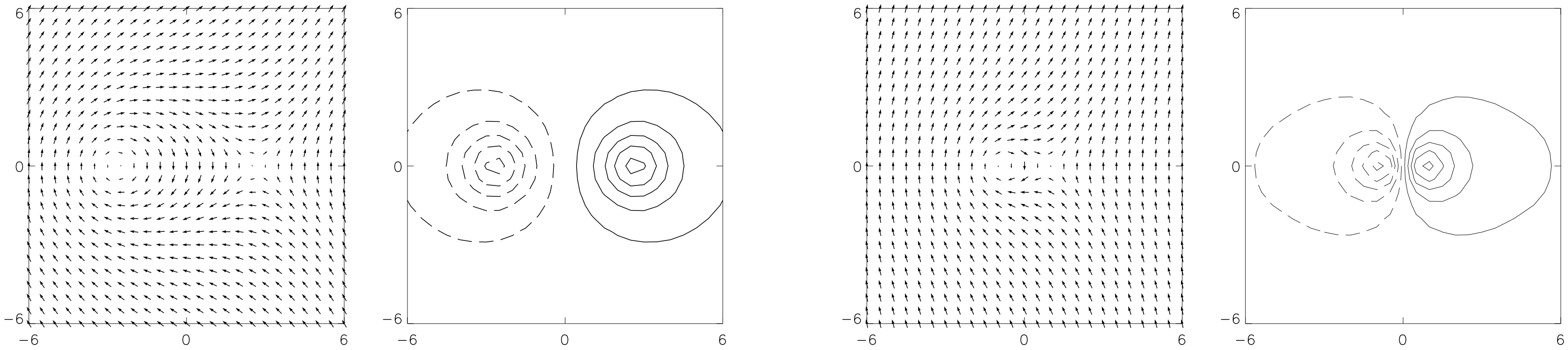
and it is proportional to the size of the dipole.
Let d the distance between vortices (size of dipole), then $\ell \sim d^2$.

Since n is positive everywhere we have $\ell > 0$. Thus the dipole should be rotating.

This is confirmed by
(i) numerical simulations of the LL equation,
(ii) the dynamics given by Thiele's (collective coordinate) equations for this system.

Rotating vortex dipoles

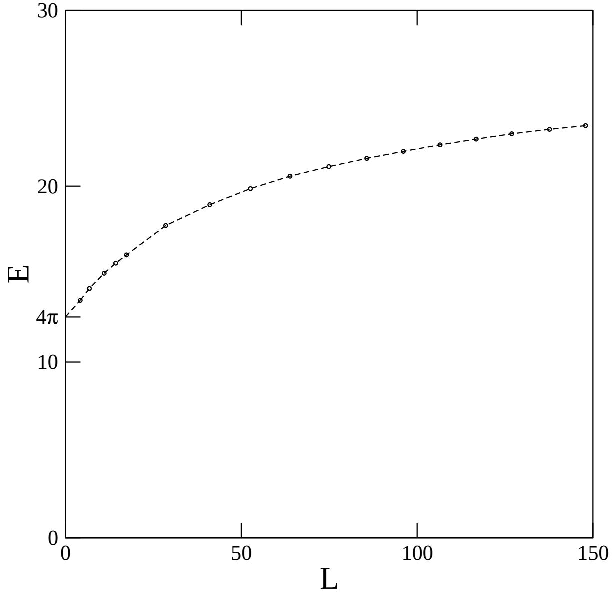
Assume a vortex dipole in *steady* coherent rotation with angular frequency ω. Such steady states can be found numerically.



Left figure: distance between vortices $d = 5.3$ and $\omega = 0.06$, $L = 64$.

Right figure: distance between vortices $d = 1.7$ and $\omega = 0.18$, $L = 11$.

Energy versus Angular momentum



Numerically calculated energy as a function of the angular momentum for vortex dipoles in steady state rotational motion.

Suppose a vortex dipole is initially created in a magnet.
Then its energy is monotonically decreasing as the dipole size (angular momentum) becomes smaller.

Well-separated vortex and antivortex

$$E \approx E_{\rm ex} = 2\pi \ln(d/d_0) = \pi \ln(\ell/\ell_0), \quad [\ell = \pi/2\, d^2,\ d_0, \ell_0 : \text{const.}]$$

The rotation frequency is

$$\omega = \frac{dE}{d\ell} = \frac{\pi}{\ell} \approx \frac{2}{d^2}.$$

A small vortex dipole

As the dipole becomes vanishingly small $(d \to 0)$ we assume the Skyrmion solution of the pure exchange model.

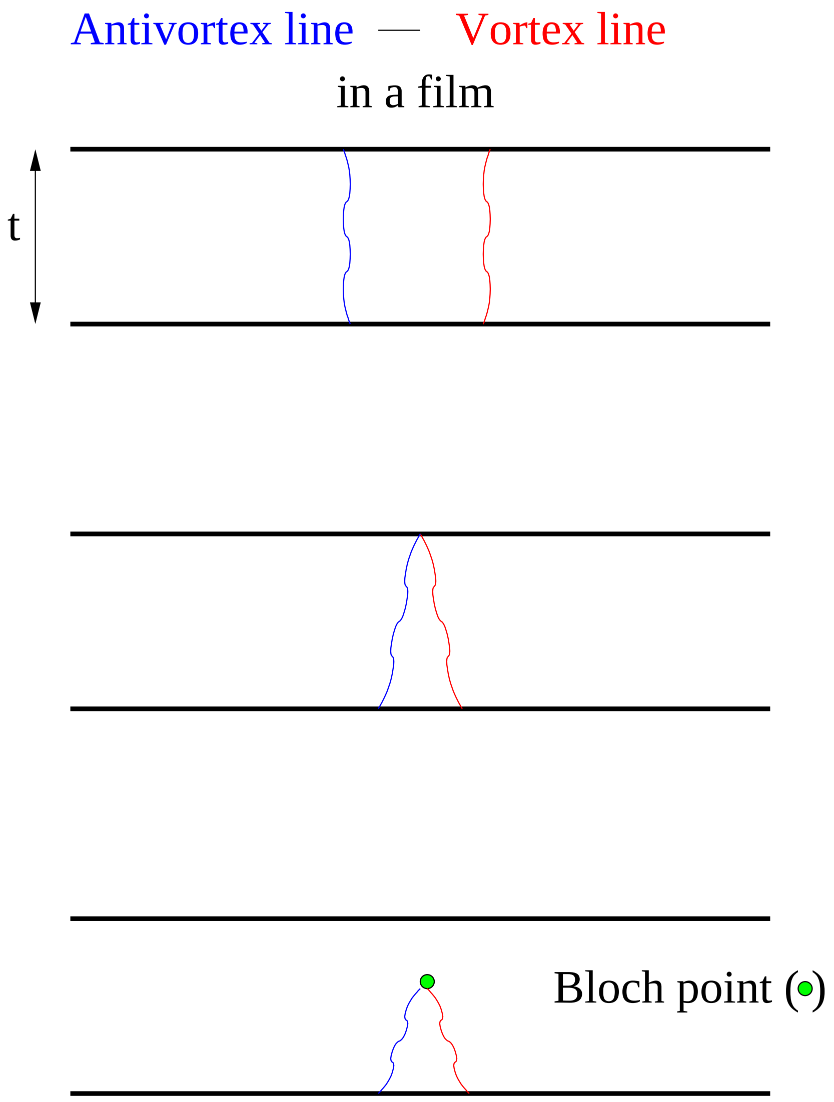
$$E \approx E_{\rm ex} = 4\pi + \ldots$$

$$\omega(d \to 0) = 1/2.$$

In the limit $(d = 0)$, we find the uniform ferromagnetic state!

There is no energy barrier to overcome as the dipole annihilates and the topological number of the configuration changes! $(\mathcal{N} = \pm 1 \to \mathcal{N} = 0)$

Bloch points in Films



- Suppose a film of finite thickness t where a vortex-antivortex line has been created.

- The vortex lines move towards one another in order to minimize energy, and they are annihilated first, say, on the upper film part.

- A Bloch Point has been created!
  What is the dynamics of the Bloch Point?
  It should be related to vortex-antivortex dynamics.

[R. Hertel, et al, cond-mat/0611668.]

Conclusions

- A vortex dipole is spontaneously created in dynamical experiments of vortex switching.
- Vortex and antivortex with opposite polarities form a rotating vortex dipole.
- Its energy is monotonically decreasing as the dipole size becomes smaller. There is no energy barrier to overcome as the dipole annihilates and the topological (Skyrmion) number of the configuration changes!
- This leads to vortex polarity switching when the vortex dipole annihilates.
- A Bloch Point is created in a film when a vortex dipole is annihilated. No energy barrier has to be overcome.