Diffusion with stochastic resetting

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Collaborators:

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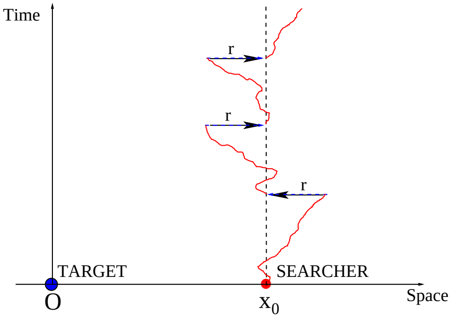
References:

- M. R. Evans and S. N. Majumdar, Phys. Rev. Lett. **106**, 160601 (2011); J. Phys. A **44**, 435001 (2011)
- SG, S. N. Majumdar, and G. Schehr, Phys. Rev. Lett. **112**, 220601 (2014)

Stochastic resetting of a single diffusing particle



- In a small time Δt,

  $x(t+\Delta t) = x_0$ with probability $r\Delta t$ (Resetting)

  $=$ Diffusion with probability $1 - r\Delta t$

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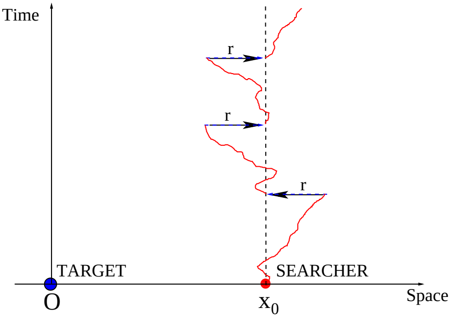

1. What is the probability to be at x at time t ?
2. What is the average time to detect the target ??

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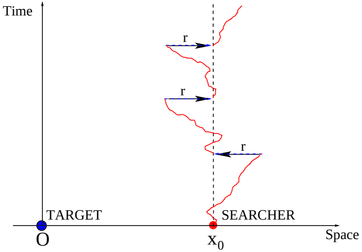
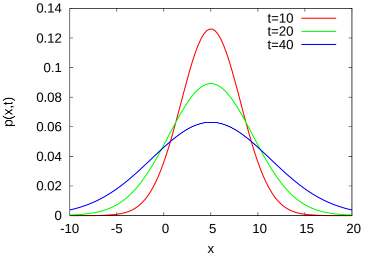


$r = 0$: Only Diffusion:

NO STATIONARY STATE,
Particle *anywhere* at long times



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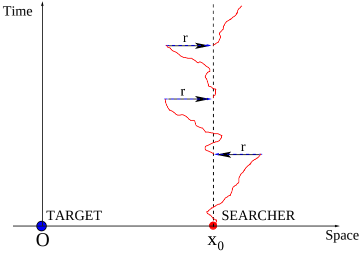
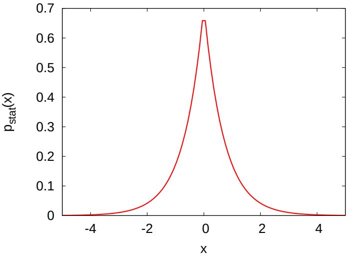


$r \neq 0$: Diffusion+resetting:

STATIONARY STATE,
Particle *cannot go very far*
even at long times
$p_{\rm stat}(x) \sim \exp(-|x|\sqrt{r/D})$



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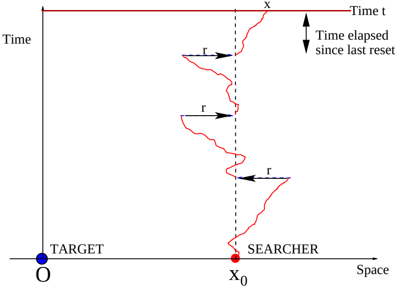
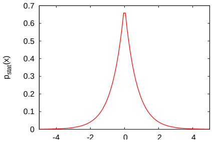


$$P(x,t) = \int_0^t d\tau \;(r\exp(-r\tau))\; G(x,\tau) + r\exp(-rt)G(x,t),$$

$G(x,t)$: Free diffusion propagator $= \frac{1}{\sqrt{4\pi Dt}} \exp\left(-x^2/(4Dt)\right)$

$t \to \infty \Rightarrow$

$p_{\rm stat}(x) \sim \exp(-|x|\sqrt{r/D})$

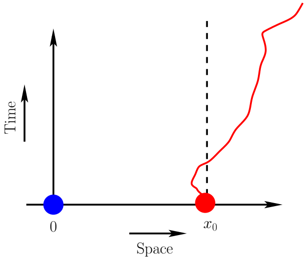


How does resetting affect the search time?

Average search time

Search time $T \leftrightarrow$ First time the searcher reaches the target

$r = 0 \rightarrow$ mean time $\overline{T} \rightarrow \infty$



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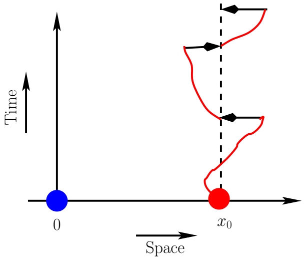
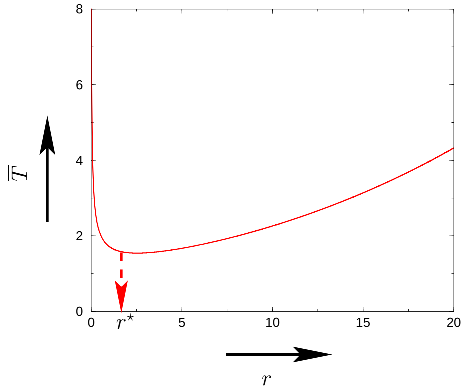


Average search time



$$\overline{T}(r, x_0) = \frac{1}{r}\left[\exp(x_0\sqrt{r/D}) - 1\right]$$

Optimal resetting rate $r^\star = \gamma^2 \frac{D}{x_0^2}$,

$$\gamma - 2(1 - \exp(-\gamma)) = 0 \to \gamma = 1.59362...$$

Stochastic resetting of multiple searchers

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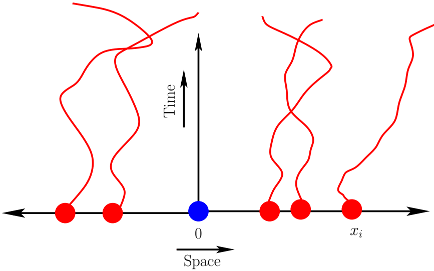
Sea of N independent searchers initially distributed with uniform density ρ.



1. Target survival probability $P_s(t) = \prod_i Q(x_i, t)$
   $Q(x_i, t)$: Probability that the i-th searcher starting at x_i does not reach the target up to time t

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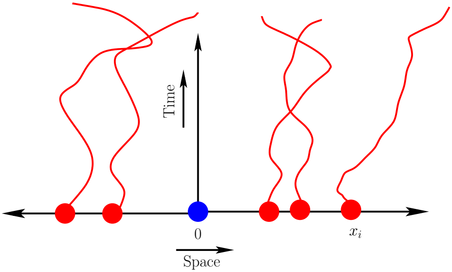

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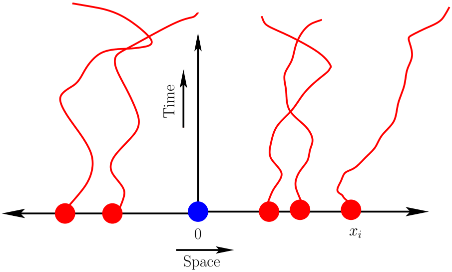

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3. Finally let $N \to \infty, L \to \infty$, keeping density $\rho = N/L$ fixed and finite

Stochastic resetting of multiple searchers

- Average $\langle P_s(t)\rangle = \langle \prod_i Q(x_i,t)\rangle = \prod_{i=1}^{N}(1 - \langle (1 - Q(x_i,t)\rangle)$

$$= \left[1 - \frac{1}{L}\int_{-L/2}^{L/2} dx\, \Big(1 - Q(x,t)\Big)\right]^N$$

$$\to \boxed{\exp\left[-2\rho\int_0^{\infty} dx\, \Big(1 - Q(x,t)\Big)\right]}$$

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- Diffusive searchers:

$$\to \langle P_s(t)\rangle = \exp(-4\rho\sqrt{Dt/\pi})$$

$$\to P_s^{\rm typ}(t) = \exp(-4\rho b\sqrt{Dt}); \quad b = 1.03442...$$

STRETCHED EXPONENTIAL DECAY !

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- Diffusive + resetting searchers:

$\to \langle P_s(t)\rangle \sim t^{-2\rho\sqrt{D/r}}$ for large t

$\to P_s^{\rm typ}(t) \sim \exp\Big(-8(1-\log 2)\sqrt{Dr}\,\rho t\Big)$ for large t

$$P_s^{\rm typ}(t) \ll \langle P_s(t)\rangle$$

Typically the target has been reached,

On the average, still not reached!

Stochastic resetting in an interacting system

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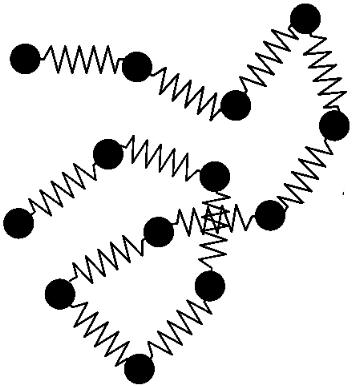

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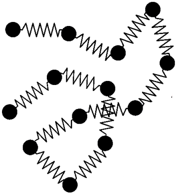

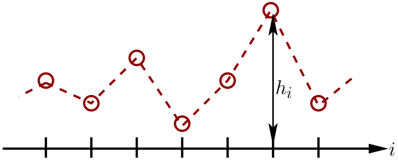

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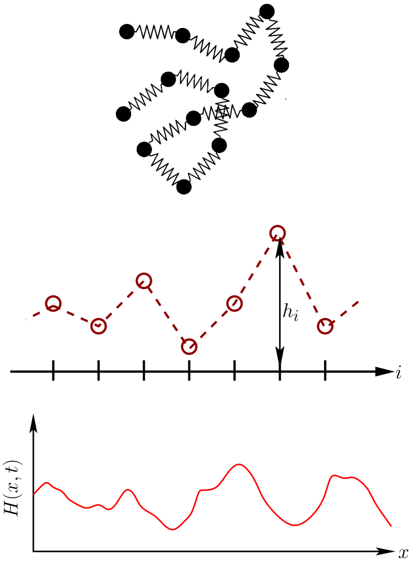

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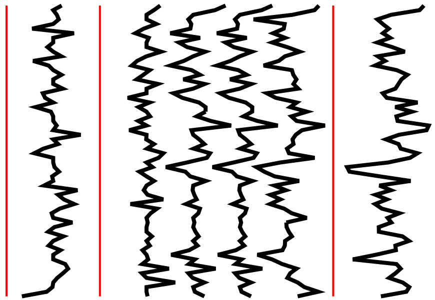

- Resetting to flat confg. at a fixed rate r

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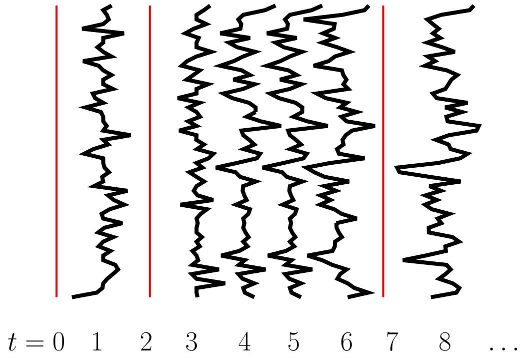

- Resetting to flat confg. at a fixed rate r
- Fluctuations $h(x,t) \equiv H(x,t) - \langle H(x,t) \rangle$

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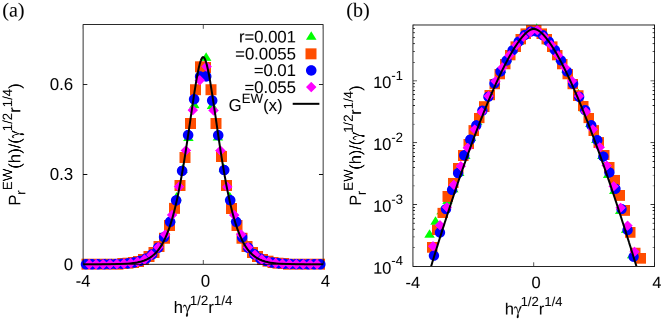
- Stationary state:



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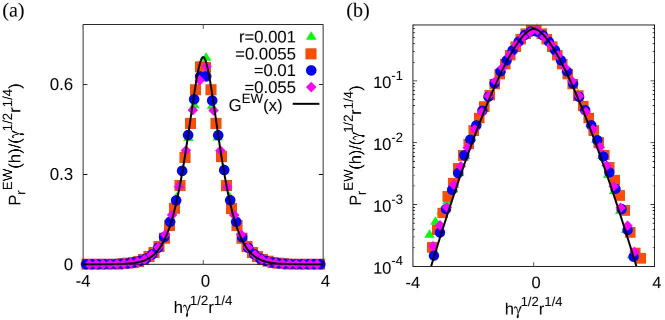

- Universal scaling form:

$$P_{\mathrm{r}}(h) \sim r^{\beta} G(hr^{\beta}) \; , \; G(x) = \int_0^{\infty} \frac{dy}{y^{\beta}} e^{-y} g\left(\frac{x}{y^{\beta}}\right).$$

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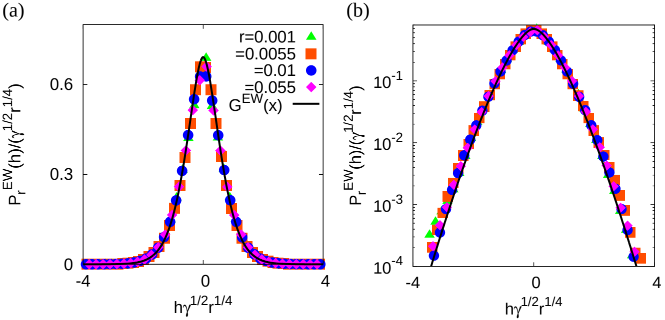

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- Highly non-Gaussian !!

Conclusions

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  Importance of rare trajectories

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- More general search strategies:
  Space-dependent resetting rate,
  Resetting of searcher distribution,
  $\ldots$