

Inelastic three-soliton collisions in a weakly discrete sine-Gordon system

A E Miroshnichenko[†], S V Dmitriev^{‡§}, A A Vasiliev[†] and T Shigenari[‡]

[†] Department of Mathematical Modelling, Tver State University, 33 Zhelyabov St, 170000, Tver, Russia

[‡] Department of Applied Physics and Chemistry, University of Electro-Communications, Chofu-shi, Tokyo 182-8585, Japan

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Abstract. The three-soliton solutions to the sine-Gordon equation describing the collision of a kink with a breather or with a kink–antikink pair are given and several separatrix three-soliton solutions are extracted from these solutions. The influence of a small perturbation on the three-soliton collisions is studied numerically. As the perturbed system the Frenkel–Kontorova model with a small degree of discreteness is considered. We show that in the three-soliton collisions, in the presence of small perturbation, energy exchange between solitons can take place. The degree of inelasticity of a three-soliton collision is extremely sensitive to the parameters of the collision.

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1. Introduction

The sine-Gordon (SG) equation plays an important role in many different fields of physics [1–3]. The completely integrable nonlinear SG equation describes, as a rule, a roughly idealized physical system. The study of many important physical phenomena requires the retaining of some additional terms which usually are considered as small perturbations of the SG equation [4–7].

The influence of the perturbation decreases when the small parameter describing the intensity of the perturbation tends to zero; however, for many-soliton collisions (more than two), even a slightly perturbed system can show quite different behaviour from the unperturbed case [4, 8–11]. It has been demonstrated that for the SG equation weakly perturbed by the discreteness of media, the three-soliton collision is strongly inelastic in the vicinity of a separatrix solution [9]. More generally, for the nonlinear Hamiltonian systems a high-frequency, small-amplitude perturbation produces a small perturbation of the invariant curves situated far from separatrices but for the invariant curves which are in the vicinity of a separatrix the influence of the perturbation results in qualitatively new dynamics, namely, in stochastic instability [12, 13]. The stochastic instability appears inside a layer in the vicinity of a separatrix and the width of the layer decreases exponentially as the amplitude of perturbation decreases [12, 13]. It means that in the vicinity of a separatrix there always exists a stochastic region, no matter how small the amplitude of the perturbation is.

§ Permanent address: Department of General Physics, Barnaul State Technical University, 656099, Barnaul, Russia.

The well known two-soliton solutions to the SG equation are the kink–antikink (KA) and breather (B) solutions, which in their centre-of-mass frame can be written as

$$u_{\text{KA}} = 4 \arctan \left[\frac{\sinh(d_{\text{KA}} \delta_{\text{KA}} t)}{d_{\text{KA}} \cosh(\delta_{\text{KA}} x)} \right] \quad (1)$$

where d_{KA} is the velocity of the kinks of a KA pair, $0 < |d_{\text{KA}}| < 1$, $\delta_{\text{KA}}^{-1} = \sqrt{1 - d_{\text{KA}}^2}$ and

$$u_{\text{B}} = 4 \arctan \left[\frac{\eta \sin(\omega t)}{\omega \cosh(\eta x)} \right] \quad (2)$$

where $0 < \omega < 1$ is the breather frequency and $\eta = \sqrt{1 - \omega^2}$. Solutions (1) and (2) have energies $E_{\text{KA}} = 16\delta_{\text{KA}} > 16$ and $E_{\text{B}} = 16\eta < 16$, respectively.

The solution (1) in the limit $d_{\text{KA}} \rightarrow 0$ coincides with the solution (2) in the limit $\omega \rightarrow 0$:

$$u_{\text{S}} = \lim_{d_{\text{KA}} \rightarrow 0} u_{\text{KA}} = \lim_{\omega \rightarrow 0} u_{\text{B}} = 4 \arctan \left[\frac{t}{\cosh x} \right]. \quad (3)$$

The solution u_{S} , given by (3), is the separatrix two-soliton solution with the energy $E_{\text{S}} = 16$.

The two-soliton solution has the separatrix solution. However, the only manifestation of a weak perturbation for the two-soliton collision is the phonon radiation, even if the collision takes place in the vicinity of the separatrix. Momentum and energy exchange between two solitons is forbidden by the momentum conservation law, which is obeyed to a high degree of accuracy by a weakly perturbed system. For the collision of more than two solitons the energy exchange between them becomes possible.

In our previous paper [9] the effect of weak discreteness on the kink–breather solution was discussed. In particular, we mentioned that the perturbed kink–breather solution in the vicinity of a separatrix SG solution can transform into the kink plus kink–antikink solution. Thus, a consideration of both of these three-soliton solutions would be interesting.

The aim of this paper is to give some separatrix three-soliton solutions to the SG equation and to demonstrate that the three-soliton collision in the vicinity of a separatrix may result in energy and momentum exchange between solitons. Moreover, this effect is noticeable even for a small perturbation. As the perturbed SG system the Frenkel–Kontorova model is considered at a small degree of discreteness.

2. Frenkel–Kontorova model and the sine-Gordon equation

We consider the Hamiltonian of the Frenkel–Kontorova model [14] in a dimensionless form

$$H = \sum_n E_n = \sum_n \left[\frac{p_n^2}{2} + \frac{1}{2h^2} (u_{n+1} - u_n)^2 + (1 - \cos u_n) \right] \quad (4)$$

where u_n is the displacement of the n th particle from an initial point with coordinate $x = nh$, p_n is the momentum of the particle with unit mass, H/h is the density of energy and h is the only parameter of the system, which gives a measure of discreteness because the equations of motion obtained from the Hamiltonian (4)

$$\frac{d^2 u_n}{dt^2} - \frac{1}{h^2} (u_{n-1} - 2u_n + u_{n+1}) + \sin u_n = 0 \quad (5)$$

in the continuum limit ($h \rightarrow 0$) are reduced to the SG equation

$$u_{tt} - u_{xx} + \sin u = 0. \quad (6)$$

3. Three-soliton solutions

The three-soliton solutions to (6), which describe the collision between a kink and a breather (K–B solution) and between a kink and a kink–antikink pair (K–KA solution) can be obtained by the Bäcklund transformations (see, e.g., [2]). Both solutions have the form of a sum

$$u_{K-B} = v_K + w_{K-B} \tag{7}$$

$$u_{K-KA} = v_K + w_{K-KA}. \tag{8}$$

Function v_K in (7) and (8) is the kink solution

$$v_K = 4 \arctan \exp B \tag{9}$$

where $B = \delta_K(x - x_K - d_K t)$, $\delta_K^{-1} = \sqrt{1 - d_K^2}$ is the width of the kink, $0 \leq |d_K| < 1$ is the velocity of the kink and x_K defines the position of the kink at the time $t = 0$. The energy of the kink is $E_K = 8\delta_K$.

The second term in the right-hand side of (7) and (8) can be written, without loss of generality, under the assumption that the centre of mass of the breather or kink–antikink pair has zero velocity. The case of a centre of mass moving with velocity $0 < |d| < 1$ can be obtained by means of a Lorentz transformation $x \rightarrow (x - dt)/\sqrt{1 - d^2}$. The origin is chosen in such a way that the kink with $d_K \neq 0$ collides with the centre of mass of the breather or kink–antikink pair at $x = 0$.

The second term in the right-hand side of (7) reads

$$w_{K-B} = 4 \arctan \left[\frac{2\omega\eta(\sinh D - \cos C \sinh B) + 2\eta\delta_K d_K \sin C \cosh B}{2\omega\eta(\cos C + \sinh D \sinh B) - 2\omega\delta_K \cosh D \cosh B} \right] \tag{10}$$

where $C = -\omega t + 2\pi m$ with an integer m , $D = \eta x$, $\eta = \sqrt{1 - \omega^2}$ and $0 < \omega < 1$ is the frequency of the breather.

The amplitude of the breather is

$$A_B = 4 \arctan(\eta/\omega). \tag{11}$$

The energy of the solution (7) is

$$E_{K-B} = E_K + E_B = 8\delta_K + 16\eta. \tag{12}$$

Collision between a kink and a breather results in the phase shift

$$\Delta_{K-B} = \frac{2}{\eta} \tanh^{-1} \left(\frac{\eta}{\delta_K} \right). \tag{13}$$

The second term on the right-hand side of (8) reads

$$w_{K-KA} = 4 \arctan \left[\frac{\delta_{KA} d_{KA} (\sinh F - \cosh G \sinh B) - \delta_K d_K \sinh G \cosh B}{\delta_{KA} d_{KA} (\sinh F \sinh B + \cosh G) - \delta_K d_{KA} \cosh F \cosh B} \right] \tag{14}$$

where $F = \delta_{KA} x$, $G = \delta_{KA} d_{KA} t$, $\delta_{KA}^{-1} = \sqrt{1 - d_{KA}^2}$, kink and antikink in the kink–antikink pair have velocities $\pm d_{KA}$, $0 < |d_{KA}| < 1$.

The energy of the solution (8) is

$$E_{K-KA} = E_K + E_{KA} = 8\delta_K + 16\delta_{KA}. \tag{15}$$

Collision between a kink and a kink–antikink pair results in the phase shift

$$\Delta_{K-KA} = 2 \tanh^{-1} \left(\frac{\delta_{KA}}{\delta_K} \right). \tag{16}$$

As has been mentioned above, the kink with the velocity $d_K \neq 0$ meets the centre of mass of a breather (solution (7)) or kink–antikink pair (solution (8)) at the origin $x = 0$. If $d_K = 0$ then, for $x_K \neq 0$, the kink never passes through the centre of mass of breather or kink–antikink pair and, for $x_K = 0$, the kink stays at the centre of mass of the breather or kink–antikink pair.

If $d_K \neq 0$, the kink passes through the centre of mass of the breather (solution (7)) or kink–antikink pair (solution (8)) at the time

$$t = -\frac{x_K}{d_K}. \quad (17)$$

The three-soliton collision, when all three kinks meet at one point, takes place at $x = t = 0$ under the following conditions. For the K–B solution (7) it happens at

$$x_K = \frac{2\pi m d_K}{\omega} \quad (18)$$

where m is an integer, and for the K–KA solution (8) at

$$x_K = 0. \quad (19)$$

4. Separatrix three-soliton solutions

Solution (7) under condition (18) describes the collision of three solitons at one point. This is the separatrix solution because it is the limiting solution for the two qualitatively different solutions, one with $x_K \rightarrow 2\pi m d_K/\omega + 0$ and another with $x_K \rightarrow 2\pi m d_K/\omega - 0$:

$$u_{S1} = u_{K-B}(x_K = 2\pi m d_K/\omega). \quad (20)$$

The separatrix solution (20) has been described in [9].

Analogously, solution (8) under condition (19) describes the collision of three solitons at one point and it is the separatrix solution for the two qualitatively different solutions, one with $x_K \rightarrow +0$ and another with $x_K \rightarrow -0$:

$$u_{S2} = u_{K-KA}(x_K = 0). \quad (21)$$

Solution (7) with $d_K = 0$ is the separatrix solution for the solutions with $d_K \rightarrow +0$ and $d_K \rightarrow -0$:

$$u_{S3} = u_{K-B}(d_K = 0). \quad (22)$$

Solution (8) with $d_K = 0$ is the separatrix solution for the solutions with $d_K \rightarrow +0$ and $d_K \rightarrow -0$:

$$u_{S4} = u_{K-KA}(d_K = 0). \quad (23)$$

It was mentioned in section 1 that the breather solution in the limit $\omega \rightarrow 0$ coincides with the kink–antikink solution in the limit $d_{KA} \rightarrow 0$. Applying this to the three-soliton solutions (7) and (8) one can obtain the following separatrix solution:

$$\begin{aligned} u_{S5} &= \lim_{\omega \rightarrow 0} u_{K-B} = \lim_{d_{KA} \rightarrow 0} u_{K-KA} \\ &= v_K + 4 \arctan \left[\frac{\sinh x - \sinh B - \delta_K d_K t \cosh B}{\sinh x \sinh B + 1 - \delta_K \cosh x \cosh B} \right]. \end{aligned} \quad (24)$$

The following separatrix solution is the limiting case of (8) when $d_{KA} \rightarrow +d_K$ or $d_{KA} \rightarrow -d_K$:

$$u_{S6} = \lim_{d_{KA} \rightarrow \pm d_K} u_{K-KA} = 4 \arctan \exp B^* + 4 \arctan \left[\frac{\mp d_K \delta_K (d_K x \mp t) \cosh[\delta_K (x - x_K)] - \sinh(d_K \delta_K t) \cosh B^*}{d_K \delta_K (d_K x \mp t) \sinh(d_K \delta_K t) \mp d_K^2 \cosh B^* \cosh[\delta_K (x - x_K \mp d_K t)]} \right] \quad (25)$$

where $B^* = \delta_K (x - x_K \mp d_K t)$.

In particular, from the separatrix (24) in the limit $d_K \rightarrow 0$ and from the separatrix (25) in the limit $d_K \rightarrow 0$ the following separatrix solution can be derived:

$$u_{S7} = \lim_{d_K \rightarrow 0} u_{S5} = \lim_{d_K \rightarrow 0} u_{S6} = 4 \arctan \exp(x - x_K) + 4 \arctan \left[\frac{x \cosh(x - x_K) - t^2 \sinh(x - x_K)}{\cosh^2(x - x_K) + t^2} \right]. \quad (26)$$

5. Inelastic collisions. Numerical results

Equations (7) and (8) predict the purely elastic collision between quasiparticles. Here we study the influence of perturbation on the elasticity of three-soliton collisions. As a perturbed SG system the Frenkel–Kontorova model (equations (4) and (5)) is investigated at $h = 0.2$. We use the analytical solutions to the SG equation, given in section 3, for setting the initial conditions for the discrete system (5), substituting $x \rightarrow nh$.

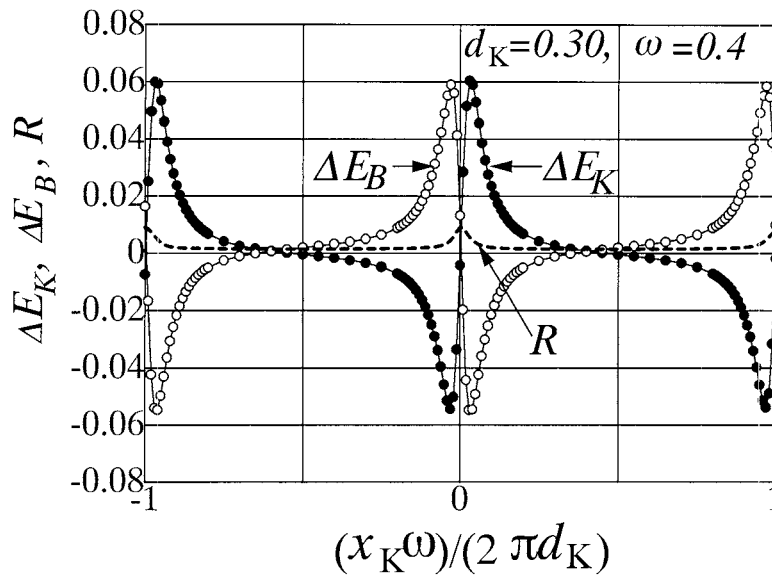


Figure 1. The energy change of the kink and breather $\Delta E_K = E_K(-T_0) - E_K(T_0)$, $\Delta E_B = E_B(-T_0) - E_B(T_0)$, and the total radiated energy $R = \Delta E_K + \Delta E_B$. The energy exchange between the kink and breather increases drastically in the vicinity of $(x_K \omega)/(2\pi d_K) = m$, where m is an integer. This condition coincides with condition (18), which is the condition for the separatrix solution (20). The total radiated energy R is considerably smaller than the maximum energy exchange.

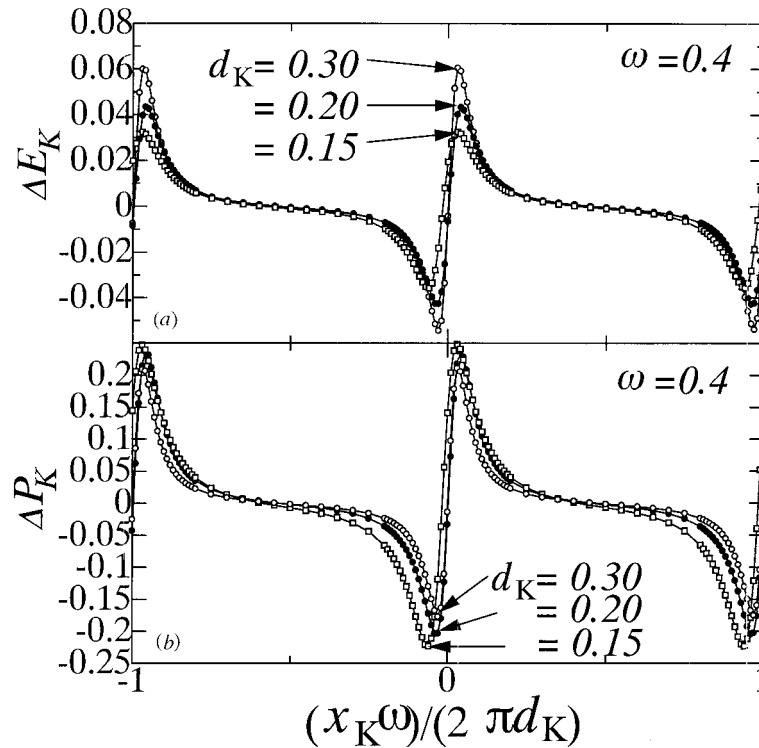


Figure 2. The influence of d_K on the inelasticity of the kink-breather collision measured by (a) ΔE_K and (b) ΔP_K .

Equations of motion (5) were integrated numerically using the Störmer method of order six [15] with the time step $\Delta t = 10^{-4}$. As one would expect, the study of the dynamics inside a stochastic layer in the vicinity of a separatrix is very sensitive to the numerical scheme. In these circumstances one has to find a compromise between the accuracy of the scheme and the width of the layer where the solution can be obtained within a given time domain. The numerical data reported in this paper did not vary essentially with a further decrease of Δt .

We carried out the numerical calculations in the time domain $-T_0 \leq t \leq T_0$ and compared the numerical solutions for the discrete system with the corresponding analytical solutions to the unperturbed SG equation. The length of the time domain was chosen in such a way that the quasiparticles are rather far from each other at the beginning ($t = -T_0$) and at the end ($t = T_0$) of the numerical run. This let us calculate and compare the individual properties of quasiparticles before the collision with that after the collision.

For $h = 0.2$, the manifestations of discreteness are usually very small and the solutions (7) and (8) to the continuous equation (6) as a rule, give a good approximation to the solutions of the discrete system (5). However, in the vicinity of the separatrix solutions, even at $h = 0.2$, the collision of solitons can be strongly inelastic.

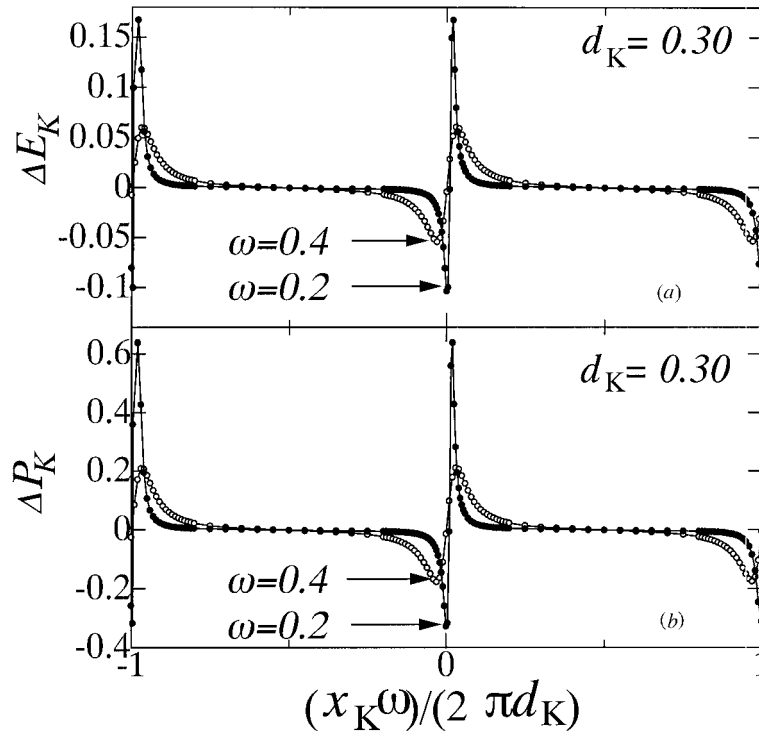


Figure 3. Influence of ω on the inelasticity of kink-breather collision measured by (a) ΔE_K and (b) ΔP_K .

5.1. Kink-breather collision

First of all, let us demonstrate that, at such a small h , the inelasticity of collision in the vicinity of a separatrix shows itself as the energy and momentum exchange between solitons rather than the radiation of phonons. In figure 1, we plot the energy change of a kink and breather

$$\Delta E_K = E_K(-T_0) - E_K(T_0) \quad \Delta E_B = E_B(-T_0) - E_B(T_0) \quad (27)$$

and the total radiated energy

$$R = \Delta E_K + \Delta E_B \quad (28)$$

as the function of the parameter $(x_K \omega)/(2\pi d_K)$. We put $\omega = 0.4$, $d_K = 0.3$ and $T_0 = 100$. The energy of solitons was calculated as the total energy of particles within the domain $\Delta x = 20$ (100 particles) for the kink and within the domain $\Delta x = 30$ (150 particles) for the breather, with the centre of the domain placed at the centre of the quasiparticle.

One can see that ΔE_K and ΔE_B are nearly periodic functions of the parameter $(x_K \omega)/(2\pi d_K)$ and that they increase drastically in the vicinity of $(x_K \omega)/(2\pi d_K) = m$, where m is an integer. Note that the latter condition coincides with condition (18), which is the condition for the separatrix solution (20). It can also be seen from figure 1 that the total radiated energy R has a background value of about 0.0015 and the peak of 0.009 at the separatrix. The background radiation is the radiation due to the inaccurate initial conditions and due to the ordinary radiation losses in a non-integrable system. The additional radiation loss in the vicinity of a separatrix comprises some 0.0075, which is considerably smaller than 0.06, the maximum of ΔE_K and ΔE_B .

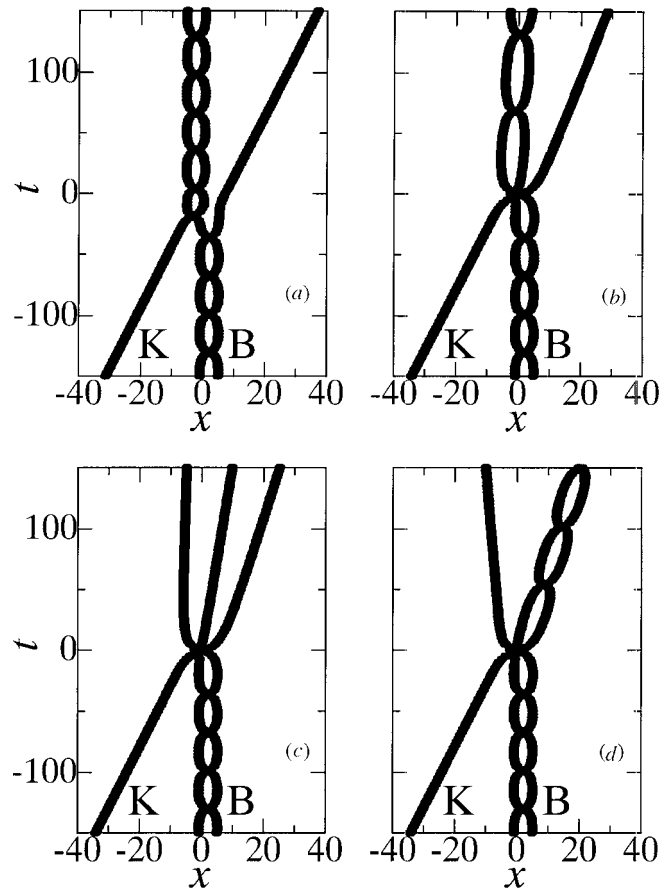


Figure 4. Different scenarios of the K–B collision observed in the discrete system at $\omega = 0.1$, $d_K = 0.2$, and (a) $x_K = 0.25(2\pi d_K/\omega)$, (b) $x_K = 0.020(2\pi d_K/\omega)$, (c) $x_K = 0.0188(2\pi d_K/\omega)$, (d) $x_K = 0.0173(2\pi d_K/\omega)$. Only the particles having energy $E_n \geq 0.5$ are shown. In (a), the collision takes place far from the separatrix (20) and it is practically elastic. In (b)–(d), all three kinks collide practically at one point (separatrix (20)). In (b) the velocity of the kink and the frequency of the breather become smaller after the collision. In (c) the breather breaks up into a kink–antikink pair. In (d) a reflection of the kink and breather takes place. The energy loss to radiation is very small for all cases.

As a measure of inelasticity of the kink–breather collision one can take the change in kink energy ΔE_K or, alternatively, the change in kink momentum

$$\Delta P_K = P_K(-T_0) - P_K(T_0) \quad (29)$$

where $P_K = 8d_K/\sqrt{1-d_K^2}$.

Let us study the influence of parameters on the inelasticity of the kink–breather collision. In figure 2, we plot ΔE_K and ΔP_K as the function of the parameter $(x_K\omega)/(2\pi d_K)$. We set $\omega = 0.4$, $d_K = 0.15, 0.2, 0.3$ and $T_0 = 30/d_K$. One can see that ΔE_K decreases, while ΔP_K increases with decrease of d_K . This is not surprising, since E_K and P_K are nonlinear functions of d_K .

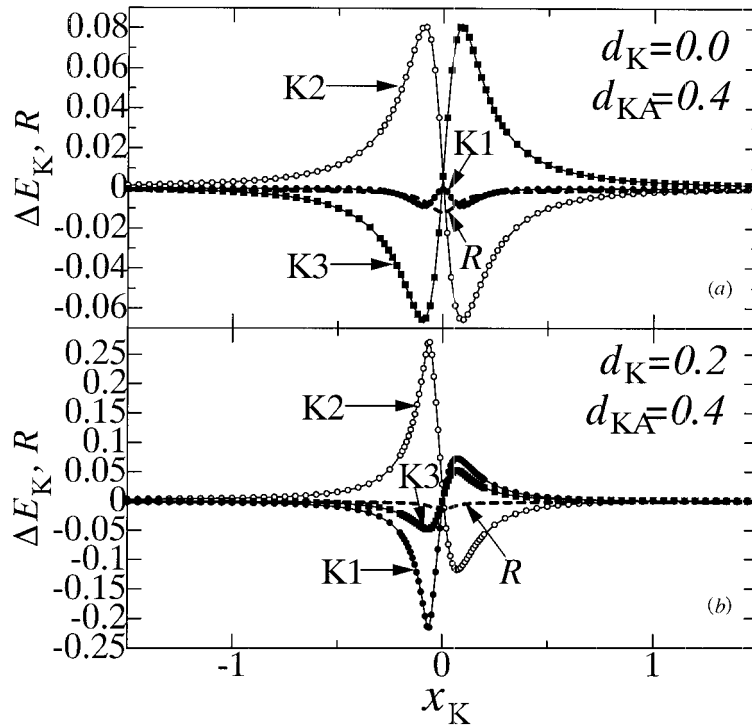


Figure 5. The energy change for each of three kinks in the kink–kink–antikink collision and the total radiated energy R as the function of x_K , (a) $d_K = 0.0$, (b) $d_K = 0.2$. K1 corresponds to the kink moving with the velocity d_K and K2, K3 correspond to the kinks of the kink–antikink pair moving toward each other with the absolute velocity d_{KA} . The inelasticity of collision increases drastically in the vicinity of $x_K = 0$, which is the condition for the separatrix solution (21).

Figure 3 shows the same as figure 2 but for $d_K = 0.3$ and $\omega = 0.2, 0.4$. As figure 3 suggests, both ΔE_K and ΔP_K increase with decreasing ω . This can be explained through the fact that $\omega \rightarrow 0$ corresponds to another separatrix solution, given by (24).

Note that ΔE_K and ΔP_K are extremely sensitive to the parameters of the collision in the vicinity of a separatrix. To the left of the separatrix the kink obtains some energy from the breather and to the right of the separatrix the kink loses its energy.

The results presented in figures 1–3 were obtained rather far from the separatrices (24) and (25). In a close vicinity to these separatrices the inelastic three-soliton collision can lead to a qualitative change of the picture of collision, namely, to the reflection of the kink and breather or to the breakup of the breather into a kink–antikink pair. These types of inelastic collisions have been observed for the SG equation with other perturbations [4], though the relation of the effect with the separatrix solutions has not been pointed out.

In figure 4 we show the different scenarios of the K–B collision observed in the discrete system at $\omega = 0.1$, $d_K = 0.2$, $T_0 = 150$, and (a) $x_K = 0.25(2\pi d_K/\omega)$, (b) $x_K = 0.020(2\pi d_K/\omega)$, (c) $x_K = 0.0188(2\pi d_K/\omega)$, (d) $x_K = 0.0173(2\pi d_K/\omega)$. The particles of the chain having energy $E_n \geq 0.5$ (see equation (4)) are shown in figure 4 by black dots and the particles with energy $E_n < 0.5$ are not shown. Thus, the black areas show the cores of solitons having a high energy. In (a), the collision takes place far from separatrix (20) and it is practically elastic. In (b)–(d), all three kinks collide practically at one point (separatrix

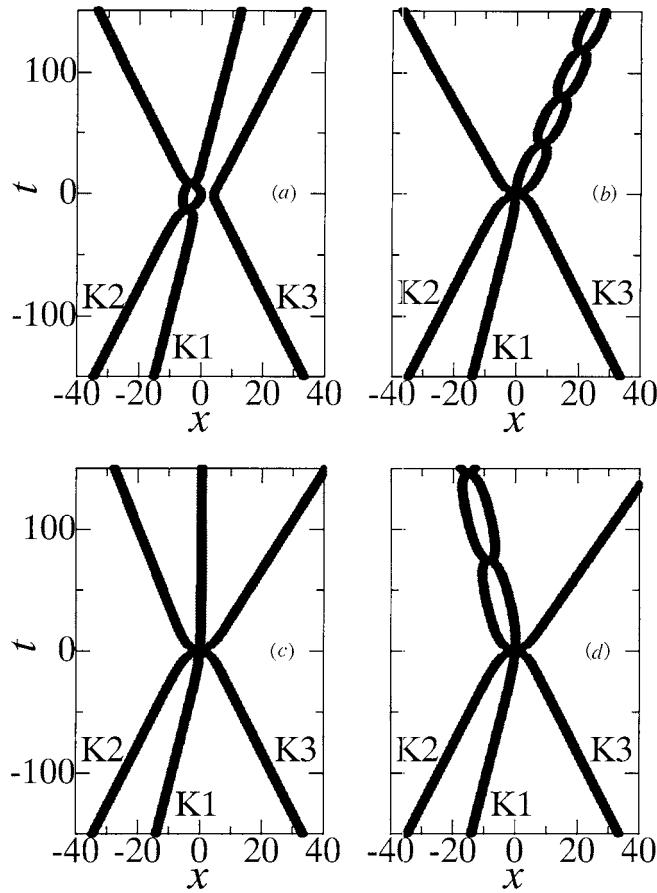


Figure 6. The space-time evolution of the K–KA collision in the discrete system for $d_K = 0.1$, $d_{KA} = 0.2$, and (a) $x_K = -1.0$, (b) $x_K = -0.04$, (c) $x_K = 0.01$, and (d) $x_K = 0.021$. In (a), the collision is practically elastic. In (b)–(d), all three kinks collide almost at one point (separatrix (21)) with a noticeable energy exchange. In (c), the momentum and energy exchange between kinks takes place. In (b) and (d), the fusion of a kink–antikink pair into a breather takes place. The energy loss to radiation is very small in all cases.

(20)) and the energy exchange between them can be easily seen. In (b) the velocity of the kink becomes smaller after the collision. The breather takes some part of the kink’s energy and, consequently, its frequency ω becomes smaller. In (c) the breather breakup into a kink–antikink pair. In (d) the reflection of kink and breather takes place. The energy loss to radiation is very small for all cases.

5.2. Kink–kink–antikink collision

Let us turn to the collision of a kink with a kink–antikink pair. In figure 5, we plot, as a function of x_K , the energy change for each of three colliding kinks, ΔE_{K1} , ΔE_{K2} , ΔE_{K3} , where K1 is a kink moving with the velocity d_K and K2, K3 are the kinks of the kink–antikink pair moving toward each other with absolute velocity d_{KA} . We also plot the total radiated energy

$$R = \Delta E_{K1} + \Delta E_{K2} + \Delta E_{K3}. \quad (30)$$

The parameters of collision are $d_{KA} = 0.4$, $T_0 = 100$, and (a) $d_K = 0.0$, (b) $d_K = 0.2$. The inelasticity of the collision increases drastically in the vicinity of $x_K = 0$, which is the condition for the separatrix solution (21). For $d_K = 0.0$, the energy exchange occurs mostly between K2 and K3, but in the case of $d_K = 0.2$, between K1 and K2.

Comparison of figures 5(a) and (b) suggests that the energy exchange increases when d_K approaches d_{KA} or, in other words, when the system is close to the separatrix (25). However, the increase of energy exchange at $d_K \rightarrow 0$ was not observed in spite of the fact that the system is close to the separatrix (23). If both $d_K \rightarrow 0$ and $d_{KA} \rightarrow 0$, which corresponds to the separatrix (26), then the inelasticity of collision increases drastically.

Closer to the separatrix (25) (or (26)) the fusion of a kink–antikink pair into a breather can take place (figures 6(b) and (d)).

In figure 6, the space–time evolution of the K–KA collision in the discrete system is presented for $d_K = 0.1$, $d_{KA} = 0.2$, and (a) $x_K = -1.0$, (b) $x_K = -0.04$, (c) $x_K = 0.01$, and (d) $x_K = 0.021$. In (a), the collision is practically elastic. In (b)–(d), all three kinks collide almost at one point (separatrix (21)) with a noticeable energy exchange. In (c), the momentum and energy exchange between kinks takes place. In (b) and (d), the fusion of a kink–antikink pair into a breather takes place. The energy loss to radiation is very small in all cases.

6. Conclusion

The three-soliton collisions in the SG system weakly perturbed by the discreteness of the media can be strongly inelastic even at a small degree of discreteness ($h < 0.5$) because the collision of three solitons at one point is a kind of separatrix solution. Inelasticity of a collision at small h shows itself as the energy and momentum exchange between solitons rather than the radiation of phonons.

The three-soliton collision in the K–B collision takes place under the condition (18) and in the K–KA collision under condition (19). The inelasticity of collision increases when the three-soliton collision takes place in the vicinity of another separatrix. It happens, for example, when condition (18) appears in combination with $\omega \rightarrow 0$ (separatrix (24)) or, when condition (19) appears in combination with $d_{KA} \rightarrow +d_K$ or $d_{KA} \rightarrow -d_K$ (separatrix (25)).

In the present paper the SG system was treated but the effect under consideration has also been observed in the other nearly integrable systems [8, 10], therefore it appears as a common many-soliton effect.

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